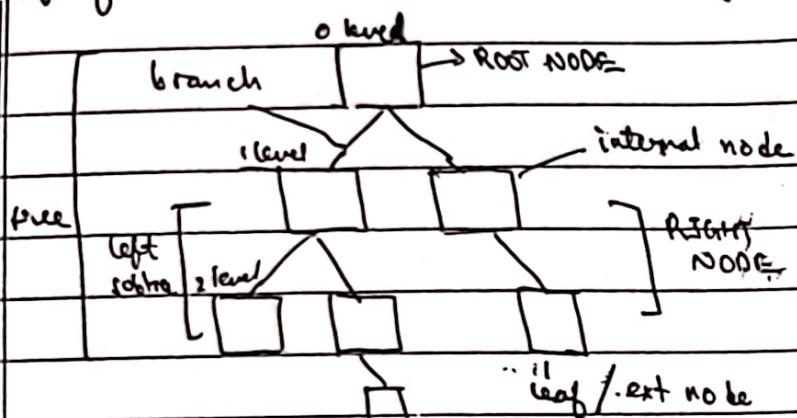


Date \_\_\_ / \_\_\_ / \_\_\_

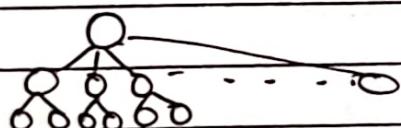
UNIT - III      TREE

eg of tree →

(binary tree)

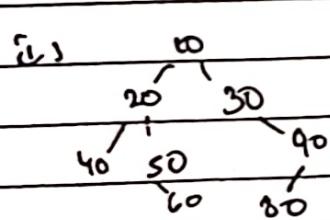


eg of parent →

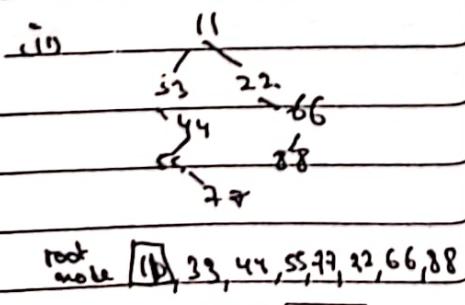
METHODS OF TRAVERSING

- 1- PRE-ORDER
- 2- IN ORDER
- 3- POST-ORDER

Q- Preorder traverse -



10, 20, 40, 50, 60, 30,  
root node 90, 80



root node 11, 33, 44, 55, 77, 22, 66, 88

Page No. \_\_\_\_\_

Date \_\_\_ / \_\_\_ / \_\_\_

Q-

Inorder traversal -

necessary to make tree from nodes  
give info about left & right subtrees

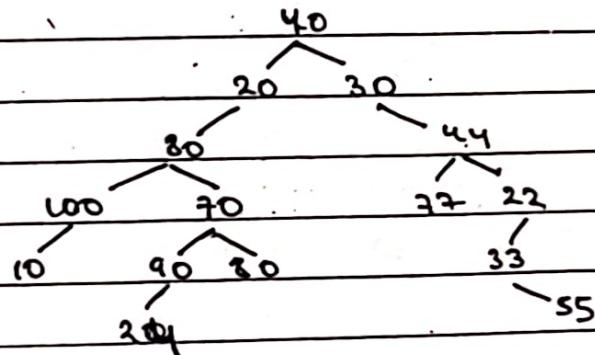
left subtree

root node

right subtree

i) 40, 20, 50, 10, 30, 80, 90.ii) 33, 55, 44, 33, 88, 66, 22, 11.Q - Post order traversal -i) 40, 60, 50, 20, 80, 90, 80, 10. root nodeii) 77, 55, 44, 33, 88, 66, 22, 11.

Q -



$\Rightarrow$  PRE  $\rightarrow$  40, 20, 80, 100, 10, 70, 90, 24, 80, 30, 44,  
77, 22, 33, 55

IN  $\rightarrow$  100, 10, 80, 24, 90, 70, 80, 20, 40, 30, 77,  
44, 33, 55, 22

POST  $\rightarrow$  10, 100, 24, 90, 80, 70, 80, 20, 77, 55,  
33, 22, 44, 30, 40

21/10/19

Date \_\_\_ / \_\_\_ / \_\_\_

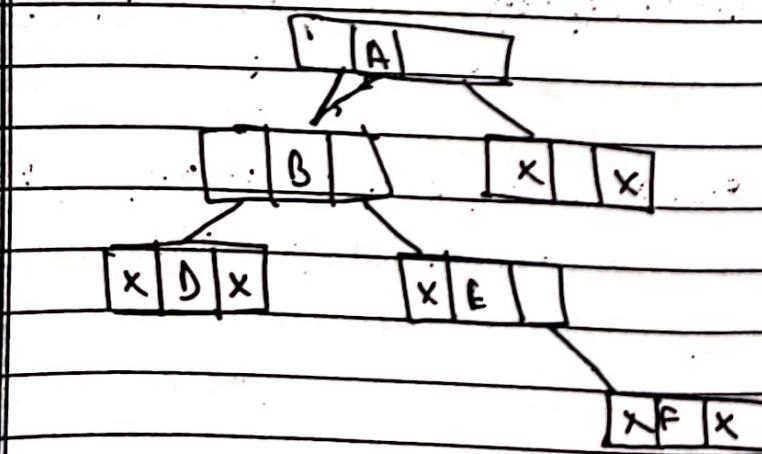
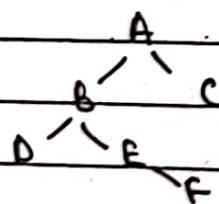
**TREE** - It is a non-linear data structure.  
 It is used mainly to represent a data with hierarchical representation with records.

**BINARY TREE - (T)** - It is defined as a finite set of elements called nodes,

(i) let 'T' be null/empty tree

(ii) 'T' contains a distinguish node 'R' called root node of 'T' & remaining node of 'T' from an order pair of disjoint trees  $T_1$  &  $T_2$ .

→ linked list -



COMPLETE BINARY TREE - It has at most 2 children, one can show the level ' $R$ ' of ' $T$ ' can have at most ' $2^R$ ' nodes.

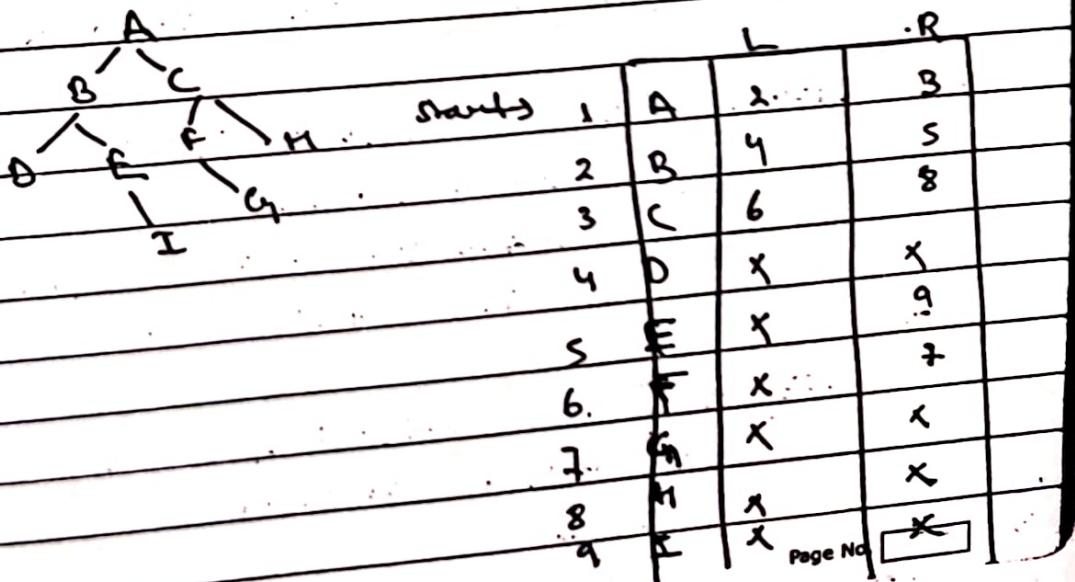
→ The tree ' $T$ ' is said to complete in all levels except possibly the last node possible levels.

→ In complete binary tree left & right children of node ' $k$ ' are ' $2k$ ' & ' $2k+1$ ' and parent of ' $k$ ' is the node ' $k/2$ '

→ The depth of complete binary node

$$D_n = \lfloor \log_2 n + 1 \rfloor, n \rightarrow \text{no of nodes}$$

~~Height~~ [Height =  $\lfloor \log_2 n \rfloor$ ]



Date \_\_\_\_\_ / \_\_\_\_\_ / \_\_\_\_\_

(to be)

**EXTENDED BINARY TREE** - A binary tree is said to be 2-tree or extended binary tree if each node 'n' has either 0 or 2 children. In such cases, the node with 2 children are called internal node & node with 0 children is called external node.

ALGO FOR PREORDER TRAVERSING  $\rightarrow$

PREORDER (INFO, LEFT, RIGHT, ROOT)

STEP 1 - Initially push NULL onto stack  
↳ initialise.

set TOP = 1, stack [1] = NULL  
↳ PTR = ROOT

STEP 2 - Repeat STEP 3 to 4 while PTR  $\neq$  NULL.  
Apply process to INFO[PTR]

STEP 3 - If RIGHT[PTR]  $\neq$  NULL then  
TOP = TOP + 1  
stack [TOP] = RIGHT[PTR]  
[end if structure]

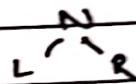
STEP 4 - If LEFT[PTR]  $\neq$  NULL then  
set PTR = LEFT[PTR]  
else

do {  
 PTR = STACK[TOP] & TOP = TOP-1  
 [end if structure]  
 STEP 5 - Exit

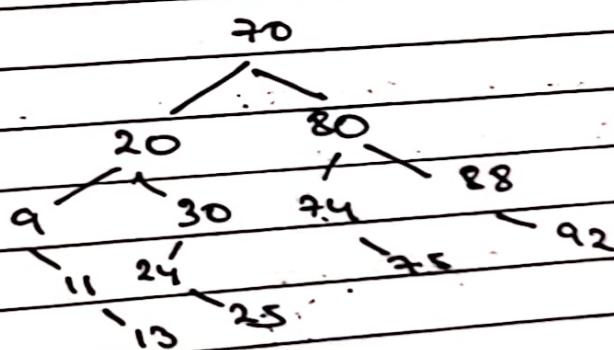
~~ALGO FOR~~

### BINARY SEARCH TREE (BST)

$$L \leq N, R \geq N$$

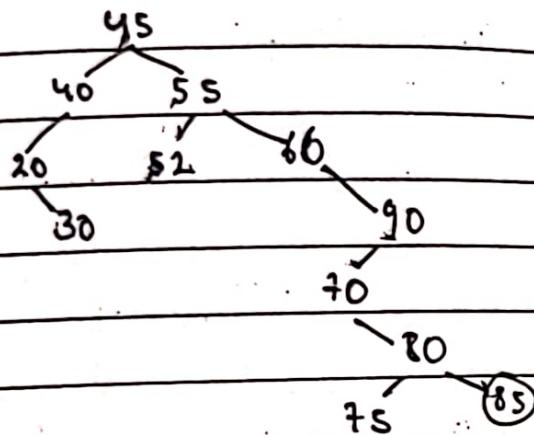


70, 20, 20, 30, 9, 88, 74, 92, 11, 13, 24, 75, 25

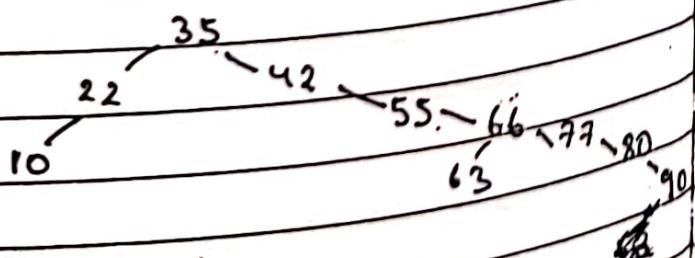


10/10/19BINARY SEARCH TREE

45 55 66 40 20 30 52 90 70 80 75

insert - 85

Q- 35 42 55 66 77 80 22 10 90 63

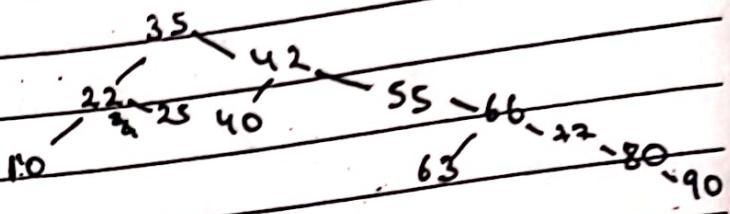
(i) insert  $\rightarrow$  40  
25(ii) delete  $\rightarrow$  55  
77  
66(iii) insert  $\rightarrow$  70  
20 $\Rightarrow$ 

\* ~~as in deletion, inorder to be selected  
is then successor to be selected.~~

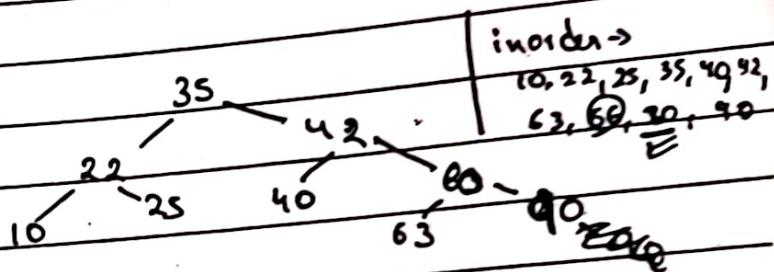
(Saathi)

Date \_\_\_ / \_\_\_ / \_\_\_

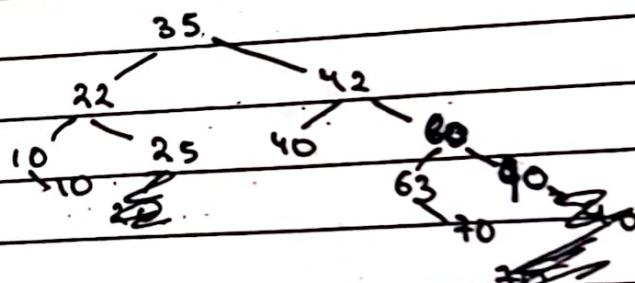
(i)



(ii)



(iii)



Q-

60 65 55 50 67 45 25 70 10 90  
77 35 9

i) insert  $\rightarrow$  69  
29  
39

ii) delete  $\rightarrow$  67  
25  
70

iii) Insert  $\rightarrow$  72  
62  
23

Page No. \_\_\_\_\_

Date \_\_\_ / \_\_\_ / \_\_\_

⇒

60  
55  
65  
69  
70  
90  
77  
50  
45  
25  
35  
10  
9

(i)

60  
55  
65  
67  
70  
69  
90  
75  
50  
45  
25  
35  
10  
9  
29  
39

(ii)

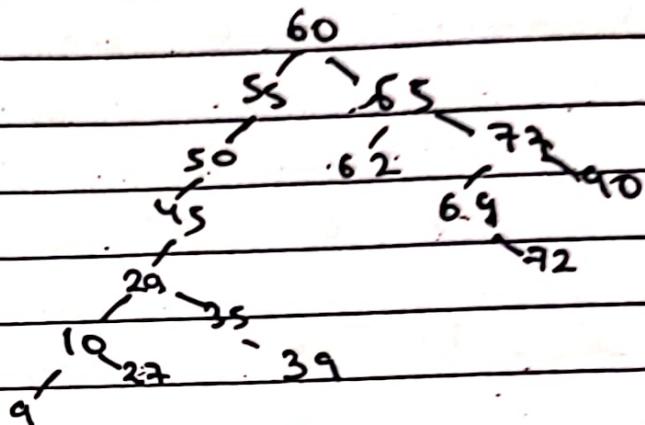
60  
55  
65  
50  
62  
67  
70  
73  
61  
90  
69  
35  
10  
9  
29  
39

19 10 (25) (29) 35 39 45 50, ... // dd 25

→ 9 10 29 35 39 45 50 55 60 65 69 70 72 <

Date \_\_\_ / \_\_\_ / \_\_\_

(iii)

ALGO FOR INORDERINORDER(INFO, LEFT, RIGHT, PTR)

STEP 1 - [push NULL onto stack]

Set TOP = 1, stack[1] = NULL, PTR = ROOT

STEP 2 - Repeat while PTR ≠ NULL

a) set TOP = TOP + 1 &amp; stack[TOP] = PTR

b) set PTR = LEFT[PTR]

[end of loop]

STEP 3 - Set PTR = stack[TOP] &amp; TOP = TOP - 1

STEP 4 - Repeat STEP 5 to 7 while PTR ≠ NULL

STEP 5 - Apply process to INFO[PTR]

~~root node~~

STEP 6 - If  $\text{RIGHT}[\text{PTR}] \neq \text{NULL}$

- Set  $\text{PTR} = \text{RIGHT}[\text{PTR}]$
- Set  $\text{PTR} = \text{RIGHT}[\text{PTR}]$
- Go to step 3

STEP 7 - Set  $\text{PTR} = \text{Stack}[\text{TOP}]$

$$\text{TOP} = \text{TOP} - 1$$

STEP 8 - Exit

### ALGO FOR INSERTION IN BST

$\text{FIND}(\text{INFO}, \text{LEFT}, \text{RIGHT}, \text{ROOT}, \text{VALUE}, \text{LOC}, \text{PAR})$

(i) ~~STEP 1~~  $\text{LOC} = \text{NULL}$  &  $\text{PAR} = \text{NULL}$ , which indicate tree is empty.

(ii)  $\text{LOC} \neq \text{NULL}$  &  ~~$\text{PAR} \neq \text{NULL}$~~   $\text{PAR} = \text{NULL}$ , which indicate that the value is the root of T.

(iii) If  $\text{LOC} = \text{NULL}$  &  $\text{PAR} \neq \text{NULL}$ , value does not exist in tree.

STEP 1 - if  $\text{ROOT} = \text{NULL}$  then set  $\text{LOC} = \text{NULL}$   
~~if  $\text{PAR} = \text{NULL}$~~

STEP 2 - if  $\text{value} = \text{INFO}[\text{ROOT}]$

then set loc = ROOT & PAR = NULL

STEP 3 - if VALUE < INFO[PTR]

set PTR = LEFT [ROOT] & SAVE = ROOT  
else

set PTR = RIGHT [ROOT] & SAVE = ROOT  
[end if]

STEP 4 - Repeat 35 & 6 : while PTR ≠ NULL

STEP 5 - If VALUE = INFO[PTR] then  
set loc = PTR & PAR = SAVE

STEP 6 - If VALUE < INFO[PTR] then set  
SAVE = PTR & PTR = LEFT [PTR]  
else

set SAVE = PTR & PTR = RIGHT [PTR]  
[end if structure]

STEP 7 - set loc = NULL & PATH = SAVE

STEP 8 - Exit

INVERTBST(INFO, LEFT, RIGHT, ROOT, Avail,  
VALUE, loc)

STEP 1 - Call FIND(INFO, LEFT, RIGHT, ROOT,  
VALUE, loc, PAR)

STEP 2 - if loc ≠ NULL

STEP 3 - if Avail = NULL they write  
"overflow"

STEP 4 - set Avail $\geq$  New = Avail &  
Avail = LEFT[Avail], INFO[New] = Value

STEP 5 - let New = loc  
set LEFT[New] = NULL &  
RIGHT[New] = NULL

STEP 6 - [add value to tree]

set ROOT = New

else if value < info[ROOT] then  
set LEFT[PAR] = New

else

set RIGHT[PAR] = New

STEP 7 - Exit

ALGO FOR DELETION IN BTs:

CASE AR (INFO, ROOT, LEFT, RIGHT, LOC, PAR)

STEP 1 - if  $\text{LEFT}[\text{LOC}] = \text{NULL}$  &  $\text{RIGHT}[\text{LOC}] = \text{NULL}$   
           set child = NULL  
       else if  $\text{LEFT}[\text{LOC}] \neq \text{NULL}$   
           set child =  $\text{LEFT}[\text{LOC}]$   
       else  
           set child =  $\text{RIGHT}[\text{LOC}]$   
       [end if structure]

STEP 2 - if  $\text{PAR} \neq \text{NULL}$   
     if  $\text{LOC} = \text{LEFT}[\text{PAR}]$  then  
         set  $\text{LEFT}[\text{PAR}] = \text{child}$   
     else  
         set  $\text{RIGHT}[\text{PAR}] = \text{child}$   
~~the end if~~ ~~else if~~ end if  
     else  
         ROOT = child

STEP 3 - Return

CASEC (INFO, LEFT, RIGHT, ROOT, LOC; PAR)

STEP 1.

- Set PTR = RIGHT[LOC] & SAVE = LOC
- Repeat while LEFT[PTR] ≠ NULL  
    ↳ set SAVE = PTR & PTR = LEFT[PTR]  
        [End of loop]
- Set SUC = PTR & PARLOC = SAVE

STEP 2. - [Delete Inorder successor using CASEB]

Call CASEB(INFO, LEFT, RIGHT, LOC, ROOT, PAR)

STEP 3 - a) if PAR ≠ NULL then

    if LOC ≠ NULL then

        set LEFT[PAR] = SUC

    else

        set RIGHT[PAR] = SUC

    else

        set ROOT = SUC

b) Set LEFT[SUC] = LEFT[LOC] &

RIGHT[SUC] = RIGHT[LOC]

STEP 4 - Return

\* max 3 node will be rotated

Saath!

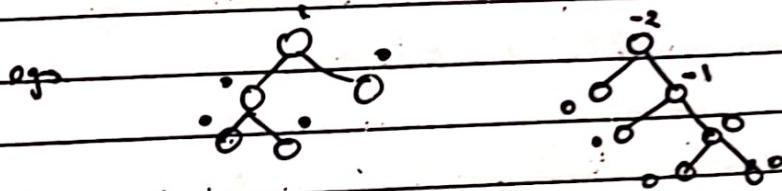
Date \_\_\_ / \_\_\_ / \_\_\_

## AVL : (BST) (HEIGHT BALANCED TREE)

AVL - balanced binary search tree  
(AVL  $\rightarrow$  Adelson Velskii Landis)

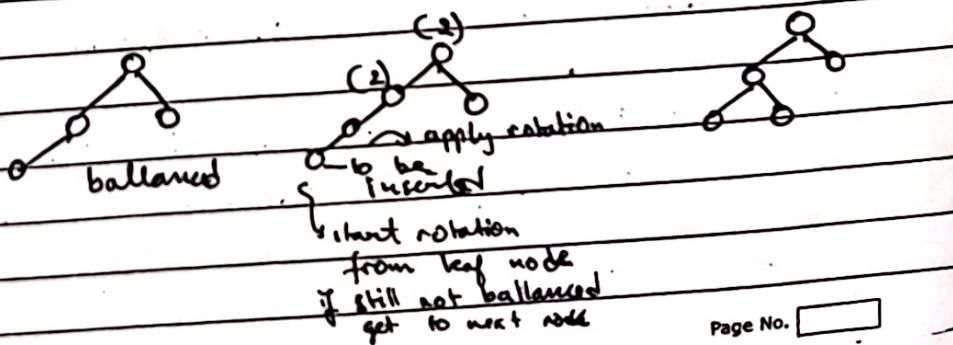
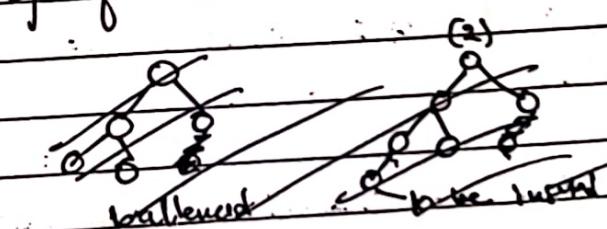
$$\text{Balanced factor} = |h_L - h_R| \quad BF \leq 1$$

$$BF = \begin{matrix} 0, 1, -1 \\ \geq \end{matrix} \quad \begin{matrix} 1 \text{ balanced} \\ 1 \text{ not balanced} \end{matrix}$$



rotations  
LL  $\rightarrow$  clockwise dir  
RR  $\rightarrow$  anti-clockwise dir  
LR  
RL

eg of LL  $\rightarrow$



Page No.

The AVL tree is a BST that has an additional balanced condition. This balanced cond. must be maintained in AVL tree.

If <sup>An</sup> empty binary tree  $B$  is an AVL tree if the non-empty binary tree with  $BL$  &  $BR$  are its left & right subtree then tree  $B$  is AVL tree iff -

(i)  $BL$  &  $BR$  are AVL trees

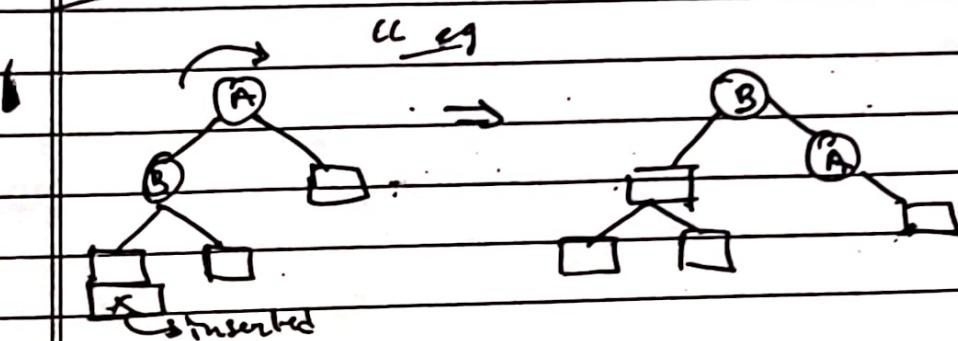
(ii)  $|h_L - h_R| \leq 1$  // balance factor.

$\rightarrow$  height of left - height of right  
sub-tree    sub-tree

- LL-rotation means inserted node is in the left subtree of left subtree of node.
- RR rotation - inserted node is in the right subtree of right subtree of node.
- LR - inserted node is in the right subtree of left subtree of node.

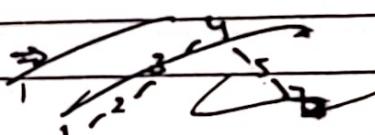
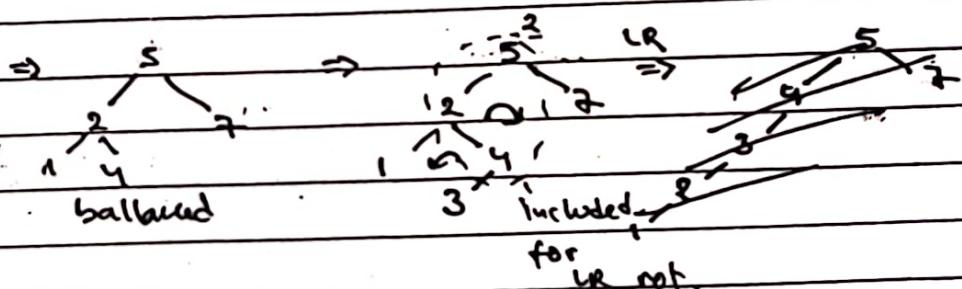
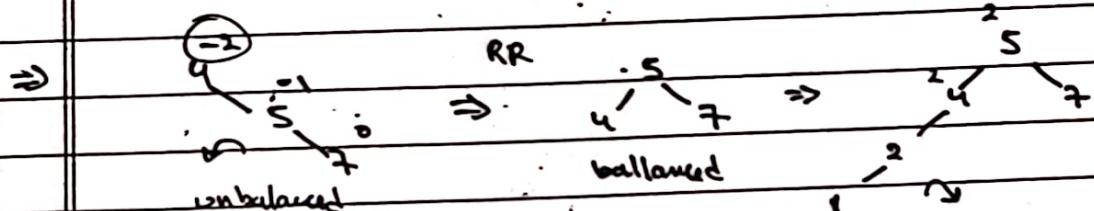
Date \_\_\_ / \_\_\_ / \_\_\_

- RL - inserted node is in the left subtree of right subtree of node.

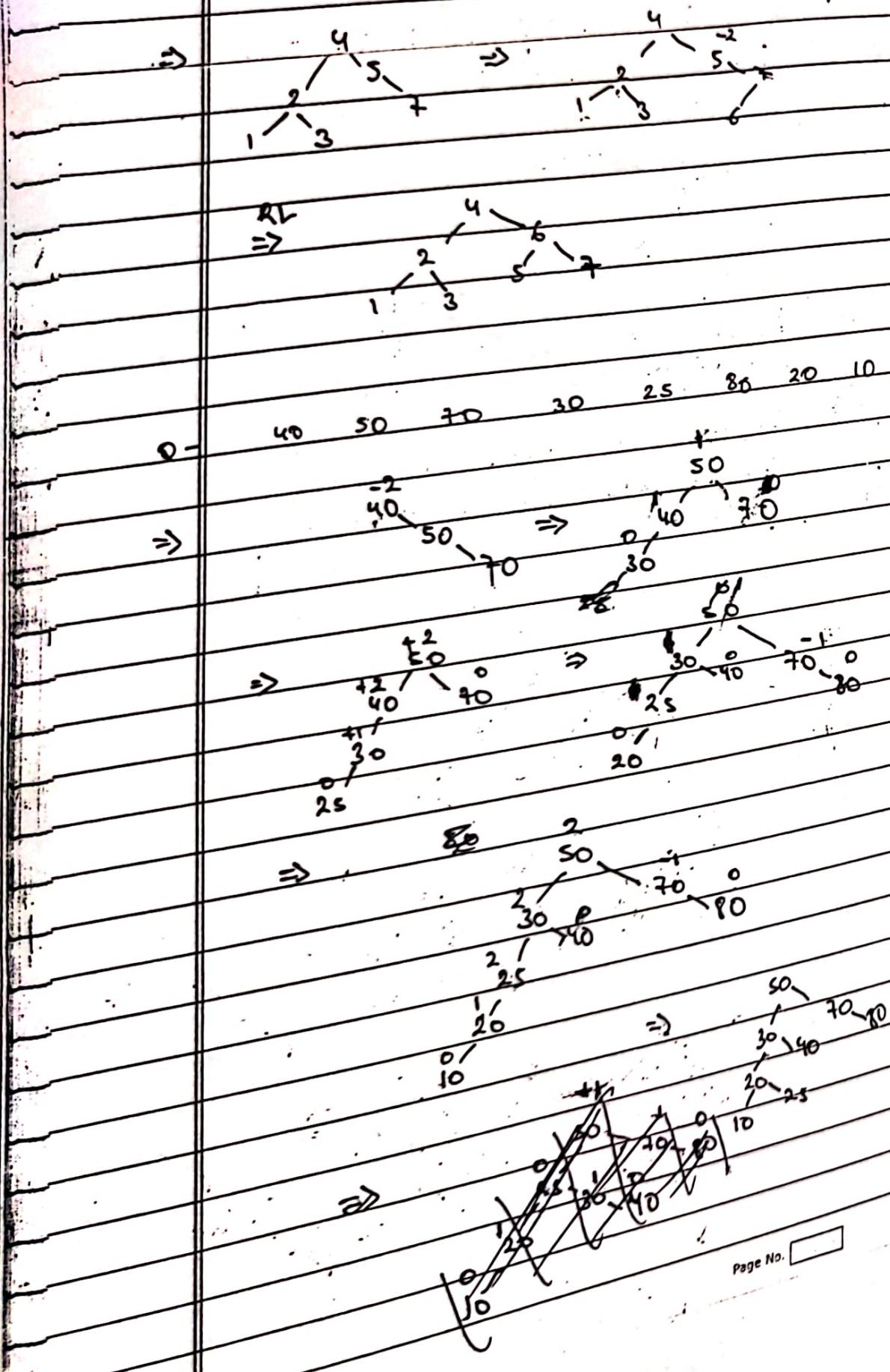


\* LL, RR are single rotation  
LR, RL are double rotation

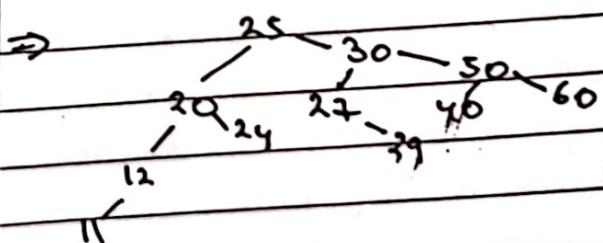
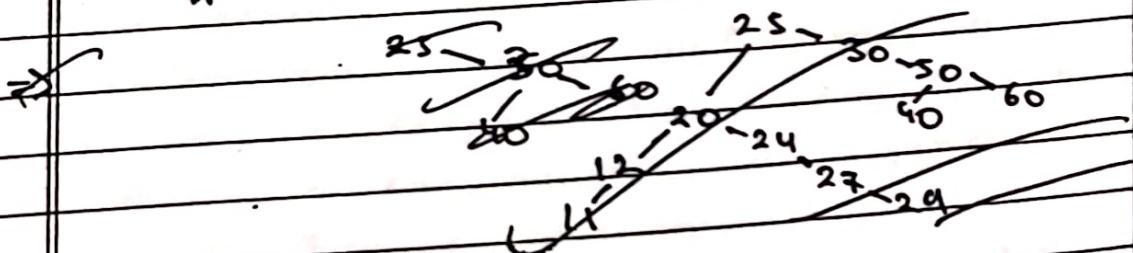
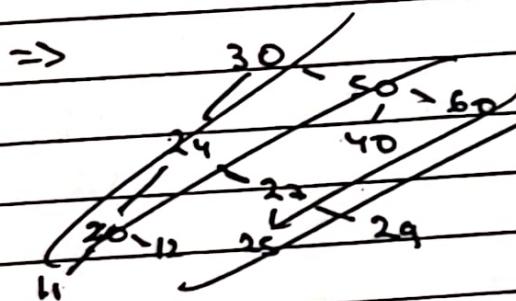
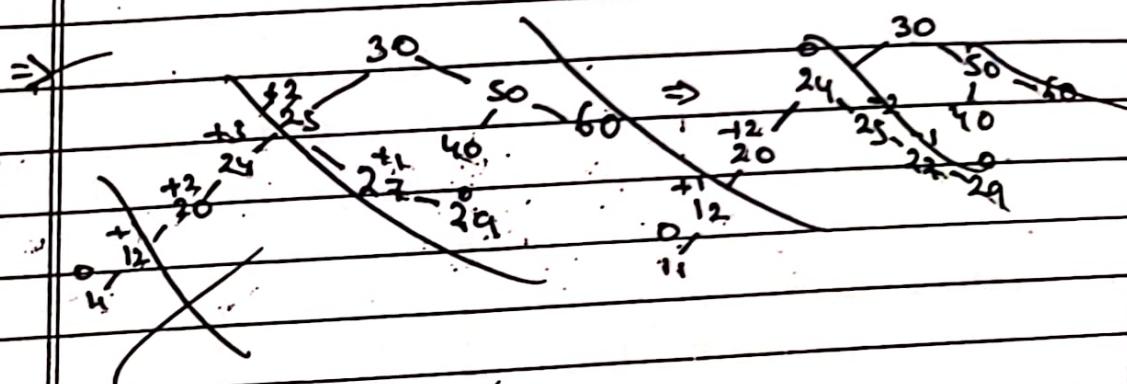
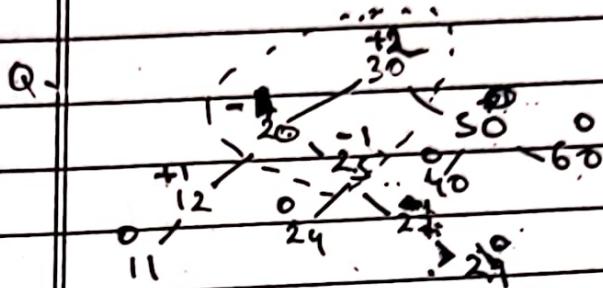
Q - 4 5 7 2 1 3 6 Σ



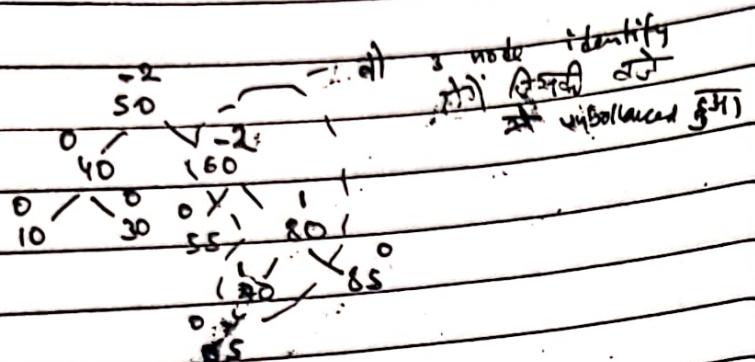
Date \_\_\_ / \_\_\_ / \_\_\_



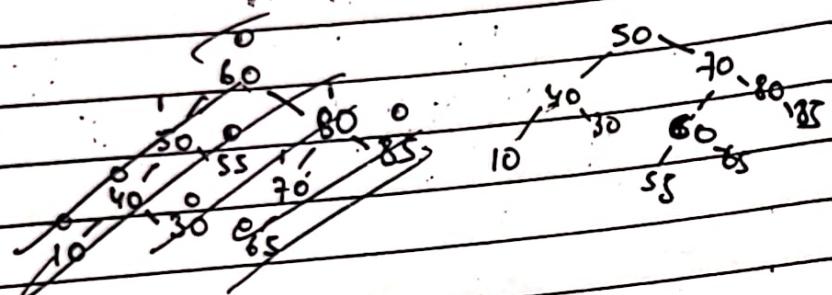
Date \_\_\_ / \_\_\_ / \_\_\_



Q -



⇒

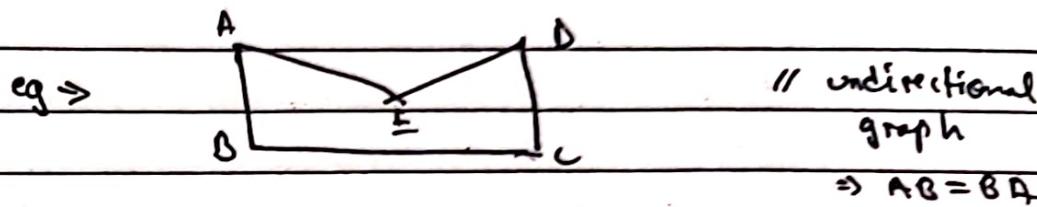


- \* In case of directed graph arrows will be present to represent the direction.

Date \_\_\_ / \_\_\_ / \_\_\_



## UNIT - IV - GRAPH



	A	B	C	D	E
A	0	1	0	0	1
B	1	0			
C			0		
D				0	
E					0

- DFS  $\rightarrow$  depth first search // stack use
- BFS  $\rightarrow$  breadth first search // queue use
- The general idea behind the BFS begins beginning at a starting node A first we examine the starting node A then we examine all the neighbours of A. Then we examine all the neighbours of the neighbours of A & so on. We need to keep track of the neighbours of a node & we need to guarantee that no node is processed more than once. This is accomplished by using a queue to hold nodes that are waiting to be processed & by change the status of the particular node.

Status  $\leftarrow$  <sup>Ready</sup>  
<sup>processes</sup>  
 waiting waiting processed

Ready  $\rightarrow$  Read/Traverse  
 $\downarrow$

waiting -  
 processed -

### ALGO FOR BFS'

STEP 1 - Initialise all nodes to the ready state.

STEP 2 - Put the starting node A in Queue and change the status to the waiting state.

STEP 3 - Repeat STEP 4 & 5 until queue is empty.

STEP 4 - Remove the front node N of queue. Process N by change the status of N to the processed state,

Q-

STEP 5 - Add to the rear of Queue all the neighbours of N that are in the ready state, and change their status to the waiting state.

निम्नीकी पारा से बहुत निकाला उसी avenue में  
include करेंगे

Date \_\_\_\_\_ / \_\_\_\_\_ / \_\_\_\_\_

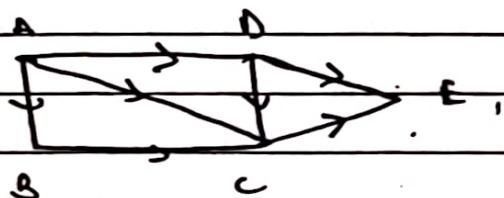
Saathi

[end of step 3 loop]

STEP 6 - Exit

BFS

eq →



Ready state → A, B, C, D, E

queue = A

source = A

queue = A, C, D

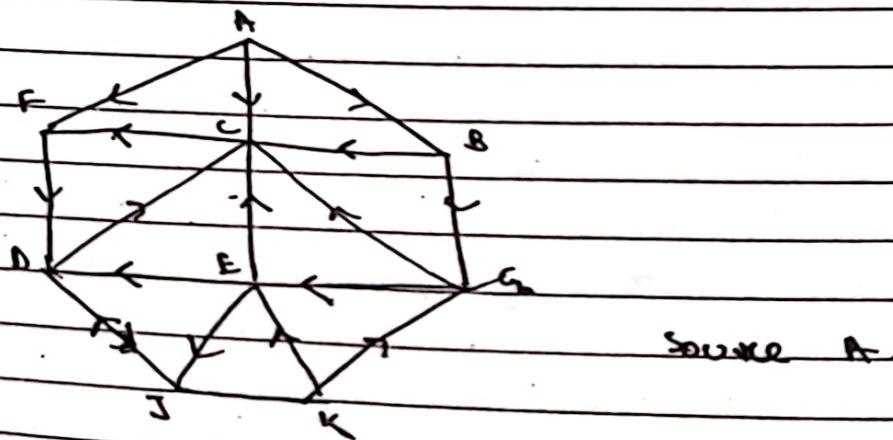
source = A, B, C

queue = D, E

source = A, B ↘

queue = E

source = A, B, C, D, E



source A.

Page No. [ ]

Date \_\_\_ / \_\_\_ / \_\_\_

$\Rightarrow$  by using adjacency list

A	F, C, B	// F, C, B in any order does not matter
B	C, G	
C	F	
D	<del>B</del> C	
E	J, D, C	
F	O	
G	E, C	
H	K, D	
I	E, G	
J		

~~Answer \*~~

Source A

Queue F C B      F का पार्ट

Source A F

Queue C B D      F का पार्ट, C का पार्ट

Source A F C

Queue B D F

Source A F C B

Queue D F \* G      already present  
in source

Source A F C B D

Queue F G \*

Source A F C B D F G

Queue G

Source A F C B D E G

Queue E

2

Page No. \_\_\_\_\_

Source A F C B D E G, E

Queue T X X

-S A F C B D E G E J

Q K

S A F C B D E G E J K

### DFS (DEPTH FIRST SEARCH)

STEP 1 - Initialize all nodes to the ready state.

STEP 2 - Push the starting node A onto stack & change the status to the waiting state.

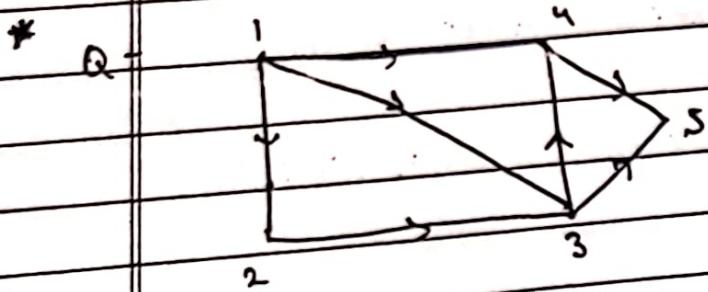
STEP 3 - Repeat STEP 4 & 5 until stack is empty.

STEP 4 - Pop the top node of stack. Process N & change its status to the processed state.

STEP 5 - Push onto stack all the neighbours of N that are still in the ready state by change their state to waiting state.

STEP 6 - Exit.

Date \_\_\_ / \_\_\_ / \_\_\_



⇒

1	2, 3, 4
2	3
3	4, 5
4	5
5	-

Stack - 1

source → 1

ste. 2, 3, 4 → top → first stack

source 1, 4 ←

stack - 2, 3, 5

source - 1, 4, 5

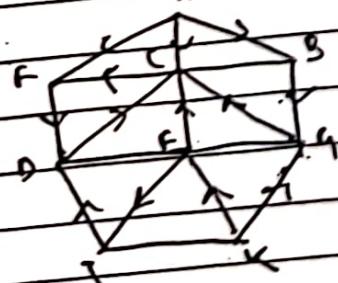
stack → 2, 3

source - 1, 4, 5, 3

1, 4, 5, 3, 2

first → 1, 4  
2nd → 5  
3rd → 4, 3  
last

Q-



Page No. \_\_\_\_\_

Stack - A

 $\Rightarrow S = A$ 

Stack - F, C, B

S - A, B

Stack - F, C, G

S - A, G, C

Stack - F, C, E, ~~B~~

S - A, B, G, E

Stack - F, C, I, D

S - A, B, G, ~~D~~ DStack - F, C, I, ~~X~~

S - A, B, G, B, F, D

Stack - F, C

S - A, B, G, ~~E~~ D, J

Stack - F, C, K

S - A, B, G, ~~E~~ D, J, K

Stack - F, C, X, E

S - F, A, B, G, ~~E~~ D, I, E

Stack - F, C

S - A, B, G, D, I, E, C

Stack - F

S - A, B, G, D, I, E, C, F

$\Rightarrow$ Stack	A	ST	A B G F
S	A	ST	F C J A
ST	F C B	ST	A B G F D
S	A B	ST	F C J
ST	F C G	ST	A B G E D J
S	A B G	J	F C K
ST	F C E	S	A B G E D F N S K C F

MINIMUM SPANNING TREE (KRUSSAL's ALGO)

STEP 1 - Initially construct a separate tree for ~~for~~ each node in a graph.

STEP 2 - Edges are placed in a priority queue, we take edges in ascending order.

(we can use a heap for the priority queue)

STEP 3 - Until we have added  $n-1$  edges-

a) Extract the cheapest edge from the queue.

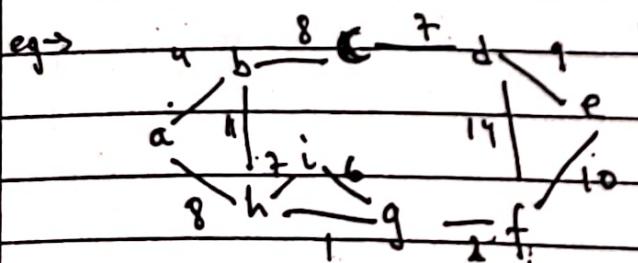
b) If it form a cycle reject it, else add it to the forest.

STEP 4 - whenever we insert an edge,

2 trees will be joined, every step will

have joined 2 trees in the ~~forest~~

forest together so that at the end there will be only 1 tree.

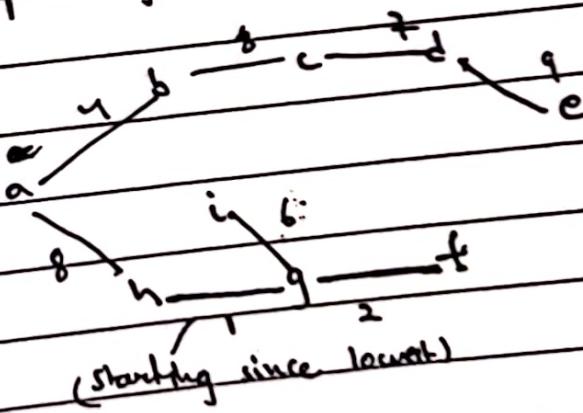


$\Rightarrow$  ~~Adjacency list~~

Arranging in circuitry order

h, q	1
g, f	2
a, b	4
i, g	6
j, h	7
c, d	7
b, c	8
a, h	8
j, d, e	9
e, f	10
b, h	11
d, f	14

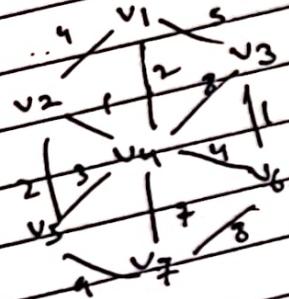
(i) plotting all nodes without edges



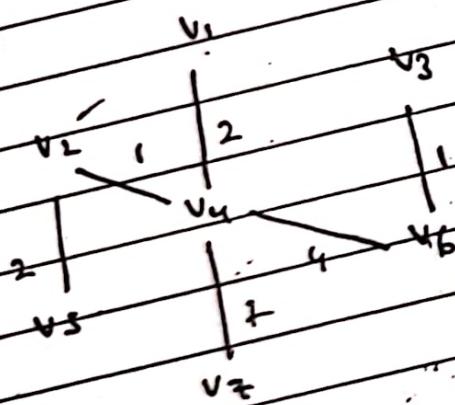
$(i, h), (e, f), (d, f), (g, h)$  are removed

Date / /

Q-



⇒



v <sub>2</sub> v <sub>4</sub>	1
v <sub>3</sub> v <sub>6</sub>	1
v <sub>1</sub> v <sub>4</sub>	2
v <sub>2</sub> v <sub>5</sub>	2
v <sub>4</sub> v <sub>5</sub>	3
v <sub>2</sub> v <sub>7</sub>	4
v <sub>4</sub> v <sub>6</sub>	5
v <sub>1</sub> v <sub>3</sub>	6
v <sub>4</sub> v <sub>3</sub>	7
v <sub>4</sub> v <sub>3</sub>	8
v <sub>7</sub> v <sub>6</sub>	9
v <sub>6</sub> v <sub>7</sub>	10

Date \_\_\_ / \_\_\_ / \_\_\_

Saathi

Memory + num  
imp

## UNIT - II

[20-25 marks]

### BINARY SEARCH

Q. 5 8 15 25 30 40 55

(i) search 30

$$\Rightarrow \text{BEG} = 0, \text{END} = 6, \text{MID} = 3$$

$$\text{if } A[M] = 30 \rightarrow A[M] < 30$$

$$\text{BEG} = \text{MID} + 1 = 4, \text{END} = 6, \text{MID} = 5$$

$$A[M] \neq > 30$$

$$\text{BEG} = 4, \text{END} = \text{MID} - 1 = 4, \text{MID} = 4$$

$$\text{if } A[M] = 30 \rightarrow 4/$$

Q- 4 15 20 35 45 55 65 75

search 20

$$\Rightarrow B=0, E=7, M=3$$

$$A[M] > 20$$

$$B=0, E=M-1=2, M=1$$

$$A[M] < 20$$

$$B=M+1=2, E=2, M=2$$

$$A[M] = 20 \rightarrow 2/$$

## SELECTION SORT

avg  $\rightarrow O(n^2)$  worst  $\rightarrow O(n^2)$

e.g.

4 7

33 44 11 88 22 66 55

11 33 44 77 88 22 66 55

" 22 44 77 88 33 66 55

" 22 33 77 88 44 66 55

" 22 33 44 88 77 66 55

" 22 33 44 55 77 66 88

" 22 33 44 55 66 77 88

" 22 33 44 55 66 77 88

## ALGO

MIN(A, k, N, loc)

STEP 1 - set MIN = A[k] & loc = k

STEP 2 - Repeat for j = k+1, k+2 ... N

if min > A[j] then

set MIN = A[j], loc = A[j], loc = j

STEP 3 - Return

SELECTION(A, N)

STEP 1 - Repeat STEP 2 & 3 for k = 1 ... N-1

STEP 2 - Call MIN(A, k, N, loc)

Interchange A[k] & A[loc]

Set Temp = A[k], A[k] = A[loc]  
A[loc] = Temp

STEP 3 - Exit

INSERTION SORT ALGO  $\Theta(n^2)$ ,  $O(n^2)$

eg  $\rightarrow$  ~~77 33 44 11 88 22 66~~  
~~( $\infty$ ) A[0] A[1] A[2] A[3] ... A[n-1]~~

~~Step~~

$\Rightarrow \rightarrow$  ~~33 77 44 ; - - - -~~  
 $\Rightarrow \rightarrow$  ~~33 44 77 11 88 22 66~~  
 $\Rightarrow \rightarrow$  ~~11 33 44 77 88 22 66~~  
 $\Rightarrow \rightarrow$  ~~11 22 33 44 77 88 66~~  
 $\Rightarrow \rightarrow$  ~~11 22 33 44 66 77 88~~

INSERTION (A, N)

STEP 1 - Set A[0] =  $-\infty$

STEP 2 - Repeat steps 3 to 5 for  $k=2, 3, \dots, N$

STEP 3 - Set Temp = A[k] & PTR = k-1

STEP 4 - Repeat while Temp < A[PTR]

a) set  $A[PTR + 1] = A[PTR]$ ,

b) set  $PTR = PTR - 1$ .

[end loop]

STEP 5 - set  $A[PTR + 1] = Temp$

STEP 6 - EXIT

### MERGE SORT

eg → 66 33 40 22 55 88 60 11 80 20

50 44 77 30, divide array into  
2 parts.

⇒ 66 33 40 22 55 88 60 11 80 20 50 44 77 30

⇒ 66 33 40 22 55 88 60

11 80 20 50 44 77 30

divide ⇒ 33 66 22 40 55 88 60

11 80 20 50 44 77 30

conquer ⇒ 22 33 40 66 55 60 88

11 20 50 80 30 44 77

⇒ 22 33 40 55 60 66 88

11 20 30 44 50 77 80

Page No. \_\_\_\_\_

$\Rightarrow 11 \ 20 \ 22 \ 30 \ 33 \ 40 \ 44 \ 50 \ 55 \ 60 \ 66$   
~~22~~ ~~33~~ ~~80~~ ~~88~~

(66) 33 40 22 55 88 60 (11) 80 20  
50 44 ~~77~~ 30; apply quick sort.

11 33 40 22 55 88 60 (66) 80 20  
50 44 ~~77~~ 30

$\Rightarrow 11 \ 33 \ 40 \ 22 \ 55 \ (66) \ 60 \ 88 \ 80 \ 20$   
50 44 ~~77~~ 30

$\Rightarrow 11 \ 33 \ 40 \ 22 \ 55 \ 20 \ 60 \ 88 \ 80 \ (66)$   
50 44 ~~77~~ 30

$\Rightarrow 11 \ 33 \ 40 \ 22 \ 55 \ 20 \ 60 \ (66) \ 80 \ 88$   
50 44 ~~77~~ 30

$\Rightarrow 11 \ 33 \ 40 \ 22 \ 55 \ 20 \ 60 \ 88 \ 80 \ 30$   
~~66~~ 80 88 50 44 ~~77~~ (66)

$\Rightarrow 11 \ 33 \ 40 \ 22 \ 55 \ 20 \ 60 \ 30$   
80 (66) 88 50 44 ~~77~~

$\Rightarrow 11 \ 33 \ 40 \ 22 \ 55 \ 20 \ 60 \ 30 \ 30$   
80 44 88 50 (66) ~~77~~

$\Rightarrow 11 \ 33 \ 40 \ 22 \ 55 \ 20 \ 60 \ 30 \ 60$   
44 (66) 50 88 ~~77~~

max heap  $\rightarrow$  data in descending order  
with  $\rightarrow$  ascending

Date \_\_\_ / \_\_\_ / \_\_\_

Saathi

HEAP SORT  $O(n \log n)$  (both avg & worst)

e.g. 44 30 50 22 60 55 77 55

$\Rightarrow$       44       $\Rightarrow$       50  
  30    50      30    44  
                22    60

$\Rightarrow$       50       $\Rightarrow$       60  
  60    44      50    44  
  22    30      22    30    55

$\Rightarrow$       60       $\Rightarrow$       77  
  50    55      50    60    65  
  22    30    10    22    30    77  
                55

$\Rightarrow$       77  $\rightarrow$  max heap  
  55    60    55  
  50    30    40    55  
  22

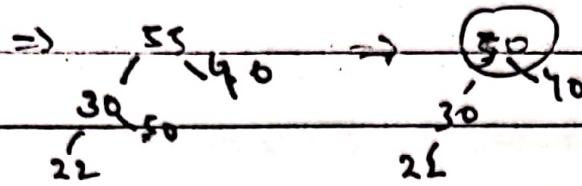
$\Rightarrow$  77 60 55 55 50 40 30 22  
  tally root

$\Rightarrow$       60       $\Rightarrow$       60  
  55    55      50    55    55    70  
  50    30    40    50    50    30  
  22

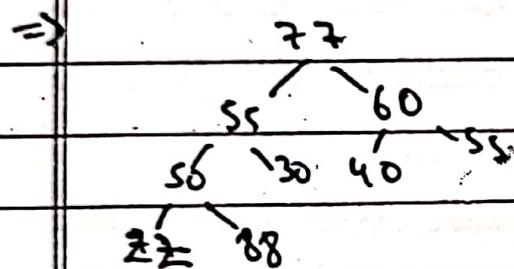
$\Rightarrow 50, 35, 44, 22, 77, 35, 60, 40$



Date \_\_\_ / \_\_\_ / \_\_\_



(ii) Insert 88  
70

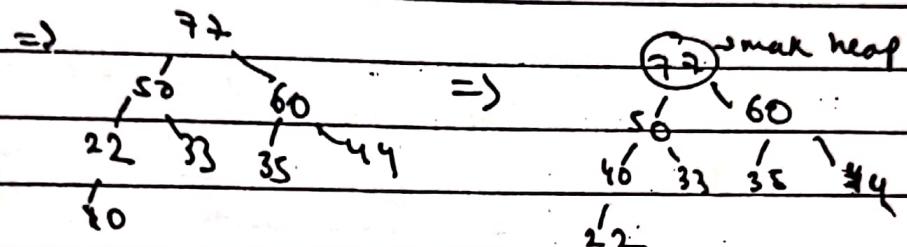
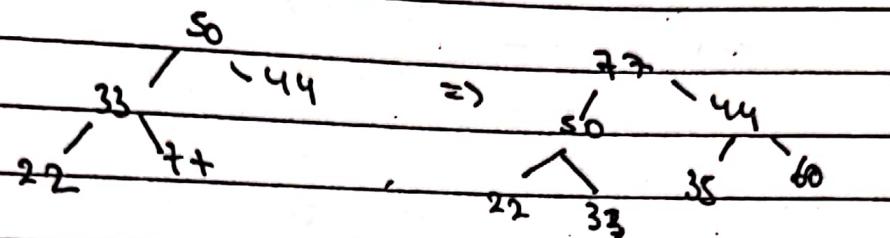


2 - 15 33 44 22 77 35 60 40

$\Rightarrow$  15  
33

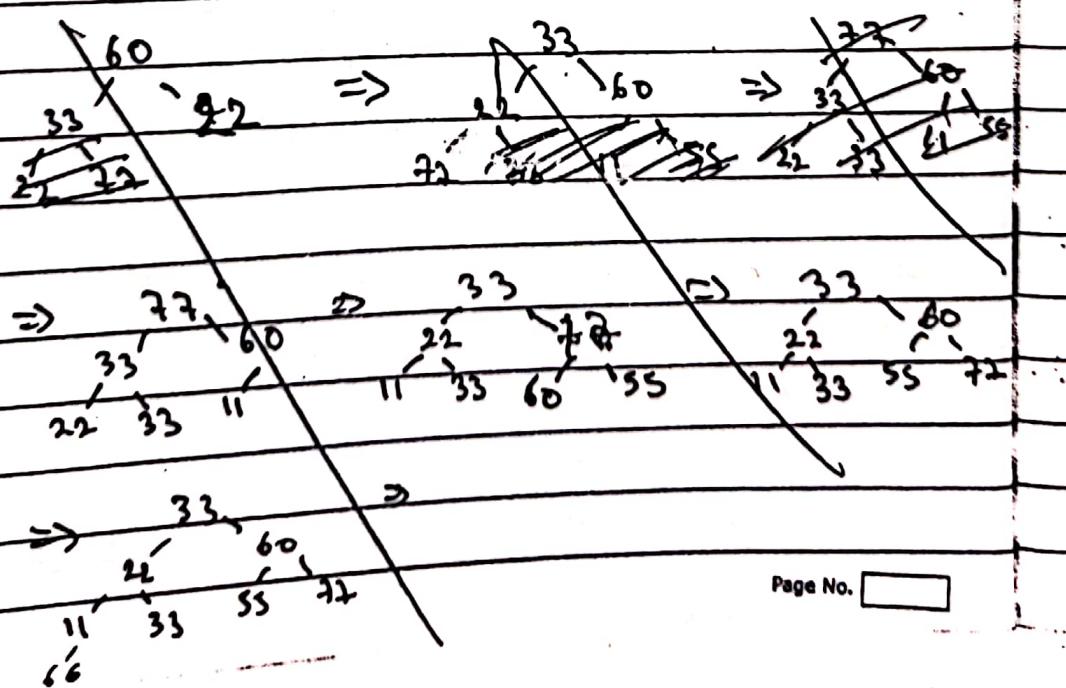
3- ~~50, 33, 44, 22, 77, 35, 60, 40~~

=>



4- ~~60, 33, 22, 77, 88, 11, 55, 66, 77~~

=>



Suppose  $H$  is a complete BT with  $n$  elements then  $H$  is called a heap or max heap if each node  $n$  of  $H$  has the following property -

- (i) The value at  $n$  is greater than or equal to the value at each node of children of  $n$ .
- (ii) for min heap  $\rightarrow$  The value at  $n$  is less than or equal to any of the children of  $n$ .

### INSERTING INTO A HEAP

Suppose  $H$  is a heap with  $n$  elements & value of info is given, we insert value into the heap  $H$  as follows -

- (i) adjoin  $\frac{1}{2}$  value at the end of the edge so that  $H$  is still a complete BT, but not necessarily a heap.
- (ii) Then let value rise to its appropriate place in  $H$  so that  $H$  is finally a heap.

\* direct mapping.

Date \_\_\_\_\_

## HASHING

$$h(k) = k \bmod 10$$

$$23 \bmod 10 = 3$$

$$45 \bmod 10 = 5$$

$$10 \bmod 10 = 0$$

0	10
1	
2	
3	23
4	
5	45
6	
7	
8	
9	

### 1- DIVISION METHOD

$$h(k) = k \bmod n$$

✓  
 hash key → number  
 fraction or data → storage  
 storage → store

### HASH VALUE

### 2- MID-SQUARE METHOD

$$\text{eq} \Rightarrow 2369$$

↓

$$\text{key} = 2(36)^2 9$$

$$2(1234)9$$

soff

$$\begin{array}{l} a b c d \\ \downarrow \\ (bcd)^2 \rightarrow xyz \\ \downarrow \\ abc(a^2y^2z)^{n-1} \end{array}$$

$$= 212349$$

$$= 21\textcircled{2}349$$

hash value

- \* div method - choose a number m larger than the no. n of keys in k. The no m is usually chosen to the prime no or a number & without small divisor. Since this frequently minimize the no of collisions.



\* we never force it to compare by  $H(S) = K \text{ mod } n$

Date \_\_\_\_\_

Saathit

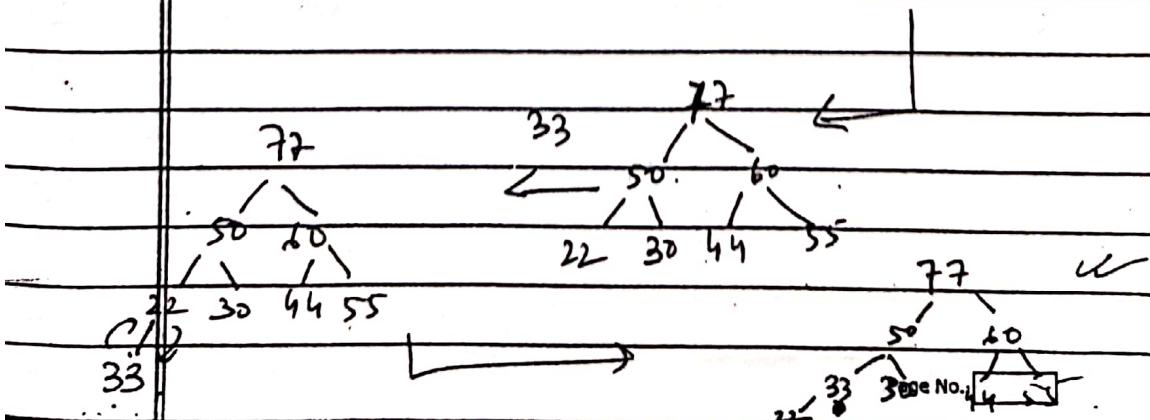
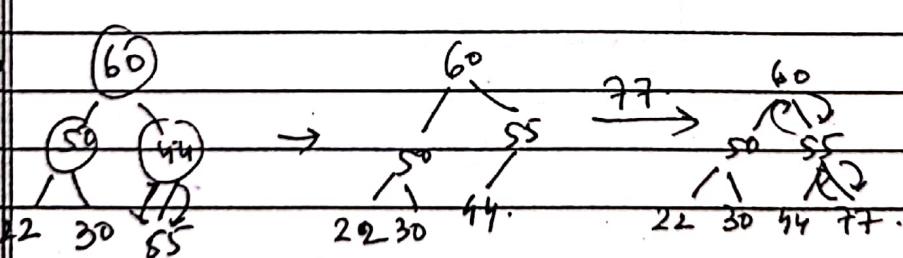
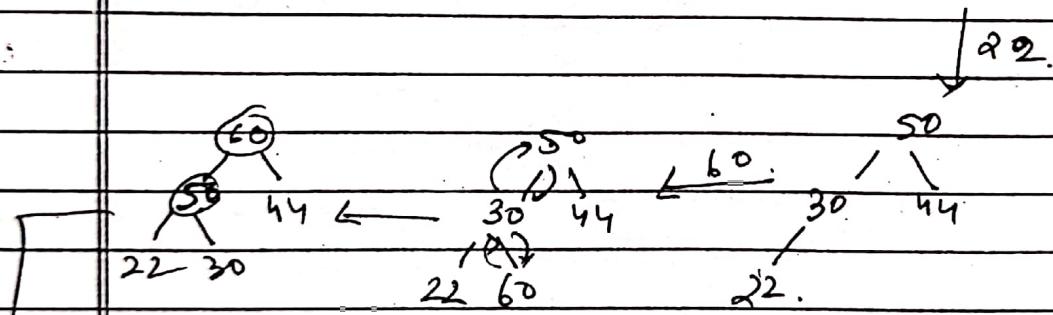
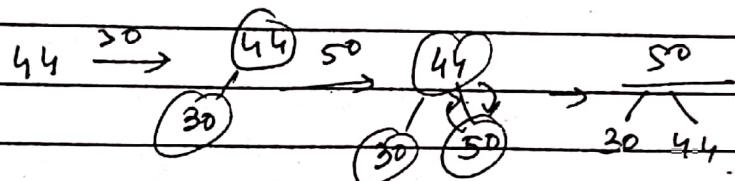
(Max/min) Heap sort  $\rightarrow$  Heap tree + partition  
(Fusion) + (Selection)

Heapify

- ① Root should be greater than all its children
- ② Left-justified tree

44, 30, 50, 22, 60, 55, 77, 33.

Max Heap



Date 1/1 Selection

Saathie

(77)

50 60

33 30 44 55

22

22

60

50 60

50 22

33 30 44 55 33 30 44 55

50 22  
33 30 44

50 55  
33 30 44

50 55  
33 30 44 22

55  
33 30 22

22  
33 30

50  
33 30 22 30  
(50)

22 30  
33 30  
22  
22 30

33 30  
22  
22

30

30

20 30 33 44 50/55 60/77

Ascending.

77 60 55

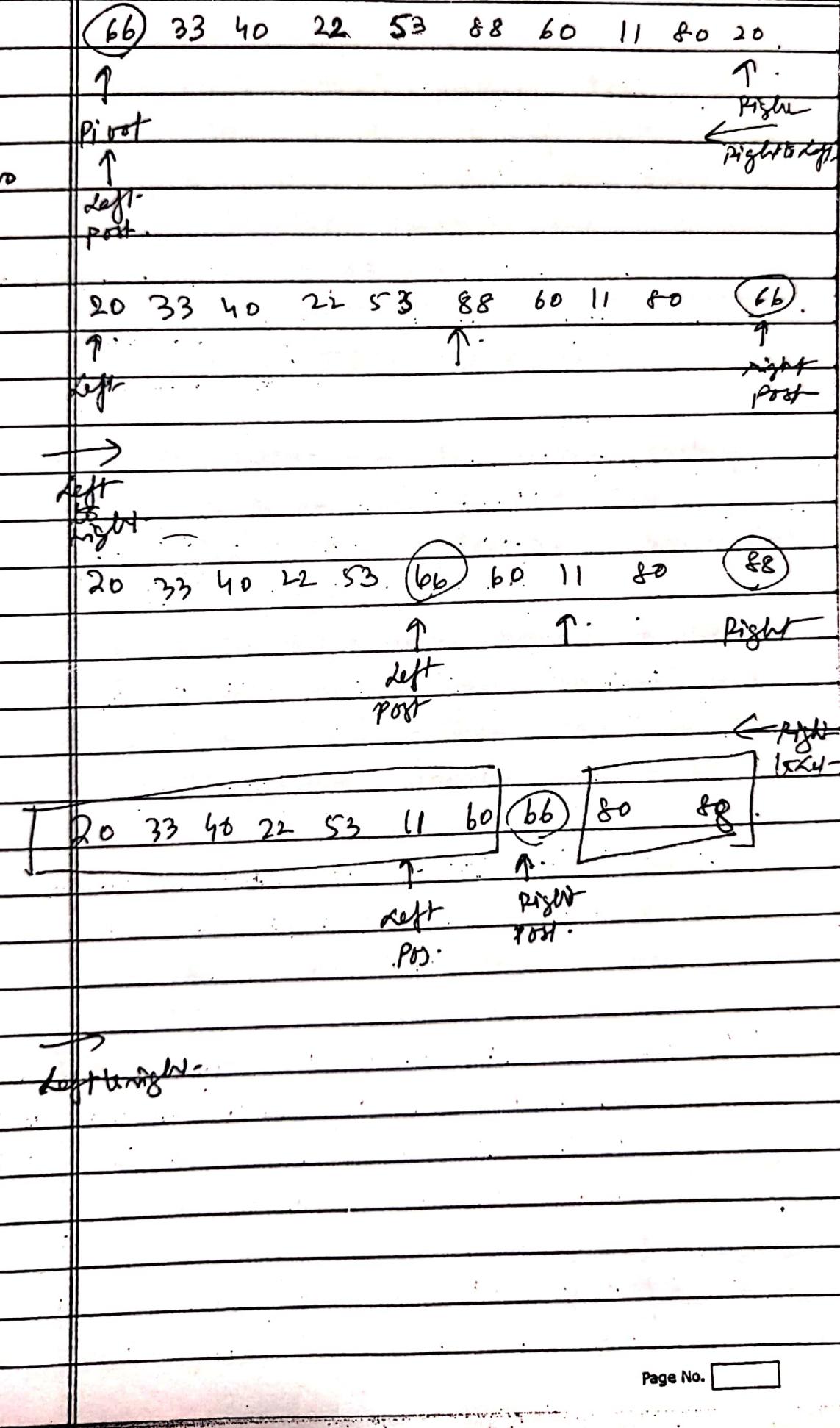
Descending.

Quick Sort → Divide & Conquer  
 Date \_\_\_\_\_

① Pivot → ② Partitioning ↴  
 ↴ nosort

**Shastri**  
 Sort

Merge



MASKING (CONTINUED)3. FOLDING METHOD

e.g. key = abcd  $\rightarrow$  // divide in 2 parts

$$\begin{array}{l} \text{hash func} \\ \text{hash} = ab + cd = n \end{array}$$

/ hash value  
at  $\approx$  index  $n$

e.g.  $\rightarrow$  1234

$$\begin{array}{rcl} 12 + 34 & = & 46 \\ 2 + 43 & = & 45 \\ 1 + 2 + 3 + 4 & = & 10 \end{array}$$

( b+d+c )

// also valid

COLLISION RESOLUTION TECHNIQUE

1. LINEAR PROBING:  $h, h+1, h+2, h+3, \dots$

2. DOUBLE MASKING: combination of 2-3 hashing methods is called double hashing methods.

\* NIN-SQUARE ME - The key is squared, the hash func is defined by  $h(k) = k^2 \mod L$  where  $L$  is obtained by deleting digits from both ends of  $k^2$ .

\* FOLDING ME - The key  $k$  is partitioned into a no. of parts  $k_1 - k_r$  where

each part except possibly the last has the same no of digits as the required address then the ~~parts~~<sup>parts</sup> are added together ignoring the last carry.

$$h(k) = k_1 + k_2 \dots$$

### COLLISION RESOLUTION TECH.

2- DOUBLE HASHING - In double hashing, 2<sup>nd</sup> hash func 'h' is used for resolving the collision,  $h, h+h', (h+h')+h'$

3- QUADRATIC PROBING - Suppose a record R with key K has the hash address ~~test~~  $h(k) = h$ , then searching the location with address with  $h, h+1, h+4, h+9, h+16 \dots$

Q- 23, 45, 43, 67, 66, 54, 65 , store in array using div method , with linear prob.

⇒ hash func

$$h(s) = k \mod 10$$

$$23 \mod 10 = 3$$

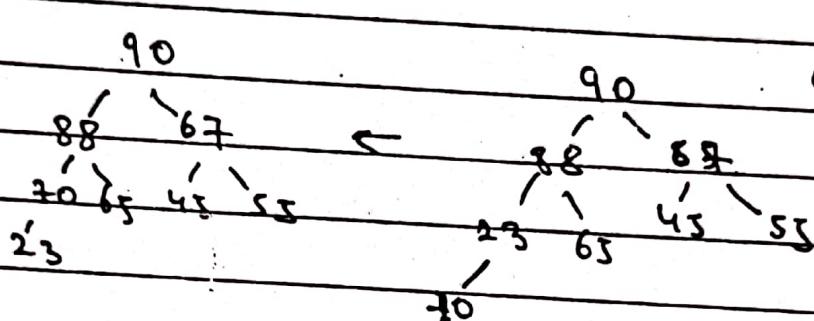
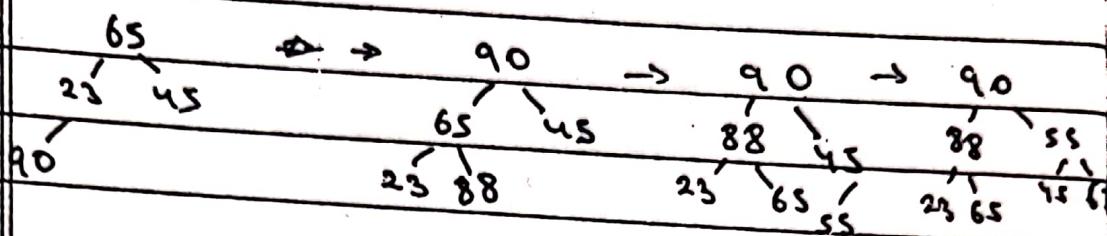
0	
1	
2	
3	23
4	43
5	45
6	66
7	67
8	54
9	65

Page No. [ ]

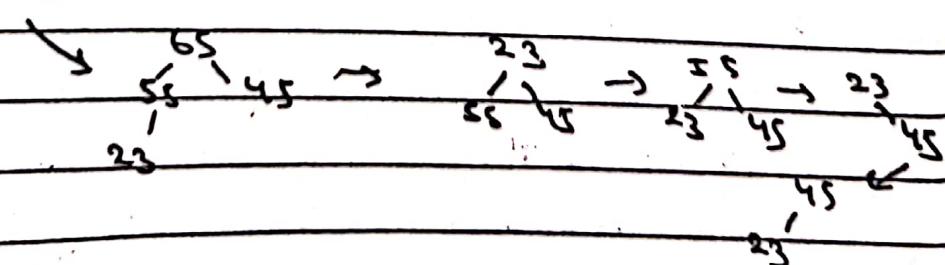
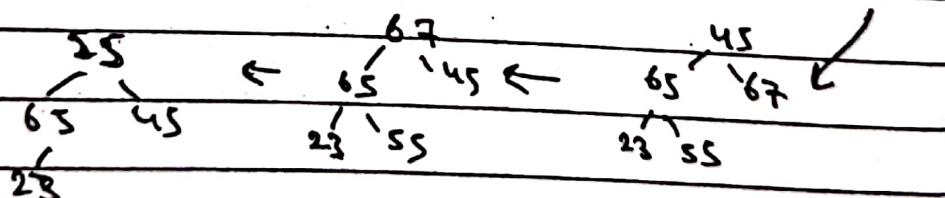
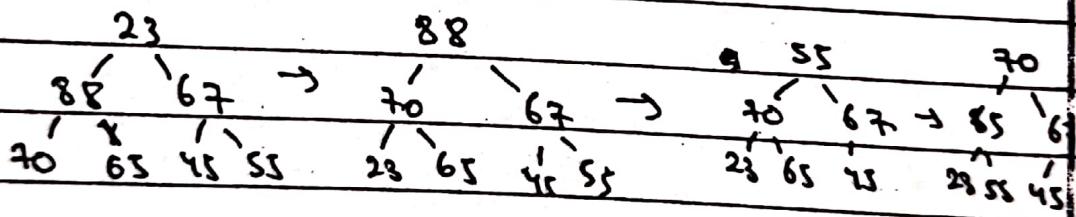
## HEAP SORT DEFINITION

eg:- 65 23 45 90 88 55 67 70

=>



deletion



90	88	70	67	65	55	45	23
----	----	----	----	----	----	----	----