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Tutorial-4

Q1 $T(n) = 3T(n/2) + n^2$

here, $a = 3$, $b = 2$

$f(n) = n^2$

so, $n^{\log_b a} = n^{\log_2 3}$

Since $n^{\log_2 3} < n^2$

So, according to master's theorem

$T(n) = O(n^2)$

Q2 $T(n) = 4T(n/2) + n^2$

here, $a = 4$, $b = 2$ & $f(n) = n^2$

So, $n^{\log_b a} = n^{\log_2 4}$ $n^{\log (2)^2}$

$= n^2$

$n^{\log_b a} = f(n)$

Acc. to master's theorem,

$T(n) = O(n^2 \log n)$

Q3 $T(n) = T(n/2) + 2^n$

here, $a = 1$, $b = 2$ and $f(n) = 2^n$

So, $n^{\log_b a} = n^{\log_2 1} \Rightarrow n^{\log_2 2^0}$

$\Rightarrow n^0 = 1$

Since, $1 < f(n)$

Acc. to Master's theorem,

$T(n) = O(2^n)$

Q4 $T(n) = 2^n T(n/2) + n^2$

Master's theorem is not applicable since 'a' is a function.

Q5.

$T(n) = 16T(n/4) + n$

here, $a = 16$, $b = 4$ and $f(n) = n$

So, $n^{\log_b a} = n^{\log_4 16} = n^{\log_4 4^2} = n^{2\log_4 4}$

$\Rightarrow n^2$

Since, $n^2 > n$

Therefore, acc. to master's theorem

$T(n) = O(n^2)$

Q6. $T(n) = 2T(n/2) + n\log n$

here, $a = 2$, $b = 2$ $f(n) = n\log n$

So, $n^{\log_b a} = n^{\log_2 2} = n$.

Acc. to master's theorem

$T(n) = O(n \log n)$

Q7 $T(n) = 2T(n/2) + n/\log n$

$a = 2$, $b = 2$, $f(n) = n/\log n$.

So, $n^{\log_b a} \Rightarrow n^{\log_2 2} = n$.

Since, $n^{\log_b a} > f(n)$

Acc. to Master's Theorem

$T(n) = O(n)$

Q8 $T(n) = 2T(n/4) + n^{0.51}$

$a = 2$ $b = 4$, $f(n) = n^{0.51}$

$\Rightarrow n^{\log_b a} \Rightarrow n^{\log_4 2} = n^{0.5}$

since $n^{\log_b a} < f(n)$

Acc. to Master's theorem:

$T(n) = O(n^{0.51})$

Q9. $T(n) = 0.51\,T(n/2) + 1/n$

not applicable master's theorem since $a < 1$.

Q10

$T(n) = 16T(n/4) + n!$

here, $a = 16$, $b = 4$ & $f(n) = n!$

so, $n^{\log_b a} = n^{\log_4 16} = n^{\log_4 (4)^2} = n^2$

Since, $n^{\log_b a} < n!$

Acc. to Master's theorem,

$T(n) = O(n!)$

Q12. $T(n) = \text{sqrt}(n)\,T(n/2) + \log n$.

Since $a \neq$ constant

Master's theorem not applicable.

Q11 $T(n) = 4T(n/2) + \log n$.

here, $a = 4$, $b = 2$ & $f(n) = \log n$.

so, $n^{\log_b a} = n^{\log_2 4} = n^2$

since, $n^{\log_b a} > f(n)$

Master's t.

$T(n) = O(n^2)$

Q13. $T(n) = 3T(n/2) + n$

where $a=3$, $b=2$ & $f(n) = n$

$n^{\log_b a} = n^{1.58} = n^{\log_2 3}$

$n^{\log_b a} > f(n)$

$T(n) = O(n^{1.58})$

Q14

$T(n) = 3T(n/3) + \sqrt{n}$

$a=3$, $b=3$ & $f(n) = \sqrt{n}$

$n^{\log_3 3} = n$

$> f(n)$

$T(n) = O(n)$

Q15. $T(n) = 4T(n/2) + n$.

$a=4$, $b=2$ & $f(n) = n$

$n^{\log_2 4} = n^2 > f(n)$

$T(n) = O(n^2)$

Q16 $T(n) = 3T(n/4) + n \log n$.

$a=3$, $b=4$ & $f(n) = n \log n$

$n^{\log_4 3} = n^{0.187} < f(n)$

$T(n) = O(n \log n)$.

Q17

$T(n) = 3T(n/3) + n/2$

$a=3$, $b=3$ & $f(n) = n/2$

$n^{\log_3 6} = n^{1.63} < n^2 \log n$.

$T(n) = O(n^2 \log n)$

Q18 $T(n) = 6T(n/3) + n^2 \log n$

$a = 6; b = 3 \quad f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63} < n^2 \log n$

$T(n) = O(n^2 \log n)$

Q19

$T(n) = 4T(n/2) + n/\log n$

$a = 4, b = 2, f(n) = n/\log n$

$n^{\log_2 4} = n^2 > f(n)$

$T(n) = O(n^2)$

Q20 $T(n) = 64T(n/8) - n^2 \log n$

here, $f(n)$ is not an increasing function. Therefore, master's theorem is not applicable.

Q21

$T(n) = 7T(n/3) + n^2$

$a = 7, b = 3 \; f(n) = n^2$

$n^{\log_3 7} = n^{1.7}$

$n^{\log_b a} < f(n)$

$\therefore$ acc. to master's method $T(n) = O(n^2)$

Q22 $T(n) = T(n/2) + n(2 - \cos n)$

here, master's theorem is not applicable due to violation of regularity condition.