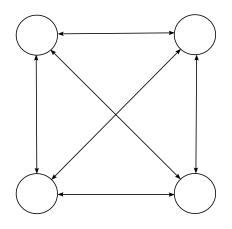
# 5. Sequence modeling with recurrent neural networks 5.1. Structure of the Elman Recursive Neural Network

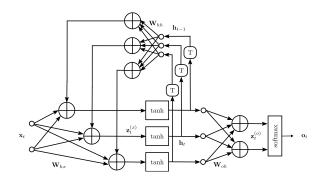
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## The Hopfield network



- Introduced in 1982 by John Hopfield.
- Structure previously introduced by W. A. Little in 1974.
- First to include the idea of recurrence.
- Used in optimization as the classic travelling salesman problem.
- Continuous state versions exist.

#### State equations



$$\mathbf{z}_{t}^{(x)} = \mathbf{W}_{hx}^{\top} \mathbf{x}_{t} + \mathbf{W}_{hh}^{\top} \mathbf{h}_{t-1} + \mathbf{b}_{h}$$

$$\mathbf{z}_{t}^{(o)} = \mathbf{W}_{oh}^{\top} \mathbf{h}_{t} + \mathbf{b}_{o}$$

$$\mathbf{h}_{t} = \tanh(\mathbf{z}_{t}^{(x)})$$

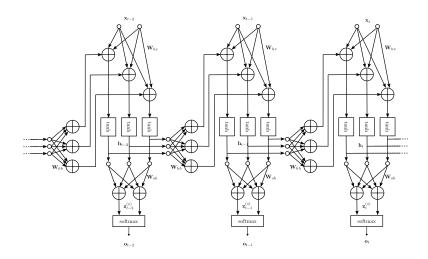
$$\mathbf{o}_{t} = \mathbf{o}(\mathbf{z}_{t}^{(o)}) = \operatorname{softmax}(\mathbf{z}_{t}^{(o)})$$
(1)

State equations

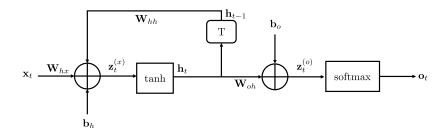
Introduced by J. L. Elman in 1990, which follows a similar structure by M. I. Jordan, 1986.

- ullet The input matrix  ${f W}_{HX}$  extracts features from the input sample.
- The output matrix  $\mathbf{W}_{oh}$  transforms the hidden state into an output response, through a softmax activation.
- The hidden state matrix  $\mathbf{W}_{hh}$  performs the feedback. It must store the dependencies between the past inputs and the present output.
- the hidden state itself is a summary of the past samples.
- The network is an infinite impulse response structure.

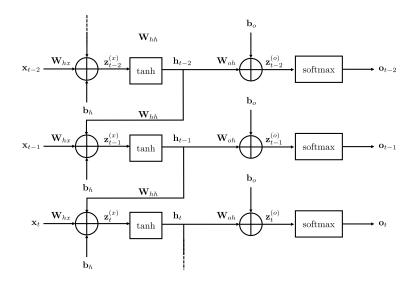
#### The Elman RNN unfolded



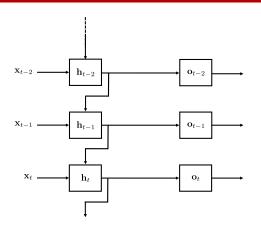
#### A more compact view of the RNN



#### Unfolded compact representation of the RNN



#### Unfolded even more compact representation



- Input  $\mathbf{x}_t$  is transformed with  $\mathbf{W}_{hx}$ ,
- Hidden state  $\mathbf{h}_{t-1}$  is transformed with  $\mathbf{W}_{hh}$  and bias  $\mathbf{b}_h$ .
- The output of this box, transformed with a tanh, is  $\mathbf{h}_t$ .