

IMAGE COMPRESSION USING SINGULAR VALUE DECOMPOSITION (SVD)

Application of Linear Algebra

MEET OUR TEAM



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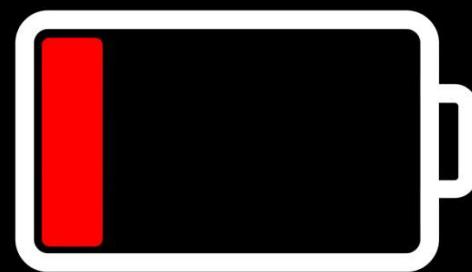
Minakshi



Sheshmani

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Introduction to Image Compression

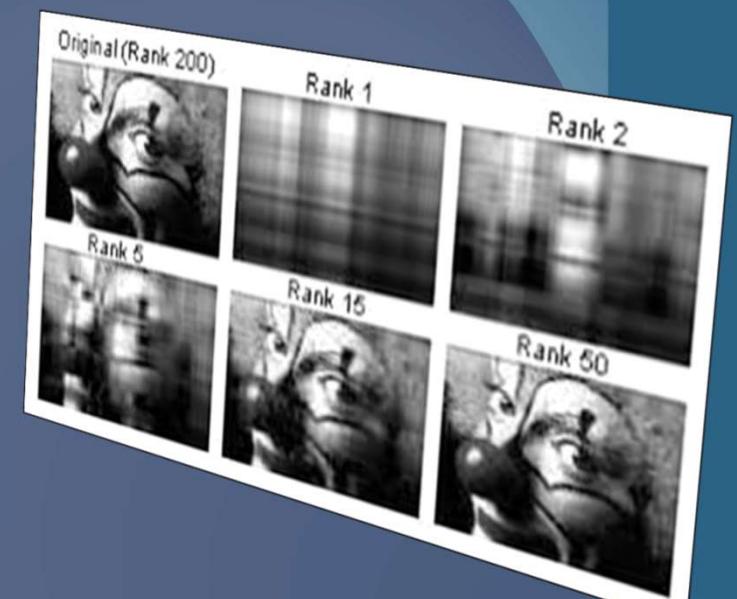
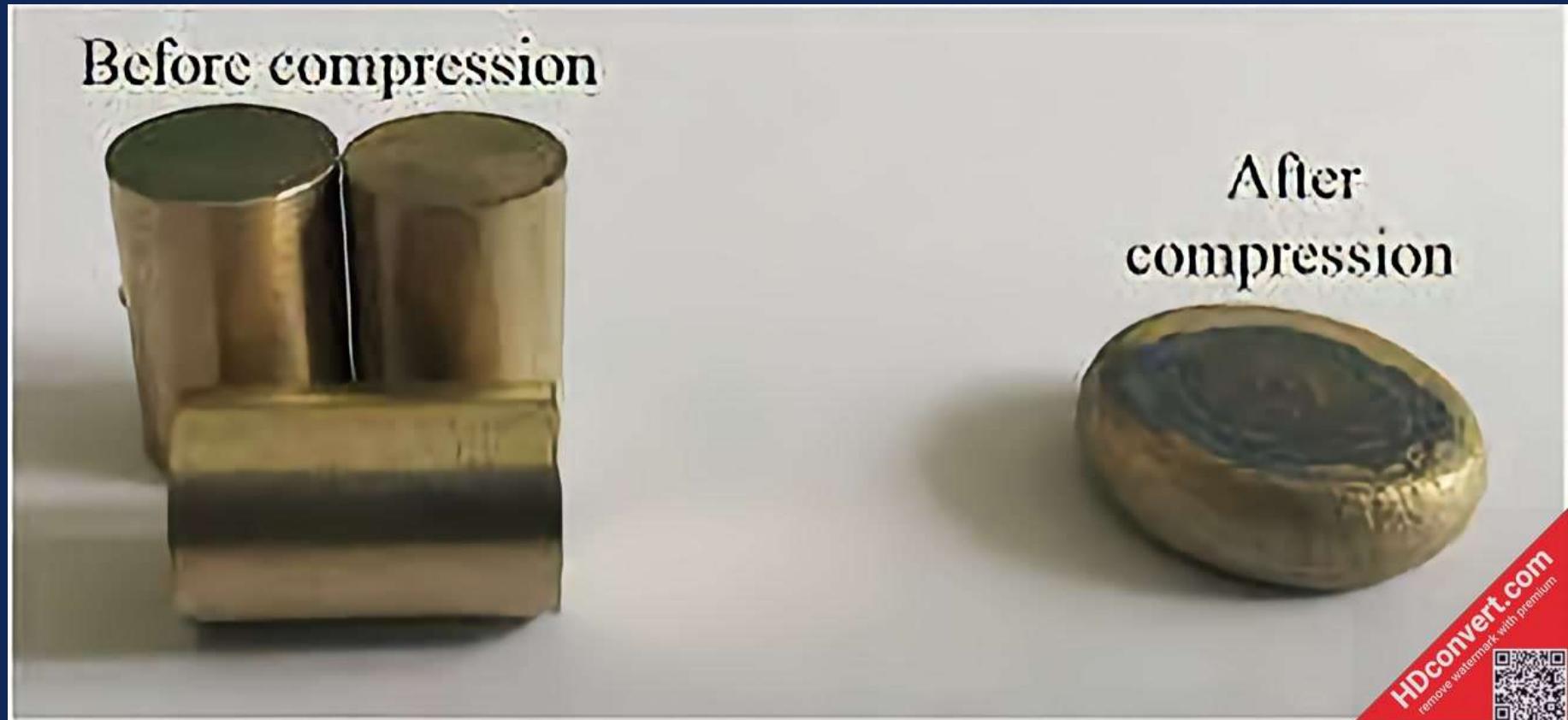
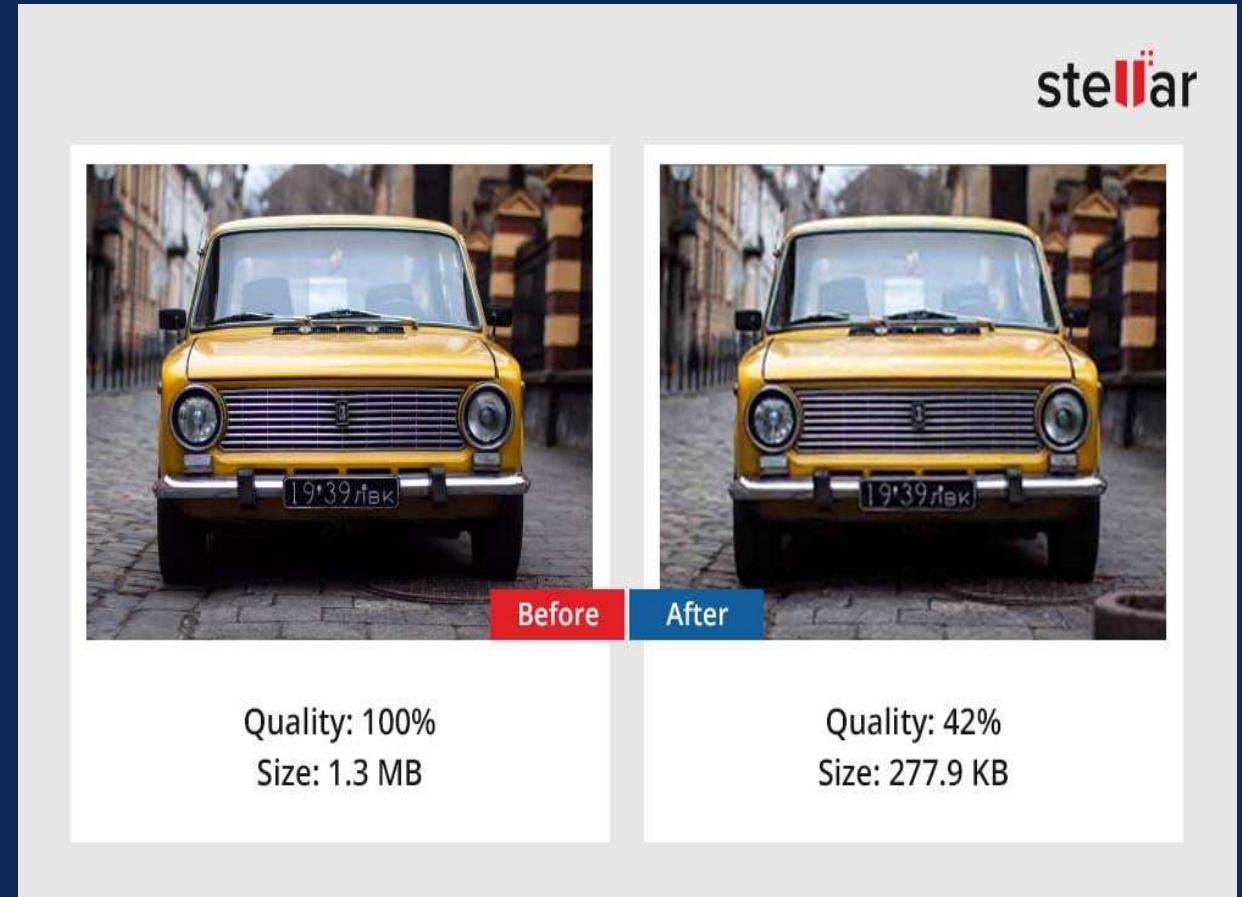


Image compression is the process of reducing the size of an image file while preserving its essential visual information.





Introduction To Image Compression

$$(1300 - 277.9) \text{ KB} = 1022.1 \text{ KB} !$$

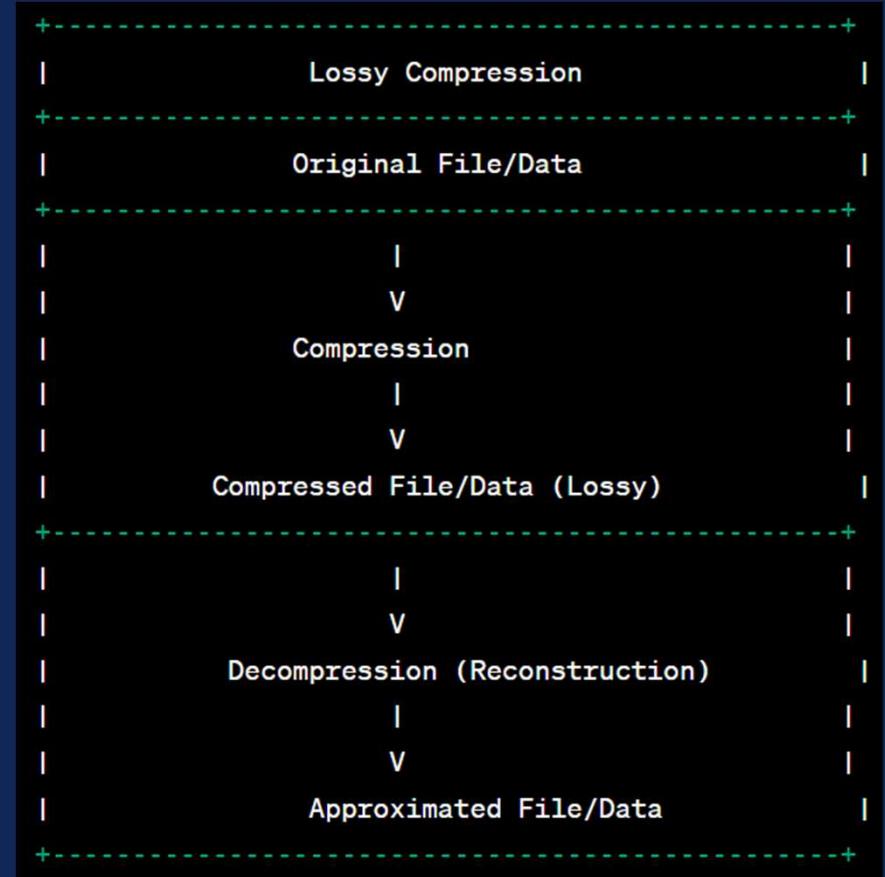
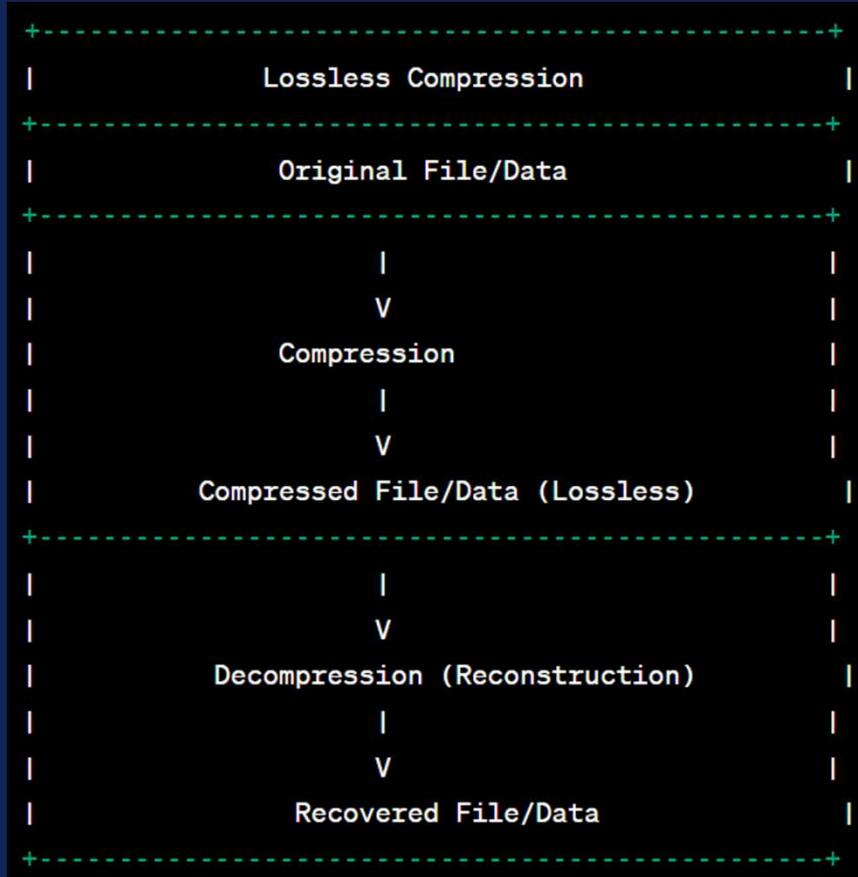
IMAGE COMPRESSION TECHNIQUE

LOSSLESS COMPRESSION

Definition	Compression technique that
	reduces file size without
	losing any data.
+-----+	
Compression	Files are compressed in a
Technique	way that allows the original
	data to be perfectly
	reconstructed.
+-----+	
File Size	Reduction in file size is
	relatively lower compared to
	lossy compression.
+-----+	
Quality	Original quality of the file
Preservation	is preserved after compression.
+-----+	
Suitable For	Ideal for text-based files,
	documents, and data storage
	where data integrity is
	crucial.
+-----+	

LOSSY COMPRESSION

Definition	Compression technique that
	reduces file size by
	permanently removing some
	information.
+-----+	
Compression	Irrelevant or less important
Technique	data is discarded to achieve
	higher compression ratios.
+-----+	
File Size	Reduction in file size is
	relatively higher compared to
	lossless compression.
+-----+	
Quality	Some loss of quality or
Sacrifice	information occurs during
	compression.
+-----+	
Suitable For	Suitable for multimedia files,
	images, videos, and audio
	files where some loss of
	quality is acceptable.
+-----+	



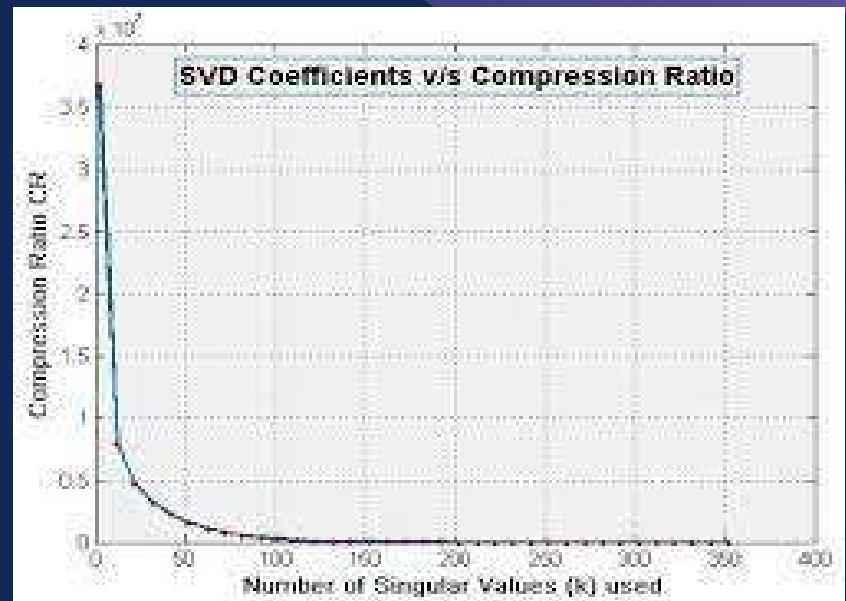
TRADE-OFF BETWEEN COMPRESSION RATIO AND IMAGE QUALITY:

- SVD-based compression techniques offer a trade-off between the compression ratio and image quality.
- Increasing the compression ratio (i.e., achieving higher compression) often leads to a loss in image quality.
- The more aggressive the compression, the more noticeable the degradation in image details and clarity.
- Users must carefully balance the desired compression ratio with acceptable image quality based on their specific needs and application requirements.



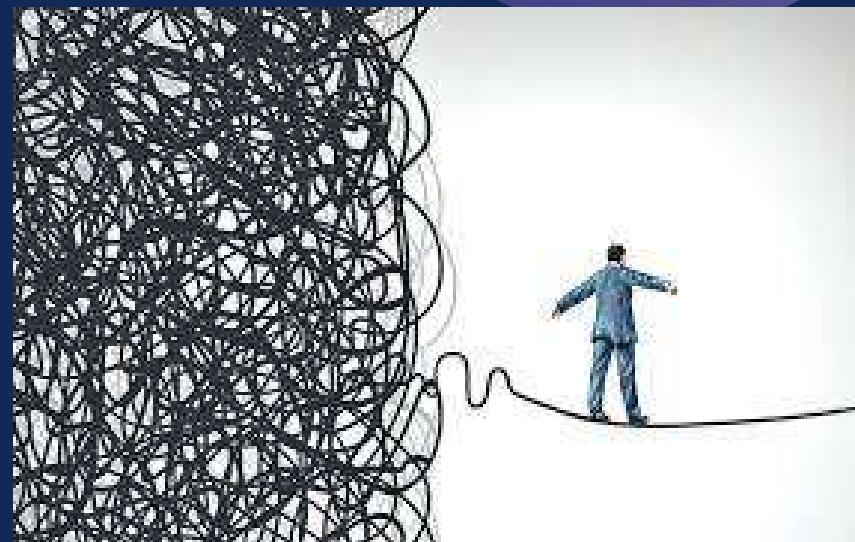
IMPACT OF SINGULAR VALUE THRESHOLDING:

- Singular value thresholding is a key step in SVD-based compression algorithms.
- It involves setting certain singular values to zero or near-zero, effectively discarding them.
- The discarded singular values represent image details that are considered less significant.
- While this reduction in singular values allows for higher compression, it can result in the loss of important image information, leading to decreased image quality.



COMPUTATIONAL COMPLEXITY:

- SVD-based compression techniques can be computationally intensive, especially for large images.
- Calculating the SVD of a large matrix requires significant computational resources and time.
- This complexity can limit the practicality of real-time or resource-constrained applications.
- Alternative methods, such as approximation algorithms or truncated SVD, can be employed to reduce computational complexity at the expense of some accuracy.



Unlocking the Power of SVD Decomposition: U , Σ , and V^T

- What is SVD Decomposition?
- Understanding the Three Main Components: U , Σ , and V^T
- Application of SVD Decomposition in Various Fields
- Challenges Faced When Working with SVD Decomposition

EXPLANATION OF EIGENVALUES AND EIGENVECTORS AND THEIR SIGNIFICANCE IN SVD-BASED IMAGE COMPRESSION.

Term	Definition	Significance in SVD-based image compression
Eigenvalue	A scaling factor that stretches or shrinks an eigenvector.	Used to represent the image in a lower-dimensional space.
Eigenvector	A direction in which the data is stretched or shrunk.	Used to represent the image in a lower-dimensional space.
SVD-based image compression	A compression technique that uses SVD to represent the image in a lower-dimensional space.	Allows the image to be compressed without losing too much information.

UNDERSTANDING THE THREE MAIN COMPONENTS: U, Σ , AND V^T

The SVD of a matrix A can be written as:

$$A = U \Sigma V^T$$

Where:

- U is an $m \times m$ matrix, where m is the number of rows in A
- Σ is a diagonal matrix with n non-negative entries, where n is the number of columns in A
- V^T is an $n \times n$ matrix, where n is the number of columns in A

What is SVD Decomposition?

The SVD can be used for a variety of tasks, including:

- **Data Compression.**

The SVD can be used to compress data by representing it in a lower-dimensional space.

- **Dimensionality Reduction.**

The SVD can be used to reduce the dimensionality of data by projecting it onto a lower-dimensional subspace.

- **Image Processing.**

The SVD can be used for image processing tasks such as denoising, deblurring, and compression.

- **Machine Learning.**

The SVD can be used for machine learning tasks such as dimensionality reduction, feature extraction, and classification.

CHALLENGES FACED WHEN WORKING WITH SVD DECOMPOSITION

Computational complexity.
SVD is a computationally expensive opera

Interpretability.
The results of SVD can be difficult to interpret, especially for high-dimensional data.

Sensitivity to noise.
SVD is sensitive to noise in the data.

Overfitting.
SVD can be prone to overfitting, especially when the number of features is large.

COMPRESSION ALGORITHM

Now lets see
how the magical
spells of SVD
works :-



Consider a matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ from an image displayed as pixel values and compute the singular value decomposition.

SOLUTION:

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

Subtracting λI from A

$$|A - \lambda I| = 0$$

$$\left| \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0$$

$$\left| \begin{matrix} 5 - \lambda & 4 \\ 1 & 2 - \lambda \end{matrix} \right| = 0$$

$$(5 - \lambda)(2 - \lambda) - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$(\lambda - 6)(\lambda - 1) = 0$$

$$\lambda = 6; \lambda = 1$$

Therefore our eigenvalues are 6 and 1. We construct the matrix S² by placing the eigenvalues along the main diagonal in decreasing order.

$$S^2 = \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$$

Therefore, taking the square root of matrix S2 gives,

$$S = \begin{bmatrix} \sqrt{6} & 0 \\ 0 & 1 \end{bmatrix}$$

Now we need to find the eigenvectors of A which are the columns of V First we will show where = 6;

$$\left(\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} \right) \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} -1 & 4 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$-x + 4y = 0 \rightarrow 1$$

$$x - 4y = 0 \rightarrow 2$$

on solving we get,

$$x = 4y; x = 4; y = 1$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Since V has an orthonormal basis, \vec{v}_1 needs to be of length one. We divide \vec{v}_1 by its magnitude to accomplish this. Thus,

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0.2 \end{pmatrix}$$

Similarly for $\lambda = 1$

$$\left(\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} 4 & 4 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$4x + 4y = 0 \rightarrow 3$$

$$x + y = 0 \rightarrow 4$$

on solving we get,

$$x = -y; x = -1; y = 1$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

Since V has an orthonormal basis, \vec{v}_2 needs to be of length one. We divide \vec{v}_2 by its magnitude to accomplish this. Thus,

$$\vec{v}_2 = \begin{pmatrix} -0.7 \\ 0.7 \end{pmatrix}$$

Now we need to construct the augmented orthogonal matrix V ,

$$V = (\overrightarrow{v_1} \quad \overrightarrow{v_2})$$

$$V = \begin{pmatrix} 1 & -0.7 \\ 0.2 & 0.7 \end{pmatrix}$$

Now we need to find the eigenvectors for A . Finding the eigenvectors using the eigen values previously found.
First we show where $\lambda = 6$;

$$\left(\begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} \right) \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 0 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$x = -5; y = 0$$

$$\overrightarrow{u_1} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$$

$$\left(\begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} 5 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$x = 0; y = 5$$

$$\overrightarrow{u_2} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

$$\textcolor{blue}{U} = (\overrightarrow{u_1} \quad \overrightarrow{u_2})$$

$$\textcolor{blue}{U} = \begin{pmatrix} -5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$A = U * S * V^T$$

$$= \begin{pmatrix} -5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} \sqrt{6} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0.2 \\ -0.7 & 0.7 \end{pmatrix}$$

$$A = \begin{pmatrix} -12 & -2.4 \\ -3.5 & 3.5 \end{pmatrix}$$



image output using 5 singular values



image output using 25 singular values



image output using 15 singular values



image output using 65 singular values

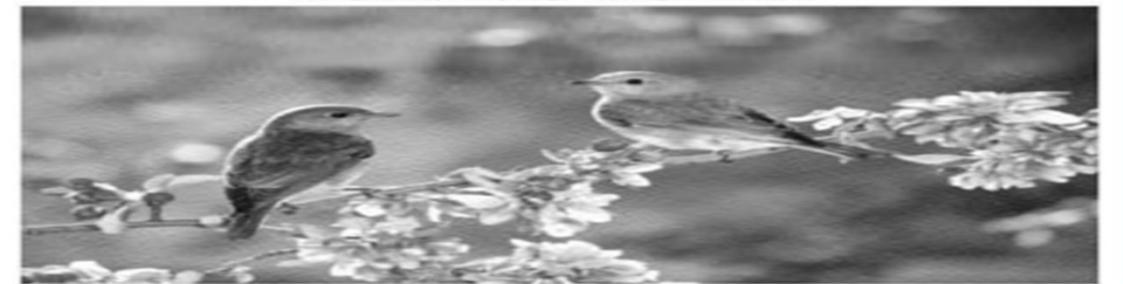


image output using 85 singular values



image output using 115 singular values



image output using 160 singular values



image output using 165 singular values



image output using 170 singular values



image output using 175 singular values



image output using 180 singular values

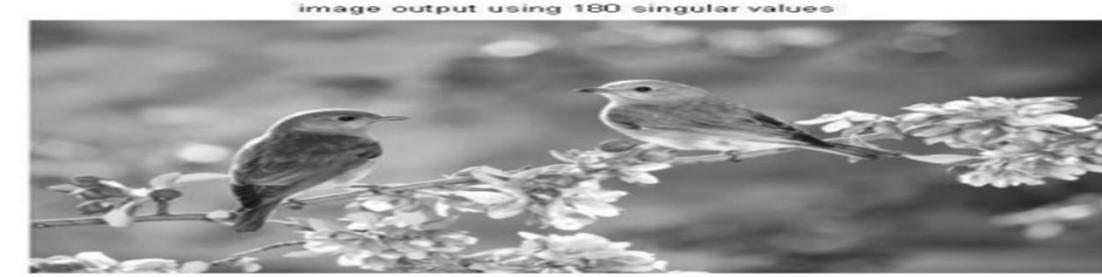
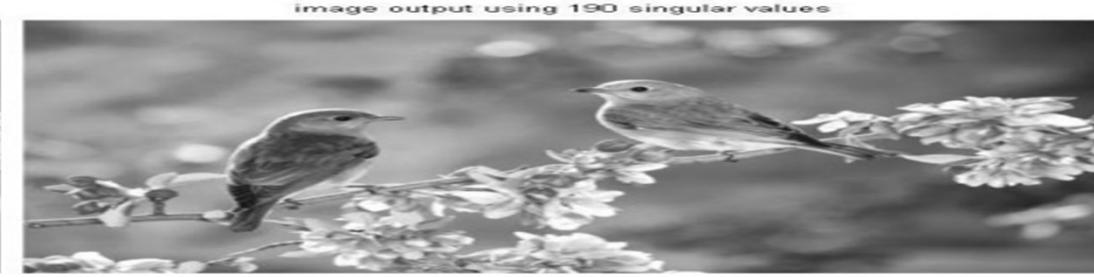
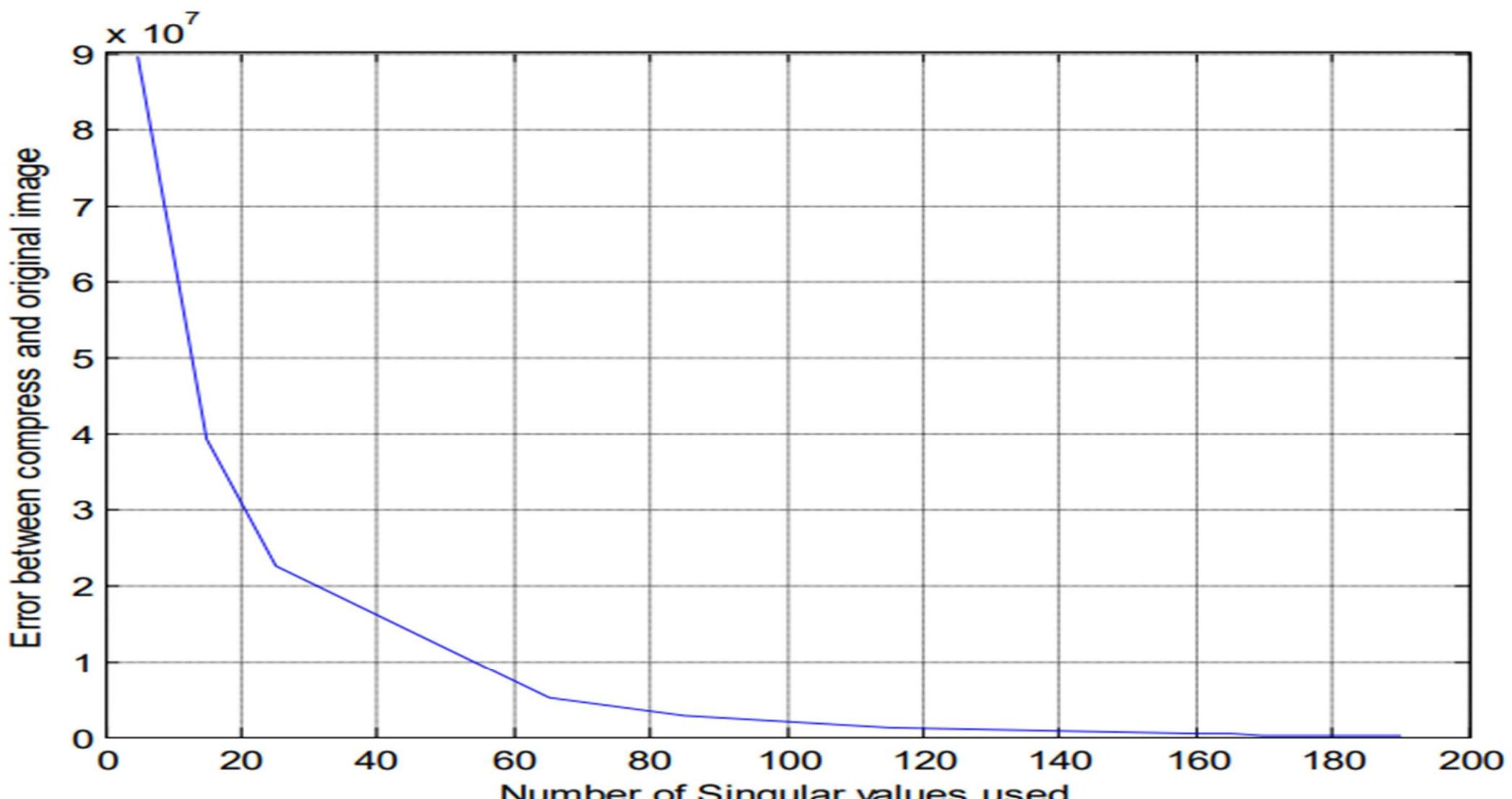


image output using 190 singular values





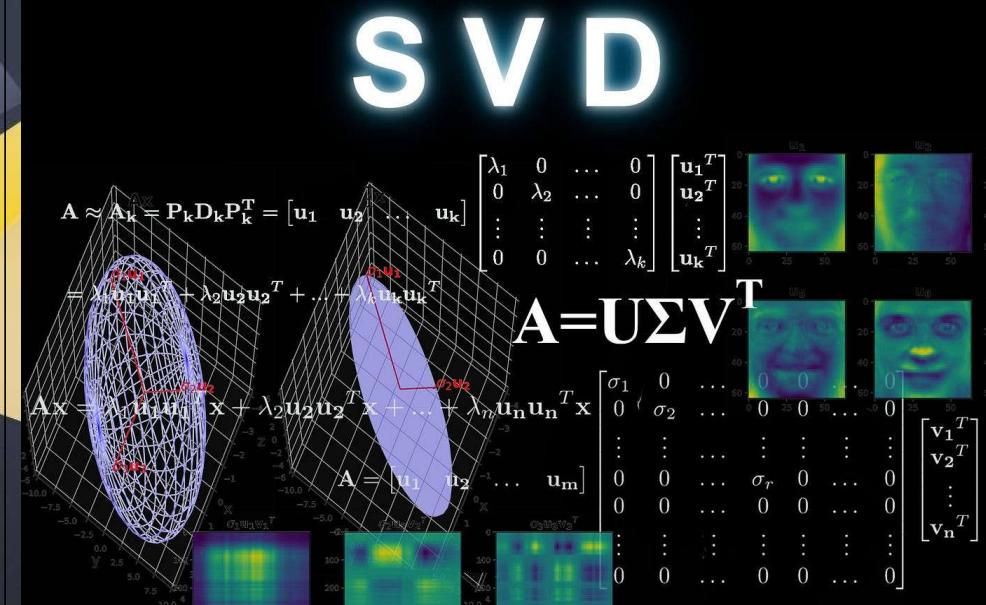
Graphical representation using MATLAB

RECONSTRUCTION PROCESS IN SVD

- Singular value decomposition (SVD) is a matrix decomposition technique that can be used to reconstruct an image from its projections.
- The reconstruction process involves three steps:
- Decomposing the image matrix into its singular value components.
 - Reconstructing the image matrix from its singular value components.
 - Applying a nonlinear regularization technique to improve the quality of the reconstructed
 - SVD is a powerful tool for image reconstruction and has been used in a variety of applications, including medical imaging, computer vision, and remote sensing.

OVERVIEW OF THE PROCESS OF RECONSTRUCTING A COMPRESSED IMAGE FROM THE COMPRESSED DATA

- The image is first decomposed into three matrices using SVD.
- The singular values in the diagonal matrix are then thresholded, which means that the smallest singular values are removed.
- The remaining singular values are then used to reconstruct the image.
- The reconstructed image is then displayed.



The SVD compression technique is a very effective way to compress images without losing too much quality. It is often used in image compression standards such as JPEG and JPEG2000.

VISUAL COMPARISON OF ORIGINAL, COMPRESSED, AND RECONSTRUCTED IMAGES TO HIGHLIGHT THE IMPACT ON IMAGE QUALITY



DISCUSSION ON METHODS FOR EVALUATING IMAGE QUALITY (E.G., PSNR, SSIM) AND HOW THEY RELATE TO THE COMPRESSION RATIO

PSNR (Peak Signal-to-Noise Ratio)

Is a measure of the quality of an image by comparing it to a reference image

SSIM (Structural Similarity Index)

Is a measure of the similarity between two images

Both PSNR and SSIM can be used to evaluate the quality of images compressed using SVD. However, SSIM is generally considered to be a more reliable measure of image quality, as it takes into account the human visual system

APPLICATION OF SVD DECOMPOSITION IN VARIOUS FIELDS

Image compression.

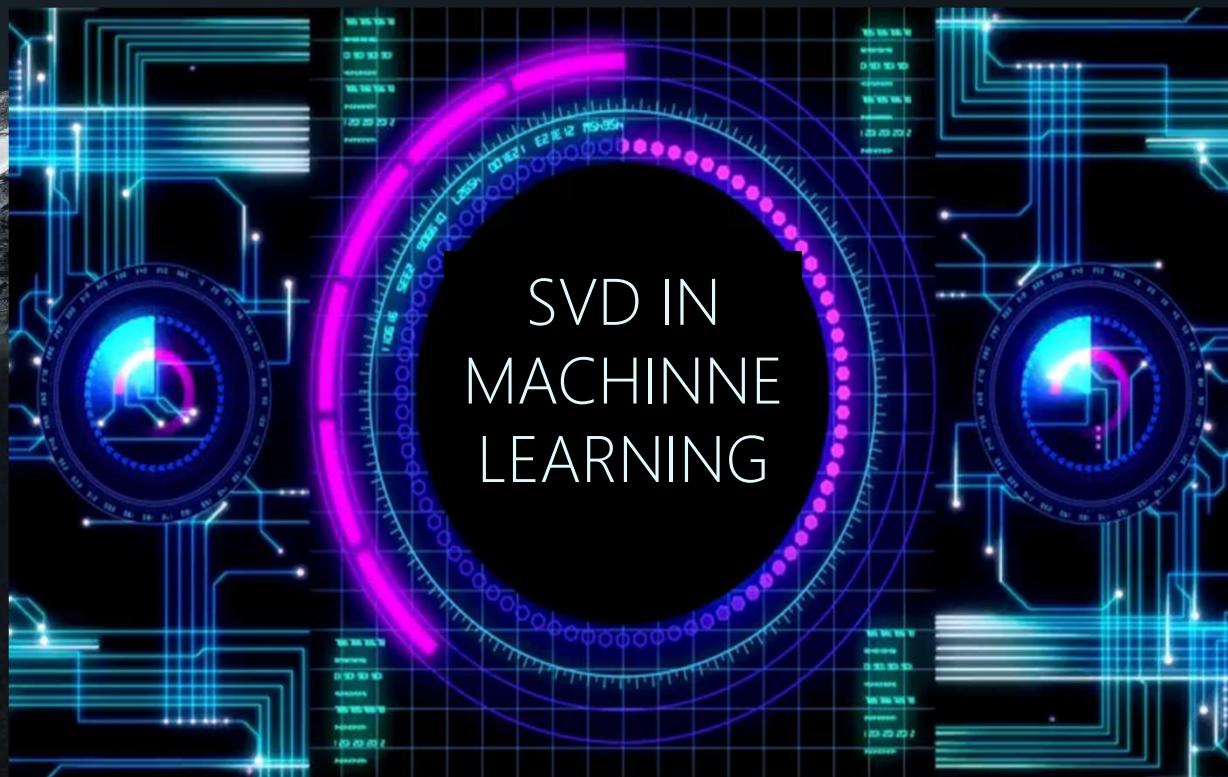
SVD can be used to compress images by reducing the number of dimensions in the image.

Machine learning.

SVD is used in machine learning algorithms such as principal component analysis (PCA) and linear regression.

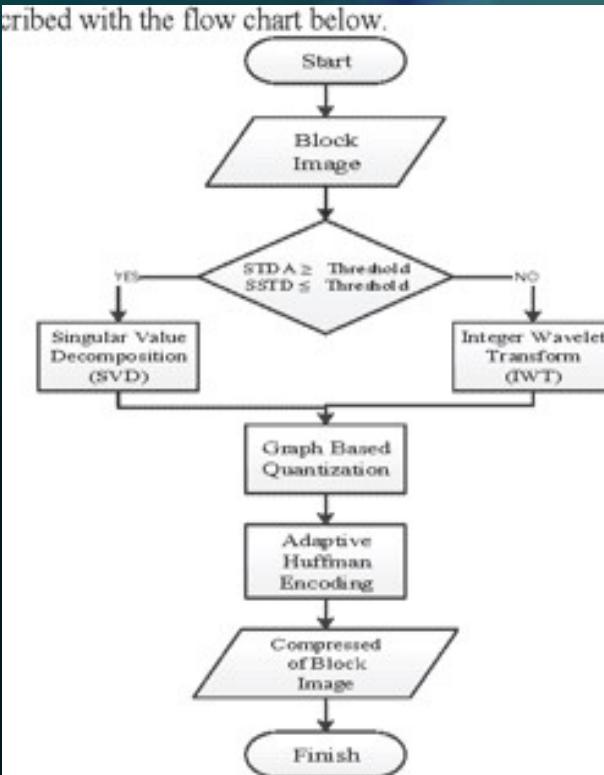
Signal processing.

SVD is used in signal processing applications such as image restoration and audio compression.

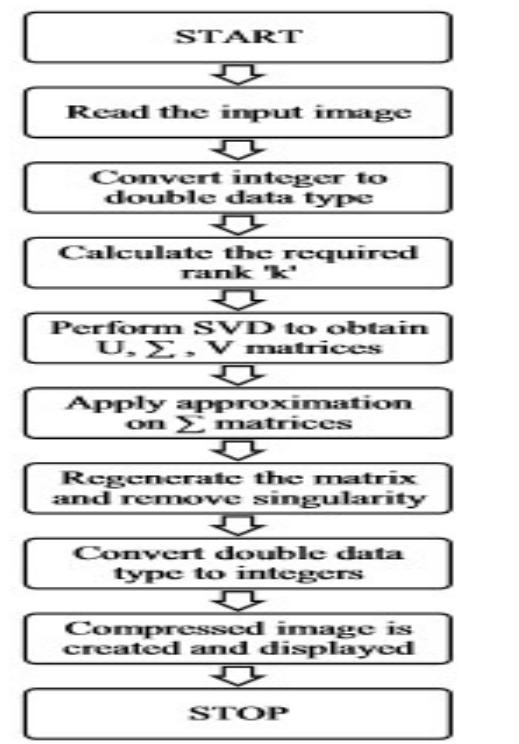


EXAMPLES OF APPLICATIONS

Satellite imaging: SVD can be used to compress satellite images, which makes them easier to store and transmit.



Video compression: SVD can be used to compress video footage, which makes it easier to store and transmit. This can be especially beneficial for applications such as streaming video and video conferencing.



Medical imaging: SVD can be used to compress medical images, which makes them easier to store and transmit.

SOME EXTENSIONS OR VARIATIONS OF THE SVD ALGORITHM USED FOR SPECIFIC PURPOSES

Randomized SVD: Randomized SVD is a faster and more scalable alternative to traditional SVD. It works by randomly sampling a subset of the data and then using SVD to decompose the sampled data.

Truncated SVD: Truncated SVD is a variation of SVD that only keeps the first few singular values. This can be useful for reducing the size of the data without sacrificing too much accuracy.

These are just a few examples of the many extensions and variations of the SVD algorithm. There are many other variations that have been developed for specific purposes.

MATHEMATICAL EXPLANATIONS OF ADVANCED CONCEPTS OR VARIATIONS RELATED TO SVD-BASED COMPRESSION TECHNIQUES

Low-rank approximation

Low-rank approximation is a technique for reducing the dimensionality of a matrix by keeping only the most important features.

Singular value thresholding

Singular value thresholding is a variation of low-rank approximation that involves setting the singular values below a certain threshold to zero

K-SVD

K-SVD is a greedy algorithm for dictionary learning. It works by iteratively updating a dictionary of atoms and a sparse representation of the data

These are just a few examples of advanced concepts or variations related to SVD-based compression techniques. There are many other techniques that have been developed for specific purposes.

**THANK YOU FOR YOUR
ATTENTION**



ANY QUESTIONS?