CA2 assignment Stream ---CSBS Submitted by--&th September 2023

Roll Nos	Problems

1-9

A morning newspaper lists the following used-car prices for a foreign compact with age x_1 measured in years and selling price x_2 measured in thousands of dollars:

x_1	1	2	3	3	4	5	6	8	9	11
x_2	18.95	19.00	17.95	15.54	14.00	12.95	8.94	7.49	6.00	3.99

(a) Construct a scatter plot of the data and marginal dot diagrams.

(b) Infer the sign of the sample covariance s_{12} from the scatter plot.

(c) Compute the sample means $\bar{x}_1$ and $\bar{x}_2$ and the sample variances s_{11} and s_{22}. Compute the sample covariance s_{12} and the sample correlation coefficient r_{12}. Interpret these quantities.

(d) Display the sample mean array $\bar{\mathbf{x}}$, the sample variance-covariance array $\mathbf{S}_n$, and the sample correlation array $\mathbf{R}$ u

The world's 10 largest companies yield the following data:

The World's 10 Largest Companies[1]

Company	x_1 = sales (billions)	x_2 = profits (billions)	x_3 = assets (billions)
Citigroup	108.28	17.05	1,484.10
General Electric	152.36	16.59	750.33
American Intl Group	95.04	10.91	766.42
Bank of America	65.45	14.14	1,110.46
HSBC Group	62.97	9.52	1,031.29
ExxonMobil	263.99	25.33	195.26
Royal Dutch/Shell	265.19	18.54	193.83
BP	285.06	15.73	191.11
ING Group	92.01	8.10	1,175.16
Toyota Motor	165.68	11.13	211.15

[1]From www.Forbes.com partially based on *Forbes* The Forbes Global 2000, April 18, 2005.

(a) Plot the scatter diagram and marginal dot diagrams for variables x_1 and x_2. Comment on the appearance of the diagrams.

(b) Compute $\bar{x}_1$, $\bar{x}_2$, s_{11}, s_{22}, s_{12}, and r_{12}. Interpret r_{12}.

2.

(a) Plot the scatter diagrams and dot diagrams for (x_2, x_3) and (x_1, x_3). Comment on the patterns.

(b) Compute the $\bar{\mathbf{x}}$, $\mathbf{S}_n$, and $\mathbf{R}$ arrays for (x_1, x_2, x_3).

10-20

Given the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$$

find the eigenvalues λ_1 and λ_2 and the associated normalized eigenvectors $\mathbf{e}_1$ and $\mathbf{e}_2$. Determine the spectral decomposition of $\mathbf{A}$.

(a) Find $\mathbf{A}^{-1}$.

(b) Compute the eigenvalues and eigenvectors of $\mathbf{A}^{-1}$.

(c) Write the spectral decomposition of $\mathbf{A}^{-1}$, and compare it with that of $\mathbf{A}$

Consider the matrices

$$\mathbf{A} = \begin{bmatrix} 4 & 4.001 \\ 4.001 & 4.002 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 4 & 4.001 \\ 4.001 & 4.002001 \end{bmatrix}$$

These matrices are identical except for a small difference in the (2, 2) position. Moreover, the columns of $\mathbf{A}$ (and $\mathbf{B}$) are nearly linearly dependent. Show that $\mathbf{A}^{-1} \doteq (-3)\mathbf{B}^{-1}$. Consequently, small changes—perhaps caused by rounding—can give substantially different inverses.

A quadratic form $\mathbf{x}'\mathbf{A}\mathbf{x}$ is said to be positive definite if the matrix $\mathbf{A}$ is positive definite. Is the quadratic form $3x_1^2 + 3x_2^2 - 2x_1x_2$ positive definite?

21-29

Determine the square-root matrix $\mathbf{A}^{1/2}$, using the matrix $\mathbf{A}$ in Also, determine $\mathbf{A}^{-1/2}$, and show that $\mathbf{A}^{1/2}\mathbf{A}^{-1/2} = \mathbf{A}^{-1/2}\mathbf{A}^{1/2} = \mathbf{I}$.

Where $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$,

	$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & -2 \\ 2 & 2 \end{bmatrix}$ (a) Calculate $\mathbf{A}'\mathbf{A}$ and obtain its eigenvalues and eigenvectors. (b) Calculate $\mathbf{A}\mathbf{A}'$ and obtain its eigenvalues and eigenvectors. Check that the nonzero eigenvalues are the same as those in part a. (c) Obtain the singular-value decomposition of $\mathbf{A}$.
30-39	$\mathbf{A} = \begin{bmatrix} 4 & 8 & 8 \\ 3 & 6 & -9 \end{bmatrix}$ (a) Calculate $\mathbf{A}\mathbf{A}'$ and obtain its eigenvalues and eigenvectors. (b) Calculate $\mathbf{A}'\mathbf{A}$ and obtain its eigenvalues and eigenvectors. Check that the nonzero eigenvalues are the same as those in part a. (c) Obtain the singular-value decomposition of $\mathbf{A}$.
	. Let $\mathbf{X}$ have covariance matrix $\boldsymbol{\Sigma} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Find (a) $\boldsymbol{\Sigma}^{-1}$ (b) The eigenvalues and eigenvectors of $\boldsymbol{\Sigma}$. (c) The eigenvalues and eigenvectors of $\boldsymbol{\Sigma}^{-1}$.
40-49	. Let $\mathbf{X}$ have covariance matrix $\boldsymbol{\Sigma} = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$ (a) Determine $\boldsymbol{\rho}$ and $\mathbf{V}^{1/2}$. (b) Multiply your matrices to check the relation $\mathbf{V}^{1/2}\boldsymbol{\rho}\mathbf{V}^{1/2} = \boldsymbol{\Sigma}$. 2. (a) Find ρ_{13}. (b) Find the correlation between X_1 and $\frac{1}{2}X_2 + \frac{1}{2}X_3$.
	You are given the random vector $\mathbf{X}' = [X_1, X_2, X_3, X_4]$ with mean vector $\boldsymbol{\mu}'_{\mathbf{X}} = [4, 3, 2, 1]$ and variance–covariance matrix $\boldsymbol{\Sigma}_{\mathbf{X}} = \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{bmatrix}$ Partition $\mathbf{X}$ as $\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \hdashline X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} \mathbf{X}^{(1)} \\ \hdashline \mathbf{X}^{(2)} \end{bmatrix}$ Let $\mathbf{A} = [1 \quad 2] \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ and consider the linear combinations $\mathbf{A}\mathbf{X}^{(1)}$ and $\mathbf{B}\mathbf{X}^{(2)}$. Find (a) $E(\mathbf{X}^{(1)})$ (b) $E(\mathbf{A}\mathbf{X}^{(1)})$ (c) $\mathrm{Cov}(\mathbf{X}^{(1)})$ (d) $\mathrm{Cov}(\mathbf{A}\mathbf{X}^{(1)})$ (e) $E(\mathbf{X}^{(2)})$ (f) $E(\mathbf{B}\mathbf{X}^{(2)})$ (g) $\mathrm{Cov}(\mathbf{X}^{(2)})$ (h) $\mathrm{Cov}(\mathbf{B}\mathbf{X}^{(2)})$ (i) $\mathrm{Cov}(\mathbf{X}^{(1)}, \mathbf{X}^{(2)})$ (j) $\mathrm{Cov}(\mathbf{A}\mathbf{X}^{(1)}, \mathbf{B}\mathbf{X}^{(2)})$

Compulsory for all group	xercise 2.32, but with $\mathbf{X}$ partitioned as $$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \hdashline X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} \mathbf{X}^{(1)} \\ \hdashline \mathbf{X}^{(2)} \end{bmatrix}$$ $\mathbf{A}$ and $\mathbf{B}$ replaced by $$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$$ and consider the linear combinations $\mathbf{AX}^{(1)}$ and $\mathbf{BX}^{(2)}$. Find (a) $E(\mathbf{X}^{(1)})$ (b) $E(\mathbf{AX}^{(1)})$ (c) $\text{Cov}(\mathbf{X}^{(1)})$ (d) $\text{Cov}(\mathbf{AX}^{(1)})$ (e) $E(\mathbf{X}^{(2)})$ (f) $E(\mathbf{BX}^{(2)})$ (g) $\text{Cov}(\mathbf{X}^{(2)})$ (h) $\text{Cov}(\mathbf{BX}^{(2)})$ (i) $\text{Cov}(\mathbf{X}^{(1)}, \mathbf{X}^{(2)})$ (j) $\text{Cov}(\mathbf{AX}^{(1)}, \mathbf{BX}^{(2)})$