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Tutorial-1

Q1 What do you understand by Asymptotic notations. Define different Asymptotic notations with examples.

=> Asymptotic notations are the mathematical notations used to describe running time of an algorithm when the input tends towards a particular value or a limiting value. Asymptotic notations are mainly categorised into 3 types :-

i) Big O-notation :- Gives worst case complexity.
ii) Omega-notation :- Gives best case complexity.
iii) Theta-notation :- Gives average case complexity.

Ex :- Bubble sort algorithm has $O(n)$ complexity in best case and $O(n^2)$ in worst case and $O(n^2)$ in average case.

Q2 for $(i = 1 \text{ to } n)$
{ $i = i * 2;$
}

$i = 1, 2, 4, 8, \ldots n \rightarrow$ G.P

$a_k = ar^{k-1}$

$a = 1, r = 2$

$a_k = 1 \cdot 2^{k-1}$

$n = 2^{k-1}$

$\log_2 n = k - 1$

$k = 1 + \log_2 n$

$\therefore T(n) = O(\log_2 n + 1)$

$= O(\log n)$ Ans.

Q3 $T(n) = \{3T(n-1)$ if $n>0$, otherwise $1\}$

$T(0)=1$

$T(n) = 3T(n-1)$ — (i)

Put $n = n-1$ ~~in~~ in eqn (i)

$T(n-1) = 3T(n-2)$ — (ii)

Put (ii) in (i)

$T(n) = 3(3T(n-2)) = 3^2 T(n-2)$ — (iv)

Put $n = n-2$ in eqn (i)

$T(n-2) = 3T(n-3)$

$T(n) = 3^2 \, 3T(n-3) = 3^3 T(n-3)$

$T(n) = 3^k T(n-k)$

let $n-k=0$

$T(n) = 3^n T(0) \Rightarrow T(n) = 3^n$

$T(n) = O(3^n)$

Q4 $T(n) = \{2T(n-1)-1$ if $n>0$, otherwise $1\}$

→ $T(n) = 2T(n-1)-1$ — (i)

$T(0) = 1$ — (ii)

Put $n = n-1$

$T(n-1) = 2T(n-2)-1$ — (iii)

Put (iii) in (i)

$T(n) = 2(2T(n-2)-1)-1$

$= 4T(n-2)-2-1 = 2^2 T(n-2) - 2^1$ — (iv)

Put $n = n-2$ in (i)

$T(n-2) = 2T(n-3)-1$

$T(n) = 2^2(2T(n-3)-1)-2-1$

$= 2^3 T(n-3) - 2^2 - 2^1 - 1$

$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - 2^{k-3} \dots 2^0$

let $n-k=0$

$n=k$

$T(n) = 2^n T(n-n) - 2^{n-1} - 2^{n-2} - 2^{n-3} \dots 2^0$

$T(n) = 2^n T(0) - 2^{n-1} - 2^{n-2} - 2^{n-3} \dots 2^0$

$T(n) = 2^n - 2^{n-1} - 2^{n-2} \dots 2^0$

$T(n) = 2^n - (2^n - 1)$.

$(\because 2^{n-1} + 2^{n-2} + \dots + 2^0 = 2^n - 1)$

$T(n) = 1$

$T(n) = O(1)$

Q5 int i=1, s=1;
while (s<=n)
{ i++;
s= s+i;
print ("#");
}

$i=1$	$s=1$	
$i=2$	$s=3$	$s=1+2$
$i=3$	$s=6$	$s=1+2+3$.
$i=4$	$s=10$	$s=1+2+3+4$

$s = 1+2+3+4+\dots+k = \frac{k(k+1)}{2} > n$ $(\because s<=n)$

$s = \frac{k^2+k}{2} > n$

$k > \sqrt{n}$

$T(n) = O(\sqrt{2})$

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Q6. void function (int n)
{ int i , count = 0;
for (i = 1; i * i <= n; i++)
count ++
}

$i = 1, 2, 3, ---n$
$i^2 = 1, 2^2, 3^2, ---n^2$
$i^2 <= n$
$\Rightarrow i <= \sqrt{n}$

$a_k = a + (k-1)d$
$a = 1, d = 1$
$a_k <= \sqrt{n}$
$\sqrt{n} = 1 + (k-1).1$
$\sqrt{n} = k$
$\therefore T(n) = O(\sqrt{n})$

Q7 void function (int n)
{ int i, j, k, count = 0;
for (i = n/2 ; i <= n; i++)
for (j = 1 ; j <= n; j = j * 2)
for (k = 1 ; k <= n; k = k * 2)
count ++
}

i	j	k
$n/2$	$\log n$	$(\log n)^2$
$n+1/2$	$\log_2 n$	$(\log_2 n)^2$
|	|	|
n	$\log n$	$(\log_2 n)^2$

$\frac{n}{2} + 1$ times

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$O(i * k) = O\left(\left(\frac{n}{2}+1\right) * (\log n)^2\right)$

$T(n) = O(n (\log n)^2)$

Q8 function (int n)
{ (if n==1) return;
for (i=1 to n)
{ for (j=1 to n)
{ print ("*");
}
} function (n-3);
}

$\Rightarrow T(n) = T(n-3) + n^2$ — (1)

$T(1) = 1$

Put $n = n-3$ in eqn (1)

~~T(n-3) in (1)~~

$T(n-3) = T(n-6) + (n-3)^2$ — (II)

Put $T(n-3)$ in (1)

$T(n) = T(n-6) + (n-3)^2 + n^2$ — (III)

Put $n = n-6$ in (1)

$T(n-6) = T(n-9) + (n-6)^2$ — (IV)

$T(n) = T(n-9) + (n-6)^2 + (n-3)^2 + n^2$

$T(n) = T(n-3k) + (n-3(k-1))^2 + (n-3(k-3))^2 + n^2 + \dots + (n-3(k-1))^2$

Put $n - 3k = 1$

$n = 1+3k \Rightarrow k = \frac{n-1}{3}$

$T(n) = T(1) + n^2 + \dots (n-3)^2 + (n-6)^2 + \dots (n-n+1)^2$

$T(n) = 1 + n^2 + (n-3)^2 + (n-6)^2 + \dots 1^2$

$T(n) = 6n^2 + k$

$T(n) = O(n^2)$

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Q9 Time +

```
void function (int n)
{ for (i=1 to n) {
    for (j=1; j<=n; j=j+i)
      print ("*")
  }
}
```

$i=1$; $j=1,2,3,4, \ldots \rightarrow n$ times
$i=2$, $j=1,3,5,7, \ldots$ $n/2$ times
$i=3$, $j=1,4,7,11, \ldots$ $n/3$ times
$\vdots$
$i=n$ $j=1 \ldots$ 1 time

$$\sum_{j=1}^{n} n+\frac{n}{2}+\frac{n}{3}+\ldots+1$$

$$\sum_{j=1}^{n} n\left(1+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{n}\right)$$

$$= n(\log n)$$
$$T(n) = (n \log n)$$
$$T(n) = O(n \log n)$$

Q10 For the functions, n^k and c^n, what is the asymptotic relationship b/w these function

$n^k = O(c^n)$
as $n^k <= 2c^n \quad \forall\, n \geq n_0$
for $n_0 = 1$
$c = 2$
$1^k <= 2$

$n_0 = 1$, $c = 2$.