EDUCATALYSTS

Class(12th)

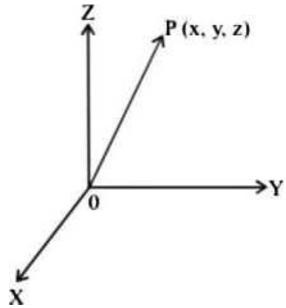
Introduction to Vector Algebra

1. **Vector -** A vector is a quantity having both magnitude and direction, such as displacement, velocity, force and acceleration. AB is a directed line segment. It is a vector AB and its direction is from A to B. A — B

Initial Points - The point A where from the vector AB starts is known as initial point. **Terminal Point -** The point B, where it ends is said to be the terminal point.

Magnitude - The distance between initial point and terminal point of a vector is the magnitude or length of the vector AB. It is denoted by | AB | or AB.

2. **Position Vector -** Consider a point p **(x,** y, z) in space. The vector OP with initial point, origin O and terminal point R is called



the position vector of P.

3. **TX pes of Vectors**

(i) **Zero Vector Or Null Vector -** A vector whose initial and terminal points coincide is known as zero vector (o).

(ii) **Unit Vector - A** vector whose magnitude is unity is said to be unit vector. It is denoted as a so that |a| = l.

(iii) **Co-initial Vectors -** Two or more vectors having the same initial point are called co-initialvectors.

(iv) **Collinear Vectors -** If two or more vectors are parallel to the same line, such vectors are known as collinear vectors.

(v) **Equal Vectors -** If two vectors a and b have the same magnitude and direction regardless of the positions of their initial points, such vectors are said to be equal *i.e.,* a = b.

(vi) **Negative of a vector - A** vector whose magnitude is same as that of a given vector AB, but the direction is opposite to that of it. is known as negative of vector AB *i.e.,* BA = - AB

4. **Sum of Vectors**

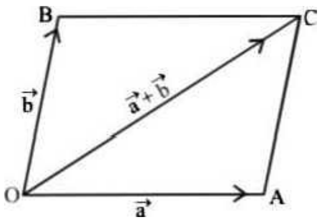
(i) **Sum of vectors a and b** let the vectors a and b be so positioned that initial point of one coincides with terminal point of the other. If a = AB, b = BC. Then the vector a + b is represented by the third side of A ABC. *i.e.,* AB + BC = AC ...(i)

AB + BC = - CA   AB+ BC + CA = 0

This is known as the triangle law of vector addition. Further AC =- CA

when sides of a triangle ABC are taken in order i.e. mitial and termmal points coincides. Then AB + BC + CA=0

(ii) **Parallelogram law of vector addition -** If the two vectors a and b are represented by the two adjacent sides OA and OB of a parallelogram OACB, then their suma + b is represented in magnitude and direction by the diagonal OC of



parallelogram through their common point O *i.e.,* OA + OB = OC

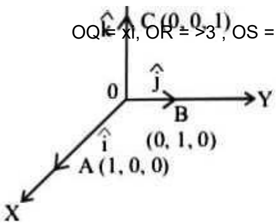
5. **Multiplication of Vector by a Scalar-** Let a be the given vector and Zbea scalar, then product of X and a = Xa

0) when X is+ve, then a and Xa are in the same direction.

(ii) when X is -ve. then a and Xa are in the opposite direction. Also *k* a| = |k| |a|.

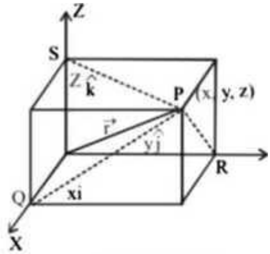
6. **Components of Vector -** Let us take the points A (1,0,0),B (0,1.0) and C (0,0, I)on the coordinate axes OX.OYandOZrespectively.Now. |OA| = 1. |OB| = 1 and |OC| = 1. Vectors OA, OB and OC each having magnitude I is known as unit vector. These are denoted by i, j and k .

OQ = xi, OR = >3 , OS = zk



Consider the vector OP , where P is the point (x, y, z). Now OQ, OR, OS are the projections of OP on coordinates axes.

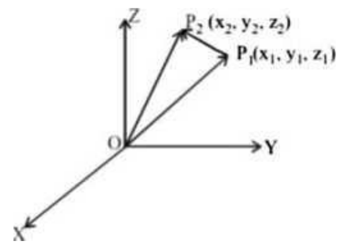
OQ = x,OR=y.OS = z



$\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$, $|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2} = |\vec{r}|$

x. y. z are called the scalar components and x i, yj , zk are called the vector components of vector OP . **Vector**

7. **joining two points - Let** P^Xp y_p zp and P,<x,,y,z,> be the two points. Then vector joining the



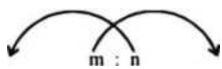
points Pj and P,is P,P_2 .Join P_p P,with O. Now OP: = OPi + P:P: (by triangle law)

... P,P_2 = OP2 - OPi

=(x_2i + y_2j+z_2k)-(X|i + y|j+Z|k) = (x_2 -x)) i + (y_2 -y)) j + (z_2 -z!) k |p^|=7(*2 - xi r+(y: - y(+(z' - zi)^2 **Section**

Formula

PR



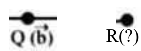
(i) A line segment PQ is divided by a point R in the ratio m: n internally *i.e.,*
If a and b arc the position vectors ofP and Q then the position vector r ofR is given by -_____ mb+ na

m + n

P?^) (K)

If R be the mid-point of PQ. then r =

(ii) when R divides PQ externally, i.e.. | a || *b| n



Then r = mb-na

9. Projection of vector along a directed line - Let the vector AB makes an angle 0 with directed line *f.*

Projection of AB on = |AB| cosG = AC = p.

l

C)

p

The vector p is called the projection vector. Its magnitudes is ,which is known as projection of vector

COS0 =

Now projection AC = | AB | cos0 = $\frac{\text{AB AC}}{\text{IACI}}$

$$= \overline{AB}\cdot\left(\frac{\overline{AC}}{|\overline{AC}|}\right), \quad \text{If } \overline{AB} = \vec{a}, \text{ then } \overline{AC} = \quad \frac{\vec{p}}{|\vec{p}|}\Big)$$

$$\vec{b} = \vec{a}\cdot\left(\frac{\vec{b}}{|\vec{b}|}\right) = \vec{a}\cdot\hat{b}$$

AB. The angle 0 between AB and AC is given by

10. Scalar Product of Two Vectors (Dot Product) - A5 AC Scalar Product of two vectors a and b is defined as

a b = |a 11 b| cos G | AB II AC

Where 0 is the angle between 3 and b (0<0< n)

(i) when 0=0. then a b = |a| Ibl = ab Also a a = |a| |a| = a.a = a^2

••• i. i = j • j = k. k = 1

Thus, the projection of a on

(ii) when 0 = —. thena-b =|a 11 b| cos y = 0 ij=jk=ki=O

11. Vector Product of two Vectors (Cross Product) - The vector product of two non-zero vectors a and b, denoted by axb is defined as

axb = |a 11 b | sin 0n,where 0 is the angle between a and b, 0 < 0< n . Unit vector n is perpendicular to both vectors a and b such that a • b and n form a right handed orthogonal system.

(i) If 0 = 0, then axb = 0, .'.axa=0 and ixi=jxj = kxk=0

(ii) If 0 = $^n/_2$,then axb=|a||b|n ixj = i,jxk = i, kxi = j

Also, jxi = -k, kxj = -i and ixk = j

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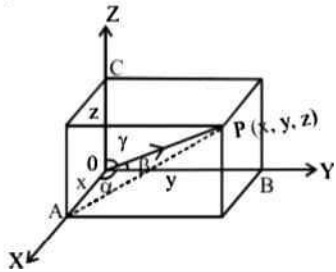
1. Direction Cosines - Let OX. OY. OZ be the positive coordinate axes, P(x, y. z) by any point in the space. Let OP makes angles a, p.y with coordinate, axes OX, OY, OZ. The angle a,p. y are known as direction angles, cosine of these angles *i.e.*,



cos a. cos p, cos y are called direction cosines ofline OP. these direction cosines are denoted by,, m, n

i. e.,f = cos a, m = cos p. n = cos y

2. **Relation Between, l, m, n and Direction Ratios -**

The perpendiculars PA, PB. PC are drawn on coordinate axes OX, OY. OZ reprectively. Let | OP I = r

In AOAP. ZA=90°cosa= x= *fr.* In AOBP. ZB = 90°.cosP=-^ = ൱ A y = mr

In A OCR Z C = 90°. cos y = — = n, ... z = nr

Thus the coordinates ofP may b expressed as (f r, mr, nr) AlsxOP = $x^2 + y^2 + z\backslash r^2 = (lr)^2 + (mr)^2 + (nr)^2$ => $f^2 + m^2 + n^2 =$ 1

Set of any there numbers, which are proportional to direction eosines are called direction ratio of the vactor. Direction ratio are denoted by a, b and c.

The numbers f r mr and nr, proportional to the direction cosines, hence, they are also direction ratios of vector OP .

3. **Properties of Vector Addition -**

   1. For two vectors a. b the sum is commutative *i.e.,* a +b = b + a

   2. For three vectors a.b and c . the sum of vectors is associative i.e., (a + b) + c = a + (b + c)

4. **Additive Inverse of Vector a** -If there exists vector- a such that a + (-a) = a- a = 0 then - a is called the additure inverse of a

5. **Some Properties -** Let a = $a_($ i + a_2 j + a_3 k and b = b(i + b_2 j + b, k

(i) a + b = (a)i + a_2 j + a_3k) + (b| i +b_2 j +b_3k) = (a(+ 6^ i + (a_2+ b_2) j + (a_3 + b_3) k

(ii) a = b or (a)i + a_2 j + a_3 k) = (b| i +b_2 j +b_3 k) => aj = b_p $a_2 = b_2$, $a_3 = b_3$

(iii) Xa = X (a, i + a_2 j + a_3 k) = (X$a_()$)i + (Ia_2)j + (Xa_3)k

(iv) a and b are parallel, if and only if there exists a non zero scalar X such that b = Xa *i.e"* bj i + b_2 j + b_3 k = X (a)i + a_2 j + a_3 k) = (X$a_{()}$i + (Xa_2)j + (Xa_3)k ... bj = Ia_p , b_2 = X$a_{2,}$$b_3$ = Xa_3 /. =

= = X

6. Properties of scalar product of two vectors (Dot Product)

(i) cos 0 =
|5||b|

If a= a, i + a_2 j + a_3 k and b = b_f i + b_2 j + b_3 k Then, a • b = (a| i + a_2 j + a_3 k) • (b(i + b_2 j + b_3 k) . a b = a-b, + a_2b_2

+ a_3b_3

|a|=7ar$_{+a}$^$_+$at|b| = 7b?$_+$b;$_+$b^

$$\therefore \cos\theta = \frac{}{|\vec{a}||\vec{b}|} = \frac{}{\sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}}$$

(ii) is commutative *i.e..* a b = b a

(iii) Ifa is a scalar, then (aa) b = a(a b) = a (ab)

7. Properties of Vector Product of two Vectors (Cross Product)-

(i) (a) If a = 0 or b = 0, then a x b = 0

(b) If a || b, then axb =0

(ii) a x b is not commutative

i.e. axb = bxa , but a xb = —bxa

(iii) If a and b represent adjacent sides of a parallelogram, then its area |a x b

(iv) If a, b represent the adjacent sides of a triangle, then its area = ^-|a^{x}b|

(v) Distributive property ax(b + c) = ax b + axc

(a) Ifa be a scalar, then a (axb) = (aa)xb = ax(ab)

(b) If a = ap + a_2j + a_3k, and b = b)i + b_2j + b_3k

j k

Then, axb = |a_1 a_2 a_3
b. b_2 b_3

8. If $a_($ pj y are the direction angles of the vector a = (aji + a_2j + a_3k) • Then direction cosines of a are given as

cos« = §i.cosp=^. cosy = g

9. **Scalar Product of Two Vectors (Dot Product) -** Scalar Product of two vectors a a -b = |a| |b| cosO b is defined as

where 0 is the angle between a and b ^0

(i) When 9= 0. then a b = |a| I bl. Also a-g a a = a- •. i i = j j = k k = 1

(ii) When9 = y ,a-b = |a| |b| cos j = 0