Module 28

Searching & Sorting

In association with SITARE Foundation and SHRI RAMSWAROOP MEMORIAL UNIVERSITY

Acknowledgement

Walker M. White
Cornell University

Linear Search

```
def linear_search(v,b):
    """Returns: first occurrence of v in b; -1 if not found

    Parameter v: The value to search for
    Precondition: None (v can be any value)

    Parameter b: The sequence to search for
    Precond: b is a sequence
    """
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How many entries do we have to look at?

Linear Search

```
def linear_search(v,b):
    """Returns: first occurrence of v in b (-1 if not found)
     Precond: b a list of number, v a number
    """

    # Loop variable
    i = 0
    while i < len(b) and b[i] != v:
        i = i + 1

    if i == len(b):  # not found
        return -1
    return i
```

Linear Search

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def linear_search(v,b):
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     Precond: b a list of number, v a number
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   # Loop variable
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   while i >= 0 and b[i] != v:
      i = i - 1

   # Equals -1 if not found
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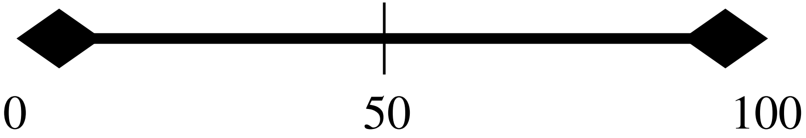
How many entries do we have to look at?

All of them!

Is There a Better Way?



- Thinking of number 0..100
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  - Continue until get it right
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  - Use my answers to help
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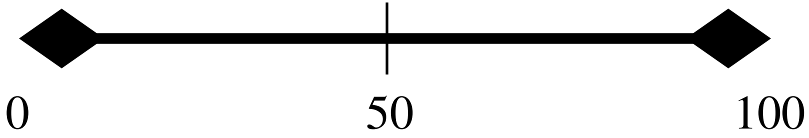

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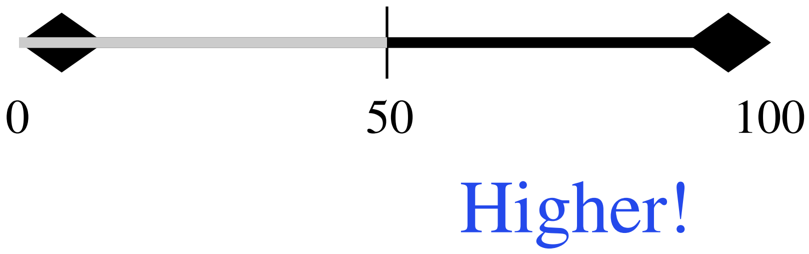

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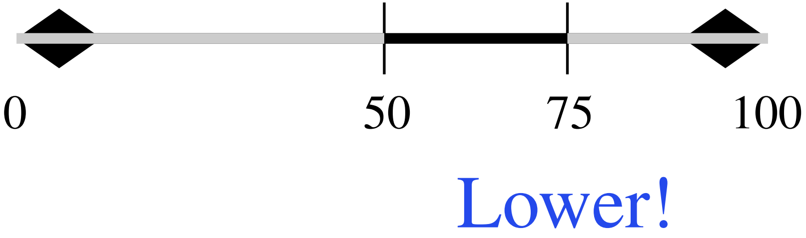

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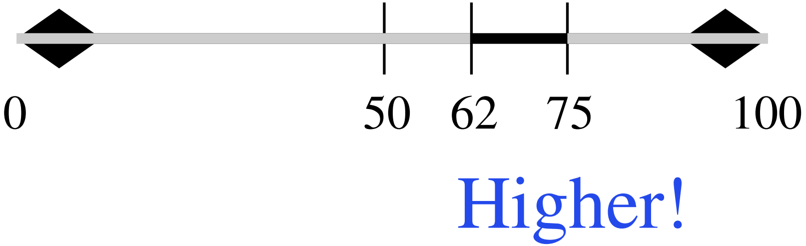

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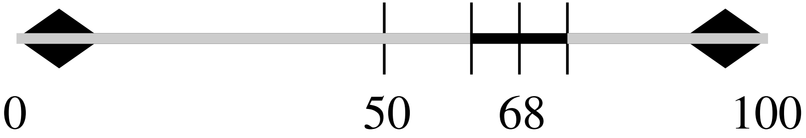

Correct!

- Thinking of number 0..100
  - You get to guess number
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- **Goal:** Keep # guesses low
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- **Strategy?**
  - Start guess in the middle
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Binary Search

```
def binary_search(v,b):
    """Returns: an occurrence of v in b

    Parameter v: The value to search for
    Precondition: None (v can be any value)

    Parameter b: The sequence to search for
    Precond: b is a sorted sequence
    """
```

Binary Search

```
def b_search(v,b):
  # Loop variable(s)
  i = 0
  j = len(b)
  mid = (i+j)//2
  while i < j:
    if b[mid] < v:
      i = mid + 1
    else:
      j = mid
    mid = (i+j)//2
  if i < len(b) and b[i] == v:
    return i
```

Requires that the data is sorted!

But few checks!

Observation About Sorting

- Sorting data can speed up searching
  - Sorting takes time, but do it once
  - Afterwards, can search many times
- Not just searching. Also speeds up
  - Duplicate elimination in data sets
  - Data compression
  - Physics computations in computer games
- Why it is a major area of computer science

The Sorting Challenge

- **Given:** A list of numbers
- **Goal:** Sort those numbers using only
  - Iteration (while-loops or for-loops)
  - Comparisons (< or >)
  - Assignment statements
- Why? For proper **analysis**.
  - Methods/functions come with hidden costs
  - Everything above has no hidden costs
  - Each comparison or assignment is “1 step”

This Requires Some Notation

- As the list is sorted…
  - Part of the list **will** be sorted
  - Part of the list will **not** be sorted
- Need a way to refer to portions of the list
  - Notation to refer to sorted/unsorted parts
- And have to do it **without** slicing!
  - Slicing makes a copy
  - Want to sort original list, not a copy

This Requires Some Notation

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    - Part of the list **will** be sorted
    - Part of the
- Need a w st
    - Notation t

But we will not be very formal!

- And have to do it **without** slicing!
    - Slicing makes a copy
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Terminology: Range Notation

- m..n is a range containing n+1-m values
    - 2..5 contains 2, 3, 4, 5. Contains 5+1 – 2 = 4 values
    - 2..4 contains 2, 3, 4. Contains 4+1 – 2 = 3 values
    - 2..3 contains 2, 3. Contains 3+1 – 2 = 2 values
    - 2..2 contains 2. Contains 2+1 – 2 = 1 values
    - 2..1 contains ???

Terminology: Range Notation

What does 2..1 contain?

A: nothing
B: 2,1
C: 1
D: 2
E: something else

Terminology: Range Notation

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B: 2,1

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  - 2..3 contains 2, 3. Contains 3+1 – 2 = 2 values
  - 2..2 contains 2. Contains 2+1 – 2 = 1 values
  - 2..1 contains ???
- The notation m..n, always implies that m <= n+1
  - So you can assume that even if we do not say it
  - If m = n+1, the range has 0 values

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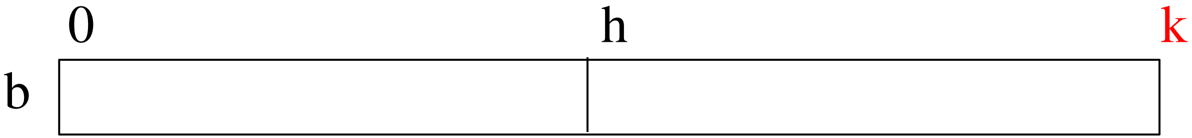
Not the same as
range(m,n)

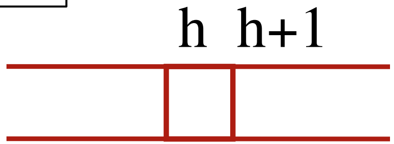
- The notation m..n, always implies that m <= n+1
  - So you can assume that even if we do not say it
  - If m = n+1, the range has 0 values

Horizontal Notation

- Want a pictorial way to visualize this sorting
  - Represent the list as long rectangle
  - We saw this idea in divide-and-conquer



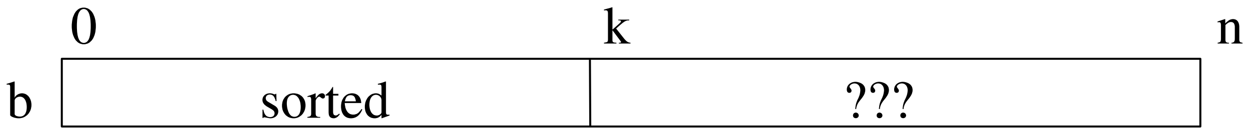
- Do **not** show individual boxes
  - Just dividing lines between regions
  - Label dividing lines with indices
  - But index is either left or right of dividing line



$(h+1) – h = 1$

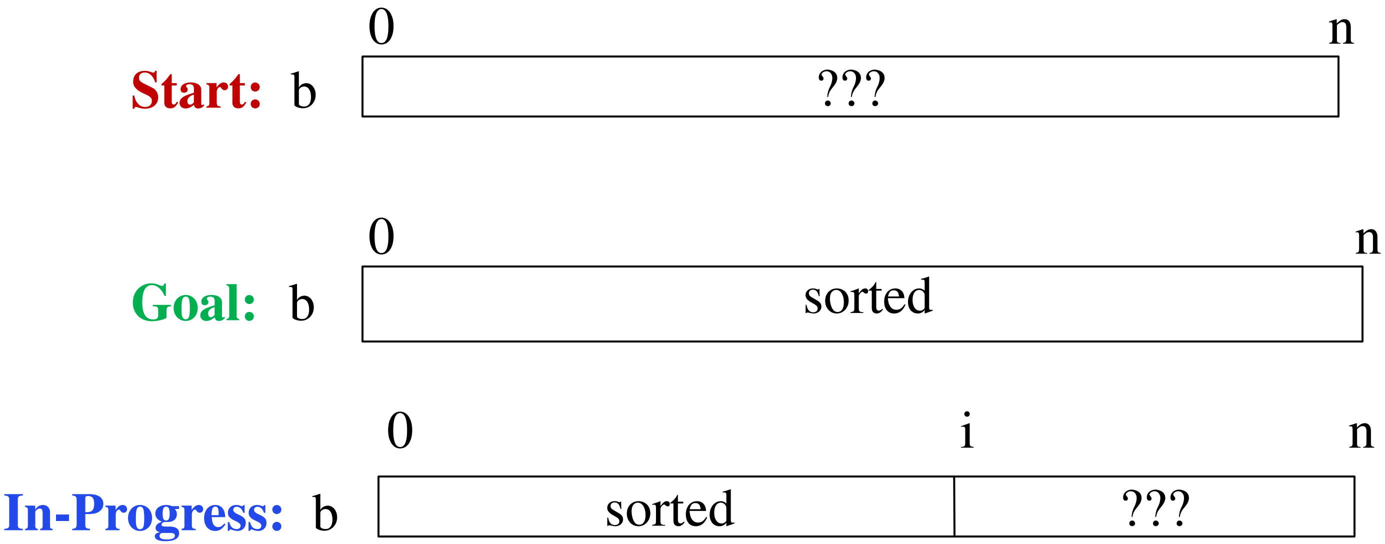
Horizontal Notation

- Label regions with properties
  - **Example:** Sorted or ???

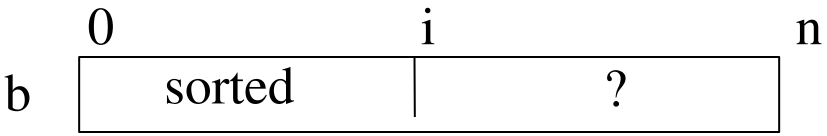


  - `b[0..k-1]` is sorted
  - `b[k..n-1]` **unknown** (might be sorted)
- Picture allows us to track progress

Visualizing Sorting

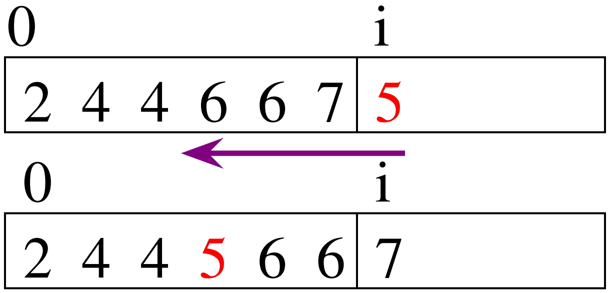


Insertion Sort



```
i = 0
while i < n:
    # Push b[i] down into its
    # sorted position in
    b[0..i]

    i = i+1
```



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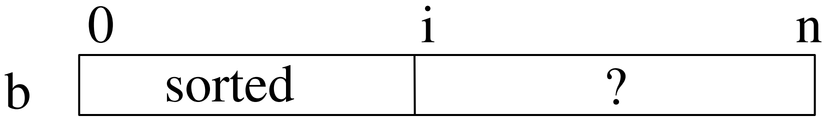

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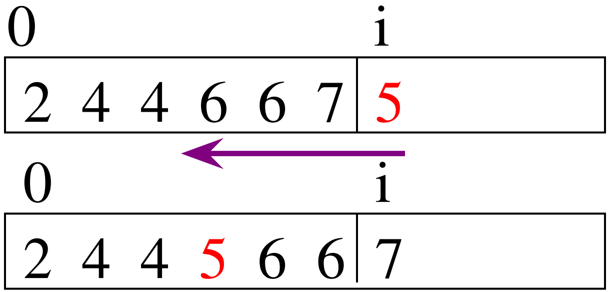
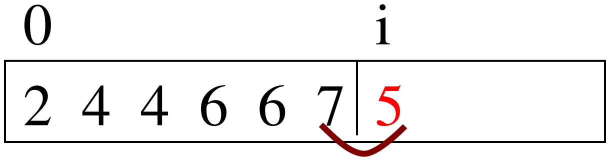


Remember the restrictions!

Insertion Sort: Moving into Position

```
i = 0
while i < n:
  push_down(b,i)
  i = i+1

def push_down(b, i):
    j = i
  while j > 0:
    if b[j-1] > b[j]:
      swap(b, j-1, j)
    j = j-1
```



swap shown in the lecture about lists

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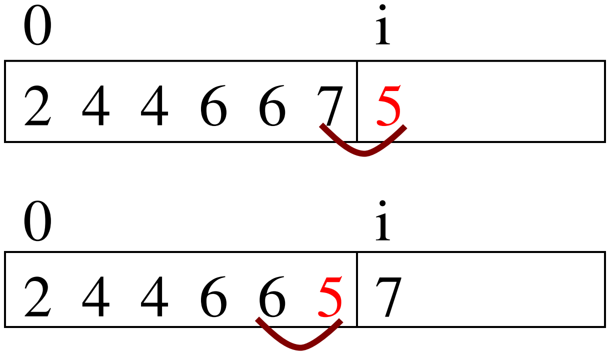

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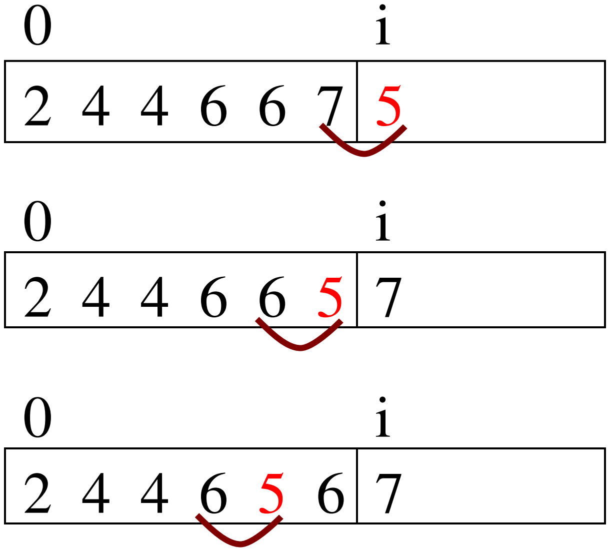

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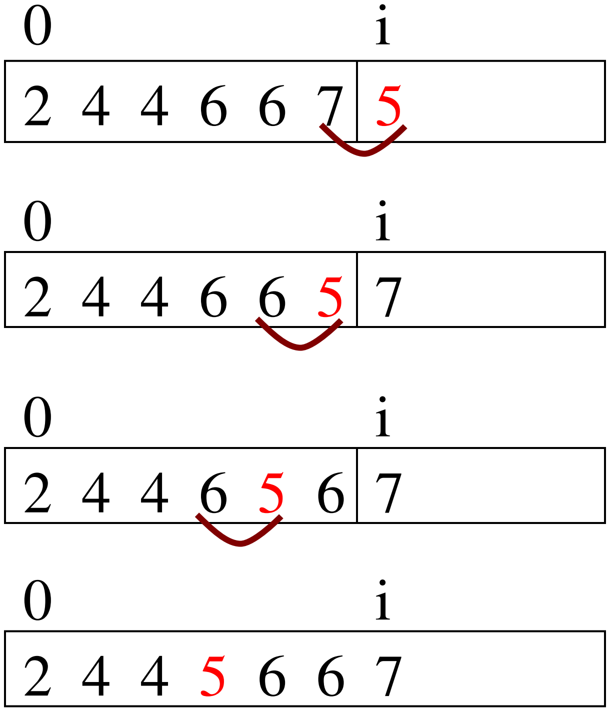

0 i
2 4 4 6 6 7 | 5

0 i
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The Importance of Helper Functions

```
i = 0
while i < n:
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def push_down(b, i):
   j = i
  while j > 0:
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    j = j-1
```

VS

```
i = 0
while i < n:
  j = i
  while j > 0:
    if b[j-1] > b[j]:
      temp = b[j]
      b[j] = b[j-1]
      b[j-1] = temp
    j = j -1
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```

The Importance of Helper Functions

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Can you understand all this code below?

Measuring Performance

- Performance is a tricky thing to measure
  - Different computers run at different speeds
  - Memory also has a major effect as well
- Need an independent way to measure
  - Measure in terms of “basic steps”
  - **Example:** Searching counted # of checks
- For sorting, we measure in terms of **swaps**
  - Three assignment statements
  - Present in all sorting algorithms

Insertion Sort: Performance

```
def push_down(b, i):
    """Push value at position i
    into sorted position in
    b[0..i-1]"""
    j = i
    while j > 0:
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- b[0..i-1]: i elements
- Worst case:
    - i = 0: 0 swaps
    - i = 1: 1 swap
    - i = 2: 2 swaps
- Pushdown is in a loop
    - Called for i in 0..n
    - i swaps each time

Total Swaps: $0 + 1 + 2 + 3 + \ldots (n-1) = (n-1)*n/2 = (n^2-n)/2$

Insertion Sort: Performance

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Insertion sort is an n^2 algorithm

Total Swaps: $0 + 1 + 2 + 3 + \ldots (n-1) = (n-1)*n/2 = (n^2-n)/2$

Algorithm "Complexity"

- **Given**: a list of length n and a problem to solve
- **Complexity**: *rough* number of steps to solve worst case
- Suppose we can compute 1000 operations a second

Complexity	n=10	n=100	n=1000
log n	0.003 s	0.006 s	0.01 s
n	0.01 s	0.1 s	1 s
n log n	0.016 s	0.32 s	4.79 s
n^2	0.1 s	10 s	16.7 m
n^3	1 s	16.7 m	11.6 d
2^n	1 s	4×10^{19} y	3×10^{290} y

Algorithm “Complexity”

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Binary Search

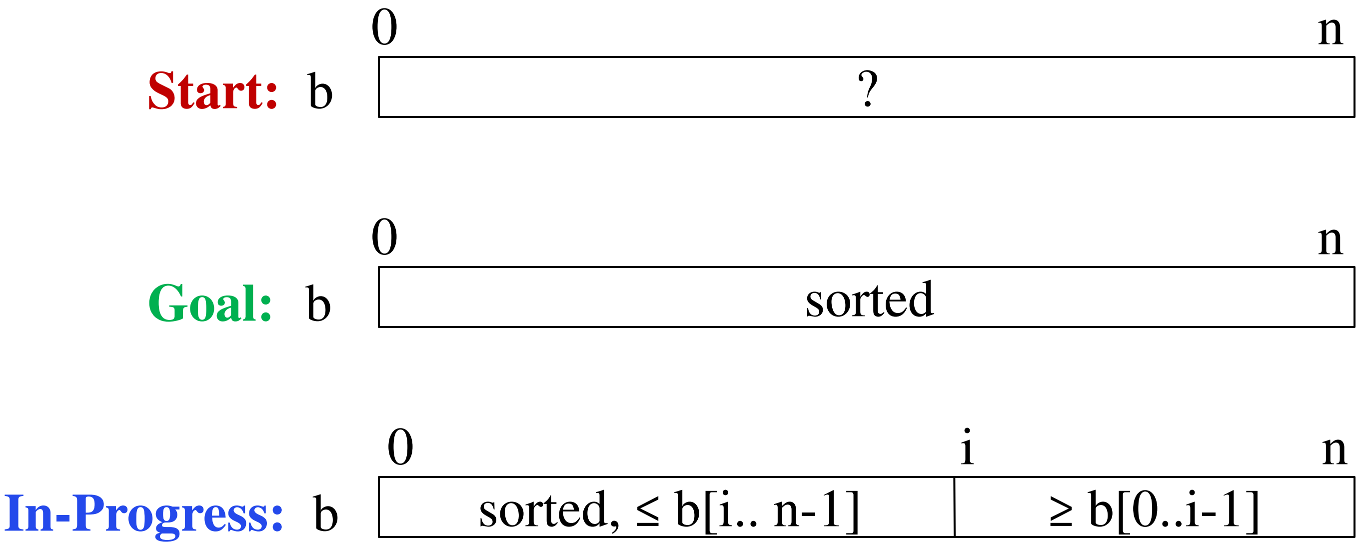
Linear Search

Insertion Sort

Insertion Sort is Not Great

- Typically n^2 is okay, but not great
  - Will perform horribly on large data
  - Very bad when performance critical (games)
- We would like to do better than this
  - Can we get n swaps (**no**)?
  - How about n log n (**maybe**)
- This will require a new algorithm
  - Let's return to horizontal notation

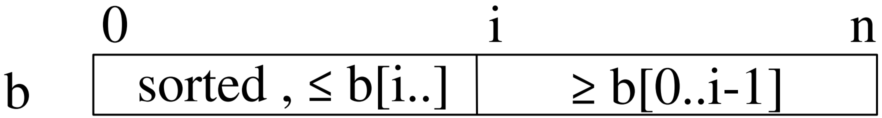
A New Algorithm

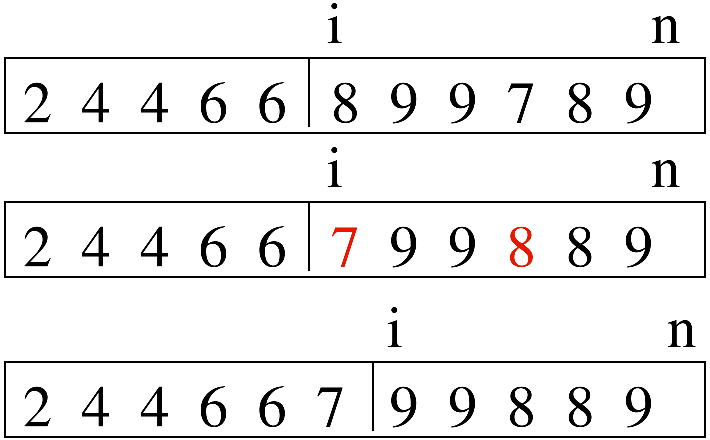


First segment always contains smaller values

Selection Sort

```
i = 0
while i < n:
    # Find minimum in b[i..n]
    # Swap it with position i
    i = i+1
```



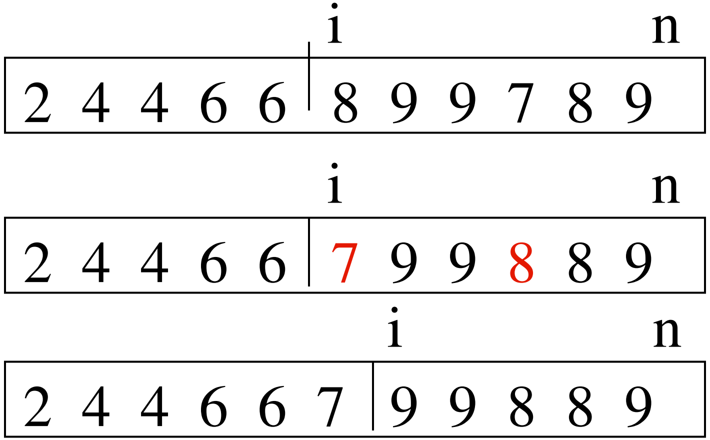


Remember the restrictions!

Selection Sort

How fast is this?

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b, i, j)
    i = i+1
```



Selection Sort

This is also n^2!

```
i = 0
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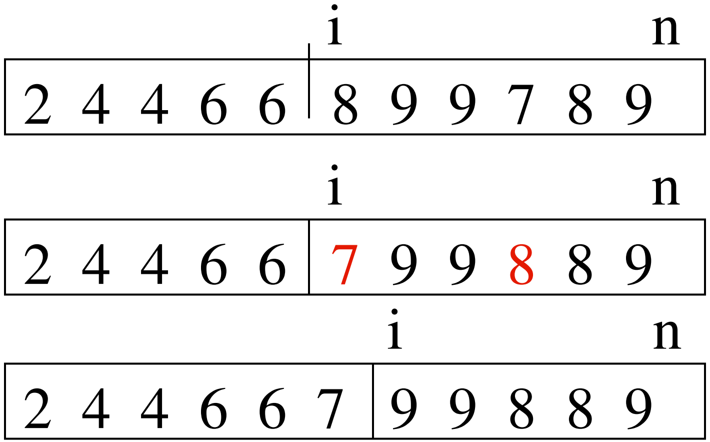
This is n steps



What is the Problem

- Both insertion, selection sort are nested loops
    - Outer loop over each element to sort
    - Inner loop to put next element in place
    - Each loop is n steps. $n \times n = n^2$
- To do better we must *eliminate* a loop
    - But with what?

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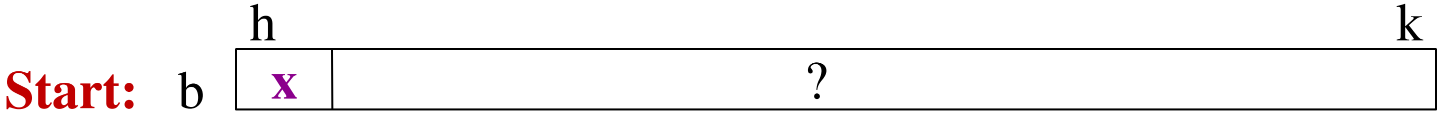
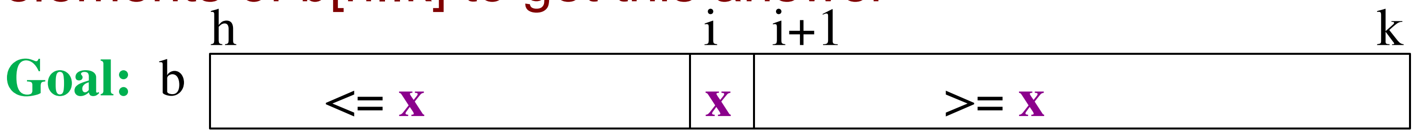
- To do better we must *eliminate* a loop
  - But with what? **Recursion!**
- But to do this we have to step back
  - Need to introduce an intermediate algorithm

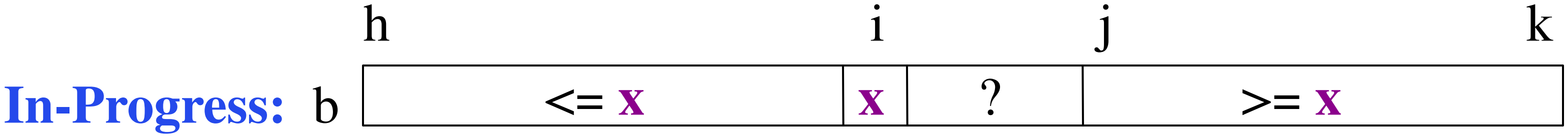
The Problem Statement

- Given a list b[h..k] with some value x in b[h]:



- Swap elements of b[h..k] to get this answer

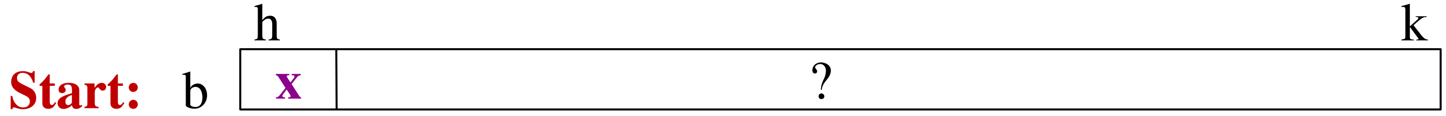




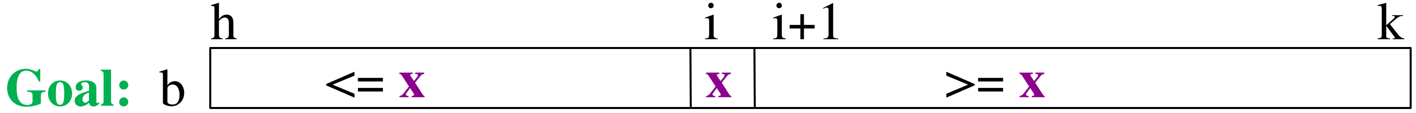
Indices h,k important!
Might partition only part

Partition Algorithm

- Given a list b[h..k] with some value x in b[h]:



- Swap elements of b[h..k] to get this answer



		h								k
Change:	b	**3**	5	4	1	6	2	3	8	1
		h			i					k
into	b	1	2	1	**3**	5	4	6	3	8
		h				i				k
or	b	1	2	3	1	**3**	4	5	6	8

- x is called the pivot value
  - x is not a program variable
  - denotes value initially in b[h]

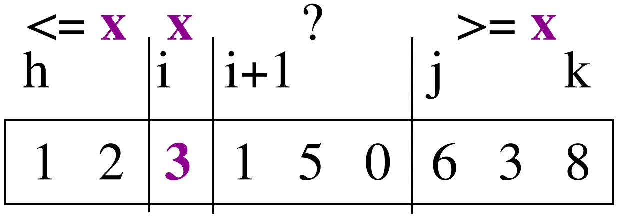
Partition Algorithm Implementation

partition(b,h,k), not partition(b[h:k+1])

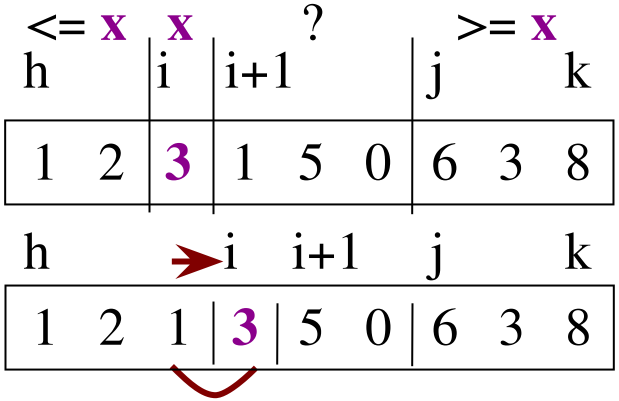
Remember, slicing always copies the list!

We want to partition the **original** list

Partition Algorithm Implementation



Partition Algorithm Implementation



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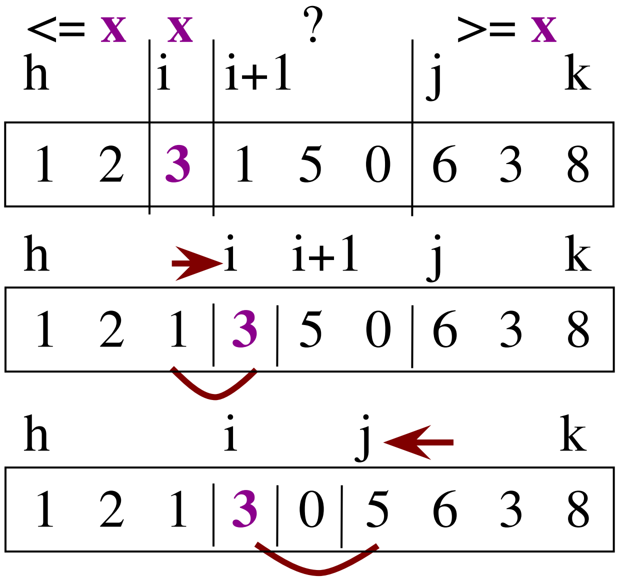
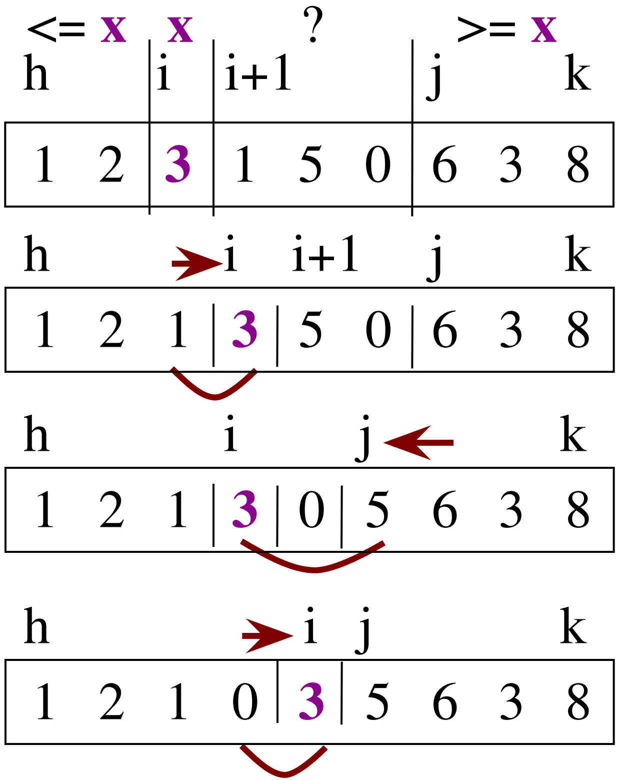


Partition Algorithm Implementation



<= x x ? >= x

h i i+1 j k

1 2 | 3 | 1 5 0 | 6 3 8

h i i+1 j k

1 2 1 | 3 | 5 0 | 6 3 8

h i j k

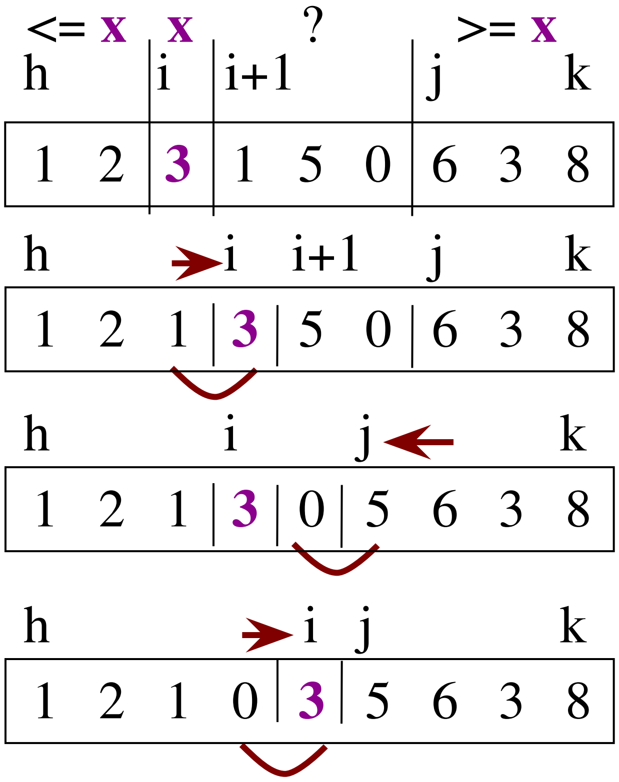
1 2 1 | 3 | 0 | 5 6 3 8

h i j k

1 2 1 0 | 3 | 5 6 3 8

Partition Algorithm Implementation

```
def partition(b, h, k):
    """ Returns the new position of
    pivot in partitioned list b[h..k].
    Partition list b[h..k] around a
    pivot x = b[h]"""
```

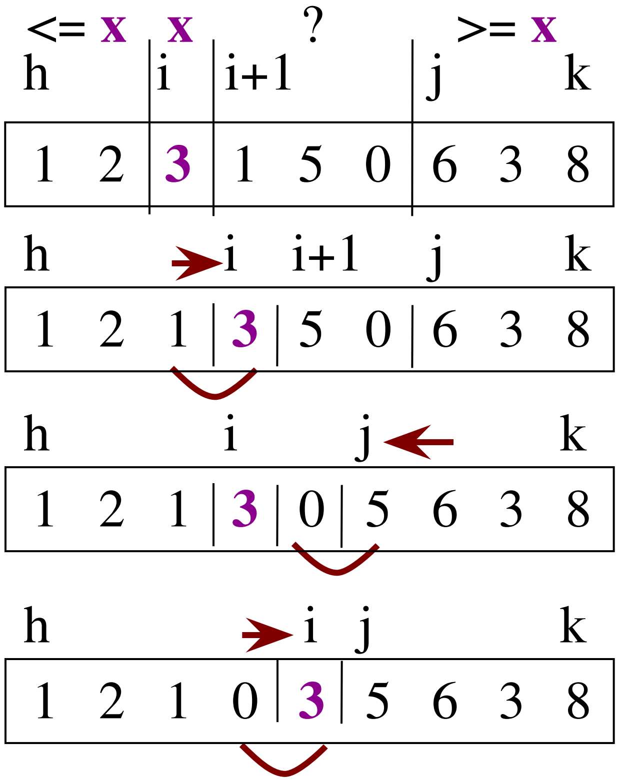


Partition Algorithm Implementation

```
def partition(b, h, k):
    """Partition list b[h..k] around a pivot x = b[h]"""
    i = h; j = k+1; x = b[h]

    while i < j-1:
        if b[i+1] >= x:
            # Move to end of block.
            swap(b,i+1,j-1)
            j = j - 1
        else:  # b[i+1] < x
            swap(b,i,i+1)
            i = i + 1

    return i
```

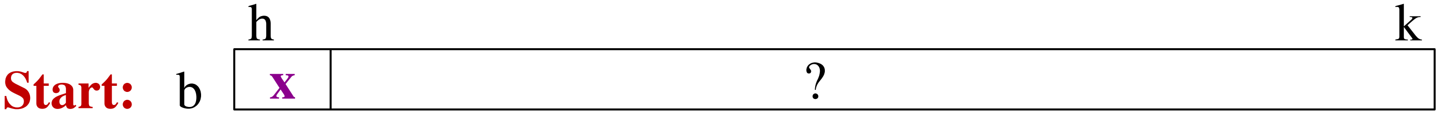


Why is this Useful?

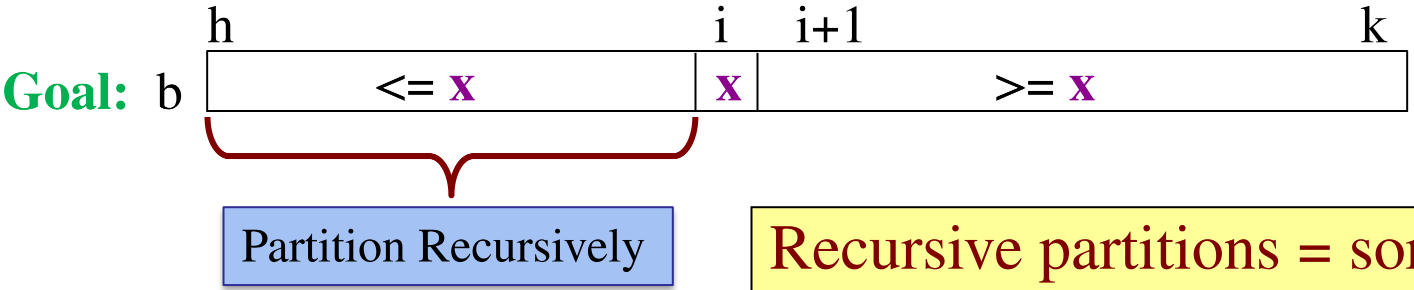
- Will use this algorithm to replace inner loop
  - The inner loop costs us n swaps every time
- Can this reduce the number of swaps?
  - Worst case is k-h swaps
  - This is n if partitioning the whole list
  - But less if only partitioning part
- **Idea:** Break up list and partition only part?
  - This is **Divide-and-Conquer!**

Sorting with Partitions

- Given a list segment b[h..k] with some value x in b[h]:



- Swap elements of b[h..k] to get this answer



Recursive partitions = sorting

- Called **QuickSort** (why???)
- Popular, fast sorting technique

Sorting with Partitions

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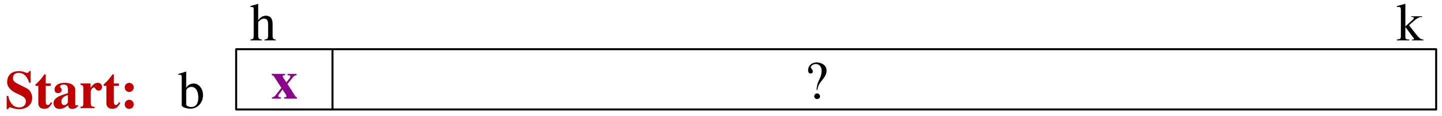
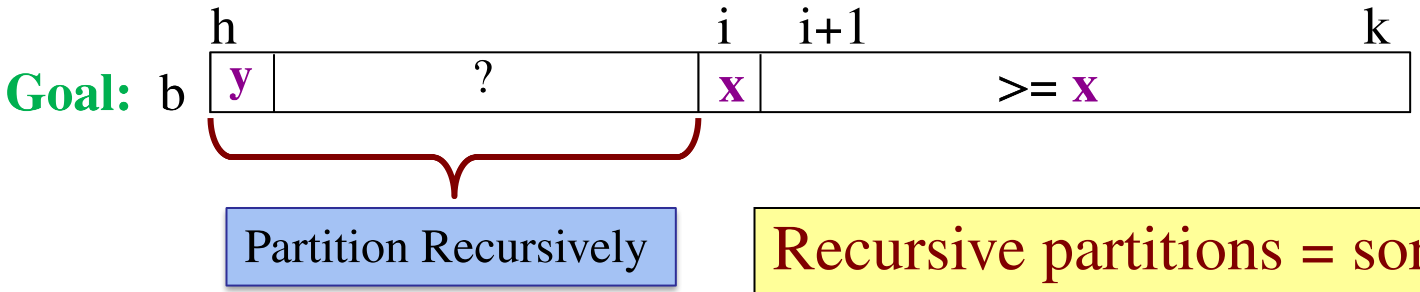


h k

Start: b | x | ?

- Swap elements of b[h..k] to get this answer



h i i+1 k

Goal: b | y | ? | x | >= x

Partition Recursively

Recursive partitions = sorting

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Sorting with Partitions

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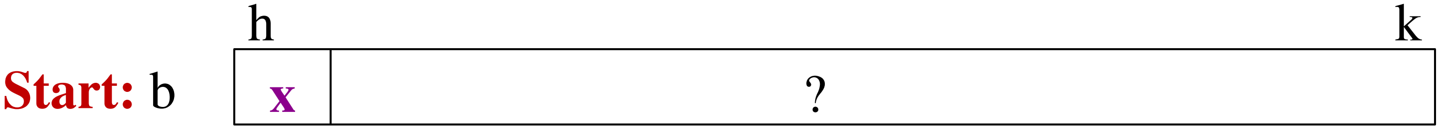

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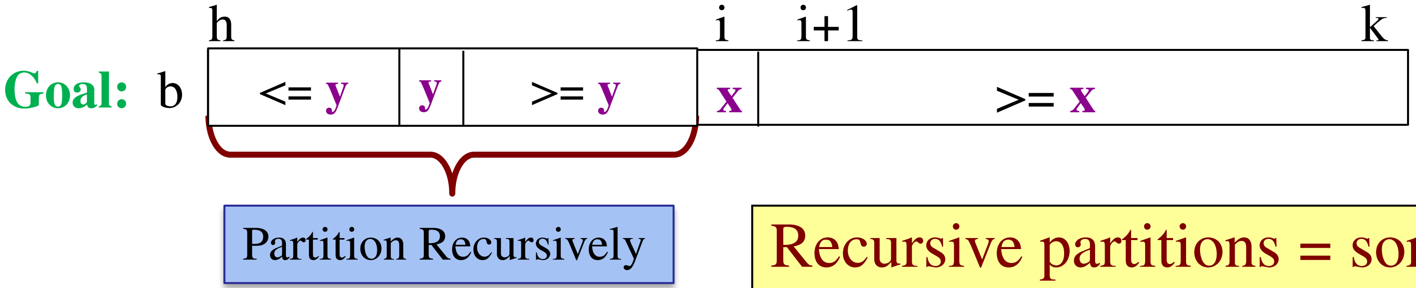


h i i+1 k

Goal: b | <= y | y | >= y | x | >= x

Partition Recursively

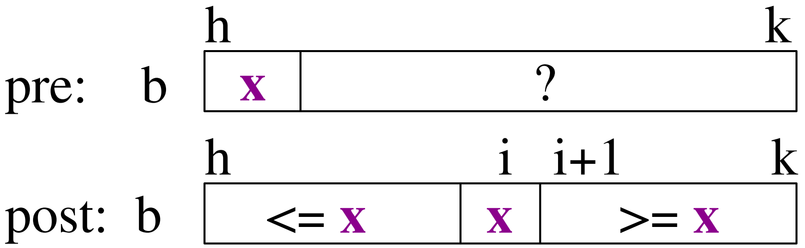
Recursive partitions = sorting

- Called **QuickSort** (why???)
- Popular, fast sorting technique

QuickSort

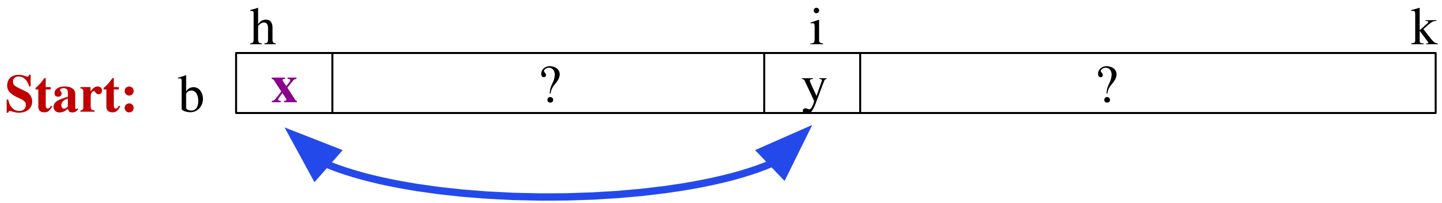
```
def quick_sort(b, h, k):
    """Sort the array fragment b[h..k]"""
    if b[h..k] has fewer than 2 elements:
        return
    j = partition(b, h, k)
    # b[h..j-1] <= b[j] <= b[j+1..k]
    # Sort b[h..j-1] and b[j+1..k]
    quick_sort (b, h, j-1)
    quick_sort (b, j+1, k)
```

- **Worst Case:** array already sorted
    - Or almost sorted
    - n^2 in that case
- **Average Case:** array is scrambled
    - n log n in that case
    - Best sorting time!



So Does that Solve It?

- Worst case still seems bad! Still n^2
    - Only happens in small number of cases
    - Just happens that case is common (already sorted)
- Can greatly reduce issue with randomization
    - Swap start with random element in list
    - Now pivot is random and already sorted unlikely



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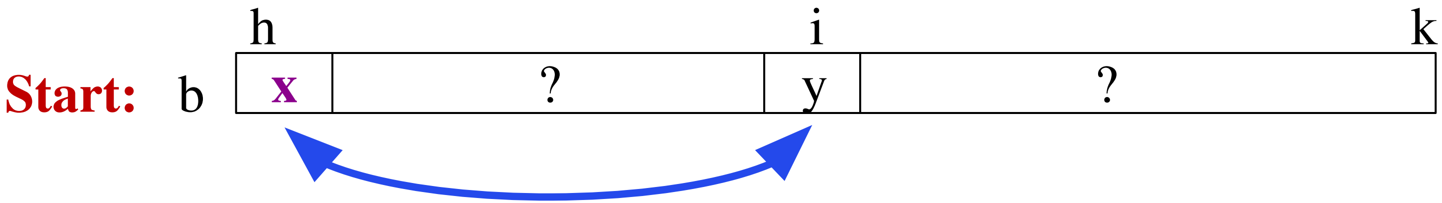
Makes it "good enough"
for most applications



Can We Do Better?

- There is guaranteed n log n sorting algorithm
    - Called **merge sort** (beyond scope of course)
    - Used heavily in large databases
    - But it has high overhead (slower on small data)
- What does the `sort()` method use?
    - Uses **Timsort** (invented by Tim Peters in 2002)
    - Combination of insertion sort and merge sort
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Quicksort is 1959!