Haskell: Monads

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- The rules of the game:
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  - Pure values can enter the impure world but cannot escape it
- We "wrap" the result of computation and impure side effects in an monadic object

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  - All functions have a single argument (made possible via currying)
- The above three properties support the *pipelined* view of computation

$$\boxed{\text{input} \Longrightarrow f_1 \Longrightarrow f_2 \Longrightarrow \ldots \Longrightarrow f_n \Longrightarrow \text{output}}$$

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  - The result of any computation in the monadic world must return a monadic value (so that we cannot escape to the pure world)
- So, the type of any function operating within the monadic world must look like:
  - f: $a \rightarrow m\ a$ (where m is a monadic type)

... completing the picture

- To enable a computation pipeline as in the pure world, we would need:
  - an ability to execute a monadic function
  - bind (>>=) operation
  - $>>=\ :\ m\ a \rightarrow (a \rightarrow m\ b) \rightarrow m\ b$

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  - an ability to execute a monadic function
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  - $>>= : \ m\ a \to (a \to m\ b) \to m\ b$

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- Enable pure values to enter the monadic world
  - return : $a \ \to m\ a$

The Monad typeclass

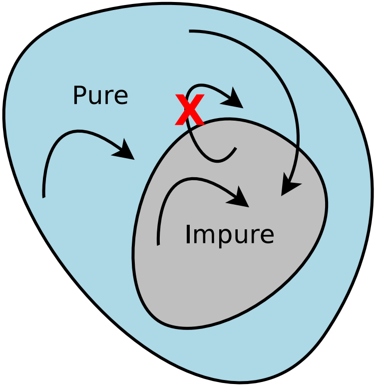
```
class Monad m where
  return :: a -> m a
  (>>=) :: m a -> (a -> m b) -> m b

  (>>) :: m a -> m b -> m b
  x >> y = x >>= \_ -> y

  fail :: String -> m a
  fail msg = error msg
```

- return and (>>=) are the interface methods to be defined by any monad; the other two you get for free.

Summary



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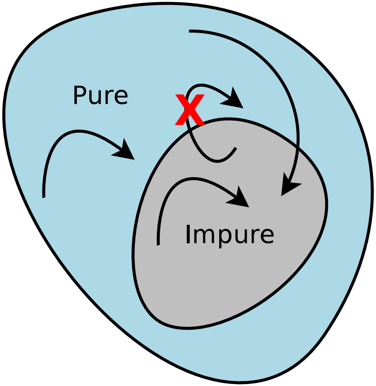

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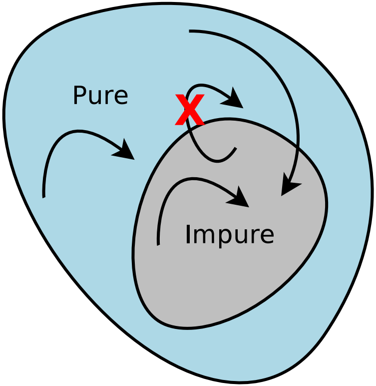

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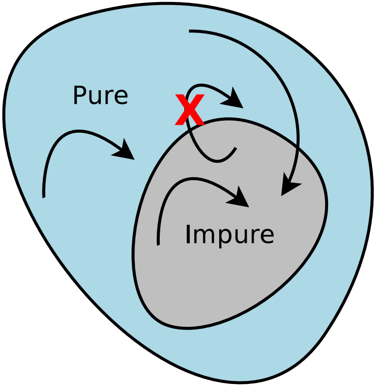

- pure → pure: function application (f a)
- impure → impure: bind (f >>= a)
- pure → impure: return (return a)

The Maybe monad

```haskell
-- define the Maybe type
data Maybe a = Just a | Nothing

-- add Maybe type as an instance of the Monad typeclass
instance Monad Maybe where
  return x = Just x
  Nothing >>= f = Nothing
  Just x >>= f = f x
  fail _ = Nothing
```

The Maybe monad

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-- define the Maybe type
data Maybe a = Just a | Nothing

-- add Maybe type as an instance of the Monad typeclass
instance Monad Maybe where
  return x = Just x
  y >>= f = case y of
    Just x -> f x
    Nothing -> Nothing
  fail _ = Nothing
```

What was the other option without monads?

```
data Maybe a = Just a | Nothing (* define the Maybe type *)

addMaybes :  Maybe a -> Maybe a
addMaybes x = case x of
  Just y -> Just y + 1
  Nothing -> Nothing

subMaybes :  Maybe a -> Maybe a
subMaybes x = case x of
  Just y -> Just y - 1
  Nothing -> Nothing
```

Notice the pattern...

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f :  Maybe a -> a -> Maybe a
f x = case x of
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and, compare with the definition of bind:

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instance Monad Maybe where
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  x >>= f = case x of
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```

That is, we simply give the pattern a shortcut: `x >>= f`

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... and the type is **explicit** of the effect (i.e. like the function can fail)

Syntax Sugar

```
x >>= f >>= g >>= h
```

is same as:

```
do
  y1 <- x
  y2 <- (f y1)
  (h y2)
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Note: Unlike pure computations, monadic computations happen in the given **sequence** (to keep an consistent effect)

IO monad

```
getChar ::  IO Char
putChar ::  Char -> IO ()
getLine :: IO String
putStr :: String -> IO ()
putStr :: String -> IO ()

main ::  IO ()
main = do
  c <- getChar
  putChar c
```

Common mistakes

`let name = getLine`

versus

do

name <- getLine

Reading multiple lines

```
main = do
  line <- getLine
  if null line then
  return ()
  else do
    putStrLn (purefun line)
  main
```

printing values

```
main = do
  print 42
  print True
  print "Hello"
  print [32,14,31]
  print 3.2

print ::  Show a => a -> IO ()
```

Higher order functions

```
mapM ::  Monad m => (a -> m b) -> [a] -> m [b]
mapM print
>> [1,2,3]

mapM_ ::  Monad m => (a -> m b) -> [a] -> m ()

fmap ::  (Functor f) => (a -> b) -> f a -> f b
fmap (+1) (Just 1)
>> Just 2
```

sequence :: [IO a] -> IO [a]

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  - (m >>= f) >>= g ≡ m >>= (\x -> f x >>= g)

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- Associativity
  - (m >>= f) >>= g $\equiv$ m >>= (\x -> f x >>= g)

Intuition

- Left and right identity say that entering the monadic space does not change the pure value in any way, other than wrapping it in a box.
- Associativity says that in monadic function, function composition (bind) associates the same way as in pure functions.

What is the output?

```
main = do
  x <- getLine
  let a = (read x ::  Int) in
    let y = case a of
          0 -> Nothing
          y -> Just y
    in print (do
          b <- y
          if (b == 20) then return "Hello" else return
"World")
```