

Singular value decomposition for approximating data, reduced order modelling, de-noise data



$r = 200$, Rel no of stored values = $4.13e-01$ Relative error: $2.30e-03$



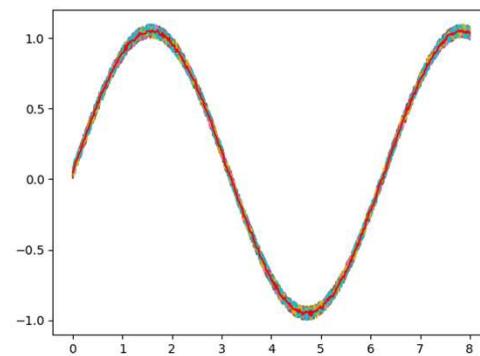
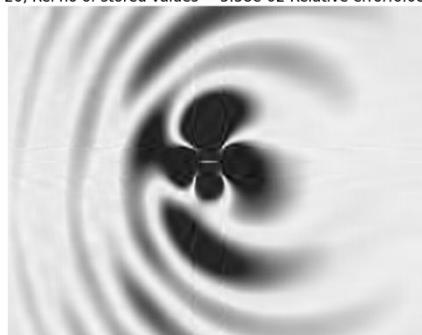
$r = 50$, Rel no of stored values = $1.03e-01$ Relative error: $1.28e-02$



$r = 5$, Rel no of stored values = $1.03e-02$ Relative error: $5.63e-02$



$r = 20$, Rel no of stored values = $5.58e-02$ Relative error: $6.08e-03$



Full SVD

$$\begin{bmatrix} \mathbf{X} \end{bmatrix} = \underbrace{\begin{bmatrix} \hat{\mathbf{U}} & \hat{\mathbf{U}}^\perp \end{bmatrix}}_{\mathbf{U}} \begin{bmatrix} \hat{\Sigma} & \\ & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}^* \end{bmatrix}$$

Economy SVD

$$= \begin{bmatrix} \hat{\mathbf{U}} \end{bmatrix} \begin{bmatrix} \hat{\Sigma} \end{bmatrix} \begin{bmatrix} \mathbf{V}^* \end{bmatrix}$$

Theorem 1 (Eckart-Young [170]) *The optimal rank- r approximation to \mathbf{X} , in a least-squares sense, is given by the rank- r SVD truncation $\tilde{\mathbf{X}}$:*

$$\underset{\tilde{\mathbf{X}}, \text{ s.t. } \text{rank}(\tilde{\mathbf{X}})=r}{\operatorname{argmin}} \|\mathbf{X} - \tilde{\mathbf{X}}\|_F = \tilde{\mathbf{U}} \tilde{\Sigma} \tilde{\mathbf{V}}^*. \quad (1.4)$$

$$\begin{aligned} \mathbf{X} &= \underbrace{\left[\begin{array}{c|c|c} \tilde{\mathbf{U}} & \hat{\mathbf{U}}_{\text{rem}} & \hat{\mathbf{U}}^\perp \end{array} \right]}_{\mathbf{U}} \left[\begin{array}{c|c} \tilde{\Sigma} & \\ \hline & \hat{\Sigma}_{\text{rem}} \\ \hline & 0 \end{array} \right] \left[\begin{array}{c} \tilde{\mathbf{V}}^* \\ \mathbf{V}_{\text{rem}} \end{array} \right] \\ &\approx \tilde{\mathbf{U}} \left[\begin{array}{c|c} \tilde{\Sigma} & \tilde{\mathbf{V}}^* \end{array} \right] \end{aligned}$$

Truncated SVD