

(6A)

(NN)

Machine Learning-3

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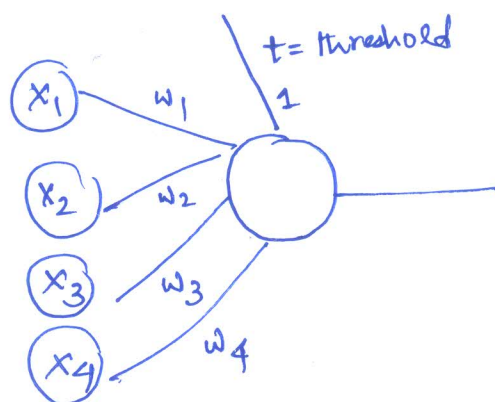
(1)

Consider the 2-class classification task that consists of the following:

$$C_1: [-1, -1], [-1, 1], [1, -1]$$

$$C_2: [1, 1]$$

Find the decision boundary b/w the two classes C_1 & C_2 using a single perceptron.



Options given:

$$(i) \quad x_1 - x_2 - 0.5 = 0$$

$$(ii) \quad -x_1 + x_2 - 0.5 = 0$$

$$(iii) \quad 0.5(x_1 + x_2) - 1.5 = 0$$

$$(iv) \quad x_1 + x_2 - 0.5 = 0$$

$$(i) \quad \underbrace{[1 \ -1 \ -0.5]}_{w_i} ; \underbrace{[-1, -1, 1]}_x \quad \leftarrow \text{bias}$$

$$[1, -1, -0.5] \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = -1 + 1 - 0.5 < 0$$

$$[1, -1, -0.5] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = -1 - 1 - 0.5 < 0 \Rightarrow \text{not working}$$

$$[1, -1, -0.5] \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 1 - 1 - 0.5 > 0$$

Simple test

 C_1 C_2

Linear separability

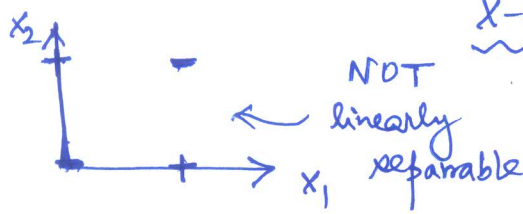
Consider the line $x + 2y = 4$
& the points $(0, 1), (1, 0), (1, 1)$
 $(3, 2), (4, 3), (0, 2)$

Plug in the points & check if;

$$x + 2y - 4 < 0$$

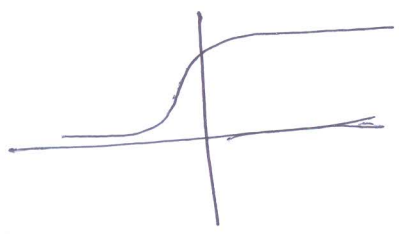
$$\& \ x + 2y - 4 > 0$$

X-OR example:

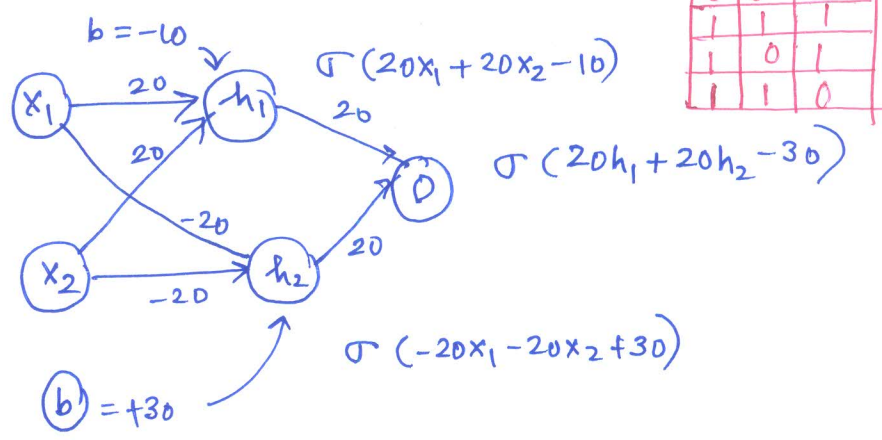


← Limitation of perceptron

Let's build a NN:



i/p



0	0	0
1	1	1
1	0	1
1	1	0

$(0,0): \begin{cases} O(h_1) = \sigma(20*0 + 20*0 - 10) \approx 0 \\ O(h_2) = \sigma(-20*0 - 20*0 + 30) \approx 1 \end{cases} \text{ hidden layers} \Rightarrow \sigma(20*0 + 20*1 - 30) \approx 0$

$(1,1): \begin{cases} h_1 = \sigma(20*1 + 20*1 - 10) \approx 1 \\ h_2 = \sigma(-20*1 + 20*1 + 30) \approx 0 \end{cases} \Rightarrow \sigma(20*1 + 20*0 - 30) \approx 0$

$(0,1): \begin{cases} h_1 = \sigma(20*0 + 20*1 - 10) \approx 1 \\ h_2 = \sigma(-20*0 + 20*1 + 30) \approx 1 \end{cases} \Rightarrow \sigma(20*1 + 20*1 - 30) \approx 1$

$(1,0): \begin{cases} h_1 = \sigma(20*1 + 20*0 - 10) \approx 1 \\ h_2 = \sigma(-20*1 + 20*0 + 30) \approx 1 \end{cases} \Rightarrow \sigma(20*1 + 20*1 - 30) \approx 1$

$\begin{matrix} h_1 \Rightarrow \text{logical OR} \\ h_2 \Rightarrow \end{matrix} \left. \vphantom{\begin{matrix} h_1 \\ h_2 \end{matrix}} \right] \text{logistic unit has an associated hyperplane.} \Rightarrow \text{two hyperplanes in space } W$

