Production relationships

Deependra Dhakal

GAASC, Baitadi

Tribhuwan University

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Outline

Product relationships

- All major production relationships can be categorized under three categories. i.e.:
  1. Factor-product relationship
  2. Factor-factor relationship
  3. Product-product relationship

Outline

Background

- When concerned with resource allocation for production optimization, an understanding of input-output or factor-product relationship is important.
- First, study of physical or technical relationship is important. Second, for decision making, application of economic choice indicators such as price ratio is required.
- In a simple scenario, we details the physical factor-product relationship of a single variable resoruce and single product.
- Many time resources or capacities of technical units, such as a ropani of land or a cow, are fixed and choice is to vary the input of only one factor – such as fertilizer OR labor.
- Other inputs such as fixed capital, buildings, implements and technical knowhow remain the same.

- Under such situation question of how much of certain input (amount of fertilizer or feed to a cow) to apply arises ?
- This situtation is dealt by single factor-product relationships. a.k.a single variable production function (in a production function various levels of input are involved with corresponding output of the product).

Inputs have several different names:

Inputs = factors = factors of production = resources = A, L, K, M

A: *Land (Natural and biological resources, climate.)*
L: *Labor (Human resources.)*
K: *Capital (Manufactured resources, which include buildings, machines, tools, and equipment.)*
M: *Management (The entrepreneur, or individual, who combines the other resources into inputs.)*

Example: Production function of wheat

To isolate the relationship between nitrogen and wheat yields, the agronomists (or other biophysical scientists) will hold constant all inputs other than the one that they are isolating, in this case nitrogen.

$$Y = f(N|L, K, M, A)$$

This relationship is highly important, since too little nitrogen means the yields will be lower than the potential, and too much nitrogen will "burn" the crop, causing smaller yields. Figure 1 shows the connection between nitrogen applications and wheat yields.

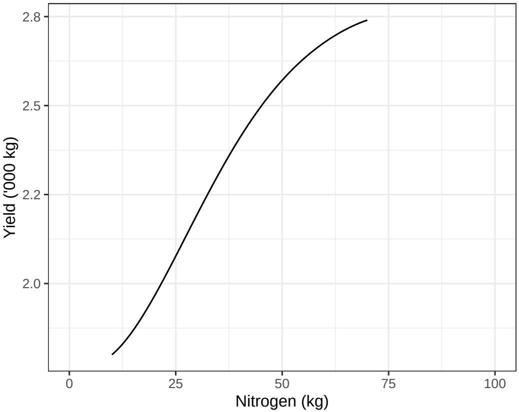


Figure 1: Relationship between nitrogen application and yield

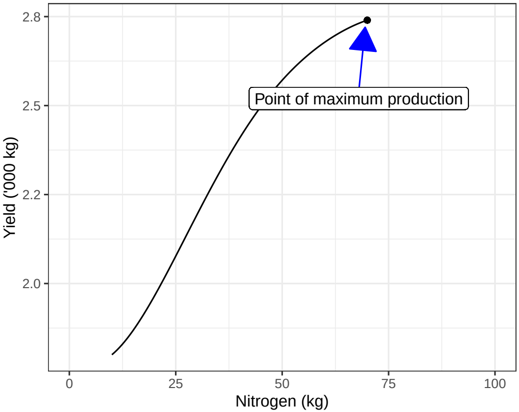


Figure 2: Relationship between nitrogen application and yield

Concept of optimum production

- The point of maximum physical wheat yield (N*) is not always the optimal economic wheat yield. This is because nitrogen is a scarce resource, and costs money to purchase. In fact, fertilizer is one of the major costs of production for farmers in most agricultural regions.
- If nitrogen were free, then the optimal application to a wheat field would always be N* in Figure 1, since this is the level of nitrogen that maximizes production.
- However, since it costs money to purchase and use fertilizer, the farmer will stop applying it at a point to the left of N*. Finding the optimal amount of nitrogen to apply requires application of economic principles. Economic reasoning will help determine the exact point where the benefits of using N minus the costs are maximized.
- Producers will not maximize production, because it costs too much. Instead, they will maximize profits.

Types of production functions

- There can be three types of input-output relationships in the production of a commodity where one input is varied and the quantities of all other inputs are fixed.
  1. Constant marginal rate of returns (Constant productivity)
  2. Increasing marginal rate of returns (Increasing productivity)
  3. Decreasing marginal rate of returns (Decreasing productivity)

Contant marginal rate of returns

- Each additional unit of the variable input when applied to fixed factors, produces an equal amount of additional product. The amount of product increases by the same magnitude for each additional unit of input.
- Not a very common relationship in agriculture and holds true only for limited range.
- Example:
  1. Addition of one acre of land (technology and other factors being same) will add the same amount of product.
  2. An addition of one tractor plus driver will do the same amount of work as previous tractor driver unit did.

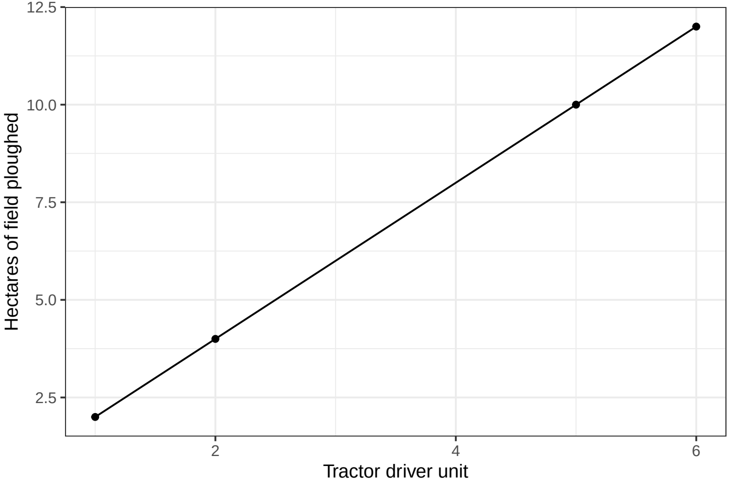


Figure 3: Constant marginal rate of returns for a input-output relationship between number of tractor plus driver unit recruits and hectares of land ploughed.

Table 1: Constant marginal rate of returns for a input-output relationship between number of tractor plus driver unit recruits and hectares of land ploughed.

tractor driver unit	field ploughed	marginal tractor driver unit	marginal field ploughed	marginal rate returns
1	2			2
2	4	1	2	2
5	10	3	6	2
6	12	1	2	2

Increasing marginal rate of returns

- Every additional or marginal unit of input adds more to the total product than the previous unit, i.e., addition to total product is at an increasing rate.
- In actual practice, the cases of purely increasing returns are rarely available except, again, in very limited range.
- This relationship is possible when the fixed factors of production are in excess capacity and addition of the small units of a variable resource makes more and more efficient use of fixed resources.
- Example:
    1. Small quanity of wheat seed applied when other factors of production such as fertilizer, irrigation and other cultural practices can be used at high levels will give low returns.

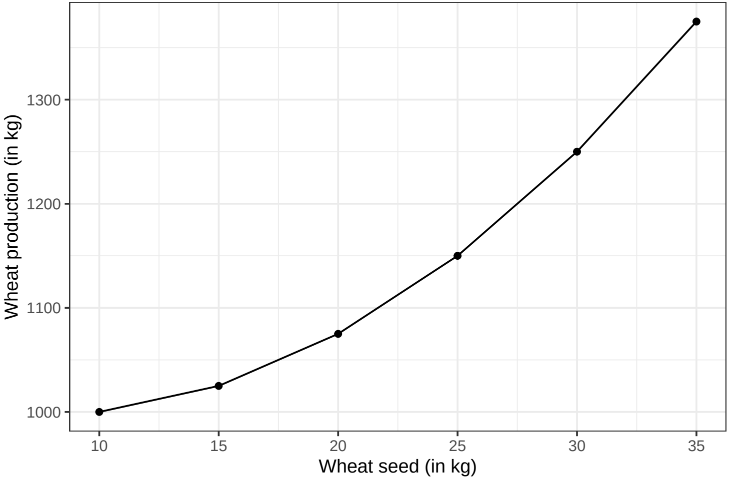


Figure 4: Increasing marginal rate of returns for hypothetical wheat production scenario.

Table 2: Increasing marginal rate of return for hypothetical wheat production scenario.

wheat seed	marginal wheat seed	wheat production	marginal wheat production	marginal rate returns
10		1000		
15	5	1025	25	5
20	5	1075	50	10
25	5	1150	75	15
30	5	1250	100	20
35	5	1375	125	25

Decreasing marginal rate of returns

- Each additional unit of input adds less to the total product than the previous unit did.
- This relationship exists in almost every practical situation in agriculture.
- Example:
    1. Response to fertilizers, insecticides, seeds, irrigation, feeds, etc. all show diminishing returns.

Figure 5: Decreasing marginal rate of returns for hypothetical wheat production scenario.

Table 3: Decreasing marginal rate of return for hypothetical wheat production scenario.

wheat fertilizer	marginal wheat fertilizer	wheat production	marginal wheat production	marginal rate returns
100		2000		
150	50	2600	600	12
200	50	3100	500	10
250	50	3500	400	8
300	50	3800	300	6
350	50	4000	200	4

Outline

Physical returns or productivity relationships

- Total, average and marginal products are related.
- Both the average and marginal product curves or relationships can be derived once total product curve has been obtained.

Table 4: Relationship between total, average and marginal products

Units of fertilizer input	Total product (TP; Y)	Average product (AP; Y/X)	Marginal product (MP; Y/X)	Remarks
0	0			
1	2	2.0	2	Increasing at increasing rate
2	5	2.5	3	
3	9	3.0	4	
4	14	3.5	5	Increasing at constant rate
5	19	3.8	5	
6	23	3.8	4	Increasing at decreasing rate
7	26	3.7	3	
8	28	3.5	2	
9	29	3.2	1	
10	29	2.9	0	
11	28	2.5	-1	Decreasing at increasing rate
12	26	2.2	-2	

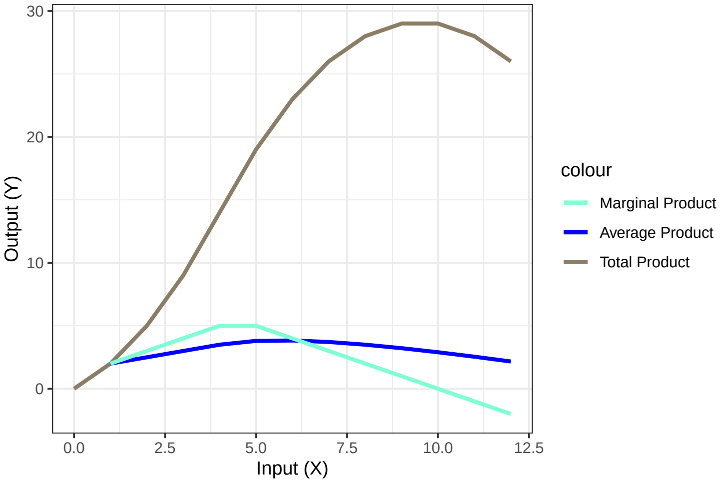


Figure 6: Relationship between TP, AP and MP.

Total product (TPC) and Marginal product (MPC)

1. Since the marginal product (MP) is a measure of rate of change therefore:
2. When the Total product (TP) is increasing, the MP will be positive.
3. When the TP remains constant, the MP will be zero, and
4. When the TP decreases, the MP will be negative.
5. So long as MP moves upwards or increases, the TP increases at an increasing rate.
6. When the MP remains constant, the TP increases at a constant rate.
7. When the MP starts declining or slopes downward, the TP will be increasing at a decreasing rate.
8. At the point when MP becomes zero or when MP intersects X-axis, the total product will be at maximum.

Marginal product (MPC) and average product (APC)

1. When the MP keeps increasing or is moving upward right from the beginning the Average product (AP) curve also keeps moving upward. So long as MP curve remains above the AP curve, the AP curve keeps increasing. This means when the AP is increasing, the MP must be greater than the average product.
2. As soon as the MP curve goes below the AP curve, the AP curve starts decreasing; i.e. when AP is decreasing, the MP is always less than the AP.
3. When AP is equal to MP, at this point AP will be at the maximum. From here onward, MP will change from greater to being less than AP, the MP curve must therefore intersect AP curve from above at its highest point.

Summary of relationship between MP and AP

1. When MP $>$ AP, AP is increasing
2. When MP $<$ AP, AP is decreasing
3. When MP $=$ AP, AP is at a maximum.

Outline

Elasticity of production

- The elasticity of production refers to the percentage change in output in response to the percentage change in input. It is be denoted by the symbol E_p and can be computed as:

$$\begin{aligned} E_p &= \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X}{X}} \\ &= \frac{X}{Y} \times \frac{\Delta Y}{\Delta X} \end{aligned}$$

- Let us consider an example, given in Table 5.

Table 5: Relationship between fertilizer input and yield of wheat.

Fertilizer doses (X)	Total yield attributable to fertilizer (Y)
0	0
1	103
2	174
3	223
4	257
5	281
6	298
7	308

As input increase from 1 to 2 units, total output increase from 103 to 174 units. Output thus increases by 71.9 percent in response to input increase of 100 percent. The elasticity of production is therefore:

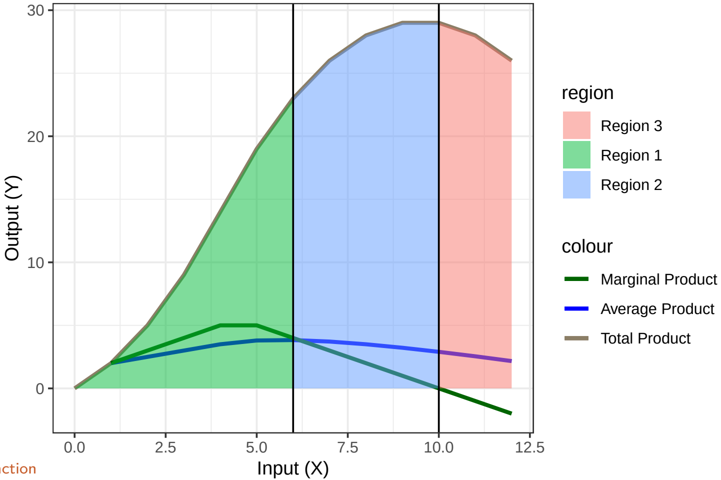
$$\begin{aligned} E_p &= \frac{71.9}{100} \\ &= 0.719 \end{aligned}$$

Similarly, between the second and third unit of input, the elasticity works out to be 0.56.

- Essential points to remember in elasticity analysis are:
  1. A production function with an elasticity of $E_p = 1.0$ indicates constant returns throughout. This means one percent increase in input is always accompanied by one percent increase in output.
  2. The elasticity is more than 1.0 up to the point of maximum average product where it becomes 1.0.
  3. The elasticity is less than 1.0 between the points of maximum average product and the maximum total product.
  4. When it becomes less than zero, total product declines.
  5. When elasticity of production is 1.0, marginal and average products are equal. This condition holds true at only one point on the classical production function as shown in Figure 6.
  6. A production function for which the elasticity is less than 1.0 throughout all ranges of input used will indicate diminishing returns.

Outline

- The classic production function (Figure 6 and Table 4) can be divided into three “regions”, “zones”, “parts” or “stages”, each important from the standpoint of decision-making on efficient resource use.
- These are (again shown in Figure 7):



Region 1 (irrational zone)

- This region hold from the point of origin up to the point the MPP remains greater than APP.
- The APP increases throughout this region indicating that the efficiency of all the units of variable input keeps increasing.
- Zone terminates as soon as APP equals MPP
- Notes:
    - once the farmer decides to produce, he must produce up to the level of input use where the APP is highest.
    - the efficiency of the variable input keeps increasing throughout the Region 1.
    - it is not reasonable to stop using an input when its efficiency on all units used is increasing.
    - reaching to the point of highest average product is always profitable

Region 3 (irrational zone)

- This region obtains where MPP crosses zero point and becomes negative.
- Negative MPP occurs when so much excessive quanity of the variable input is used that total output begins to decrease.
- Notes:
  - in the third region of production, the TP is decreasing.
  - additional quantities of input reduce the total output in this region, hence it is not profitable zone even if the additional quantities of resources are available free of cost.
  - e.g., if a farmer operates in this region – a farmer growing local variety and applying fertilizer without restraint might suffer loss of yield due to lodging and inefficient nutrient utilization – he will incurr double loss:

1. reduced production
2. unnecessary additional cost of inputs

Region 2 (rational zone)

- This region obtains when MPP is decreasing and is less than APP. In this region at the starting point, MPP is equal to APP and it extends to the point where MPP becomes zero.
- The APP is also decreasing.
- Within the boundaries of this region is the area of economic relevance. Optimum point of input-use must be somewhere in this rational zone.
- Optimum point can, however, be located only when input and output prices are known.
- This region of production embodies diminishing returns phase – both AP and MP are decreasing.

Knowing the point of operation

- In order to determine what level in Region 2 one should operate, one needs to know the product as well as the input prices. Multiplying the quantities of product and inputs with the respective prices we can convert the physical production function into revenue and cost functions.
- Thus the optimum level of input use, will be where the additional cost of an input is equal to the additional revenue which the input yields.
- Hence in the rational zone of production, factor-product price ratio will be used as a choice indicator.

Table 6: Factor-product relationship and economic decision analysis.

Physical input units nitrogen ropani	Physical output units ropani	Physical marginal input units	Physical marginal product units	Physical average product	Economic marginal cost unit	Economic marginal returns unit	Economic total returns	Economic total cost inputs	Economic profit
0	2						6	0	6
1	5	1	3	5.0	4	9	15	4	11
2	9	1	4	4.5	4	12	27	8	19
3	14	1	5	4.7	4	15	42	12	30
4	21	1	7	5.2	4	21	63	16	47
5	26	1	5	5.2	4	15	78	20	58
6	30	1	4	5.0	4	12	90	24	66
7	33	1	3	4.7	4	9	99	28	71
8	35	1	2	4.4	4	6	105	32	73
9	36	1	1	4.0	4	3	108	36	72
10	36	1	0	3.6	4	0	108	40	68
11	35	1	-1	3.2	4	-3	105	44	61
12	33	1	-2	2.8	4	-6	99	48	51

- Take for example Table 6, the price per unit of input to be Rs. 4 and price per unit of product Rs. 3. Multiply the various quantities of product at different levels of input-use by Rs. 3. Assuming the producer's purchases are low enough not to cause the price of the input to rise, each input unit will cost Rs. 4. This is shown in 6th column of the Table 6.
- We can see from the Table 6 that the highest net revenue can be obtained when 8 units of the input are used yielding net revenue at Rs 73. If the producer applies less number of input units, say 7 units, the net revenue will be less. Here the returns from the last unit of input are greater than the costs, indicating a scope of further increasing the revenue by pushing up input use.

- On the other hand, if inputs are added to the point where the price of the input exceeds the value of the marginal product, the added costs go greater than the added returns.
- By adding the 9th unit of input, the additional returns will be only Rs 3 whereas the cost of this unit of input will be the same Rs 4. This means it does not pay the producer to add more than 8 units of the input.
- Optimum level of input use will thus be where the value of the additional product will be equal to the cost of additional input.

- We can write this condition for maximization of net revenue as:

$$P_{Y1}\Delta Y_1 = P_{X1}\Delta X_1$$

Where $P_{Y1}\Delta Y_1$ is added revenue and $P_{X1}\Delta X_1$ is added cost.

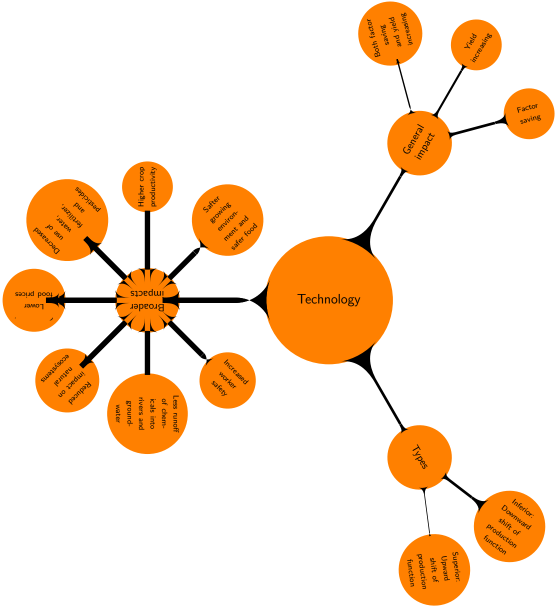
- This can be written in ratio form as well.

$$\frac{\Delta Y_1}{\Delta X_1} = \frac{P_{X1}}{P_{Y1}}$$

Outline

Impact of technological change on production function

- Technical advances may result in such development as evolution of new crop varieties, breeds of animals and birds, invention of new fertiliers and their use methods, improved rations, new farm machinery etc.
- So far we have made or point about production function with assumption that technology remains constant.
- Application of different technology results in a different production function.


Technology
General impact
Both factor saving and yield increasing
Yield increasing
Factor saving
Types
Superior: Upward shift of production function
Inferior: Downward shift of production function
Broader impacts
Higher crop productivity
Safter growing environment and safer food
Decreased use of water, fertilizer, and pesticides
Lower food prices
Reduced impact on natural ecosystems
Less runoff of chemicals into rivers and groundwater
Increased worker safety

Outline

For more information