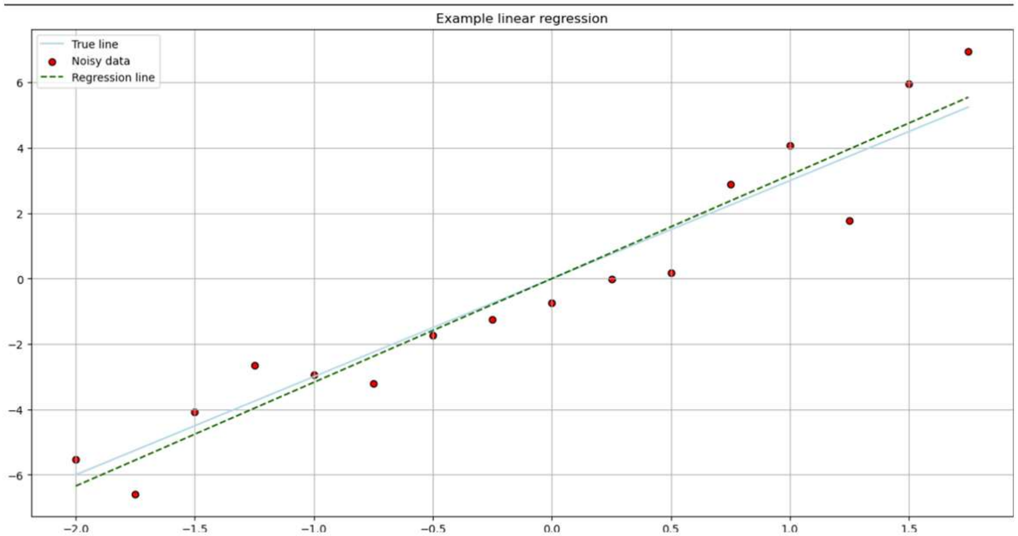
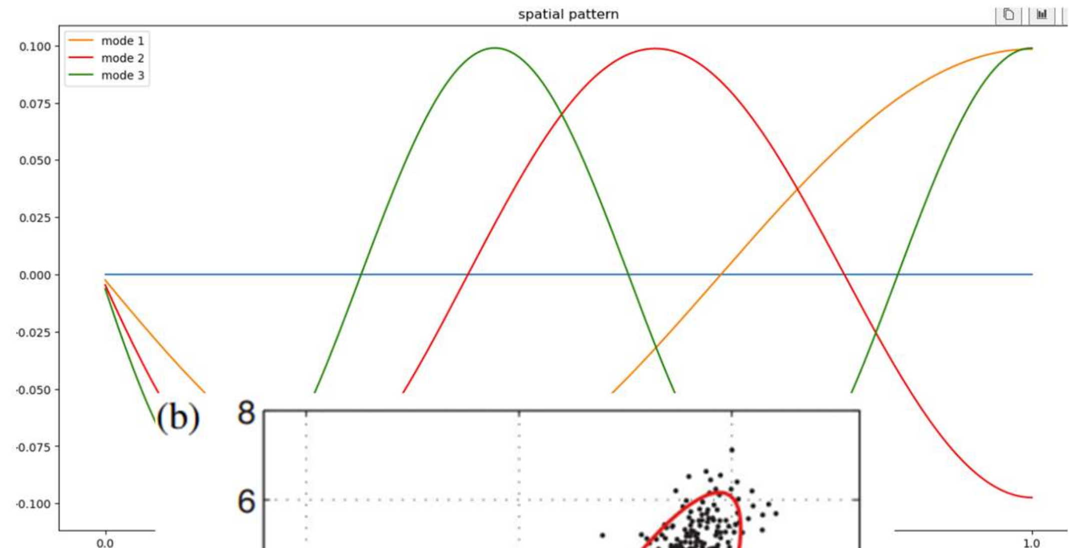
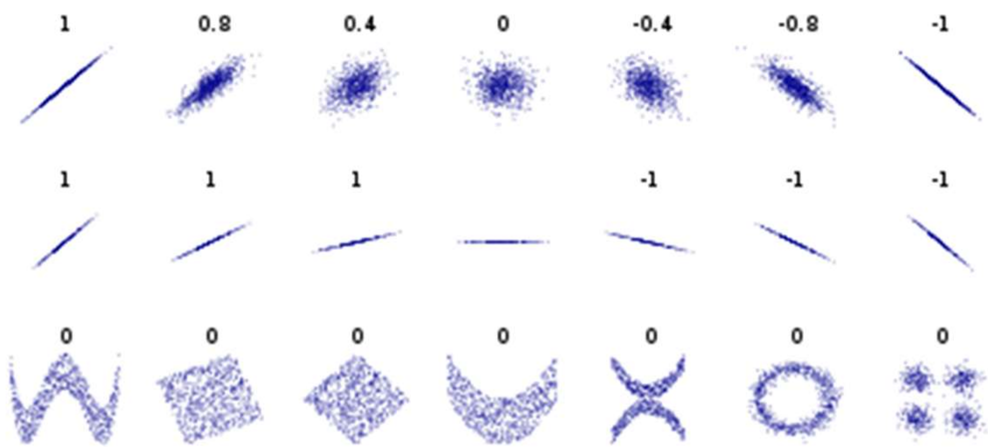
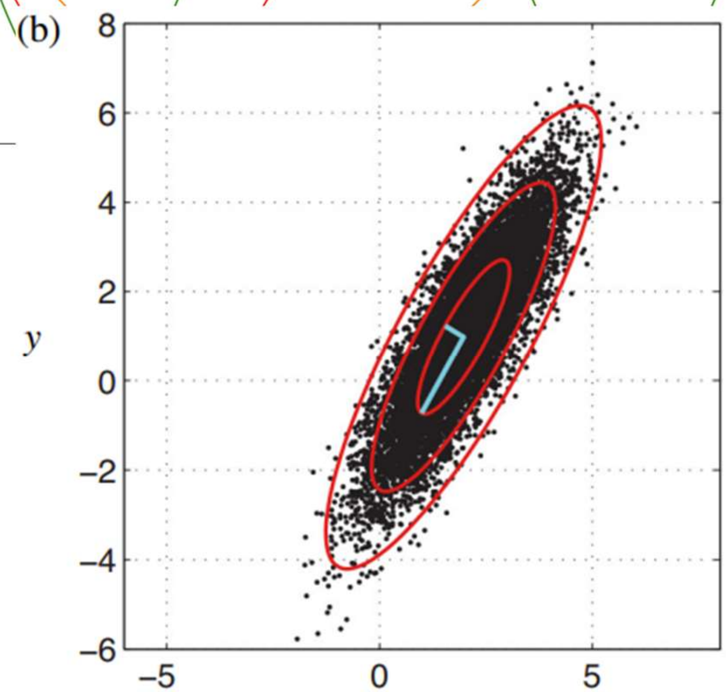

Example linear regression
True line
Noisy data
Regression line


spatial pattern
mode 1
mode 2
mode 3


1
0.8
0.4
0
-0.4
-0.8
-1
1
1
1
-1
-1
-1
0
0
0
0
0
0
0


(b)
y

$$\underline{\Sigma} = \begin{bmatrix} \begin{matrix} \sigma_1 & & 0 \\ & \ddots & \\ 0 & & \sigma_r \end{matrix} & 0 \\ & \begin{matrix} \ddots & \\ & \sigma_n \end{matrix} \\ 0 & \end{bmatrix}$$

$\hat{\Sigma}$ economy

$\tilde{\Sigma}$ = exact truncated $\Rightarrow \tilde{\underline{\Sigma}}^{-1}$ exist

all these = 0

$$\underline{A} = \underset{n \times r}{\tilde{\underline{U}}}\ \tilde{\underline{\Sigma}}\ \underset{r \times m}{\tilde{\underline{V}}^T}$$

where

$$\tilde{\underline{U}} = \begin{bmatrix} | & & | \\ \underline{u}_1 & \cdots & \underline{u}_r \\ | & & | \end{bmatrix} \Rightarrow \tilde{\underline{U}}^T \tilde{\underline{U}} = \begin{bmatrix} - & \underline{u}_1 & - \\ & \vdots & \\ - & \underline{u}_r & - \end{bmatrix} \begin{bmatrix} | & & | \\ \underline{u}_1 & \cdots & \underline{u}_r \\ | & & | \end{bmatrix} = \underline{I}_{r \times r}$$

but $\tilde{\underline{U}}\ \tilde{\underline{U}}^T \neq \underline{I}_{n \times n}$

$$\tilde{\underline{V}}^T = \begin{bmatrix} - & \underline{v}_1 & - \\ & \vdots & \\ - & \underline{v}_r & - \end{bmatrix}$$

$\Rightarrow \tilde{\underline{V}}^T \tilde{\underline{V}} = \underline{I}_{r \times r}$

but

$\tilde{\underline{V}}\ \tilde{\underline{V}}^T \neq \underline{I}_{m \times m}$

Pseudoinverse defined as:

$$\underline{A}^\dagger = \tilde{\underline{V}}\ \tilde{\underline{\Sigma}}^{-1}\ \tilde{\underline{U}}^T$$

$$\Longrightarrow \quad \mathbf{A}^\dagger \mathbf{A} = \tilde{\mathbf{V}} \tilde{\mathbf{V}}^*$$

Define $\underset{\sim}{\tilde{x}}$ from: $\underset{\sim}{\tilde{x}} = \underset{\sim}{A}^{+} \underset{\sim}{b}$

insert into original eqn

$$\underset{\sim}{A}\,\underset{\sim}{\tilde{x}} = \underset{\sim}{A}\,\underset{\sim}{A}^{+}\underset{\sim}{b} = \underset{\sim}{\tilde{U}}\,\underset{\sim}{\tilde{\Sigma}}\,\underbrace{\underset{\sim}{\tilde{V}}^{T}\underset{\sim}{\tilde{V}}}_{\underset{\sim}{I}}\,\underset{\sim}{\tilde{\Sigma}}^{-1}\,\underset{\sim}{\tilde{U}}^{T}\underset{\sim}{b} = \underbrace{\underset{\sim}{\tilde{U}}\,\underset{\sim}{\tilde{U}}^{T}}_{\neq \underset{\sim}{I}_{n\times n}}\underset{\sim}{b}$$

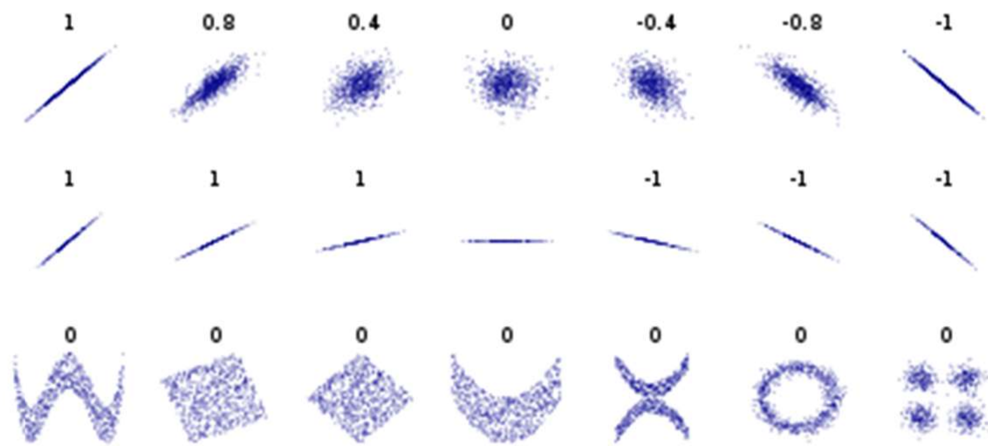
i.e. not fully fulfilled

$\Rightarrow \underset{\sim}{\tilde{x}}$ is an approximation.

2025-09-03

Sample correlation coefficient

$$r_{xy} \stackrel{\text{def}}{=} \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$



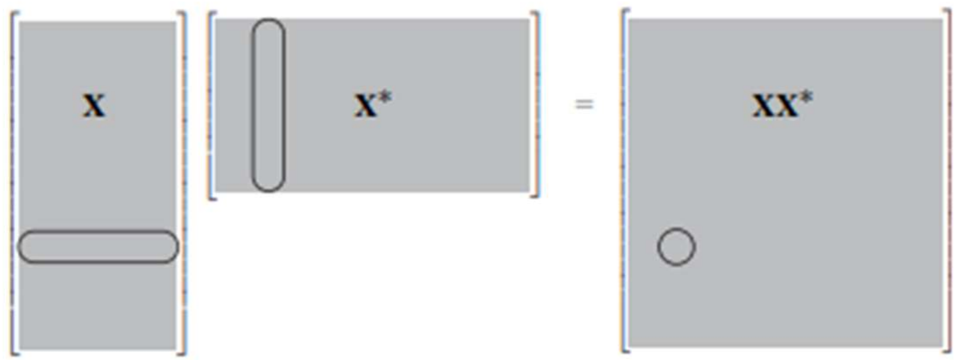


Figure 1.6 Correlation matrix $\mathbf{XX}^*$ is formed by taking the inner product of rows of $\mathbf{X}$.

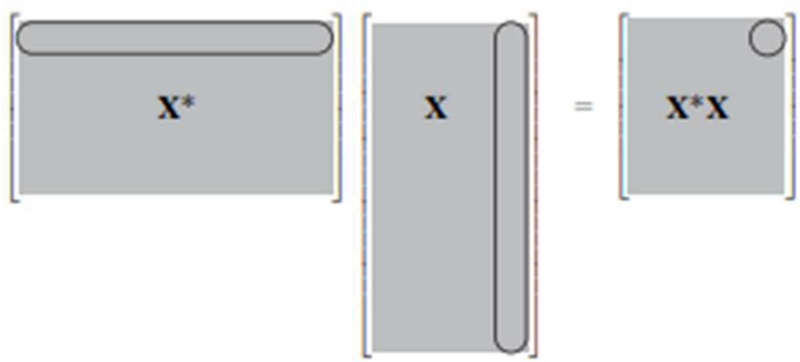


Figure 1.7 Correlation matrix $\mathbf{X}^*\mathbf{X}$ is formed by taking the inner product of columns of $\mathbf{X}$.