Asansol Engineering College

CA-2 Assignment

Name : Pampa Malakar
Dept. : CSBS Sem. : 3rd
Roll : 10831122030
Subject Name : Computational Statistics
Subject Code : BSC-301

Q.1. $A = \begin{bmatrix} 4 & 8 & 8 \\ 3 & 6 & -9 \end{bmatrix}$

(i) calculate AA' and obtain its eigen values and eigen vectors.

(ii) calculate $A'A$ and obtain its eigenvalues and eigenvectors check that the nonzero eigenvalues are the same as those in part a.

(iii) Obtain the singular value decomposition of A.

$\Rightarrow AA' = \begin{bmatrix} 4 & 8 & 8 \\ 3 & 6 & -9 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 8 & 6 \\ 8 & -9 \end{bmatrix} = \begin{bmatrix} 16+64+64 & 12+48-72 \\ 12+48-72 & 9+36+81 \end{bmatrix}$

$= \begin{bmatrix} 144 & -12 \\ -12 & 126 \end{bmatrix}$

$\therefore \quad AA' - \lambda I = 0$

or, $\begin{bmatrix} 144 & -12 \\ -12 & 126 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$ [where λ is the eigen values of AA']

or, $\begin{bmatrix} 144-\lambda & -12 \\ -12 & 126-\lambda \end{bmatrix} = 0 \quad \ldots (i)$

or, $(144-\lambda)(126-\lambda) - 144 = 0$

or, $\lambda^2 + 18144 - 144\lambda - 126\lambda - 144 = 0$

or, $\lambda(\lambda - 150) - 120(\lambda - 150) = 0$

or, $(\lambda - 150)(\lambda - 120) = 0$

or, $\lambda = 150 \quad | \quad \lambda = 120 \qquad \therefore \boxed{\lambda_1 = 150 \text{ and } \lambda_2 = 120}$

Let, the eigen vectors of AA' are $\tilde{e}_1$ and $\tilde{e}_2$

Let, put $\lambda_1 = 150$ in equ.(i)

$\therefore \begin{bmatrix} 144-150 & -12 \\ -12 & 126-150 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \lambda_1 \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$

or, $\begin{bmatrix} -6 & -12 \\ -12 & -24 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \cancel{\begin{bmatrix} 150\alpha_1 \\ 150\alpha_2 \end{bmatrix}} \; 0$

or, $\cancel{-6\alpha_1 - 12\alpha_2 = 150\alpha_1}$
$\cancel{-12\alpha_1 - 24\alpha_2 = 150\alpha_2}$

$$\Rightarrow \begin{array}{l} -6\alpha_1 - 12\alpha_2 = 0 \\ -12\alpha_1 - 24\alpha_2 = 0 \end{array}$$

$$\Rightarrow \begin{array}{l} \alpha_1 + 2\alpha_2 = 0 \\ \alpha_1 + 2\alpha_2 = 0 \end{array} \qquad \therefore \alpha_1 = -2\alpha_2$$

$$\therefore \tilde{e}_1 = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$$

when put $\lambda_2 = 120$ in eq.(i)

$$\begin{bmatrix} 24 & -12 \\ -12 & 6 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = 0$$

$$\Rightarrow \begin{array}{l} 24\alpha_1 - 12\alpha_2 = 0 \\ -12\alpha_1 + 6\alpha_2 = 0 \end{array}$$

$$\Rightarrow \begin{array}{l} \alpha_2 = 2\alpha_1 \\ \alpha_2 = 2\alpha_1 \end{array} \qquad \therefore \tilde{e}_2 = \begin{bmatrix} -1/\sqrt{5} \\ -2/\sqrt{5} \end{bmatrix}$$

$\therefore$ The corresponding eigenvectors are :

$$\tilde{e}_1 = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}, \quad \tilde{e}_2 = \begin{bmatrix} -1/\sqrt{5} \\ -2/\sqrt{5} \end{bmatrix}.$$

(ii) $$A'A = \begin{bmatrix} 4 & 3 \\ 8 & 6 \\ 8 & -9 \end{bmatrix} \begin{bmatrix} 4 & 8 & 8 \\ 3 & 6 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 50 & 5 \\ 50 & 100 & 10 \\ 5 & 10 & 145 \end{bmatrix}$$

Using eigen equations :-

$$A'A - \lambda I = 0$$

$$\text{or, } \begin{bmatrix} 25 & 50 & 5 \\ 50 & 100 & 10 \\ 5 & 10 & 145 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\text{or, } \begin{bmatrix} 25-\lambda & 50 & 5 \\ 50 & 100-\lambda & 10 \\ 5 & 10 & 145-\lambda \end{bmatrix} = 0 \quad \cdots \text{(i)}$$

or, $(25-\lambda)\{(100-\lambda)(145-\lambda)-100\} - 50\{50(145-\lambda)-50\} + 5\{500-5(100-\lambda)\} = 0$

or, $(25-\lambda)(\lambda^2 - 245\lambda + 14400) - 50(7200-50\lambda) + 5\lambda = 0$

or, $25\lambda^2 - 6125\lambda + 360000 - \lambda^3 + 245\lambda^2 - 14400\lambda - 360000 + 2500\lambda + 25\lambda = 0$

or, $-\lambda^3 + 270\lambda^2 - 18000\lambda = 0$

or, $\lambda(\lambda^2 - 270\lambda - 18000) = 0$

or, $\lambda(\lambda-150)(\lambda-120) = 0$ $\quad \therefore \lambda = 0,\ \lambda = 150, \lambda = 120$

$\therefore \lambda_1 = 150, \lambda_2 = 120, \lambda_3 = 0$

The eigenvalues are $\lambda_1 = 150, \lambda_2 = 120, \lambda_3 = 0$

The eigenvectors are,

put $\lambda_1 = 150$ in eq.(i)

$$\begin{bmatrix} 25-\lambda & 50 & 5 \\ 50 & 100-\lambda & 10 \\ 5 & 10 & 145-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow \begin{aligned} -125x_1 + 50x_2 + 5x_3 &= 0 \\ 50x_1 - 50x_2 + 10x_3 &= 0 \\ 5x_1 + 10x_2 - 5x_3 &= 0 \end{aligned}$$

$$\Rightarrow \begin{aligned} -25x_1 + 10x_2 + x_3 &= 0 \quad \cdots (ii) \\ 5x_1 - 5x_2 + x_3 &= 0 \quad \cdots (iii) \\ x_1 + 2x_2 - x_3 &= 0 \quad \cdots (iv) \end{aligned}$$

Adding eq. (ii) & (iv)

$$\begin{aligned} -25x_1 + 10x_2 + \cancel{x_3} &= 0 \\ x_1 + 2x_2 - \cancel{x_3} &= 0 \\ \hline -24x_1 + 12x_2 &= 0 \end{aligned}$$

$$\boxed{x_2 = 2x_1}$$

Again adding (iii) & (iv) × 2

$$-25x_1 + 10\cancel{x_2} + x_3 = 0$$
$$10x_1 - 1\cancel{0}x_2 + 2x_3 = 0$$
$$-15x_1 + 3x_3 = 0$$
$$\boxed{x_3 = 5x_1}$$

$$\tilde{e}_1' = \begin{bmatrix} 1/\sqrt{30} \\ 2/\sqrt{30} \\ 5/\sqrt{30} \end{bmatrix}$$

when again put $\lambda_2 = 120$ in eq. (i)

$$\begin{bmatrix} -95 & 50 & 5 \\ 50 & -20 & 10 \\ 5 & 10 & 25 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\Rightarrow -95x_1 + 50x_2 + 5x_3 = 0$
$50x_1 - 20x_2 + 10x_3 = 0$
$5x_1 + 10x_2 + 25x_3 = 0$

$\Rightarrow -19x_1 + 10x_2 + x_3 = 0$ --- (v)
$5x_1 - 20x_2 + x_3 = 0$ --- (vi)
$x_1 + 2x_2 + 5x_3 = 0$ --- (vii)

Adding (v) & (vii) × 19

$$-1\cancel{9x_1} + 10x_2 + x_3 = 0$$
$$1\cancel{9x_1} + 38x_2 + 95x_3 = 0$$
$$48x_2 + 96x_3 = 0$$
$$\boxed{x_2 = -2x_3}$$

Again Adding (vi) & (vii)

$$5x_1 - 2x_2 + x_3 = 0$$
$$x_1 + 2x_2 + 5x_3 = 0$$
$$6x_1 + 6x_3 = 0$$
$$\boxed{x_1 = -x_3}$$

∴ eigen vector, $\tilde{e}_2' = \begin{bmatrix} -1/\sqrt{6} \\ -2/\sqrt{6} \\ \cdot 1/\sqrt{6} \end{bmatrix}$

Again put, $\lambda_3 = 0$ in eq. (i)

$$\begin{bmatrix} -25 & 50 & 5 \\ 50 & 100 & 10 \\ 5 & 10 & 145 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\Rightarrow -25x_1 + 50x_2 + 5x_3 = 0$
$50x_1 + 100x_2 + 10x_3 = 0$
$5x_1 + 10x_2 + 145x_3 = 0$

$-5x_1 + 10x_2 + x_3 = 0$ --- (viii)
$\Rightarrow 5x_1 + 10x_2 + x_3 = 0$ --- (ix)
$x_1 + 2x_2 + 29x_3 = 0$ --- (x)

Subtracting (ix) & (x) × 5

$$\cancel{5x_1} + 1\cancel{0x_2} + x_3 = 0$$
$$\cancel{5x_1} + 1\cancel{0x_2} + 145x_3 = 0$$
$$-144x_3 = 0$$
$$\boxed{x_3 = 0}$$

put $x_3 = 0$ in eq. (viii)

$$-5x_1 + 10x_2 = 0$$
$$\boxed{x_1 = 2x_2}$$

$\therefore$ eigen vector v_3, $\tilde{e}_3' = \begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \\ 0 \end{bmatrix}$

$\therefore$ The eigen values of $A'A$ matrix are, $\lambda_1 = 150, \lambda_2 = 120, \lambda_3 = 0$
and the eigen vectors are, $\tilde{e}_1' = \begin{bmatrix} 1/\sqrt{30} \\ 2/\sqrt{30} \\ 5/\sqrt{30} \end{bmatrix}$, $\tilde{e}_2' = \begin{bmatrix} -1/\sqrt{6} \\ -2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$,

$\tilde{e}_3' = \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{bmatrix}$

Notice that the nonzero eigenvalues are the same as those of AA'.

(iii) The singular value decomposition of A is,

$$A = \lambda_1 \tilde{e}_1(\tilde{e}_1')' + \lambda_2 \tilde{e}_2(\tilde{e}_2')' + \lambda_3 \tilde{e}_3(\tilde{e}_3')'$$

$$= \sqrt{150} \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{30}} & \frac{2}{\sqrt{30}} & \frac{5}{\sqrt{30}} \end{bmatrix} + \sqrt{120} \begin{bmatrix} -1/\sqrt{5} \\ -2/\sqrt{5} \end{bmatrix} \begin{bmatrix} \frac{-1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix} + 0$$

$[\because \lambda_3 = 0]$

Q.2. Let X have covariance matrix, $\Sigma = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
find, (i) Σ^{-1}

(ii) The eigen values and eigen vectors of Σ

(iii) The eigen values and eigen vectors of Σ^{-1}

$\Rightarrow$ (i) $\det(\Sigma) = 4(9-0) + 0(0-0) + 0(0-0) = \cancel{45}\,36 \neq 0$

so Σ^{-1} exists.

$$\text{Adj}(\Sigma) = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 36 \end{bmatrix}$$

$$\therefore \Sigma^{-1} = \frac{\text{Adj}(\Sigma)}{|A|} = \frac{1}{36} \begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 36 \end{bmatrix} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/9 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(ii) Let, λ be the eigen value of matrix Σ

$$\Sigma - \lambda I = 0$$

$$\Rightarrow \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 4-\lambda & 0 & 0 \\ 0 & 9-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (4-\lambda)(9-\lambda)(1-\lambda) = 0$$

$$\therefore \lambda_1 = 4, \quad \lambda_2 = 9, \quad \lambda_3 = 1$$

$\therefore$ The eigen vectors of Σ are, $\tilde{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\tilde{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\tilde{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(iii) Let, λ be the eigen value of matrix Σ^{-1}

$$\Sigma^{-1} - \lambda I = 0$$

$$\Rightarrow \begin{bmatrix} 1/4 - \lambda & 0 & 0 \\ 0 & 1/9 - \lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (1/4 - \lambda)(1/9 - \lambda)(1-\lambda) = 0$$

$$\therefore \lambda_1 = \frac{1}{4}, \quad \lambda_2 = \frac{1}{9}, \quad \lambda_3 = 1$$

$\therefore$ The eigen vectors of Σ^{-1} are,

$$\tilde{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \tilde{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \tilde{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Q.3. You are given the random vector $X' = [x_1, x_2, x_3, x_4]$ with mean vector $\mu'_x = [4, 3, 2, 1]$ and variance-covariance matrix, $\Sigma_x = \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{bmatrix}$

Partition X as, $X = \begin{bmatrix} x_1 \\ x_2 \\ \hdashline x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} X^{(1)} \\ \hdashline X^{(2)} \end{bmatrix}$

Let, $A = \begin{bmatrix} 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ and consider the linear combinations $AX^{(1)}$ and $BX^{(2)}$. Find

(i) $E(X^{(1)})$ (ii) $E(AX^{(1)})$ (iii) $Cov(X^{(1)})$

(iv) $Cov(AX^{(1)})$ (v) $E(X^{(2)})$ (vi) $E(BX^{(2)})$

(vii) $Cov(X^{(2)})$ (viii) $Cov(BX^{(2)})$ (ix) $Cov(X^{(1)}, X^{(2)})$

(x) $Cov(AX^{(1)}, BX^{(2)})$

$\Rightarrow X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$, $\mu'_x = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} = \begin{bmatrix} E(x_1) \\ E(x_2) \\ E(x_3) \\ E(x_4) \\ E(x_5) \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -1 \\ 3 \\ 0 \end{bmatrix}$

$$\Sigma_x = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} & \sigma_{25} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} & \sigma_{35} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & \sigma_{45} \\ \sigma_{51} & \sigma_{52} & \sigma_{53} & \sigma_{54} & \sigma_{55} \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 & 1/2 & -1/2 & 0 \\ -1 & 3 & 1 & -1 & 0 \\ 1/2 & 1 & 6 & 1 & -1 \\ -1/2 & -1 & 1 & 4 & 0 \\ 0 & 0 & -1 & 0 & 2 \end{bmatrix}$$

Let, $X = \left.\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}\right\} p = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \hline x_4 \\ x_5 \end{bmatrix} \begin{matrix} \}q \\ \}p-q \end{matrix} = \begin{bmatrix} X^{(1)} \\ \hline X^{(2)} \end{bmatrix}$

Let, $p = 5$, $q = 3$, $p - q = 2$

$$\mu_x = \begin{bmatrix} \mu^{(1)} \\ \hline \mu^{(2)} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \hline \mu_4 \\ \mu_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -1 \\ \hline 3 \\ 0 \end{bmatrix}$$

$$\Sigma_x = \left[\begin{array}{c|c} \Sigma_{11} & \Sigma_{12} \\ \hline \Sigma_{21} & \Sigma_{22} \end{array}\right] = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} & \sigma_{15} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} & \sigma_{25} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} & \sigma_{35} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} & \sigma_{45} \\ \sigma_{51} & \sigma_{52} & \sigma_{53} & \sigma_{54} & \sigma_{55} \end{bmatrix}$$

(i) $\mu^{(1)} = E\left(X^{(1)}\right) = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$

(ii) $E\left(A X^{(1)}\right) = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 - 4 + 0 \\ 2 + 4 - 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

(iii) $Cov(x^{(1)}) = \Sigma_{11} = \begin{bmatrix} 4 & -1 & 1/2 \\ -1 & 3 & 1 \\ 1/2 & 1 & 6 \end{bmatrix}$

(iv) $Cov(A x^{(1)}) = A \, \Sigma_{11} \, A'$

$$= \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 4 & -1 & 1/2 \\ -1 & 3 & 1 \\ 1/2 & 1 & 6 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 9 & 9/2 \\ -5 & 5 \\ 0 & 39/2 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 4 \\ 4 & 68 \end{bmatrix}$$

(v) $E(x^{(2)}) = \begin{bmatrix} \mu_4 \\ \mu_5 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

(vi) $E(B x^{(2)}) = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

(vii) $Cov(x^{(2)}) = \Sigma_{22} = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

(viii) $Cov(B x^{(2)}) = B \, \Sigma_{22} \, B'$

$$= \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 12 & 0 \\ 0 & 6 \end{bmatrix}$$

(ix) $\mathrm{Cov}\left(X^{(1)}, X^{(2)}\right) = \Sigma_{12}$

$$= \begin{bmatrix} -1/2 & 0 \\ -1 & 0 \\ 1 & -1 \end{bmatrix}$$

(x) $\mathrm{Cov}\left(AX^{(1)}, BX^{(2)}\right) = A\Sigma_{12}B'$

$$= \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1/2 & 0 \\ -1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1/2 & -1/2 \\ -1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ -9/2 & 9/2 \end{bmatrix}$$