Learning Objective (Jo Hum Is Chapter Mein Seekhenge)

Is chapter mein hum **Linear Algebra** ke basic concepts aur **Data Science** mein uska role samajhne wale hain. Hum **scalars, vectors, aur matrices** ke beech ka difference jaanenge aur matrices par important operations perform karna seekhenge. Specifically, hum yeh seekhenge:

✅ **Linear Algebra ke fundamentals aur Data Science mein uski importance**
✅ **Scalars, Vectors, aur Matrices ka difference aur unki properties**
✅ **Matrices ke operations** jaise:

- Addition (जोड़)
- Multiplication (गुणा)
- Transposition (पलटना)
- Determinant nikalna
- Inverse aur Rank calculate karna

Linear Algebra Kya Hai?

Definition (Paribhasha)

Linear Algebra ek **important branch** hai **mathematics** ka, jo mainly **vectors, vector spaces, aur linear transformations** ka study karta hai. Yeh ek **powerful tool** hai jo hume **linear equations solve** karne, **linear functions samajhne**, aur **inhe matrices aur determinants ke through represent karne** mein madad karta hai.

Iska **sirf theoretical mathematics** tak hi use nahi hai, balki **bahut saare fields** mein iska **broad application** hai, jaise:

✅ **Machine Learning**
✅ **Computer Graphics**
✅ **Data Science**
✅ **Physics & Engineering**

Simple Example

Agar aapke paas ek **linear equation** hai:

$2x+3y=52x + 3y = 52x+3y=5$ $x−y=2x - y = 2x−y=2$

Toh Linear Algebra ke tools ka use karke hum **inhe easily solve** kar sakte hain. Yeh **matrix aur vector representation** ko use karta hai jo complex calculations ko **simple aur fast** banata hai.

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Common Linear Algebra Symbols (Aam Linear Algebra Symbols aur Unka Matlab)

Linear Algebra mein **different symbols** ka use hota hai jo **mathematical operations** ko represent karte hain. Neeche kuch important symbols aur unke meanings diye gaye hain:

Symbol Name	Matlab / Usage
· (Dot Product)	Do vectors ka **scalar product** (Ek number milega result mein)
× (Cross Product)	Do vectors ka **vector product** jo dono vectors ke **orthogonal** hoga
ATA^TAT (Transpose of Matrix A)	**Matrix ke rows ko columns aur columns ko rows** mein convert karna
ϕ (Null Set)	Aisa **set jismein koi bhi element nahi hota**
A−1A^{-1}A−1 (Inverse of Matrix A)	Ek aisi **matrix jo original matrix ke saath multiply ho kar Identity Matrix de**
**(	A
ϱ(A)ϱ(A)ϱ(A) (Matrix Rank)	Matrix ke **linearly independent rows ya columns ka count**
**(	
λλλ (Eigenvalue)	Ek **scalar jo batata hai ki Linear Transformation vectors ko kaise scale karega**
Eigenvector	**Ek non-zero vector jo sirf scale hota hai, direction change nahi karta**

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Role of Linear Algebra in Data Science (Data Science mein Linear Algebra ka Role)

Linear Algebra **Data Science ka backbone** hai. Har jagah **data ko represent karna, analyze karna aur process karna** matrices aur vectors ke through hota hai. Yeh **Machine Learning, NLP, aur Recommender Systems** jaise advanced areas mein bhi use hota hai.

1️⃣ Data Representation (Data ko Represent Karna)

- **Data ko store aur manipulate** karne ke liye **matrices ka use** hota hai.
- **Example:** Agar ek company ke **1000 customers aur 50 products** hain, toh unka data **1000 × 50 matrix** mein store ho sakta hai.

2️⃣ Machine Learning Algorithms (ML Algorithms mein Istemaal)

- **Support Vector Machines (SVM), Neural Networks** jaise ML models **matrix operations** ka use karte hain.
- **Example:** Neural Networks **weight matrices aur activation functions** ka use karke predictions karta hai.

3️⃣ Dimensionality Reduction (Data ka Size Kam Karna - PCA Technique)

- **Principal Component Analysis (PCA)** jaise techniques **linear algebra ka use** karke **features reduce** karti hain.

- **Example:** Agar ek dataset **1000 features** ka hai, toh PCA **sirf important features** ko select karke **data ko chhota aur fast banata hai**.

4️⃣ Recommender Systems (Netflix, Amazon Recommendation Engines)

- **Matrix Factorization** methods jaise **Singular Value Decomposition (SVD)** ka use **recommendations generate** karne ke liye hota hai.
- **Example:** Netflix aur Amazon **past user preferences ke basis par naye movies/products recommend karte hain** using **matrix operations**.

5️⃣ Natural Language Processing (NLP - Language Processing mein Role)

- **Words aur documents** ko **vectors aur matrices** mein represent kiya jata hai.
- **Example:**
    - **Word Embeddings (Word2Vec, GloVe)**: Har word ko **vector format** mein convert karte hain.
    - **Document-Term Matrix (DTM)**: Ek matrix **jo batata hai ki kisi document mein kaunse words kitni baar aaye hain**.

Branches of Linear Algebra (Linear Algebra ke Vibhaag)

Linear Algebra ko **3 main branches** mein divide kiya gaya hai, jo **basic se lekar advanced aur practical applications** tak cover karte hain.

1️⃣ Elementary Linear Algebra (बुनियादी Linear Algebra)

🔹 **Basic Concepts**: Is branch mein **fundamental concepts** cover kiye jate hain jaise:

✅ **Scalars, Vectors, Matrices**

✅ **Matrix Addition, Multiplication, Transposition**

✅ **Linear Equations ka solution using Matrices**

📌 **Example:**

Agar ek **matrix A** hai:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Toh hum **iska transpose** A^T easily nikal sakte hain:

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

2️⃣ Advanced Linear Algebra (Advanced Level Concepts)

🔹 Yeh **complex topics** ko cover karta hai jaise:

✅ **Eigenvalues & Eigenvectors**

✅ **Vector Spaces aur Subspaces**

✅ **Linear Transformations aur their properties**

📌 **Example:**
Agar ek **linear transformation T(x)T(x)T(x)** diya ho, toh **Eigenvalue (λ) aur Eigenvector (v) nikalne** ke liye equation hoti hai:

Av=λvA v = λ vAv=λv

Jisme **λ ek scalar hota hai jo transformation ka effect batata hai**, aur **v ek special vector hota hai jo sirf scale hota hai, direction change nahi karta**.

3️⃣ Applied Linear Algebra (Practical Implementation & Applications)

🔹 Yeh **Linear Algebra ke real-world applications** par focus karta hai, jaise:
✅ **Machine Learning aur Deep Learning**
✅ **Matrix Factorization (Singular Value Decomposition - SVD)**
✅ **Computer Graphics aur Image Processing**

📌 **Example:**

- **Google Search Algorithm**: Web pages ki ranking **Eigenvalues aur Eigenvectors** ka use karke hoti hai.
- **Face Recognition Systems**: **Principal Component Analysis (PCA)** ka use karke face detection hoti hai.

🔹 **Conclusion:**

Linear Algebra **ek important subject** hai jo **basic se lekar advanced aur practical applications** tak use hoti hai. **Data Science, AI, Physics, aur Engineering** jaise fields mein iska **bahut bada role** hai! 🚀

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Industrial Applications of Linear Algebra (Linear Algebra ka Vyavsayik Upyog)

Linear Algebra **industry ke kai important areas** mein use hoti hai, jaise **Finance aur Computer Vision**. Chaliye inhe detail mein samajhte hain.

1️⃣Finance (Vitta aur Investment Sector)

- **Investors aur financial analysts** Linear Algebra ka use **risk aur return ko balance** karne ke liye karte hain.
- **Covariance Analysis aur Linear Equations** ka use **best asset allocation** decide karne mein hota hai.

✅ **Example:**
Agar ek investor **2 assets (Stock A aur Stock B)** mein invest kar raha hai, toh uska **portfolio optimization** Linear Algebra se hota hai:

Portfolio Risk=wA2σA2+wB2σB2+2wAwBσAB\text{Portfolio Risk} = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_{AB}Portfolio Risk=wA2σA2+wB2σB2+2wAwBσAB

Jisme:

- wA,wBw_A, w_BwA,wB = investment weights
- σA,σB\sigma_A, \sigma_BσA,σB = individual stock volatility
- σAB\sigma_{AB}σAB = covariance (dono stocks ka risk correlation)

🔹 **Isse investors yeh decide kar sakte hain ki kitna paisa kis asset mein lagana chahiye!** 📈

2️⃣ Computer Vision (Image Processing aur AI Vision)

🔹 **Linear Algebra ke powerful matrix operations** ka use **image processing techniques** jaise:
✅ **Image Compression (JPEG, PNG Formats)**
✅ **Image Enhancement (Brightness aur Contrast Improve karna)**
✅ **Edge Detection (Face aur Object Recognition Systems)**

✅ **Example - Edge Detection (Convolution Technique)**
Image ko **matrix ke form mein represent** kiya jata hai aur **convolution operation** se **edges detect** kiye jate hain.

Agar ek **3×3 filter matrix (Kernel)** diya jaye:

K=[−1−1−1−18−1−1−1−1]K = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}K=−1−1−1−18−1−1−1−1

Toh isko **image matrix par apply** karke **sharp edges detect** kiye ja sakte hain.

- **Iska use Face Recognition, Object Detection, aur Medical Imaging jaisi fields mein hota hai!** 🖼️👁️

- **Conclusion:**

Linear Algebra **finance aur computer vision jaise industrial fields** mein **essential role play** karti hai. **Risk optimization, investment planning, aur AI-based image processing** sab Linear Algebra ke bina possible nahi hote! 🚀

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Industrial Applications of Linear Algebra (Linear Algebra ka Vyavsayik Upyog - Part 2)

Linear Algebra **Robotics, Engineering, Big Data, aur AI** jaise advanced fields mein bhi use hoti hai. Chaliye inhe detail mein samajhte hain.

1️⃣Robotics and Engineering (Robotics aur Engineering mein Linear Algebra ka Use)

🔹 **Robots ka movement aur control** Linear Algebra se model kiya jata hai.

🔹 **State-Space Representation** ka use system behavior **predict aur optimize** karne ke liye hota hai.

✅ **Example - Robot Arm Movement**
Agar ek **robotic arm** ko ek specific position par move karna ho, toh uske motion ko **matrix equations** se solve kiya jata hai:

$Ax=BAx = BAx=B$

Jisme:

- AAA = **Transformation Matrix** (Rotation, Scaling, Translation)
- xxx = **Robot ke joint angles ka vector**
- BBB = **Target Position ka vector**

🔹 **Yeh technique autonomous robots aur industrial automation mein bahut use hoti hai!** 🤖⚙️

2️⃣ Big Data and Artificial Intelligence (AI)

🔹 **Data analysis aur AI models** mein Linear Algebra **core role** play karti hai.

🔹 **PCA (Principal Component Analysis)** ka use **data dimension reduce** karne aur **important patterns detect** karne ke liye hota hai.

✅ **Example - PCA in Big Data**

Maan lijiye ek dataset hai jisme **1000 features hain**, toh **ML algorithms slow ho sakte hain**. PCA ka use karke **sabse important features select** kiye jate hain, jisse **data fast process ho** aur **accuracy bhi bani rahe**.

Step-by-Step PCA Working:

1️⃣ Data ko **standardize** karo (mean = 0, variance = 1).

2️⃣ **Covariance matrix** banao.

3️⃣ **Eigenvalues aur Eigenvectors** calculate karo.

4️⃣ **Top k Eigenvectors** ko choose karke **new reduced dataset** banao.

🔹 **PCA ka use Face Recognition, Fraud Detection, aur Stock Market Analysis jaise areas mein hota hai!** 📊📉

🔹 **Conclusion:**

Linear Algebra **Robotics, Engineering, Big Data aur AI** jaise modern applications mein **essential tool** hai. **Autonomous robots, industrial automation aur data-driven AI models** Linear Algebra ke bina impossible hote! 🚀

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Real-Life Applications of Linear Algebra (Linear Algebra ka Rozmarra Jeevan me Upyog)

Linear Algebra sirf theoretical concept nahi hai, balki **bahut saari real-world applications** mein iska **major role** hai. Chaliye kuch **important examples** ko detail mein samajhte hain.

1️⃣ Predicting House Prices (Ghar ke Prices ka Prediction) 🏠

🔹 **House price prediction** ke liye **feature vectors** ka use hota hai, jaise:

✅ **Square footage (kitna bada hai ghar?)**
✅ **Number of bedrooms (kitne kamre hain?)**
✅ **Locality Score (area kitna acha hai?)**

➡️ Yeh sab **Linear Regression Model** ke andar ek **Matrix Equation** me represent kiya jata hai:

$\hat{y} = W X + b$

Jisme:

- X = **Feature matrix** (ghar ke features)
- W = **Weight vector** (kitna impact hai har feature ka)
- b = **Bias term**
- $\hat{y}$ = **Predicted house price**

🔹 **Yeh method real estate companies aur banks use karte hain loan approval ke liye!** 🏡📈

2️⃣ Image Compression (Tasveeron ka Size Kam Karna) 🖼️

🔹 **High-resolution images ka size reduce** karne ke liye **Singular Value Decomposition (SVD)** ka use hota hai.

🔹 **Bina quality lose kiye** image compress hoti hai.

✅ **Example:**
Agar ek **1000x1000 pixels ki image** ko compress karna ho, toh **SVD algorithm** use karke isko **chhoti size wali matrix mein convert** kiya jata hai.

➡️ Formula:

$A = USV^T$

Jisme:

- A = **Original Image Matrix**
- U, S, V^T = **Compressed matrices**

🔹 **JPEG format aur video streaming platforms (Netflix, YouTube) yahi technique use karte hain!** 🎥

3️⃣ Stock Market Prediction (Share Bazaar ka Analysis) 📊📉

🔹 **Covariance Matrices** ka use **multiple stocks ke trend aur correlation ko analyze** karne ke liye hota hai.

🔹 **High-risk aur low-risk stocks ko identify** karne ke liye **investment strategies** me Linear Algebra ka use hota hai.

✅ **Example:**
Agar **Stock A aur Stock B** ek-dusre se related hain, toh unka **covariance positive hoga**, warna **negative hoga**.

🔹 **Mutual Funds aur Portfolio Management ke liye yeh technique important hai!** 📈 💰

4️⃣ Movie Recommendation Systems (Netflix/Amazon me Suggestions kaise milte hain?) 🎬 🍿

🔹 **Matrix Factorization** ka use **movie recommendation** systems me hota hai.

🔹 **Netflix, Amazon, aur Spotify jaise platforms yeh technique use karte hain!**

✅ **Example - User Preference Prediction using SVD**
Maan lo ek **user ke pasand na pasand ka matrix** diya hai:

User	Movie 1	Movie 2	Movie 3	Movie 4
A	5	?	3	?
B	?	4	?	5

➡️ **"?" wale missing ratings ko predict karne ke liye SVD algorithm** ka use hota hai!

🔹 **Spotify aur YouTube bhi isi concept par song/video recommendations dete hain!** 🎶

5️⃣ Self-Driving Cars (Khud chalne wali Gaadiyan - Tesla, Waymo) 🚗 🤖

🔹 **Image Recognition, Object Detection, aur Decision-Making** ke liye **Linear Algebra ke transformations** use hote hain.

✅ **Example - Camera ka Input Process Karna**

Agar ek **self-driving car** ko samne **pedestrian ya traffic light** dikh rahi hai, toh uska data **matrix transformation** se process hota hai:

➡️ **Image Matrix → Edge Detection → Object Recognition → Decision Making**

🔹 **Tesla, Google Waymo, aur Uber Automated Vehicles Linear Algebra ka use karte hain!** 🚦

🔹 **Conclusion:**

Linear Algebra **har industry me use hoti hai**, chahe **real estate, finance, AI, robotics ya entertainment ho**. **Data Science aur AI ke bina aaj ke modern applications possible nahi hain!** 🚀

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Scalars: Definition and Representation (Scalars Kya Hote Hain?)

🔹 **Definition (Paribhasha)**

Scalar ek **single numerical value** hota hai jo **sirf magnitude (maatra)** batata hai, **direction (disha) nahi**.

- Scalar quantities **simple mathematical operations**

(addition, subtraction, multiplication, division) ko follow karti hain.

✅ **Example:**
Agar **temperature 20°C** hai, toh yeh **sirf ek numerical value hai** aur iski koi **direction nahi** hai.

🔹 **Representation of Scalars (Scalars ko kaise dikhaya jata hai?)**

✅ Scalars ko **chhoti (lowercase) letters** se represent kiya jata hai, jaise:

a,b,c,Ta, b, c, Ta,b,c,T

✅ **Example:**

- **Temperature** → T=20°CT = 20°CT=20°C
- **Speed** → s=50s = 50s=50 km/h (direction nahi bataya gaya)
- **Height** → h=170h = 170h=170 cm

🔹 **Examples of Scalar Quantities (Scalars ke Real-Life Examples)**

1️⃣ **Mass (Vajan)** → **10 kg** → Sirf weight batata hai, koi direction nahi.
2️⃣ **Time (Samay)** → **5 seconds** → Sirf ek number hai, direction nahi.
3️⃣ **Distance (Doori)** → **100 meters** → Kitni doori cover ki gayi,

lekin kis direction me, yeh nahi batata.
4️⃣ **Temperature (Taapmaan) → 30°C** → Yeh sirf **garmi ya thandi batata hai**, koi direction nahi.

✅ **Scalar ka Simple Rule:** Agar koi value **direction specify nahi karti** toh woh **scalar quantity hoti hai!** 🎯

🔹 **Conclusion:**

Scalars **simple numerical values** hote hain jo **magnitude specify** karte hain **lekin direction nahi** dete.
Ab hum **Vectors** ke baare mein detail mein samjhenge, jo **magnitude ke saath direction bhi show karte hain!** 🚀

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Vectors: Definition, Operations, and Interpretations

🔹 **Definition (Paribhasha)**

Vector ek **ordered collection of numbers** hota hai jo **magnitude (maatra) aur direction (disha)** dono ko represent karta hai.

✅ **Scalar aur Vector ka Basic Difference:**

- **Scalar** → Sirf **magnitude** hota hai, koi **direction nahi** (e.g., Mass, Time, Temperature).

- **Vector → Magnitude + Direction** dono hoti hain (e.g., Velocity, Force, Position).

🔹 **Geometric Interpretation (Vector ka Visual Representation)**

✅ **Vectors ko ek arrow (teer) se represent kiya jata hai:**

- **Arrow ki length** → Vector ka **magnitude** batati hai.
- **Arrow ki direction** → Vector kis disha me ja raha hai, yeh batata hai.

✅ **Example:**
Agar ek vector **v = [3,4]** diya hai, toh iska matlab:
➡️ **Origin (0,0) se ek arrow (vector) point (3,4) tak ja raha hai**
➡️ Yeh ek **2D plane** me position batata hai.

📌 **Graphically yeh aise dikhai deta hai:**
📍 **(0,0) → (3,4) (Arrow Pointing Towards [3,4])**

🔹 **Examples of Vectors (Vectors ke Real-Life Examples)**

1️⃣ Velocity (Raftaar aur Disha)

- Velocity ek **vector quantity** hai kyunki yeh **speed ke saath direction bhi batata hai**.
- **Example:** "Ek car **60 km/h East** ja rahi hai."
    - **Speed (60 km/h) = Magnitude**

- **East = Direction**

2️⃣ Force (Bal)

- Jab hum ek object ko **push ya pull** karte hain, toh usme ek **force lagta hai** jo **direction aur magnitude** dono specify karta hai.
- **Example:** Koi object **10 Newton force se right direction me move ho raha hai.**

3️⃣ Position (Sthiti ya Location)

- Kisi bhi object ki **position specify karne ke liye coordinates ka use hota hai**, jo ek **vector** represent karta hai.
- **Example:**
  - Agar ek point **P = (2,5,7)** diya hai, toh yeh ek **3D space me position** batata hai.

🔹 **Conclusion:**

Vectors **magnitude aur direction** dono ko represent karte hain. Yeh **Physics, Engineering, aur Data Science** me **bahut important role** play karte hain.

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Examples of Scalar and Vector

🔹 **Scalar Examples (Magnitude Only, No Direction)**

✅ Scalar quantities sirf **magnitude** batati hain, **direction nahi** hota.

1️⃣ **Speed (Raftaar)** → **60 km/h** (Direction specify nahi kiya)
2️⃣ **Mass (Vajan)** → **10 kg** (Sirf wajan hai, direction nahi)
3️⃣ **Volume (Aayatan)** → **2 liters** (Sirf kitna space cover kiya, direction nahi)
4️⃣ **Time (Samay)** → **5 seconds** (Bas duration hai, koi disha nahi)

🔹 **Vector Examples (Magnitude + Direction)**

✅ Vectors **magnitude ke saath direction bhi specify** karte hain.

1️⃣ **Velocity (Raftaar + Disha)** → **60 km/h East** (Speed ke saath direction bhi)
2️⃣ **Weight (Bhar ya Taakat)** → **500 N downward** (Gravity ke karan neeche ki taraf force)
3️⃣ **Friction (Gharshan Bal)** → **Opposite direction me lagta hai** (Jab ek object move karta hai)
4️⃣ **Force (Bal)** → **Ek object par 10 N ka force Right me lag raha hai**

✅ **Simple Rule:**

- **Sirf number hai, toh Scalar!**
- **Number + Direction hai, toh Vector!** 🚀

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Differences Between Scalar and Vector Quantities

Feature	**Scalar Quantity (Keval Magnitude)**	**Vector Quantity (Magnitude + Direction)**
Definition	Sirf **magnitude** hota hai.	**Magnitude + Direction** dono hote hain.
Dimensions	Sirf **1D (one dimension)** me exist karta hai.	**1D, 2D ya 3D kisi bhi dimension** me ho sakta hai.
Change Effect	Sirf **magnitude change** hota hai.	**Magnitude ya direction, dono change ho sakte hain.**
Resolution	Components me **nahi toda ja sakta**.	**Components me toda ja sakta hai** using trigonometric functions.
Mathematical Operations	Operations ka result **hamesha scalar** hota hai.	Operations ka result **scalar ya vector dono ho sakta hai** (Dot product → Scalar, Cross product → Vector).

Feature	Scalar Quantity (Keval Magnitude)	Vector Quantity (Magnitude + Direction)
Examples	Mass, Speed, Time, Area, Volume.	Velocity, Force, Displacement, Acceleration.

✅ **Short Trick to Remember:**

- **Sirf number → Scalar**
- **Number + Direction → Vector** 🚀

Slide-18

Lab

Slide-19

Vector Operations (Vector par Kiya Jane Wale Calculation)

Vectors par **direct arithmetic operations** (jaise normal numbers ka addition ya division) **possible nahi** hote. Isliye, vectors ke saath kaam karne ke liye **special vector operations** use kiye jate hain.

- 🔹 **1. Vector Addition (Vectors ko jodna)**

✓ **Do ya zyada vectors ko combine** karne ke liye use hota hai.
✓ Example:
A = (2, 3)
B = (4, 1)
A + B = (2+4, 3+1) = (6, 4)

🔹 2. Vector Subtraction (Vectors ka antar nikalna)

✓ **Do vectors ke beech difference** nikalne ke liye use hota hai.
✓ Example:
A = (5, 7)
B = (2, 3)
A - B = (5-2, 7-3) = (3, 4)

🔹 3. Scalar Multiplication (Vector ko Scalar se Multiply karna)

✓ **Ek vector ko kisi scalar (single number) se multiply karna.**
✓ Example:
Vector A = (3, 5)
Scalar k = 2
k * A = (2 × 3, 2 × 5) = (6, 10)

🔹 4. Product of Two Vectors (Do Vectors ka Guna Karna)

Vectors ka division **nahi hota**, lekin **multiplication do tarike se ho sakta hai**:

✅ **Dot Product (•) → Scalar Output**

✓ **Result hamesha ek scalar (number) hota hai.**

✓ **Formula:** $A \bullet B = |A|\,|B| \cos(\theta)$

✓ Example:

$A = (2, 3), B = (4, 1)$

$A \bullet B = (2 \times 4) + (3 \times 1) = 8 + 3 =$ **11**

✅ **Cross Product (×) → Vector Output**

✓ **Result ek naya vector hota hai jo dono input vectors ke perpendicular hota hai.**

✓ **Formula:** $A \times B = |A|\,|B| \sin(\theta)$

✓ Example:

$A = (i, j, k) = (1, 2, 3)$

$B = (i, j, k) = (4, 5, 6)$

$A \times B$ ka result ek **naya vector** hoga.

✅ **Short Trick to Remember:**

- **Dot Product → Scalar Output**
- **Cross Product → Vector Output** 🚀

Slide-19

Vector Addition: Triangle Law of Vector Addition

📌 **Concept:**

Agar **do vectors** ko **ek triangle ke do sides** ke roop me represent kiya jaye, toh **unka sum (resultant vector)** triangle ki **teesri side** hoti hai.

✅ **Triangle Law ke Mutabik:**
Agar **do vectors** ko **sequence me rakha jaye**, toh unka **sum ek naya vector** hoga jo **first vector ke start point se second vector ke end point tak jayega.**

🔹 **Triangle Law Explanation with Figure:**

1️⃣ **Vector "a"** ko **AB line** represent karti hai.
2️⃣ **Vector "b"** ko **BC line** represent karti hai.
3️⃣ **Resultant Vector (a + b)** ko **AC line** represent karti hai.
4️⃣ **AC ka direction** hamesha **A se C ki taraf hota hai**.

✅ **Magnitude (Length) of Resultant Vector:**
|R|=|a|2+|b|2+2|a||b|cos⁡(θ)|R| = \sqrt{|a|^2 + |b|^2 + 2 |a| |b| \cos(\theta)}|R|=|a|2+|b|2+2|a||b|cos(θ)

✅ **Yahan θ\thetaθ do vectors ke beech ka angle hota hai.**

🔹 **Simple Example:**

Maan lo, ek aadmi **east ki taraf 5 km chalta hai (Vector A)** aur fir **north ki taraf 12 km chalta hai (Vector B)**.

✓ Triangle Law ke hisaab se, **aadmi ka total displacement ek diagonal line (hypotenuse) hoga:**

R=52+122=25+144=169=13kmR = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13 \text{ km}R=52+122=25+144=169 =13km

📌 **Toh aadmi ka net displacement 13 km hoga, jo directly start point se end point tak jata hai.**

✅ **Short Trick to Remember:**

- **Triangle ke 2 sides → Vectors**
- **Triangle ka 3rd side → Resultant Vector** 🚀

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