Singular value decomposition for approximating data, reduced order modelling, de-noise data

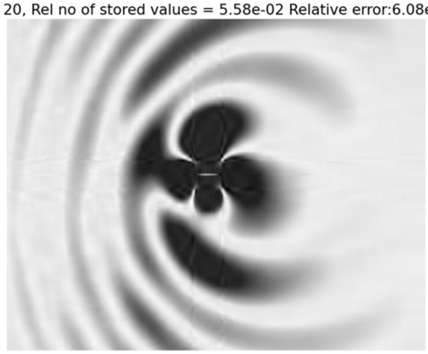
Original picture

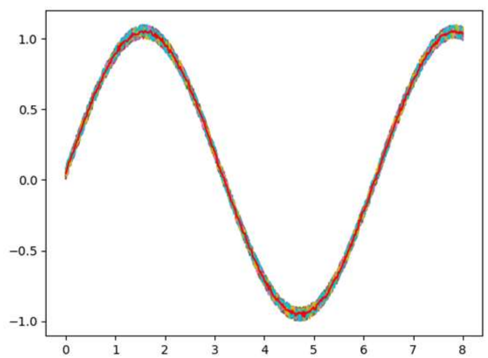
r = 200, Rel no of stored values = 4.13e-01 Relative error: 2.30e-03

r = 50, Rel no of stored values = 1.03e-01 Relative error: 1.28e-02

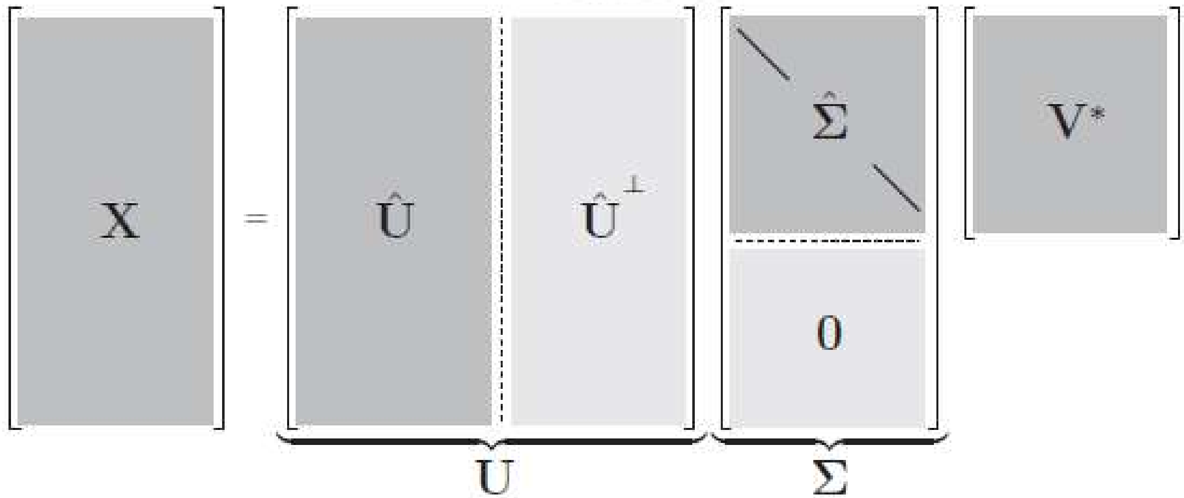
r = 5, Rel no of stored values = 1.03e-02 Relative error: 5.63e-02

r = 20, Rel no of stored values = 5.58e-02 Relative error:6.08e-03

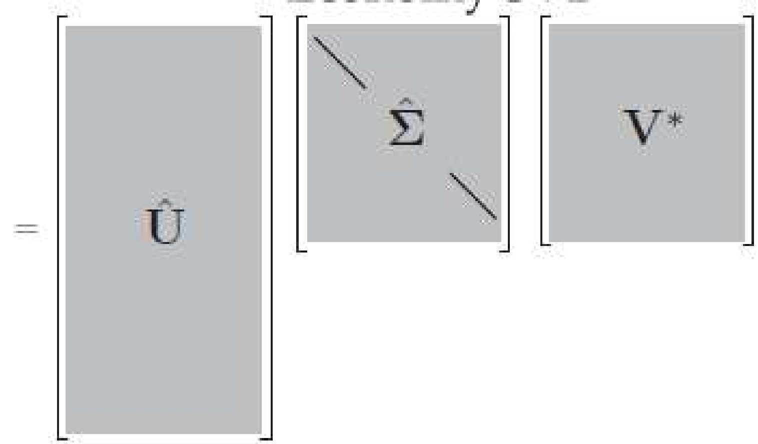




Full SVD



Economy SVD



Theorem 1 (Eckart-Young [170]) *The optimal rank-r approximation to* $\mathbf{X}$*, in a least-squares sense, is given by the rank-r SVD truncation* $\tilde{\mathbf{X}}$*:*

$$\underset{\tilde{\mathbf{X}},\ s.t.\ \operatorname{rank}(\tilde{\mathbf{X}})=r}{\operatorname{argmin}} \|\mathbf{X}-\tilde{\mathbf{X}}\|_F = \tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^*. \tag{1.4}$$