Distributed lag model for multi-year enterprise appraisal under incremental production unit regime: A simulation in coffee crop

Deependra Dhakal
Agriculture and Forestry University
ddhakal@afu.edu.np

Background

Modeling multi-year crop production enterprise is a challenging but essential task for informed decision-making. The choice of method depends on the specific context, available data, and the level of detail required. Ideally, modeling the cost and returns of such ventures requires accounting for the biological growth cycle, varying yields, and potential expansion over the course of enterprise operations.

Cost variables of such processes may be defined as either being fixed or variable in light of following context:

- Fixed costs are non-contingent upon the nature of production units (biological growth). Instead, their costs are predefined, as to be incurred at fixed points in time.
- Variable cost heads are relevant during every consecutive or periodic accounting times. They are affected by the nature of production units – biological growth generally has multiplicative impact on cost.
- Growth lag and lead cycles of production unit during enterprise duration and the requirement for them to be incorporated on regular accounting periods requires variable costs to be dynamically modeled[1].

In business of coffee fruit production, for instance, it takes coffee sapling 3 years to reach age at which it first starts producing cherry; the plant normally thrives for 4 years with increasing fruit yields and for further 4 years with receding yields. The enterprises' only production units are the coffee plants and the duration should be no less than life span of the crop itself. An entrepreneur involved in coffee production might add certain number of coffee plants in some years during the enterprise duration, provided that s/he has enough land holding for the venture. For example, a farmer may begin with 0.2 hectares planted in the first year and only during later years he may choose to plant remaining parcels (that too may be planted periodically) – a management scheme, hereon, referred to as production unit allocation profile.

In lieu addition of units and given the properties of these units such as growth, fixed life span and time dependent expenditure requirements, costs vary over the accounting periods of enterprise. Decisions in one period (e.g., planting new coffee plants versus retaining of aging plants) impact future periods (e.g., future yields and future expenditures). Simulating cost and return progression under context of myriad of unit allocation schemes, one can optimize their enterprise run for securing profitable financial turnovers.

Substantial discussions are offered in Johnson (1960) and Anderson (1974) for rationale behind long term farm pricing and agricultural supply planning, wherein the latter invokes a case of barley acerage response analysis using distributed lag (DL) model. Readers are referred to early pioneering works of Fisher (1933), Fisher (1892) and Koyck (1954) for an derivation and properties of DL models. For discussion about autoregressive variations of the model Pesaran, Shin, et al. (1995) or more recent works may be consulted.

Distributed lag cost and return accumulation method

DL (also referred to as, progressive weighted) cost/return accumulation is defined with following recursion operation (Equation 1 for cost and Equation 2 for return) on input matrices $\mathbf{P_u}$, $\mathbf{P_r}$ and $\mathbf{Q}$ of sizes $h \times y$,

[1] Kennedy (1986) is a dedicated text showcasing applications of dynamic programming in agricultural resource allocation

$n \times y$ and $n \times y$, respectively (for cost) and matrices $\mathbf{P_u}$, $\mathbf{P_w}$ both of sizes $h \times y$, and vector p_r of length y (for return).

For each row (of unit allocation profiles) $m \in \{1, 2, \ldots, h\}$ of $\mathbf{P_u}$, compute the matrix $\mathbf{Y}$ of size $m \times j$:

$$\mathbf{Y}_{(i,j)} = \sum_{k=1}^{\min(j,y)} \mathbf{P}_{\mathbf{u}(m,k)} \cdot \mathbf{P}_{\mathbf{r}(i,j)} \cdot \mathbf{Q}_{(i,j-(k-1))}, \quad \forall i \in \{1, 2, \ldots, n\} \text{ and } j \in \{1, 2, \ldots, y\}. \tag{1}$$

$$\mathbf{R}_{(:,j)} = \sum_{k=1}^{j} \mathbf{P}_{\mathbf{u}(:,k)} \cdot \mathbf{P}_{\mathbf{w}(:,j-k+1)} \cdot p_{r}(j), \quad \forall j \in \{1, ..., y\} \tag{2}$$

where each column of matrix $\mathbf{R}$ is the result of cumulative sum of weighted products of element-wise products between preceding columns of $\mathbf{P_u}$ and corresponding "reversed" columns of $\mathbf{P_w}$, scaled by the j-th element of p_r.

Production units allottment profiles simulation

Production profile represents an option of production planning through altered temporal allocation of production units. Owing to its efficiency, in terms of maximizing returns or minimizing the cost or both, each profile has a unique implication to enterprise operations. A farm enterprise operation may be thought of as a run of a specific production unit – *cetarius paribus* – profile. Distributed lag model was fitted to obtain both cost and return calculations of following six 1 simulated profiles.

Table 1. A representative sample of seven production unit allottment profiles amongst several possible. An interpretation of each profile is provided alongside.

Profile	Yr1	Yr2	Yr3	Yr4	Yr5	Yr6	Yr7	Yr8	Yr9	Yr10	Yr11	Yr12	Yr13	Yr14	Yr15
Pr 1(Leading additive rise with trailing zeros)	35.5	71.1	106.7	142.2	177.8	213.3	248.8	284	319.9	0.0	0.0	0.0	0.0	0.0	0.0
Pr 2(Leading additive rise throughout run)	13.3	26.7	40.0	53.3	66.7	80.0	93.3	107	120.0	133.3	146.6	160.0	173.3	186.6	199.9
Pr 3(Leading multiplicative rise throughout run)	11.4	23.3	35.5	48.2	61.2	74.7	88.6	103	117.6	132.7	148.2	164.1	180.4	197.2	214.3
Pr 4(Leading additive fall throughout run)	199.9	186.6	173.3	160.0	146.6	133.3	120.0	107	93.3	80.0	66.7	53.3	40.0	26.7	13.3
Pr 5(Leading multiplicative fall throughout run)	214.3	197.2	180.4	164.1	148.2	132.7	117.6	103	88.6	74.7	61.2	48.2	35.5	23.3	11.4
Pr 6(Lead zeros with additive fall)	25.6	51.2	76.8	102.4	128.1	153.7	179.3	166	179.3	153.7	128.1	102.4	76.8	51.2	25.6
Pr 7(Constant and distributed allocation)	106.7	106.7	106.7	106.7	106.7	106.7	106.7	107	106.7	106.7	106.7	106.7	106.7	106.7	106.7

A hypothetical scenario of a coffee crop cultivation in limited holding size of 1 hectare for a 15 year enterprise course is discussed henceforth. Two variable cost factors, each requiring different expenditure schedule, are chosen for profile calculation. Cost calculations of fixed input variables (such as cost of plant/seedling purchase, land preparation, land rent, interest on loan, depreciation, etc.) entail lag-only model (without distribution), hence more simpler to fit. Since augmenting those heads would only shift or scale the profiles without affecting the shape of distribution, I have skipped them entirely, in order to spare extra steps of total cost calculations. Following assumptions are made for structuring costs and returns and calculating their nominal values:

- Maximum number of plants (production units) that can be accomodated: 1600
- Enterprise accounting duration: 1 year
- Annual compound factor price inflation rate: 0.1
- Annual fruit yield of coffee (as function of biological growth)

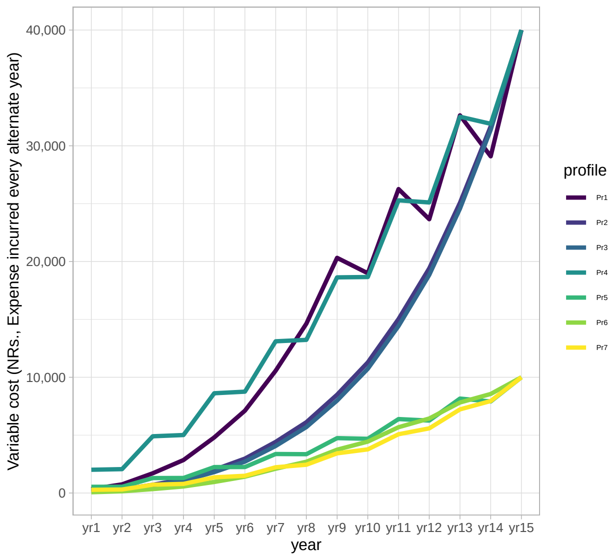
Yr1	Yr2	Yr3	Yr4	Yr5	Yr6	Yr7	Yr8	Yr9	Yr10	Yr11	Yr12	Yr13	Yr14	Yr15

0	0	0	3	4.5	6	7	8	9	10	9	8	8	8	6

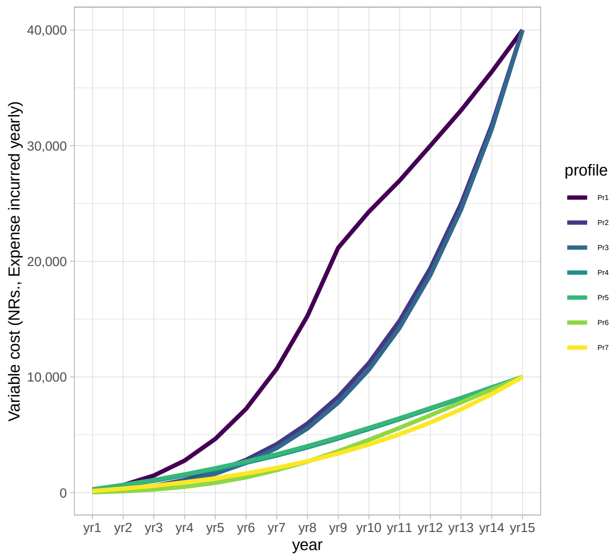
- Annual fixed price addition of product (nominal value, per kg): NRs. 4.5
- Price of product at current/last operational year of enterprise (nominal value, per kg): NRs. 90

Although production units in agricultural enterprise are discrete entities, proposed model is numerically stable for scenarios with fractional unit allocation.

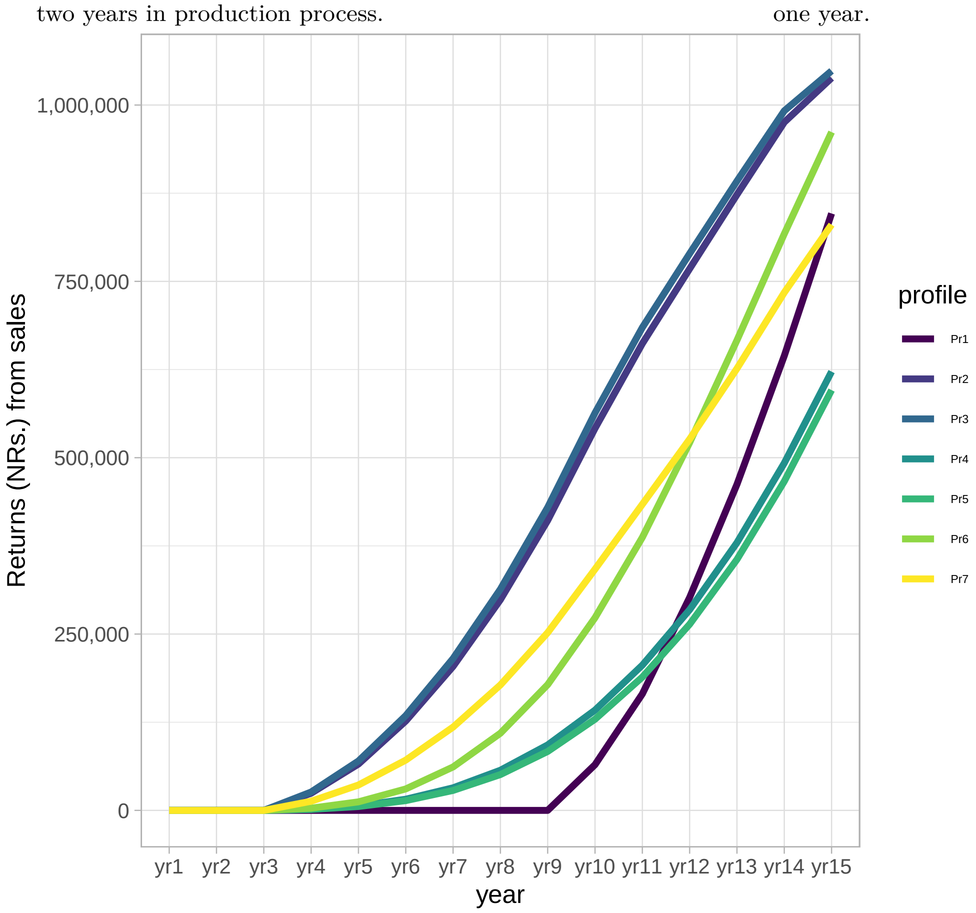
Cost and return profile over enterprise duration



(a) Cost of variable factor 1 with input expense cyle of two years in production process.



(b) Cost of variable factor 2 with input expense cycle of one year.



(c) Returns

Figure 1. Cost and return profile calculations (in nominal values) of seven different production units allottment profiles over enterprise run period of 15 years.

Discussion and conclusion

Table 3. Relative performance indicators of various production unit allocation profiles based on two input factors and single product enterprise.

Profile	Base year value of total: Cost of Variable 1	Cost of Variable 2	Returns	Benefit cost ratio
Pr1	108.4	117.3	947	4.20
Pr2	72.6	71.6	2644	18.34
Pr3	70.2	69.2	2728	19.57
Pr4	120.5	31.8	952	6.25
Pr5	30.5	32.4	888	14.12
Pr6	24.8	24.5	1663	33.70
Pr7	24.3	24.9	1799	36.56

For long term enterprise with preponderence of returns over cost, “Profile 7” (Constant and evenly distributed allocation) followed by “Profile 6” (Allocation with initial additive or multiplicative increase, peaking around midway, and additive or multiplicative decrease through to the enterprise life) scheme of unit allocation are best suited. Note here, however, that numerical values of indicators, particularly that of BC ratio (ranging as high as 36.6 due to large gap in addressing input cost heads, while including only two) is only to be taken as qualitative measure. In essence, the profile largely emulates the biological growth property of the crop: increasing during first half and tapering during the second half of the crop life cycle.

References

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