SC 631: Games and Information

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Lecture 1: August 14

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This lecture's notes illustrate some uses of various LATEX macros. Take a look at this and imitate.

1.1 Some Definitions

1.1.1 Event

An event is a subset of Y if w=Y is the state of the world then we say event A obtains in w, if w=A.

1.1.2 $F_i(\omega)$

 $F_i(\omega) \triangleq \text{that event F of Fi s.t. } \omega \in F$

1.1.3 Knowing an Event

 P_i knows event A in state of the world ω if $F_i(\omega) \subseteq A$ $\Longrightarrow \forall \omega' \in F_i(\omega) \Longrightarrow \omega' \in A$

1.1.4 Knowledge Operator $K_i(A)$

 $K_i(A) = \{ \omega \in Y \mid F_i(A) \subseteq A \} K_i(A)$ is the set of all those states of the world when player P_i knows A

1.1.5 Common Knowledge

An Event A is common knowledge in state of the world ω if for all $i_1, i_2, i_n \in N, n < \infty$ $\omega \in K_{i_1} K_{i_2} K_{i_3} K_{i_n}(A)$

1.2 Some theorems and stuff

- If $\omega^* \in Y$ is the state of the world then P_i knows A in ω^* iff $\omega^* \in K_i(A)$
- $K_j(K_i(A)) = \text{Event that } P_j \text{ knows that } P_i \text{knows A}$

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1.3 Important Properties

- $K_i(A) \subseteq A$
- $A \subseteq B$ $\Longrightarrow K_i(A) \subseteq K_i(B)$
- $\forall A \subseteq Y, K_iK_i(A) = K_i(A)$
- $K_i(A) \cap K_i(B) = K_i(A \cap B)$
- $K_i((K_i(A))^c) = (K_i(A))^c$

Lemma 1.1 If A is common knowledge in ω , and $B \supseteq A$, then B is also common knowledge in ω (follows from property 2)

Lemma 1.2 If A is common knowledge in ω , then $\omega \in K_i(A)$ and $F_i(\omega) \subseteq A$ for i

Theorem 1.3 If A is common knowledge in ω and $\omega' \in F_i(\omega)$ for some $i \in N$, then A is common knowledge in ω'

A is common knowledge in ω $\Longrightarrow \omega \in K_i K_{i_1} K_{i_2} K_{i_n}(A) \, \forall i_1, i_2, i_n \in N$ Suppose $\omega' \in F_i(\omega)$, $\omega \in K_i (K_{i_1} K_{i_2} K_{i_n}(A))$ $\Longrightarrow F_i(\omega) \subseteq K_{i_1} K_{i_2} K_{i_n}(A)$ $\Longrightarrow \omega' \in K_{i_1} K_{i_2} K_{i_n}(A)$

1.4 Next topic

Here is a citation, just for fun [CW87].

References

[CW87] AUTHOR1 and AUTHOR2, Title of the paper Name of the Journal, 2014, pp. 1–6.