EDUCATALYSTS

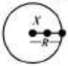
Class(12th)

Introduction to Electrostatics Potential and Capacitance

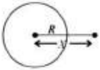
ELECTRIC POTENTIAL

Definition– Work done to bring a unit positive charge from infinite distance to a point in the electric field is called electric potential at that point.

- It represents the electrical condition or state of the body and it is similar to temperature.
- +vely charged body is considered to be at higher potential and –vely charged body is considered to be at lower potential.
- Electric potential at a point is a relative value but not an absolute value.
- Potential at a point due to a point charge $= \dfrac{1}{4\pi \epsilon_0} \dfrac{Q}{r}$
- Potential due to a group of charges is the algebraic sum of their individual potentials.
  i.e. $V = V_1 + V_2 + V_3 + \ldots\ldots\ldots$
- Two charges $+Q$ and $-Q$ are separated by a distance d, the potential on the perpendicular bisector of the line joining the charges is zero.
- When a charged particle is accelerated from rest through a p.d. 'V', work done, $W = Vq = \dfrac{1}{2}mv^2$ (or) $v = \sqrt{\dfrac{2Vq}{m}}$
- The work done in moving a charge of q coulomb between two points separated by p.d. $V_2 - V_1$ is $q(V_2 - V_1)$.
- The work done in moving a charge from one point to another point on an equipotential surface is zero.
- A hollow sphere of radius R is given a charge Q the potential at a distance x from the centre is $\dfrac{1}{4\pi \epsilon_0} \cdot \dfrac{Q}{R} (x \le R)$



- The potential at a distance when $x > R$ is $\dfrac{1}{4\pi \epsilon_0} \cdot \dfrac{Q}{x}$.

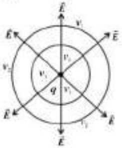


- A sphere is charged to a potential. The potential at any point inside the sphere is same as that of the surface.
- Inside a hollow conducting spherical shell, $E = 0$, $v \neq 0$.
- Relation among E, V and d in a uniform electric field is $E = \dfrac{V}{d}$ (or) $E = -\dfrac{dV}{dx}$

— KEY NOTE —

- Electric field is always in the direction of decreasing potential.
  The component of electric field in any direction is equal to the negative of potential gradient in that direction.
  $$\overline{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

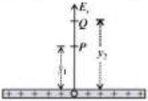
- An equipotential surface has a constant value of potential at all points on the surface.
  For single charge q



— KEY NOTE —

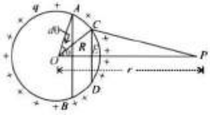
- Electric field at every point is normal to the equipotential surface passing through that point
- No work is required to move a test charge on unequipotential surface.
- The electric force is conservative in nature.
- Coulomb force is central.

Potential due to a line Charge of Infinite Length :



- The potential difference between two points P and Q is $\dfrac{\lambda}{2\pi\varepsilon_0}\log_e\left(\dfrac{y_2}{y_1}\right)$
  Where $\lambda \rightarrow$ linear charge density

Potential due to Charged Spherical Shell



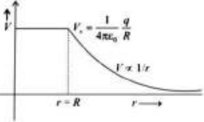
When point P lies outside the shell : $V = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{r}$

When *P* lies on the surface :

In this case $r = R$

$\therefore$ Potential at the surface $= \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{R}$

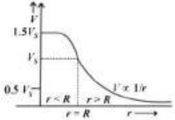
When P lies inside the shell : $V = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{R}$



Potential due to spherical volume distribution of charge

In case of spherical volume distribution of charge (e.g. nucleus or uniformly charged dielectic sphere) for an internal point (i.e. $r < R$) Potential varies non-linearly with r as,

$$V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{q}{2R}\left[3 - \frac{r^2}{R^2}\right]$$



$V_{outside} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$; $V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

Potential of a charged ring

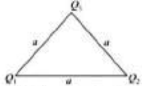
A charge q is distributed over the circumference of ring (either uniformly or non-uniformly) , then electric potential at the centre of the ring is $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R}$.

At distance 'r' from the centre of ring on its axis would be $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{\sqrt{R^2 + r^2}}$

Potential Energy of System of Charges

- Two charges Q_1 and Q_2 are separated by a distance 'd'. The P.E. of the system of charges is $U = \frac{1}{4\pi\epsilon_0}.\frac{Q_1Q_2}{d}$ from $U = W = Vq$

- Three charges Q_1, Q_2, Q_3 are placed at the three vertices of an equilateral triangle of side 'a'. The P.E. of the system of charges is $U = \frac{1}{4\pi\epsilon_0}\left[\frac{Q_1Q_2}{a} + \frac{Q_2Q_3}{a} + \frac{Q_3Q_1}{a}\right]$ or $U = \frac{1}{4\pi\epsilon_0}\frac{\sum Q_1Q_2}{a}$



- A charged particle of charge Q_2 is held at rest at a distance 'd' from a stationary charge Q_1. When the charge is released, the K.E. of the charge Q_2 at infinity is $\frac{1}{4\pi\epsilon_0}.\frac{Q_1Q_2}{d}$.
- If two like charges are brought closer, P.E of the system increases.
- If two unlike charges are brought closer, P.E of the system decrease.
- For an attractive system U is always NEGATIVE.

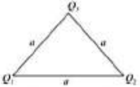
  For a repulsive system U is always POSITIVE.

  For a stable system U is MINIMUM.

  i.e. $= F = -\frac{dU}{dx} = 0$ (for stable system)

  The P.E. of the system of charges is $U = \frac{1}{4\pi\epsilon_0}.\frac{Q_1Q_2}{d}$ from $U = W = qv$

- Three charges Q_1, Q_2, Q_3 are placed at the three vertices of an equilateral triangle of side 'a'. The P.E. of the system of charges is $U = \frac{1}{4\pi\epsilon_0}\left[\frac{Q_1Q_2}{a} + \frac{Q_2Q_3}{a} + \frac{Q_3Q_1}{a}\right]$ or $U = \frac{1}{4\pi\epsilon_0}\frac{\sum Q_1Q_2}{a}$.



- A charged particle of charge Q_2 is held at rest at a distance 'd' from a stationary charge Q_1. When the charge is released, the K.E. of the charge Q_2 at infinity is $\frac{1}{4\pi\epsilon_0}.\frac{Q_1Q_2}{d}$.

— KEY NOTE —

- If two like charges are brought closer, P.E of the system increases.
- If two unlike charges are brought closer, P.E of the system decreases.

For an attractive system U is always NEGATIVE.

For a repulsive system U is always POSITIVE.

For a stable system U is MINIMUM.

i.e. $F = -\frac{dU}{dx} = 0$ (for stable system)

Potential Due To An Electric Dipole

The potential due to the dipole is the sum of potentials due to the charges q and -q

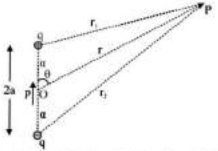


Fig.: Quantities involved in the calculation of potential due to a dipole

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) \quad ...(1)$$

where r_1 and r_2 are the distances of the point P from q and -q, respectively.

Now, by geometry,

$$r_1^2 = r^2 + a^2 - 2ar\cos\theta$$
$$r_2^2 = r^2 + a^2 + 2ar\cos\theta \quad (2)$$

We take r much greater than a (r >> a) and retain terms only upto the first order in a/r

$$r_1^2 = r^2\left(1 - \frac{2a\cos\theta}{r} + \frac{a^2}{r^2}\right)$$
$$\cong r^2\left(1 - \frac{2a\cos\theta}{r}\right) \quad (3)$$

Similarly,

$$r_2^2 \cong r^2\left(1 + \frac{2a\cos\theta}{r}\right) \quad (4)$$

Using the Binomial theorem and retaining terms upto the first order in a/r ; we obtain,

$$\frac{1}{r_1} \cong \frac{1}{r}\left(1 - \frac{2a\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right) \quad (5)$$

$$\frac{1}{r_2} \cong \frac{1}{r}\left(1 + \frac{2a\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right) \quad (6)$$

Using Eqs. 1, 5 and 6 and p = 2qa, we get

$$V = \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \quad (7)$$

Now, $p\cos\theta = \mathbf{p}.\hat{\mathbf{r}}$

where $\hat{\mathbf{r}}$ is the unit vector along the position vector OP. The electric potential of a dipole is then given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}.\hat{\mathbf{r}}}{r^2}; \quad (r >> a) \quad (8)$$

Potential Energy of a Dipole in an External Field

Consider a dipole with charges $q_1 = +q$ and $q_2 = -q$ placed in a uniform electric field E, as shown in Fig.

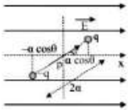


Fig.: Potential energy of a dipole in a uniform external field

As seen in the last chapter, in a uniform electric field, the dipole experiences no net force; but experiences a torque τ given by

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \quad (1)$$

which will tend to rotate it (unless p is parallel or antiparallel to E). Suppose an external torque τ_{ext} is applied in such a manner that it just neutralises this torque and rotates it in the plane of paper from angle θ_0 to angle θ_1 at an infinitesimal angular speed and without angular acceleration. The amount of work done by the external torque will be given by

$$W = \int_{\theta_0}^{\theta_1} \tau_{ext}(\theta)d\theta = \int_{\theta_0}^{\theta_1} pE\sin\theta d\theta$$
$$= pE(\cos\theta_0 - \cos\theta_1) \quad (2)$$

This work is stored as the potential energy of the system. We can then associate potential energy U(θ) with an inclination θ of the dipole. Similar to other potential energies, there is a freedom in choosing the angle where the potential energy U is taken to be zero. A natural choice is to take $\theta_0 = \pi/2$. (An explanation for it is provided towards the end of discussion.) We can then write,

$$U(\theta) = pE\left(\cos\frac{\pi}{2} - \cos\theta\right) = pE\cos\theta = -\mathbf{p}.\mathbf{E} \quad (3)$$

We apply Eq. to the present system of two charges +q and –q. The potential energy expression then reads

$$U'(\theta) = q\left[V(\mathbf{r}_1) - V(\mathbf{r}_2)\right] - \frac{q^2}{4\pi\varepsilon_0 \times 2a} \quad (4)$$

Here, $\mathbf{r}_1$ and $\mathbf{r}_2$ denote the position vectors of +q and -q. Now, the potential difference between positions $\mathbf{r}_1$ and $\mathbf{r}_2$ equals the work done in bringing a unit positive charge against field from $\mathbf{r}_2$ to $\mathbf{r}_1$. The displacement parallel to the force is 2a cosθ. Thus, $[V(\mathbf{r}_1) - V(\mathbf{r}_2)] = -E \times 2a\cos\theta$. We thus obtain,

$$U'(\theta) = -qE\cos\theta - \frac{q^2}{4\pi\varepsilon_0 \times 2a} = -\mathbf{p}.\mathbf{E} - \frac{q^2}{4\pi\varepsilon_0 \times 2a} \quad (5)$$

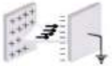
Capacitor

A capacitor is a two-terminal electric component. It has the ability or capacity to store energy in the form of electric charge. Capacitors are also known as Electric-condensers. Capacitors are usually designed to enhance and increase the effect of capacitance. Therefore, they take into account properties like size and shape. The storing capacity of capacitance may vary from small storage to high storage.

Properties of Capacitor or Condenser

i. A capacitor is a device that stores electric energy or a capacitor is a pair of two conductors of any shape, which are close to each other and have equal and opposite charge.

ii. The capacitance of a capacitor is defined as the magnitude of the charge Q on the positive plate divided by the magnitude of the potential difference V between the plates i.e., $C = \frac{Q}{V}$



iii A capacitor get's charged when a battery is connected across the plates. Once capacitor get's fully charged, flow of charge carriers stops in the circuit and in this condition potential difference across the plates of capacitor is same as the potential difference across the terminals of battery.

iv net charge on a capacitor is always zero, but when we speak of the charge Q on a capacitor, we are referring to the magnitude of the charge on each plate.

v. energy stored : When a capacitor is charged by a voltage source (say battery) it stores the electric energy. If C = Capacitance of capacitor ; Q = Charge on capacitor and V = Potential difference across capacitor then energy stored in capacitor

$$\boxed{U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}}$$

In charging capacitor by battery half the energy supplied is stored in the capacitor and remaining half energy $(1/2QV)$ is lost in the form of heat.

Capacitance

Capacitance is nothing but the ability of a capacitor to store the energy in form of electric charge. In other words, the capacitance is the storing ability of a capacitor. It is measured in farads.

Characteristics of capacitor and capacitance

(i) Charge given to a conductor increases it's potential $Q \propto V$

$\Rightarrow Q = CV$

where C is a proportionality constant, called capacity or capacitance of conductor. Hence capacitance is the ability of conductor to hold the charge.

(ii) It's S.I. unit is $\frac{\text{Coulomb}}{\text{Volt}} = \text{Farad}(F)$

Smaller S.I. units are mF, μF and μF (1mF = $10^{-3}F$, 1μF = $10^{-6}F$, 1nF = $10^{-9}F$, 1pF = 1μμF = $10^{-12}F$)

(iii) It's C.G.S. unit is Stat Farad 1F = 9×10^{11} Stat Farad.

(iv) It's dimension : $[C] = [M^{-1}L^{-2}T^4A^2]$.

(v) Capacity of a body is independent of charge given to the body or it's potential raised and decreased depends on shape and size only.

Capacity Of An Isolated Spherical Conductor

When charge Q is given to a spherical conductor of radius R, then potential at the surface of sphere is

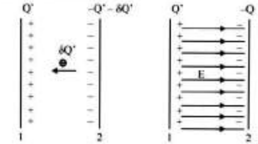
$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \Rightarrow \frac{Q}{V} = 4\pi\varepsilon_0 R$$

$$\boxed{C = 4\pi\varepsilon_0 R = \frac{1}{9\times10^9}.R}$$

If earth is assumed to be a conducting sphere having radius R = 6400 km. It's theoretical capacitance C = 711 μF. But for all practical purpose capacitance of earth is taken infinity and its potential $V = 0$.

Energy Stored In A Capacitor

To calculate the total work done, we first calculate the work done in a small step involving transfer of an infinitesimal (i.e., vanishingly small) amount of charge. Consider the intermediate situation when the conductors 1 and 2 have charges Q' and –Q' respectively. At this stage, the potential difference V' between conductors 1 to 2 is Q'/C, where C is the capacitance of the system. Next imagine that a small charge δQ' is transferred from conductor 2 to 1. Work done in this step (δW), resulting in charge Q' on conductor 1 increasing to Q' + δQ', is given by



(a) (b)

Fig.: (a) Work done in a small step of building charge on conductor 1 from Q' to Q' + δQ'. (b) Total work done in charging the capacitor may be viewed as stored in the energy of electric field between the plates.

$$\delta W = V'\delta Q' = \frac{Q'}{C}\delta Q' \qquad (1)$$

Since $\delta Q'$ can be made as small as we like, Eq. (2.68) can be written as

$$\delta W = \frac{1}{2C}\left[(Q'+\delta Q')^2 - Q'^2\right] \qquad (2)$$

Equations (1) and (2) are identical because the term of second order in $\delta Q'$, i.e., $\delta Q'^2/2C$, is negligible, since $\delta Q'$ is arbitrarily small. The total work done (W) is the sum of the small work (δW) over the very large number of steps involved in building the charge Q' from zero to Q.

$$W = \sum_{\text{sum over all steps}} \delta W$$

$$= \sum_{\text{sum over all steps}} \frac{1}{2C}\left[(Q'+\delta Q')^2 - Q'^2\right] \qquad (3)$$

$$= \frac{1}{2C}\left[\begin{array}{l}\{\delta Q'^2 - 0\} + \{(2\delta Q')^2 - \delta Q'^2\} + \{(3\delta Q')^2 - (2\delta Q')^2\} + ... \\ + \{Q^2 - (Q-\delta Q')^2\}\end{array}\right] \qquad (4)$$

$$= \frac{1}{2C}\left[Q^2 - 0\right] = \frac{Q^2}{2C} \qquad (5)$$

The same result can be obtained directly from Eq. (1) by integration

$$W = \int_0^Q \frac{Q'}{C}\delta Q' = \frac{1}{C}\frac{Q'^2}{2}\Big|_0^Q = \frac{Q^2}{2C}$$

This is not surprising since integration is nothing but summation of a large number of small terms.

We can write the final result, Eq. (5) in different ways

$$W = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \qquad (6)$$

Energy stores in the capacitor

$$= \frac{1}{2}\frac{Q^2}{C} = \frac{(A\sigma)^2}{2} \times \frac{d}{\varepsilon_0 A} \qquad (7)$$

The surface charge density σ is related to the electric field E between the plates.

$$E = \frac{\sigma}{\varepsilon_0} \qquad (8)$$

From Eqs. (7) and (8), we get

Energy stored in the capacitor

$$U = (1/2)\varepsilon_0 E^2 \times Ad \qquad (9)$$

Train Your Brain

Q. Can a sphere of radius 1 cm, placed in air be given a charge of 1 coulomb.

Ans. Potential of sphere of 1 cm radius is

$$V = k\frac{q}{r}$$

$$= 9 \times 10^9 \times \frac{1}{10^{-2}} = 9 \times 10^{11} \text{ volt}$$

This potential is so high that the surrounding air gets ionized, thereby charge leaks to the medium. [air gets ionized at 3×10^6 volt]

Hence charge of 1 coulomb can't be given on it.

Combination of Charged Drops

Suppose we have n identical drops each having Radius (r), Capacitance (c), Charge (q), Potential (v) and Energy (u).

If these drops are combined to form a big drop of Radius (R), Capacitance (C), Charge (Q), Potential (V) and energy (U) then

i. **Charge on big drop :** $Q = nq$

ii. **Radius of big drop :** volume of big drop = n × volume of a single drop i.e.,

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3, R = n^{1/3} r$$

iii. **Capacitance of big drop :** $C = n^{1/3}c$

iv. **Potential of big drop :** $V = \frac{Q}{C} = \frac{nq}{n^{1/3}c} \Rightarrow V = n^{2/3}v$

v. **Energy of big drop :** $U = \frac{1}{2}CV^2 = \frac{1}{2}(n^{1/3}c)(n^{2/3}v)^2$

vi. **Energy difference :** Total energy of big drop is greater than the total energy of all smaller drop. Hence energy difference

$$\Delta U = U - nu = U - n \times \frac{U}{n^{5/3}} = U\left(1 - \frac{1}{n^{2/3}}\right)$$

vii. **Surface charge density of big drop (σ') :** $\sigma = \frac{q}{4\pi r^2}$

$$\therefore \sigma' = \frac{Q}{4\pi R^2} \Rightarrow \sigma' = \frac{nq}{4\pi(n^{1/3}r)^2} = \frac{nq}{4\pi r^2} \times \frac{1}{n^{2/3}} = \frac{\sigma}{n^{2/3}} \cdot n$$

$$\Rightarrow \sigma' = \sigma n^{1/3}$$

Train Your Brain

Q. 64 water drops having equal charges and equal radius combine to form one bigger drop. The capacity of bigger drop, as compared to that of smaller drop will be:

a. 8 times
b. 64 times
c. 4 times
d. 16 times

Ans. (c) $\because \frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$

$$\Rightarrow R = 4r \Rightarrow C \propto R$$

Redistribution of Charges And Loss of Energy

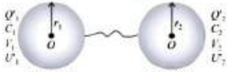
When two charged conductors are joined together through a conducting wire, charge begins to flow from one conductor to another from higher potential to lower potential.

This flow of charge stops when they attain the same potential. Due to flow of charge, loss of energy also takes place in the form of heat through the connecting wire.

Suppose there are two spherical conductors of radii r_1 and r_2, having charge Q_1 and Q_2, potential V_1 and V_2, energies U_1 and U_2 and capacitance C_1 and C_2 respectively.

$Q_1 = C_1V_1$ $\qquad$ $Q_2 = C_2V_2$

If these two spheres are connected through a conducting wire, then alteration of charge, potential and energy takes place.



$Q'_1 = C_1V$ $\qquad$ $Q'_2 = C_2V$

(1) New charge : According to the conservation of charge

$Q_1 + Q_2 = Q'_1 + Q'_2 = Q$ (say), also $\dfrac{Q'_1}{Q'_2} = \dfrac{C_1}{C_2} = \dfrac{r_1}{r_2}$

$\Rightarrow Q'_2 = Q\left[\dfrac{r_2}{r_1 + r_2}\right]$ and similarly $Q'_1 = Q\left[\dfrac{r_1}{r_1 + r_2}\right]$

(2) Common potential : Common potential

$$(V) = \frac{Total\ charge}{Total\ capacity} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{Q'_1 + Q'_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

(3) Energy loss: The loss of energy due to redistribution of charge is given by

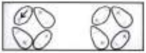
$$\Delta U = U_i - U_f = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

Dielectric

Dielectric are insulating (non-conducting) materials which transmits electric effect without conducting.

Dielectrics are of two types :

(i) Polar dielectrics : A polar molecule has permanent electric dipole moment ($\vec{p}$) in the absence of electric field also. But a polar dielectric has net dipole moment zero in the absence of electric field because polar molecules are randomly oriented as shown in figure.



In the presence of electric field polar molecules tends to line up on the direction of electric field, and the substance has finite dipole moment e.g. water, Alcohol, CO_2, NH_3, HCl etc. are made of polar atoms/molecules.

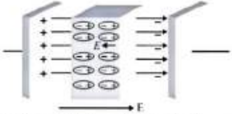
(ii) Non polar dielectric : In non-polar molecules, Each molecule has zero dipole moment in its normal state.

When electric field is applied, molecules becomes induced electric dipole e.g. N_2, O_2, Benzene, Methane etc. are made of non-polar atoms/molecules.

— *KEY NOTE* —

In general, any non-conducting material can be called as a dielectric but broadly non-conducting material having non-polar molecules are referred to as dielectric.

Polarization of a dielectric slab : It is the process of inducing equal and opposite charges on the two faces of the dielectric on the application of electric field.



(a) Electric field between the plates in the presence of dielectric medium is $E' = E - E_i$ where E = Main field, E' = Induced field.

(b) dielectric constant of dielectric medium is defined as

$$\frac{E}{E'} = \frac{\text{Electric field between the plate in vaccum / air}}{\text{Electric field between the plates with medium}} = K$$

(c) K is also known as relative permittivity (ε_r) of the material

Dielectric breakdown and dielectric strength: If a very high electric field is created in a dielectric. The dielectric then behaves like a conductor. This phenomenon is known as **dielectric breakdown.**

The maximum value of electric field (or potential gradient) that a dielectric material can tolerate without it's electric breakdown is called it's **dielectric strength.**

S.I. unit of dielectric strength of a material is $\dfrac{V}{m}$ but practical unit is $\dfrac{kV}{mm}$

Effect of Dielectric on Capacitance

As before, we have two large plates, each of area A, separated by a distance d. The charge on the plates is ±Q, corresponding to the charge density ±σ (with σ = Q/A). When there is vacuum between the plates.

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

and the potential difference V_0 is

$$V_0 = E_0 d$$

The capacitance C_0 in this case is

$$C_0 = \frac{Q}{V_0} = \varepsilon_0 \frac{A}{d} \quad (1)$$

Consider next a dielectric inserted between the plates fully occupying the intervening region. The dielectric is polarised by the field the effect is equivalent to two charged sheets (at the surfaces of the dielectric normal to the field) with surface charge densities σ_p and $-\sigma_p$. The electric field in the dielectric then corresponds to the case when the net surface charge density on the plates is $\pm(\sigma - \sigma_p)$. That is,

$$E = \frac{\sigma - \sigma_p}{\varepsilon_0} \quad (2)$$

so that the potential difference across the plates is

$$V = Ed = \frac{\sigma - \sigma_p}{\varepsilon_0} d \quad (3)$$

For linear dielectries, we expect σ_p to be proportional to E_0, i.e., to σ. Thus, $(\sigma - \sigma_p)$ is proportional to σ and we can write

$$\sigma - \sigma_p = \frac{\sigma}{K} \quad (4)$$

where K is a constant characteristic of the dielectric. Clearly, $K > 1$. We

then have

$$V = \frac{\sigma d}{\varepsilon_0 K} = \frac{Qd}{A\varepsilon_0 K} \quad (5)$$

The capacitance C, with dielectric between the plates, is then

$$C = \frac{Q}{V} = \frac{\varepsilon_0 KA}{d} \quad (6)$$

The product $\varepsilon_0 K$ is called the permittivity of the medium and is denoted by ε

$$\varepsilon = \varepsilon_0 K \quad (7)$$

For vacuum $K = 1$ and $\varepsilon = \varepsilon_0$; ε_0 is called the *permittivity of the vacuum*.

The dimensionless ratio

$$K = \frac{\varepsilon}{\varepsilon_0} \quad (8)$$

is called the dielectric constant of the substance. As remarked before,

from Eq. (4), it is clear that K is greater than 1. From Eqs. (1) and (6)

$$K = \frac{C}{C_0} \quad (9)$$

The dielectric constant of a substance is the factor (>1) by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor

Capacity of Various Capacitor

Parallel plate capacitor

It consists of two parallel metallic plates (may be circular, rectangular, square) separated by a small distance. If A = Effective overlapping area of each plate.

(i) Electric field between the plates : $\boxed{E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}}$

(ii) Potential difference between the plates : $V = E \times d = \frac{\sigma d}{\varepsilon_0}$

(iii) Capacitance : $\boxed{C = \frac{\varepsilon_0 A}{d}}$

(iv) If a dielectric medium of dielectric constant K is filled completely between the plates then capacitance increases by K times i.e. $\boxed{C' = \frac{K\varepsilon_0 A}{d}} \Rightarrow C' = KC$

(v) The capacitance of parallel plate capacitor depends on A $(C \propto A)$ and d $\left(C \propto \frac{1}{d}\right)$. It does not depend on the charge on the plates or the potential difference between the plates.

(vi) If a dielectric slab is partially filled between the plates

$$\Rightarrow \boxed{C' = \frac{\varepsilon_0 A}{\left(d - t + \frac{t}{K}\right)}}$$

Capacitance of a Parallel Plate Capacitor With a Dielectric Slab

The capacitance of a parallel plate capacitor of plate area A and plate separation d with vacuum/air in between is

$$C_0 = \frac{\varepsilon_0 A}{d}$$

Suppose + Q are the charges on the capacitor plate which produce a uniform electric field E_0 in the space between the plates, Fig.

When a dielectric slab of thickness $t < d$ is introduced between the plates, the molecules in the slab get polarized in the direction of E_0 induces an electric field E_p opposite of E_0. Therefore the effective field inside the dielectric is $E = E_0 - E_p$.

Outside the dielectric, field remains E_0 only. Therefore potential difference between the two plates is $V = E_0(d - t) + Et$

But $\frac{E_0}{E} = \epsilon_r$ or K $\therefore E = \frac{E_0}{K}$ $\therefore V = E_0(d - t) + \frac{E_0}{K}t$

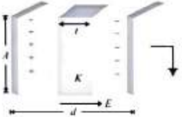
$$V = E_0\left[d - t + \frac{t}{K}\right]$$

As $\quad E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0} \quad \therefore V = \frac{Q}{A\epsilon_0}\left[d - t + \frac{t}{K}\right]$

$\therefore$ Capacitance of the capacitor with dielectric inbetween is

$$C = \frac{Q}{V} = \frac{A\epsilon_0}{d - t + \frac{t}{K}} = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}, \text{ i.e., } \quad C = \frac{\epsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

Clearly, $C > C_0$, i.e., on introduction of a dielectric slab inbetween the plates of a capacitor, the capacitance increases.



(vii) If a number of dielectric slabs are inserted between the plate as shown.

$$C' = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + t_3 + \ldots\ldots) + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} + \ldots\ldots\right)}$$

(viii) When a metallic slab is inserted between the plates

$$C' = \frac{\varepsilon_0 A}{(d - t)}$$

If metallic slab fills the complete space between the plates (i.e. $t = d$) or both plates are joined through a metallic wire then capacitance becomes infinite.

(ix) Force between the plates of a parallel plate capacitor.

$$|F| = \frac{\sigma^2 A}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A} = \frac{CV^2}{2d}$$

(x) Energy density between the plates of a parallel plate capacitor

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \boxed{\frac{1}{2}\varepsilon_0 E^2}$$

Train Your Brain

Q. If $50\mu F$ be the capacity of a capacitor in air, and $110\mu F$ in oil then the dielectric constant of oil will be

a. 0.45 b. 0.55

c. 1.10 d. 2.20

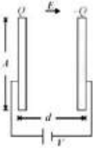
Ans. (d) $C \propto K \Rightarrow \frac{C_1}{C_2} = \frac{K_1}{K_2}$

$$\Rightarrow \frac{50 \times 10^{-6}}{110 \times 10^{-6}} = \frac{1}{K_2} \text{ [for air } K_1 = 1]$$

$$\Rightarrow K_2 = 2.20$$

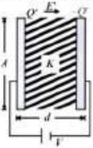
Variation of different variable (Q, C, V, E and U) of parallel plate capacitor when dielectric is introduced

Initial Condition



Battery is Removed

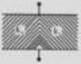
Battery Remains connected



	Battery is Removed	Battery Remains connected
Capacity	$C' = KC$	$C' = KC$
Charge	$Q' = Q$	$Q' = KQ$
Potential	$V' = V/K$	$V' = V$
Electric field	$E' = E/K$	$E' = E$
Energy	$U' = U/K$	$U' = KU$

Train Your Brain

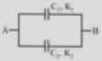
Q. Two materials of dielectric constant k_1 and k_2 are filled between two parallel plates of a capacitor as shown in figure. The capacity of the capacitor is:



a. $\dfrac{A\epsilon_0(k_1+k_2)}{2d}$ b. $\dfrac{2A\epsilon_0}{d}\left(\dfrac{k_1k_2}{k_1+k_2}\right)$

c. $\dfrac{A\epsilon_0}{d}\left(\dfrac{k_1k_2}{k_1+k_2}\right)$ d. $\dfrac{A\epsilon_0}{2d}\left(\dfrac{k_1+k_2}{k_1k_2}\right)$

Ans. (a) Two capacitors are connected in parallel combination.



$$C_1=\frac{\epsilon_0 K_1 A/2}{d}=\frac{\epsilon_0 K_1 A}{2d}$$

$$C_2=\frac{\epsilon_0 K_2 A}{2d}$$

$$\therefore C_{AB}=C_1+C_2=\frac{\epsilon_0 A}{2d}(K_1+K_2)$$

Train Your Brain

Q. The radii of two metallic spheres are 5 cm and 10 cm and both carry equal charge of 75μC. If the two spheres are shorted then charge will be transferred

a. 25 μC from smaller to bigger

b. 25 μC from bigger to smaller

c. 50 μC from smaller to bigger

d. 50 μC from bigger to smaller

Ans. (a) $Q_1=Q_1+Q_2=150\mu C$

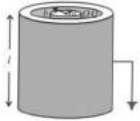
$$\frac{Q'1}{Q'2}=\frac{C_1}{C_2}=\frac{1}{2}\quad\Rightarrow Q1'=50\mu C$$

$$Q_2'=100\mu C$$

25 μC Charge will flow from smaller to bigger sphere.

Cylindrical Capacitor

It consists of two co-axial cylinders of radii a and b $(a<b)$, inner cylinder is given charge $+Q$ while outer cylinder is earthed. Common length of the cylinders is ℓ then $C=\dfrac{2\pi\varepsilon_0\ell}{\log_e\left(\dfrac{b}{a}\right)}$



Spherical capacitor

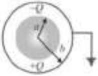
It consists of two concentric conducting spheres of radii a and b $(a<b)$. Inner sphere is given charge $+Q$, while outer sphere is earthed

(i) Potential difference : Between the spheres is

$$V=\frac{Q}{4\pi\varepsilon_0 a}-\frac{Q}{4\pi\varepsilon_0 b}$$

(ii) Capacitance : $C=4\pi\varepsilon_0\left(\dfrac{ab}{b-a}\right)$

(iii) If outer sphere is given a charge $+Q$ while inner sphere is earthed Induced charge on the inner sphere $Q'=-\dfrac{a}{b}Q$ and capacitance of the system $C'=4\pi\varepsilon_0\cdot\dfrac{b^2}{b-a}$



This arrangement is not a capacitor. But it's capacitance is equivalent to the sum of capacitance of spherical capacitor and spherical conductor i.e. $4\pi\varepsilon_0\cdot\dfrac{b^2}{b-a}=4\pi\varepsilon_0\dfrac{ab}{b-a}+4\pi\varepsilon_0 b$

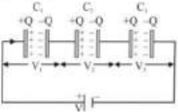
Grouping of capacitors

Series grouping

(i) Charge on each capacitor remains same and equals to the main charge supplied by the battery but potential difference distributes i.e. $V=V_1+V_2+V_3$

(ii) Equivalent capacitance

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$



(iii) In series combination potential difference and energy distributes in the reverse ratio of capacitances i.e.

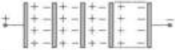
$V \propto \frac{1}{C}$ and $U \propto \frac{1}{C}$

(iv) If two capacitors having capacitances C_1 and C_2 are connected in series then $C_{eq} = \frac{C_1C_2}{C_1+C_2}$

$V_1 = \left(\frac{C_2}{C_1+C_2}\right)V$ and $V_2 = \left(\frac{C_1}{C_1+C_2}\right)V$

(v) If n identical capacitors each having capacitances C are connected in series with supply voltage V then Equivalent capacitance $C_{eq} = \frac{C}{n}$ and potential difference across each capacitor $V' = \frac{V}{n}$

(vi) If n identical plates are arranged as shown below, they constitute $(n-1)$ capacitors in series. If each capacitor has capacitance $\frac{\varepsilon_0 A}{d}$ then $C_{eq} = \frac{\varepsilon_0 A}{(n-1)d}$



In this situation except two extreme plates each plate is common to adjacent capacitor.

(vii) Here, effective capacitance C_{eq} is even less than the least of the individual capacitance.

Train Your Brain

Q. Two condensers C_1 and C_2 in a circuit are joined as shown in fig. The potential of point A is V_1 and that of B is V_2. The potential of point D will be

a. $\frac{1}{2}(v_1+v_2)$ b. $\frac{C_2V_1+C_1V_2}{C_1+C_2}$

c. $\frac{C_1V_1+C_2V_2}{C_1+C_2}$ d. $\frac{C_2V_1-C_1V_2}{C_1+C_2}$

Ans. (c) Q same r $Q = C_1V_{AD} = C_1(V_1 - V_D)$

and $Q = C_2V_{DB} = C_2(V_D - V_2)$

$C_1(V_1 - V_D) = C_2(V_D - V_2)$

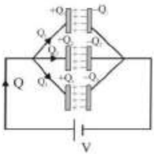
$C_1V_1 + C_2V_2 = (C_1 + C_2)V_D$

$V_D = \frac{C_1V_1 + C_2V_2}{C_1+C_2}$

Parallel Grouping

(i) Potential difference across each capacitor remains same and equal to the applied potential difference but charge distributes

i.e., $Q = Q_1 + Q_2 + Q_3$



(ii) $C_{eq} = C_1 + C_2 + C_3$

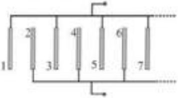
(iii) In parallel combination charge and energy distributes in the ratio of capacitance i.e. $Q \propto C$ and $U \propto C$

(iv) If two capacitors having capacitance C_1 and C_2 respectively are connected in parallel then $C_{eq} = C_1 + C_2$

$Q_1 = \left(\frac{C_1}{C_1+C_2}\right)Q$ and $Q_2 = \left(\frac{C_2}{C_1+C_2}\right)Q$

(v) If n identical capacitors are connected in parallel, then Equivalent capacitance $C_{eq} = nc$ and Charge on each capacitor $Q' = \frac{Q}{n}$

If n identical plates are arranged such that even numbered plates are connected together and odd numbered plates are connected together, then $(n-1)$ capacitors will be formed and they will be in parallel grouping.



Equivalent capacitance $C' = (n-1)C$

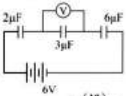
where C = capacitance of a capacitor $= \frac{\varepsilon_0 A}{d}$

— KEY NOTE —

If C_p is the effective capacity when n identical capacitors are connected in parallel and C_s is their effective capacity when connected in series, then $\frac{C_p}{C_s} = n^2$

Train Your Brain

Q. In the combination shown in the figure, the ideal voltmeter reading will be



a. 4.5 V b. $\left(\frac{18}{11}\right)V$

c. 3 V d. 2 V

Ans. (d) $\frac{1}{2} = \left(\frac{1/C_2}{1/C_1+1/C_2+1/C_3}\right)^{V_0}$

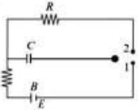
$= \left(\frac{1/3}{1/2+1/3+1/6}\right)^{6}$

$= \frac{(1/3)}{\left(\frac{3+2+1}{6}\right)}.6 = 2V$

Charging And Discharging

Charging of a condenser:

(i) In the following circuit. If key 1 is closed then the condenser gets charged. Finite time is taken in the charging process. The quantity of charge at any instant of time t is given by $q = q_0 [1 - e^{-(t/RC)}]$



Where q_0 = maximum final value of charge at $t = \infty$.

According to this equations the quantity of charge on the condenser increases exponentially with increase of time.

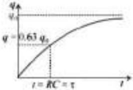
(ii) If $t = RC = \tau$ then

$q = q_0 [1 - e^{-(RC/RC)}]$

$= q_0 \left[1 - \frac{1}{e}\right]$

or $q = q_0 (1 - 0.37) = 0.63\, q_0$

$= 63\%$ of q_0



(iii) Time $t = RC$ is known as time constant.

i.e. the time constant is that time during which the charge rises on the condenser plates to 63% of its maximum value.

(iv) The potential difference across the condenser plates at any instant of time is given by

$V = V_0 [1 - e^{-(t/RC)}]$ volt

(v) The potential curve is also similar to that of charge. During charging process an electric current flows in the circuit for a small interval of time which is known as the transient current. The value of this current at any instant of time is given by

$I = I_0 [e^{-(t/RC)}]$ ampere

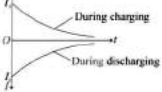
According to this equation the current falls in the circuit exponentially.

(vi) If $t = RC = \tau$ = Time constant

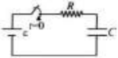
$I = I_0 e^{(-RC/RC)} = \frac{I_0}{e} = 0.37\, I_0$

$= 37\%$ of I_0

i.e., time constant is that time during which current in the circuit falls to 37% of its maximum value.

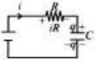


Derivation of formulae for charging of capacitor



It is given that initially capacitor is uncharged.

let at any time



Applying kirchoff voltage law

$\varepsilon - iR - \frac{q}{C} = 0 \quad \Rightarrow \quad iR = \frac{\varepsilon C - q}{C}$

$i = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{dq}{dt} = \frac{\varepsilon C - q}{CR}$

$\frac{dq}{dt} = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{CR}{\varepsilon C - q} . dq = dt.$

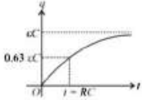
$\int_0^q \frac{dq}{\varepsilon C - q} = \int_0^t \frac{dt}{RC} \quad \Rightarrow \quad -\ln(\varepsilon C - q) + \ln \varepsilon C = \frac{t}{RC}$

$$\ln \frac{\varepsilon C}{\varepsilon C - q} = \frac{t}{RC}$$

$$\varepsilon C - q = \varepsilon C \cdot e^{-t/RC}$$

$$q = \varepsilon C(1 - e^{-t/RC})$$

RC = time constant of the RC series circuit.



After one time constant

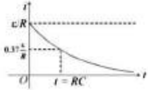
$$q = \varepsilon C\left(1 - \frac{1}{e}\right)$$

$$= \varepsilon C\,(1 - 0.37) = 0.63\,\varepsilon C$$

Current at any time *t*

$$i = \frac{dq}{dt} = \varepsilon C\left(-e^{-t/RC}\left(-\frac{1}{RC}\right)\right)$$

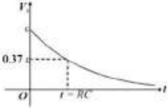
$$= \frac{\varepsilon}{R} e^{-t/RC}$$



Voltage across capacitor after one time constant $V = 0.63\,\varepsilon$

$Q = CV$

$V_C = \varepsilon(1 - e^{-t/RC})$



Voltage across the resistor

$V_R = iR$

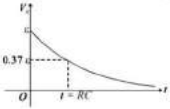
$= \varepsilon e^{-t/RC}$

By energy conservation,

Heat dissipated = work done by battery − ΔUcapacitor

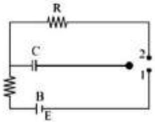
$$= C\varepsilon(\varepsilon) - \left(\frac{1}{2}C\varepsilon^2 - 0\right)$$

$$= \frac{1}{2}C\varepsilon^2$$



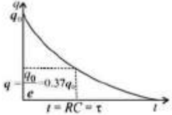
Discharging of a condenser:

(i) In the given circuit if key 1 is opened and key 2 is closed then the condenser gets discharged.



(ii) The quantity of charge on the condenser at any instant of time t is given by $q = q_0 e^{-(t/RC)}$

i.e. the charge falls exponentially.



(iii) If $t = RC = \tau$ = time constant, then

$$q = \frac{q_0}{e} = 0.37q_0 = 37\% \text{ of } q_0$$

i.e. the time constant is that time during which the charge on condenser plates discharge process falls to 37%.

(iv) The dimensions of RC are those of time i.e. $M^0L^0T^1$ and the dimensions of $\frac{1}{RC}$ are those of frequency i.e. $M^0L^0T^{-1}$.

(v) The potential difference across the condenser plates at any instant of time t is given by

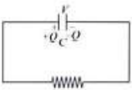
$V = V_0 e^{-(t/RC)}$ Volt.

— KEY NOTE —

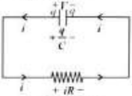
The transient current at any instant of time is given by $I = -I_0 e^{-(t/RC)}$ ampere.

i.e., the current in the circuit decreases exponentially but its direction is opposite to that of charging current.

Derivation of equation of discharging circuit :



Initially



At any time t

Applying K.V.L.

$$+\frac{q}{C} - iR = 0$$

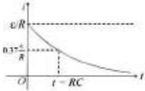
$$i = \frac{q}{CR}$$

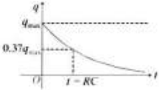
$$\int_Q^q \frac{-dq}{q} = \int_0^t \frac{dt}{CR}$$

$$-\ln\frac{q}{Q} = +\frac{t}{RC}$$

$$q = Qe^{-t/RC}$$

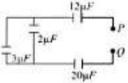
$$i = -\frac{dq}{dt} = \frac{Q}{RC}e^{-t/RC}$$





Train Your Brain

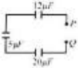
Q. In the circuit diagram shown in the adjoining figure, the resultant capacitance between P and Q is:



a. 47 μF b. 3 μF

c. 60 μF d. 10 μF

Ans. (b) The given circuit can be drawn as



Where $C = (3+2)\ \mu F = 5\mu F$

$$\frac{1}{C_{PQ}} = \frac{1}{5} + \frac{1}{20} + \frac{1}{12} = \frac{20}{60} = \frac{1}{3}$$

$$\Rightarrow C_{PQ} = 3\ \mu F$$

Q. A capacitor is connected to a 12 V battery through a resistance of 10Ω. It is found that the potential difference across the capacitor rises to 4.0 V in 1μs. Find the capacitance of the capacitor.

Ans. The charge on the capacitor during charging is given by

$Q = Q_0(1 - e^{-t/RC})$.

Hence, the potential difference across the capacitor is

$V = Q/C = Q_0/C\,(1 - e^{-t/RC})$.

Here, at $t = 1\ \mu s$, the potential difference is $4V$ whereas the steady potential difference is

$Q_0/C = 12V$. So, $\Rightarrow$ $4V = 12V(1 - e^{-t/RC})$

or, $1 - e^{-t/RC} = \frac{1}{3}$ or, $e^{-t/RC} = \frac{2}{3}$

or, $\frac{t}{RC} = \ln\left(\frac{3}{2}\right) = 0.405$

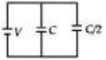
or, $RC = \frac{t}{0.405} = \frac{1\ \mu s}{0.45} = 2.469\ \mu s$

or, $C = \frac{2.469\mu s}{10\Omega} = 0.25\ \mu F$.

1. Electric potential is:
   a. Scalar and dimensionless quantity
   b. Vector and dimensionless quantity
   c. Scalar and dimensional quantity
   d. Vector and dimensional quantity

Ans. (c) Electric potential is a scalar and dimensional quantity.

2. Two condensers, one of capacity C and other of capacity $C/2$ are connected to a V volt battery, as shown in the figure. The work done in charging fully both the condensers is:



a. $\frac{1}{4}CV^2$ b. $\frac{3}{4}CV^2$

c. $\frac{1}{2}CV^2$ d. $2\ CV^2$

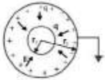
Ans. (b) As the capacitors are connected in parallel, therefore potential difference across both the condensers remains the same.

$\therefore q_1 = CV,\ q_2 = \frac{C}{2}V$ Also $q = q_1 + q_2 = CV + \frac{C}{2}V = \frac{3}{2}CV$

Work done in charging fully both the condensers is given by

$$W = \frac{1}{2}qV = \frac{1}{2}\times\left(\frac{3}{2}CV\right)V = \frac{3}{4}CV^2$$

3. A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports as shown in figure. The capacitance C, of this spherical capacitor is:



a. $\frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$ b. $\frac{4\pi\varepsilon_0 (r_2 - r_1)}{r_1 r_2}$

c. $\frac{r_1 r_2}{4\pi\varepsilon_0 (r_2 - r_1)}$ d. $\frac{(r_1 - r_2)}{4\pi\varepsilon_0 r_1 r_2}$

Ans. (a) As shown in figure, +q charge spreads uniformly on inner surface of outer sphere of radius r_1. The induced charge -q spreads uniformly on the outer surface of inner sphere of radius r_2.

The outer surface of outer sphere is earthed. Due to electrostatic shielding E = 0 for $r < r_2$ and E = 0 for $r > r_1$.

In the space between the two spheres,

Potential difference between two spheres,

$$V = \frac{q}{4\pi\varepsilon_0 r_2} - \frac{q}{4\pi\varepsilon_0 r_1} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right];$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_1 - r_2}{r_1 r_2}\right) \qquad \text{...... (i)}$$

Also $C = \frac{q}{V}\ \therefore C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$ (using (i))

4. A large solid sphere with uniformly distributed positive charge has a smooth narrow tunnel through its centre. A small particle with negative charge, initially at rest far from the sphere, approaches it along the line of the tunnel, reaches its surface with a speed v, and passes through the tunnel. Its speed at the centre of the sphere will be
   a. 0 b. v c. $\sqrt{2}v$ d. $\sqrt{1.5}v$

Ans. (d) Potential at $\infty = V_\infty = 0$, Potential at the surface of the sphere $= V_s = k\frac{Q}{R}$

Potential at the centre of the sphere $= V_c = \frac{3}{2}k\frac{Q}{R}$

Let m and -q be the mass and the charge of the particle respectively. Let v_0 = speed of the particle at the centre of the sphere

$$\frac{1}{2}mv^2 = -q[V_\infty - V_s] = qk\frac{Q}{R}$$

$$\frac{1}{2}mv_0^2 = -q[V_\infty - V_c] = q.\frac{3}{2}k\frac{Q}{R}$$

Dividing, $\frac{v_0^2}{v^2} = \frac{3}{2} = 1.5$ or $v_0 = \sqrt{1.5}\,v$

5. If the earth's surface is treated as a conducting surface with some charge, what should be the order of magnitude of the charge per unit area, in C/m^2, so that a proton remains suspended in space near the earth's surface?
   a. 10^{-18} b. 10^{-12} c. 10^{-6} d. 1

Ans. (a) Let σ = charge per unit area on the earth's surface

Then, the field near the surface $= E = \frac{\sigma}{\varepsilon_0}$

The upward force on a proton $= eE = \frac{e\sigma}{\varepsilon_0}$

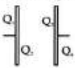
The downward force, for equilibrium $mg = \frac{e\sigma}{\varepsilon_0}$ or

$$\sigma = \frac{\varepsilon_0 mg}{e}$$

$$\sigma = \frac{8.85\times10^{-12}\times1.67\times10^{-27}\times10}{1.6\times10^{-19}}C/m^2$$

$= 0.92\times10^{-18}$ cm $\sim 10^{-18}\ C/m^2$

6. In an isolated parallel-plate capacitor of capacitance C, the four surfaces have charges Q_1, Q_2, Q_3 and Q_4, as shown. The potential difference between the plates is:



a. $\frac{Q_1+Q_2+Q_3+Q_4}{2C}$ b. $\frac{Q_2+Q_3}{2C}$

c. $\frac{Q_2-Q_3}{2C}$ d. $\frac{Q_1+Q_4}{2C}$

Ans. (c) Plane conducting surfaces facing each other must have equal and opposite charge densities. Here, as the plate areas are equal, $Q_2 = -Q_3$.

The charge on a capacitor means the charge on the inner surface of the positive plate - in this case, Q_2.

Potential difference between the plates = charge on the capacitor / capacitance.

$\therefore$ potential difference $= \frac{Q_2}{C} = \frac{2Q_2}{2C} = \frac{Q_2-(-Q_3)}{2C} = \frac{Q_2-Q_3}{2C}$

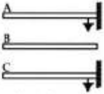
7. Two points are at distances a and b (a < b) from a long string of charge per unit length λ. The potential difference between the points is proportional to

a. b/a b. b^2/a^2 c. $\sqrt{b/a}$ d. ln (b/a)

Ans. (d) $E = \frac{\lambda}{2\pi\varepsilon_0 r} = -\frac{\partial V}{\partial r}$

or $\int_{V_b}^{V_a} dV = -\int_b^a \frac{\lambda}{2\pi\varepsilon_0}\cdot\frac{dr}{r}$ or $V_a - V_b = \frac{\lambda}{2\pi\varepsilon_0}\times \ln\left(\frac{b}{a}\right)$.

8. A, B and C are three large, parallel conducting plates, placed horizontally. A and C are rigidly fixed and earthed. B is given some charge. Under electrostatic and gravitational forces, B may be:



a. In equilibrium midway between A and C
b. In equilibrium if it is closer to A than to C
c. In equilibrium if it is closer to C than to A
d. B can never be in stable equilibrium

Ans. (b,d) As A and C are earthed, they are connected to each other. Hence, 'A + B' and 'B + C' are two capacitors with the same potential difference. If B is closer to A than to C then the capacitance C_{AB} is > C_{BC}. The upper surface of B will have greater charge than the lower surface. As the force of attraction between the plates of a capacitor is proportional to Q^2, there will be a net upward force on B. This can balance its weight.

9. A conducting sphere of radius R, carrying charge Q, lies inside an uncharged conducting shell of radius 2R. If they are joined by a metal wire:

a. Q/3 amount of charge will flow from the sphere to the shell

b. 2Q/3 amount of charge will flow from the sphere to the shell

c. Q amount of charge will flow from the sphere to the shell

d. $k\frac{Q^2}{4R}$ amount of heat will be produced

Ans. (c,d) The capacitances of the two are $C_1 = 4\pi\varepsilon_0 R$ and $C_2 = 4\pi\varepsilon_0(2R)$

The initial energy, $E_i = \frac{Q^2}{2C_1}$. The final energy, $E_f = \frac{Q^2}{2C_2}$

The heat produced $= E_i - E_f = \frac{Q^2}{2}\left[\frac{1}{4\pi\varepsilon_0 R} - \frac{1}{2\times 4\pi\varepsilon_0 R}\right]$

$$= k\frac{Q^2}{2R}\left[1-\frac{1}{2}\right] = \frac{kQ^2}{4R}$$

10. A parallel-plate air capacitor has capacity C. A dielectric slab of dielectric constant K, whose thickness is half of the air gap between the plates, is now inserted between the plates. The capacity of the plate will now be:

a. KC/2 b. 2KC/(K + 1)

c. 2KC/(K – 1) d. C(K + 1)/(K – 1)

Ans. (b) $C = \varepsilon_0 A/d$

Let capacity with dielectric be C'

$\sigma = \frac{Q}{A}$ $V = \frac{\sigma}{\varepsilon_0}\cdot\frac{d}{2} + \frac{\sigma}{K\varepsilon_0}\cdot\frac{d}{2} = \frac{Q}{A}\frac{d}{2\varepsilon_0}\left(1+\frac{1}{K}\right)$

$$C' = \frac{Q}{V} = \frac{2\varepsilon_0 A}{d\left(1+\frac{1}{K}\right)} = C.\frac{2K}{K+1}$$

11. A simple pendulum has time period T. Charges are now fixed at the point of suspension of the pendulum and on the bob. If the pendulum continues to oscillate, its time period will now be:

a. Greater than T
b. Equal to T
c. Less than T
d. Either (a) or (c) depending on whether the charges attract or repel each other

Ans. (b) The additional electrostatic force acts along the string. This does not have a component in the direction of motion of the bob.

12. A capacitor of capacitance C_1 is charged to a potential V and then connected in parallel to an uncharged capacitor of capacitance C_2. The final potential difference across each capacitor will be:

a. $\frac{C_1 V}{C_1 + C_2}$ b. $\frac{C_2 V}{C_1 + C_2}$

c. $1 + \frac{C_2}{C_1}$ d. $1 - \frac{C_2}{C_1}$

Ans. (a) $Q_1 = C_1 V, Q_2 = 0$

When the 2 capacitors are connected in parallel, then final charge on capacitor C_1 is $Q_1 = C_1 V_C$. Similarly, $Q_2' = C_2 V_C$ where V_C is the common potential of C_1 and C_2.

$\therefore$ Charge is conserved $\therefore Q_1 + Q_2 = Q_1' + Q_2'$

$$\Rightarrow C_1 V = C_1 V_C + C_2 V_C \Rightarrow V_C = \frac{C_1 V}{C_1 + C_2}$$

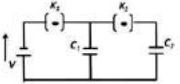
13. A parallel plate capacitor has a uniform electric field E in the space between the plates. If the distance between the plates is d and area of each plate is A, the energy stored in the capacitor is:

a. $\frac{1}{2}\varepsilon_0 E^2$ b. $\frac{E^2 Ad}{\varepsilon_0}$

c. $\frac{1}{2}\varepsilon_0 E^2 Ad$ d. $\varepsilon_0 E^2 Ad$

Ans. (c) Energy, $E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{A\varepsilon_0}{d}(Ed)^2$

14. In the given circuit, initially K_1 is closed and K_2 is open. The K_1 is opened and K_2 is closed. If q_1' and q_2' are charges on C_1 and C_2 and V_1 and V_2 are the voltages respectively, then:



a. Charge on C_1 gets redistributed such that $V_1 = V_2$
b. Charge on C_1 gets redistributed such that $q_1' = q_2'$
c. Charge on C_1 gets redistributed such that $C_1 V_1 = C_2 V_2$
d. Charge on C_1 gets redistributed such that $q_1' + q_2' = 2q$

Ans. (a) The charge on capacitor C_1 gets redistributed assuming C_1 & C_2, such that $V_1 = V_2$.

15. Two spherical conductors each of capacity C are charged to potential V and –V. These are then connected by means of a fine wire. The loss of energy is:

a. Zero b. $\frac{1}{2}CV^2$

c. CV^2 d. $2CV^2$

Ans. (c) Loss of energy $= \frac{1}{2}\frac{C_1 C_2}{(C_1 + C_2)} \times (V_1 - V_2)^2$

$$= \frac{1}{2} \times \frac{C^2}{2C} \times (2V)^2 = CV^2$$

16. The work done in carrying a charge q once around a circle with a charge Q at its centre is:

a. $\frac{qQ}{4\pi\varepsilon_0 a}$ b. $\frac{qQ}{4\pi\varepsilon_0 a^2}$

c. $\frac{q}{4\pi\varepsilon_0 a}$ d. Zero

Ans. (d) Work done $= q\,\Delta V$

$\because$ The initial and final position around the circle are same

$\therefore$ change in potential, $\Delta V = 0$

$\therefore W = 0$

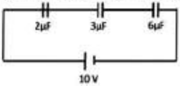
17. When air is replaced by a dielectric medium of constant K, the maximum force of attraction between two charges separated by a distance:
a. Increases K times
b. Remains unchanged
c. Decreases K times
d. Increases K^{-1} times

Ans. (c) $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

$$F' = \frac{1}{4\pi\varepsilon_m}\frac{q_1 q_2}{r^2}$$

$$\therefore \frac{F'}{F} = \frac{\varepsilon_0}{\varepsilon_m} = \frac{1}{K} \Rightarrow F' = \frac{F}{K}$$

18. The charge on 3 µF capacitor shown in the figure is:



a. 2 µC b. 10 µC
c. 6 µC d. 8 µC

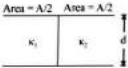
Ans. (b) $\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{6}{6} = 1$

$\Rightarrow C_{eq} = 1\mu F$

$\therefore Q = CV = 10\mu C$

The capacitors are connected in series $\therefore$ same charge will be stored in the 3 capacitors. $\therefore$ charge on $3\mu F$ capacitor $= 10\mu C$.

19. Two materials of dielectric constants κ_1 and κ_2 are filled between two parallel plates of a capacitor as shown in figure:



The capacitance of the capacitor is:

a. $\dfrac{A\varepsilon_0(\kappa_1 + \kappa_2)}{2d}$ b. $\dfrac{A\varepsilon_0}{2d}\left(\dfrac{\kappa_1 + \kappa_2}{\kappa_1\kappa_2}\right)$

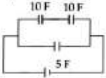
c. $\dfrac{A\varepsilon_0}{d}\left(\dfrac{\kappa_1\kappa_2}{\kappa_1 + \kappa_2}\right)$ d. $\dfrac{2A\varepsilon_0}{d}\left(\dfrac{\kappa_1\kappa_2}{\kappa_1 + \kappa_2}\right)$

Ans. (a) The arrangement is equivalent to a parallel combination of two capacitors, each with plate area $A/2$ and separation d. Total capacitance is

$$C = C_1 + C_2 = \frac{\varepsilon_0(A/2)\kappa_1}{d} + \frac{\varepsilon_0(A/2)\kappa_2}{d}$$

$$= \frac{\varepsilon_0 A(\kappa_1 + \kappa_2)}{2d}.$$

20. The equivalent capacitance of the following combination is:



a. $10\ \mu F$ b. $4\ \mu F$

c. $25\ \mu F$ d. $15\ \mu F$

Ans. (a) Both $10\mu F$ capacitances are in series. Hence, equivalent capacitance can be calculated as that is,

$$\frac{1}{C_1} = \frac{1}{10} + \frac{1}{10} = \frac{1}{5} \Rightarrow C_1 = 5\ \mu F$$

Now, $C_1 = 5\mu F$ and $C_2 = 5\mu F$, both are in parallel

Hence, equivalent capacitance,

$$C = C_1 + C_2 = 5 + 5 = 10\ \mu F$$