**Learning Objective (Jo Hum Is Chapter Mein Seekhenge)**

Is chapter mein hum **Linear Algebra** ke basic concepts aur **Data Science** mein uska role samajhne wale hain. Hum **scalars, vectors, aur matrices** ke beech ka difference jaanenge aur matrices par important operations perform karna seekhenge. Specifically, hum yeh seekhenge:

✅ **Linear Algebra ke fundamentals aur Data Science mein uski importance**  
✅ **Scalars, Vectors, aur Matrices ka difference aur unki properties**  
✅ **Matrices ke operations** jaise:

* Addition (जोड़)
* Multiplication (गुणा)
* Transposition (पलटना)
* Determinant nikalna
* Inverse aur Rank calculate karna

**Linear Algebra Kya Hai?**

**Definition (Paribhasha)**

Linear Algebra ek **important branch** hai **mathematics** ka, jo mainly **vectors, vector spaces, aur linear transformations** ka study karta hai. Yeh ek **powerful tool** hai jo hume **linear equations solve** karne, **linear functions samajhne**, aur **inhe matrices aur determinants ke through represent karne** mein madad karta hai.

Iska **sirf theoretical mathematics** tak hi use nahi hai, balki **bahut saare fields** mein iska **broad application** hai, jaise:  
✅ **Machine Learning**  
✅ **Computer Graphics**  
✅ **Data Science**  
✅ **Physics & Engineering**

**Simple Example**

Agar aapke paas ek **linear equation** hai:

2x+3y=52x + 3y = 52x+3y=5 x−y=2x - y = 2x−y=2

Toh Linear Algebra ke tools ka use karke hum **inhe easily solve** kar sakte hain. Yeh **matrix aur vector representation** ko use karta hai jo complex calculations ko **simple aur fast** banata hai.

Slide-8

**Common Linear Algebra Symbols (Aam Linear Algebra Symbols aur Unka Matlab)**

Linear Algebra mein **different symbols** ka use hota hai jo **mathematical operations** ko represent karte hain. Neeche kuch important symbols aur unke meanings diye gaye hain:

| **Symbol Name** | **Matlab / Usage** |
| --- | --- |
| **⋅ (Dot Product)** | Do vectors ka **scalar product** (Ek number milega result mein) |
| **× (Cross Product)** | Do vectors ka **vector product** jo dono vectors ke **orthogonal** hoga |
| **ATA^TAT (Transpose of Matrix A)** | **Matrix ke rows ko columns aur columns ko rows** mein convert karna |
| **ϕ (Null Set)** | Aisa **set jismein koi bhi element nahi hota** |
| **A−1A^{-1}A−1 (Inverse of Matrix A)** | Ek aisi **matrix jo original matrix ke saath multiply ho kar Identity Matrix de** |
| \*\*( | A |
| **ϱ(A)ϱ(A)ϱ(A) (Matrix Rank)** | Matrix ke **linearly independent rows ya columns ka count** |
| \*\*( |  |
| **λλλ (Eigenvalue)** | Ek **scalar jo batata hai ki Linear Transformation vectors ko kaise scale karega** |
| **Eigenvector** | **Ek non-zero vector jo sirf scale hota hai, direction change nahi karta** |

Slide-9

**Role of Linear Algebra in Data Science (Data Science mein Linear Algebra ka Role)**

Linear Algebra **Data Science ka backbone** hai. Har jagah **data ko represent karna, analyze karna aur process karna** matrices aur vectors ke through hota hai. Yeh **Machine Learning, NLP, aur Recommender Systems** jaise advanced areas mein bhi use hota hai.

**1️⃣ Data Representation (Data ko Represent Karna)**

* **Data ko store aur manipulate** karne ke liye **matrices ka use** hota hai.
* **Example:** Agar ek company ke **1000 customers aur 50 products** hain, toh unka data **1000 × 50 matrix** mein store ho sakta hai.

**2️⃣ Machine Learning Algorithms (ML Algorithms mein Istemaal)**

* **Support Vector Machines (SVM), Neural Networks** jaise ML models **matrix operations** ka use karte hain.
* **Example:** Neural Networks **weight matrices aur activation functions** ka use karke predictions karta hai.

**3️⃣ Dimensionality Reduction (Data ka Size Kam Karna - PCA Technique)**

* **Principal Component Analysis (PCA)** jaise techniques **linear algebra ka use** karke **features reduce** karti hain.
* **Example:** Agar ek dataset **1000 features** ka hai, toh PCA **sirf important features** ko select karke **data ko chhota aur fast banata hai**.

**4️⃣ Recommender Systems (Netflix, Amazon Recommendation Engines)**

* **Matrix Factorization** methods jaise **Singular Value Decomposition (SVD)** ka use **recommendations generate** karne ke liye hota hai.
* **Example:** Netflix aur Amazon **past user preferences ke basis par naye movies/products recommend karte hain** using **matrix operations**.

**5️⃣ Natural Language Processing (NLP - Language Processing mein Role)**

* **Words aur documents** ko **vectors aur matrices** mein represent kiya jata hai.
* **Example:**
  + **Word Embeddings (Word2Vec, GloVe)**: Har word ko **vector format** mein convert karte hain.
  + **Document-Term Matrix (DTM)**: Ek matrix **jo batata hai ki kisi document mein kaunse words kitni baar aaye hain**.

Slide-10

**Branches of Linear Algebra (Linear Algebra ke Vibhaag)**

Linear Algebra ko **3 main branches** mein divide kiya gaya hai, jo **basic se lekar advanced aur practical applications** tak cover karte hain.

**1️⃣ Elementary Linear Algebra (बुनियादी Linear Algebra)**

🔹 **Basic Concepts**: Is branch mein **fundamental concepts** cover kiye jate hain jaise:  
✅ **Scalars, Vectors, Matrices**  
✅ **Matrix Addition, Multiplication, Transposition**  
✅ **Linear Equations ka solution using Matrices**

**📌 Example:**  
Agar ek **matrix A** hai:

A=[1234]A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}A=[13​24​]

Toh hum **iska transpose** ATA^TAT easily nikal sakte hain:

AT=[1324]A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}AT=[12​34​]

**2️⃣ Advanced Linear Algebra (Advanced Level Concepts)**

🔹 Yeh **complex topics** ko cover karta hai jaise:  
✅ **Eigenvalues & Eigenvectors**  
✅ **Vector Spaces aur Subspaces**  
✅ **Linear Transformations aur their properties**

**📌 Example:**  
Agar ek **linear transformation T(x)T(x)T(x)** diya ho, toh **Eigenvalue (λ) aur Eigenvector (v) nikalne** ke liye equation hoti hai:

Av=λvA v = λ vAv=λv

Jisme **λ ek scalar hota hai jo transformation ka effect batata hai**, aur **v ek special vector hota hai jo sirf scale hota hai, direction change nahi karta**.

**3️⃣ Applied Linear Algebra (Practical Implementation & Applications)**

🔹 Yeh **Linear Algebra ke real-world applications** par focus karta hai, jaise:  
✅ **Machine Learning aur Deep Learning**  
✅ **Matrix Factorization (Singular Value Decomposition - SVD)**  
✅ **Computer Graphics aur Image Processing**

**📌 Example:**

* **Google Search Algorithm**: Web pages ki ranking **Eigenvalues aur Eigenvectors** ka use karke hoti hai.
* **Face Recognition Systems**: **Principal Component Analysis (PCA)** ka use karke face detection hoti hai.

**🔹 Conclusion:**

Linear Algebra **ek important subject** hai jo **basic se lekar advanced aur practical applications** tak use hoti hai. **Data Science, AI, Physics, aur Engineering** jaise fields mein iska **bahut bada role** hai! 🚀

Slide-11

**Industrial Applications of Linear Algebra (Linear Algebra ka Vyavsayik Upyog)**

Linear Algebra **industry ke kai important areas** mein use hoti hai, jaise **Finance aur Computer Vision**. Chaliye inhe detail mein samajhte hain.

**1️⃣ Finance (Vitta aur Investment Sector)**

🔹 **Investors aur financial analysts** Linear Algebra ka use **risk aur return ko balance** karne ke liye karte hain.  
🔹 **Covariance Analysis aur Linear Equations** ka use **best asset allocation** decide karne mein hota hai.

✅ **Example:**  
Agar ek investor **2 assets (Stock A aur Stock B)** mein invest kar raha hai, toh uska **portfolio optimization** Linear Algebra se hota hai:

Portfolio Risk=wA2σA2+wB2σB2+2wAwBσAB\text{Portfolio Risk} = w\_A^2 \sigma\_A^2 + w\_B^2 \sigma\_B^2 + 2 w\_A w\_B \sigma\_{AB}Portfolio Risk=wA2​σA2​+wB2​σB2​+2wA​wB​σAB​

Jisme:

* wA,wBw\_A, w\_BwA​,wB​ = investment weights
* σA,σB\sigma\_A, \sigma\_BσA​,σB​ = individual stock volatility
* σAB\sigma\_{AB}σAB​ = covariance (dono stocks ka risk correlation)

🔹 **Isse investors yeh decide kar sakte hain ki kitna paisa kis asset mein lagana chahiye!** 📈

**2️⃣ Computer Vision (Image Processing aur AI Vision)**

🔹 **Linear Algebra ke powerful matrix operations** ka use **image processing techniques** jaise:  
✅ **Image Compression (JPEG, PNG Formats)**  
✅ **Image Enhancement (Brightness aur Contrast Improve karna)**  
✅ **Edge Detection (Face aur Object Recognition Systems)**

✅ **Example - Edge Detection (Convolution Technique)**  
Image ko **matrix ke form mein represent** kiya jata hai aur **convolution operation** se **edges detect** kiye jate hain.

Agar ek **3×3 filter matrix (Kernel)** diya jaye:

K=[−1−1−1−18−1−1−1−1]K = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}K=​−1−1−1​−18−1​−1−1−1​​

Toh isko **image matrix par apply** karke **sharp edges detect** kiye ja sakte hain.

🔹 **Iska use Face Recognition, Object Detection, aur Medical Imaging jaisi fields mein hota hai!** 🖼️👁️

**🔹 Conclusion:**

Linear Algebra **finance aur computer vision jaise industrial fields** mein **essential role play** karti hai. **Risk optimization, investment planning, aur AI-based image processing** sab Linear Algebra ke bina possible nahi hote! 🚀

Slide-12

**Industrial Applications of Linear Algebra (Linear Algebra ka Vyavsayik Upyog - Part 2)**

Linear Algebra **Robotics, Engineering, Big Data, aur AI** jaise advanced fields mein bhi use hoti hai. Chaliye inhe detail mein samajhte hain.

**1️⃣ Robotics and Engineering (Robotics aur Engineering mein Linear Algebra ka Use)**

🔹 **Robots ka movement aur control** Linear Algebra se model kiya jata hai.  
🔹 **State-Space Representation** ka use system behavior **predict aur optimize** karne ke liye hota hai.

✅ **Example - Robot Arm Movement**  
Agar ek **robotic arm** ko ek specific position par move karna ho, toh uske motion ko **matrix equations** se solve kiya jata hai:

Ax=BAx = BAx=B

Jisme:

* AAA = **Transformation Matrix** (Rotation, Scaling, Translation)
* xxx = **Robot ke joint angles ka vector**
* BBB = **Target Position ka vector**

🔹 **Yeh technique autonomous robots aur industrial automation mein bahut use hoti hai!** 🤖⚙️

**2️⃣ Big Data and Artificial Intelligence (AI)**

🔹 **Data analysis aur AI models** mein Linear Algebra **core role** play karti hai.  
🔹 **PCA (Principal Component Analysis)** ka use **data dimension reduce** karne aur **important patterns detect** karne ke liye hota hai.

✅ **Example - PCA in Big Data**  
Maan lijiye ek dataset hai jisme **1000 features hain**, toh **ML algorithms slow ho sakte hain**. PCA ka use karke **sabse important features select** kiye jate hain, jisse **data fast process ho** aur **accuracy bhi bani rahe**.

**Step-by-Step PCA Working:**  
1️⃣ Data ko **standardize** karo (mean = 0, variance = 1).  
2️⃣ **Covariance matrix** banao.  
3️⃣ **Eigenvalues aur Eigenvectors** calculate karo.  
4️⃣ **Top k Eigenvectors** ko choose karke **new reduced dataset** banao.

🔹 **PCA ka use Face Recognition, Fraud Detection, aur Stock Market Analysis jaise areas mein hota hai!** 📊📉

**🔹 Conclusion:**

Linear Algebra **Robotics, Engineering, Big Data aur AI** jaise modern applications mein **essential tool** hai. **Autonomous robots, industrial automation aur data-driven AI models** Linear Algebra ke bina impossible hote! 🚀

Slide-13

**Real-Life Applications of Linear Algebra (Linear Algebra ka Rozmarra Jeevan me Upyog)**

Linear Algebra sirf theoretical concept nahi hai, balki **bahut saari real-world applications** mein iska **major role** hai. Chaliye kuch **important examples** ko detail mein samajhte hain.

**1️⃣ Predicting House Prices (Ghar ke Prices ka Prediction) 🏠**

🔹 **House price prediction** ke liye **feature vectors** ka use hota hai, jaise:  
✅ **Square footage (kitna bada hai ghar?)**  
✅ **Number of bedrooms (kitne kamre hain?)**  
✅ **Locality Score (area kitna acha hai?)**

➡️ Yeh sab **Linear Regression Model** ke andar ek **Matrix Equation** me represent kiya jata hai:

y^=WX+b\hat{y} = W X + by^​=WX+b

Jisme:

* XXX = **Feature matrix** (ghar ke features)
* WWW = **Weight vector** (kitna impact hai har feature ka)
* bbb = **Bias term**
* y^\hat{y}y^​ = **Predicted house price**

🔹 **Yeh method real estate companies aur banks use karte hain loan approval ke liye!** 🏡📈

**2️⃣ Image Compression (Tasveeron ka Size Kam Karna) 🖼️**

🔹 **High-resolution images ka size reduce** karne ke liye **Singular Value Decomposition (SVD)** ka use hota hai.  
🔹 **Bina quality lose kiye** image compress hoti hai.

✅ **Example:**  
Agar ek **1000x1000 pixels ki image** ko compress karna ho, toh **SVD algorithm** use karke isko **chhoti size wali matrix mein convert** kiya jata hai.

➡️ Formula:

A=USVTA = U S V^TA=USVT

Jisme:

* AAA = **Original Image Matrix**
* U,S,VTU, S, V^TU,S,VT = **Compressed matrices**

🔹 **JPEG format aur video streaming platforms (Netflix, YouTube) yahi technique use karte hain!** 🎥

**3️⃣ Stock Market Prediction (Share Bazaar ka Analysis) 📊📉**

🔹 **Covariance Matrices** ka use **multiple stocks ke trend aur correlation ko analyze** karne ke liye hota hai.  
🔹 **High-risk aur low-risk stocks ko identify** karne ke liye **investment strategies** me Linear Algebra ka use hota hai.

✅ **Example:**  
Agar **Stock A aur Stock B** ek-dusre se related hain, toh unka **covariance positive hoga**, warna **negative hoga**.

🔹 **Mutual Funds aur Portfolio Management ke liye yeh technique important hai!** 📈💰

**4️⃣ Movie Recommendation Systems (Netflix/Amazon me Suggestions kaise milte hain?) 🎬🍿**

🔹 **Matrix Factorization** ka use **movie recommendation** systems me hota hai.  
🔹 **Netflix, Amazon, aur Spotify jaise platforms yeh technique use karte hain!**

✅ **Example - User Preference Prediction using SVD**  
Maan lo ek **user ke pasand na pasand ka matrix** diya hai:

| **User** | **Movie 1** | **Movie 2** | **Movie 3** | **Movie 4** |
| --- | --- | --- | --- | --- |
| A | 5 | ? | 3 | ? |
| B | ? | 4 | ? | 5 |

➡️ **"?" wale missing ratings ko predict karne ke liye** **SVD algorithm** ka use hota hai!  
🔹 **Spotify aur YouTube bhi isi concept par song/video recommendations dete hain!** 🎶

**5️⃣ Self-Driving Cars (Khud chalne wali Gaadiyan - Tesla, Waymo) 🚗🤖**

🔹 **Image Recognition, Object Detection, aur Decision-Making** ke liye **Linear Algebra ke transformations** use hote hain.

✅ **Example - Camera ka Input Process Karna**  
Agar ek **self-driving car** ko samne **pedestrian ya traffic light** dikh rahi hai, toh uska data **matrix transformation** se process hota hai:

➡️ **Image Matrix → Edge Detection → Object Recognition → Decision Making**

🔹 **Tesla, Google Waymo, aur Uber Automated Vehicles Linear Algebra ka use karte hain!** 🚦

**🔹 Conclusion:**

Linear Algebra **har industry me use hoti hai**, chahe **real estate, finance, AI, robotics ya entertainment ho**. **Data Science aur AI ke bina aaj ke modern applications possible nahi hain!** 🚀

Slide-14

**Scalars: Definition and Representation (Scalars Kya Hote Hain?)**

**🔹 Definition (Paribhasha)**

Scalar ek **single numerical value** hota hai jo **sirf magnitude (maatra)** batata hai, **direction (disha) nahi**.  
🔹 Scalar quantities **simple mathematical operations** (addition, subtraction, multiplication, division) ko follow karti hain.

✅ **Example:**  
Agar **temperature 20°C** hai, toh yeh **sirf ek numerical value hai** aur iski koi **direction nahi** hai.

**🔹 Representation of Scalars (Scalars ko kaise dikhaya jata hai?)**

✅ Scalars ko **chhoti (lowercase) letters** se represent kiya jata hai, jaise:

a,b,c,Ta, b, c, Ta,b,c,T

✅ **Example:**

* **Temperature** → T=20°CT = 20°CT=20°C
* **Speed** → s=50s = 50s=50 km/h (direction nahi bataya gaya)
* **Height** → h=170h = 170h=170 cm

**🔹 Examples of Scalar Quantities (Scalars ke Real-Life Examples)**

1️⃣ **Mass (Vajan)** → **10 kg** → Sirf weight batata hai, koi direction nahi.  
2️⃣ **Time (Samay)** → **5 seconds** → Sirf ek number hai, direction nahi.  
3️⃣ **Distance (Doori)** → **100 meters** → Kitni doori cover ki gayi, lekin kis direction me, yeh nahi batata.  
4️⃣ **Temperature (Taapmaan)** → **30°C** → Yeh sirf **garmi ya thandi batata hai**, koi direction nahi.

✅ **Scalar ka Simple Rule:** Agar koi value **direction specify nahi karti** toh woh **scalar quantity hoti hai!** 🎯

**🔹 Conclusion:**

Scalars **simple numerical values** hote hain jo **magnitude specify** karte hain **lekin direction nahi** dete.  
Ab hum **Vectors** ke baare mein detail mein samjhenge, jo **magnitude ke saath direction bhi show karte hain!** 🚀

Slide-15

**Vectors: Definition, Operations, and Interpretations**

**🔹 Definition (Paribhasha)**

Vector ek **ordered collection of numbers** hota hai jo **magnitude (maatra) aur direction (disha)** dono ko represent karta hai.

✅ **Scalar aur Vector ka Basic Difference:**

* **Scalar** → Sirf **magnitude** hota hai, koi **direction nahi** (e.g., Mass, Time, Temperature).
* **Vector** → **Magnitude + Direction** dono hoti hain (e.g., Velocity, Force, Position).

**🔹 Geometric Interpretation (Vector ka Visual Representation)**

✅ **Vectors ko ek arrow (teer) se represent kiya jata hai:**

* **Arrow ki length** → Vector ka **magnitude** batati hai.
* **Arrow ki direction** → Vector kis disha me ja raha hai, yeh batata hai.

✅ **Example:**  
Agar ek vector **v = [3,4]** diya hai, toh iska matlab:  
➡️ **Origin (0,0) se ek arrow (vector) point (3,4) tak ja raha hai**  
➡️ Yeh ek **2D plane** me position batata hai.

📌 **Graphically yeh aise dikhai deta hai:**  
📍 **(0,0) → (3,4) (Arrow Pointing Towards [3,4])**

**🔹 Examples of Vectors (Vectors ke Real-Life Examples)**

1️⃣ **Velocity (Raftaar aur Disha)**

* Velocity ek **vector quantity** hai kyunki yeh **speed ke saath direction bhi batata hai**.
* **Example:** "Ek car **60 km/h East** ja rahi hai."
  + **Speed (60 km/h) = Magnitude**
  + **East = Direction**

2️⃣ **Force (Bal)**

* Jab hum ek object ko **push ya pull** karte hain, toh usme ek **force lagta hai** jo **direction aur magnitude** dono specify karta hai.
* **Example:** Koi object **10 Newton force se right direction me move ho raha hai.**

3️⃣ **Position (Sthiti ya Location)**

* Kisi bhi object ki **position specify karne ke liye coordinates ka use hota hai**, jo ek **vector** represent karta hai.
* **Example:**
  + Agar ek point **P = (2,5,7)** diya hai, toh yeh ek **3D space me position** batata hai.

**🔹 Conclusion:**

Vectors **magnitude aur direction** dono ko represent karte hain. Yeh **Physics, Engineering, aur Data Science** me **bahut important role** play karte hain.

Slide-16

**Examples of Scalar and Vector**

**🔹 Scalar Examples (Magnitude Only, No Direction)**

✅ Scalar quantities sirf **magnitude** batati hain, **direction nahi** hota.

1️⃣ **Speed (Raftaar)** → **60 km/h** (Direction specify nahi kiya)  
2️⃣ **Mass (Vajan)** → **10 kg** (Sirf wajan hai, direction nahi)  
3️⃣ **Volume (Aayatan)** → **2 liters** (Sirf kitna space cover kiya, direction nahi)  
4️⃣ **Time (Samay)** → **5 seconds** (Bas duration hai, koi disha nahi)

**🔹 Vector Examples (Magnitude + Direction)**

✅ Vectors **magnitude ke saath direction bhi specify** karte hain.

1️⃣ **Velocity (Raftaar + Disha)** → **60 km/h East** (Speed ke saath direction bhi)  
2️⃣ **Weight (Bhar ya Taakat)** → **500 N downward** (Gravity ke karan neeche ki taraf force)  
3️⃣ **Friction (Gharshan Bal)** → **Opposite direction me lagta hai** (Jab ek object move karta hai)  
4️⃣ **Force (Bal)** → **Ek object par 10 N ka force Right me lag raha hai**

✅ **Simple Rule:**

* **Sirf number hai, toh Scalar!**
* **Number + Direction hai, toh Vector!** 🚀

Slide-17

**Differences Between Scalar and Vector Quantities**

| **Feature** | **Scalar Quantity (Keval Magnitude)** | **Vector Quantity (Magnitude + Direction)** |
| --- | --- | --- |
| **Definition** | Sirf **magnitude** hota hai. | **Magnitude + Direction** dono hote hain. |
| **Dimensions** | Sirf **1D (one dimension)** me exist karta hai. | **1D, 2D ya 3D kisi bhi dimension** me ho sakta hai. |
| **Change Effect** | Sirf **magnitude change** hota hai. | **Magnitude ya direction, dono change ho sakte hain.** |
| **Resolution** | Components me **nahi toda ja sakta**. | **Components me toda ja sakta hai** using trigonometric functions. |
| **Mathematical Operations** | Operations ka result **hamesha scalar** hota hai. | Operations ka result **scalar ya vector dono ho sakta hai** (Dot product → Scalar, Cross product → Vector). |
| **Examples** | **Mass, Speed, Time, Area, Volume.** | **Velocity, Force, Displacement, Acceleration.** |

✅ **Short Trick to Remember:**

* **Sirf number → Scalar**
* **Number + Direction → Vector** 🚀

Slide-18

Lab

Slide-19

**Vector Operations (Vector par Kiya Jane Wale Calculation)**

Vectors par **direct arithmetic operations** (jaise normal numbers ka addition ya division) **possible nahi** hote. Isliye, vectors ke saath kaam karne ke liye **special vector operations** use kiye jate hain.

**🔹 1. Vector Addition (Vectors ko jodna)**

✔ **Do ya zyada vectors ko combine** karne ke liye use hota hai.  
✔ Example:  
A = (2, 3)  
B = (4, 1)  
A + B = (2+4, 3+1) = (6, 4)

**🔹 2. Vector Subtraction (Vectors ka antar nikalna)**

✔ **Do vectors ke beech difference** nikalne ke liye use hota hai.  
✔ Example:  
A = (5, 7)  
B = (2, 3)  
A - B = (5-2, 7-3) = (3, 4)

**🔹 3. Scalar Multiplication (Vector ko Scalar se Multiply karna)**

✔ **Ek vector ko kisi scalar (single number) se multiply karna.**  
✔ Example:  
Vector A = (3, 5)  
Scalar k = 2  
k \* A = (2 × 3, 2 × 5) = (6, 10)

**🔹 4. Product of Two Vectors (Do Vectors ka Guna Karna)**

Vectors ka division **nahi hota**, lekin **multiplication do tarike se ho sakta hai**:

**✅ Dot Product (•) → Scalar Output**

✔ **Result hamesha ek scalar (number) hota hai.**  
✔ **Formula:** A • B = |A| |B| cos(θ)  
✔ Example:  
A = (2, 3), B = (4, 1)  
A • B = (2 × 4) + (3 × 1) = 8 + 3 = **11**

**✅ Cross Product (×) → Vector Output**

✔ **Result ek naya vector hota hai jo dono input vectors ke perpendicular hota hai.**  
✔ **Formula:** A × B = |A| |B| sin(θ)  
✔ Example:  
A = (i, j, k) = (1, 2, 3)  
B = (i, j, k) = (4, 5, 6)  
A × B ka result ek **naya vector** hoga.

✅ **Short Trick to Remember:**

* **Dot Product → Scalar Output**
* **Cross Product → Vector Output** 🚀

Slide-19

**Vector Addition: Triangle Law of Vector Addition**

**📌 Concept:**

Agar **do vectors** ko **ek triangle ke do sides** ke roop me represent kiya jaye, toh **unka sum (resultant vector)** triangle ki **teesri side** hoti hai.

✅ **Triangle Law ke Mutabik:**  
Agar **do vectors** ko **sequence me rakha jaye**, toh unka **sum ek naya vector** hoga jo **first vector ke start point se second vector ke end point tak jayega.**

**🔹 Triangle Law Explanation with Figure:**

1️⃣ **Vector "a"** ko **AB line** represent karti hai.  
2️⃣ **Vector "b"** ko **BC line** represent karti hai.  
3️⃣ **Resultant Vector (a + b)** ko **AC line** represent karti hai.  
4️⃣ **AC ka direction** hamesha **A se C ki taraf hota hai**.

✅ **Magnitude (Length) of Resultant Vector:**  
∣R∣=∣a∣2+∣b∣2+2∣a∣∣b∣cos⁡(θ)|R| = \sqrt{|a|^2 + |b|^2 + 2 |a| |b| \cos(\theta)}∣R∣=∣a∣2+∣b∣2+2∣a∣∣b∣cos(θ)​

✅ **Yahan θ\thetaθ do vectors ke beech ka angle hota hai.**

**🔹 Simple Example:**

Maan lo, ek aadmi **east ki taraf 5 km chalta hai (Vector A)** aur fir **north ki taraf 12 km chalta hai (Vector B)**.

✔ Triangle Law ke hisaab se, **aadmi ka total displacement ek diagonal line (hypotenuse) hoga:**

R=52+122=25+144=169=13kmR = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13 kmR=52+122​=25+144​=169​=13km

📌 **Toh aadmi ka net displacement 13 km hoga, jo directly start point se end point tak jata hai.**

✅ **Short Trick to Remember:**

* **Triangle ke 2 sides → Vectors**
* **Triangle ka 3rd side → Resultant Vector** 🚀

Slide-20