EDUCATALYSTS

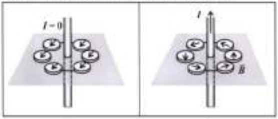
Class(12th)

Introduction to Magnetism

Magnetism Due To Electricity

- A current carrying wire produces a magnetic field of its own. This was first discovered by Oersted. Oersted's discovery which linked the motion of electric charges with the creation of a magnetic field marked the beginning of an important discipline called electro magnetism.
- The magnetic lines due to current carrying conductor lie in a plane perpendicular to the conductor.
- The field due to current carrying conductor is non uniform i.e. both in magnitude and direction changes.

  We have seen that currents (fundamentally moving charges) are the source of magnetism. This can be readily demonstrated by placing compass needles near a wire. As shown in figure (a), all compass needles point in the same direction in the absence of current (in the direction of earth's magnetic field). However, when a strong current passes through (so that earth's magnetic field becomes negligible), the needles will be deflected along the tangential direction of the circular path (figure (b)).



(a) (b)

Fig.: Deflection of compass needles near a current-carrying wire

Following are the some of the ways to find out the direction of line of forces

Ampere's right hand grip rule:

- If a straight conductor is held in right hand such that the thumb points in the direction of the current then the direction of the curl of the remaining fingers gives the direction of the line of force.

Ampere's swimming rule:

- Imagine that a man is swimming along the conductor in the direction of the current facing a magnetic needle placed below the conductor, then the north pole of the needle will deflect towards his left hand side.

Maxwell's cork screw rule:

- If the direction of the movement of the tip of the right handed cork screw represents the direction of current in a wire then the direction of rotation of the head of the screw gives the direction of the line of force, i.e., direction of the magnetic field around it.

Biot-Savart Law

- Currents which arise due to the motion of charges are the source of magnetic fields. When charges move in a conducting wire and produce a current I, the magnetic field at any point P due to the current can be calculated by adding up the magnetic field contributions, $d\vec{B}$, from small segments of the wire $d\vec{s}$ (Figure).

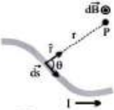


Fig.: Magnetic field $d\vec{B}$ at point P due to a current-carrying element $Id\vec{s}$

- These segments can be thought of as a vector quantity having a magnitude of the length of the segment and pointing in the direction of the current flow. The infinitesimal current source can then be written as $Id\vec{s}$.
- Let r denote the distance from the current source to the field point P and $\hat{r}$ the corresponding unit vector. The Biot-Savart law gives an expression for the magnetic field contribution, $d\vec{B}$, from the current source, $Id\vec{s}$,

  $$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}$$

  where μ_0 is a constant called the permeability of free space.

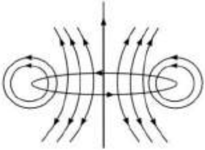
  $\mu_0 = 4\pi\times10^{-7}$ T.m/A here Tesla (T) is SI unit of $\vec{B}$
- Notice that the expression is remarkably similar to the Coulomb's law for the electric field due to a charge element dq :

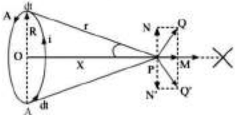
  $$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$
- Adding up these contributions to find the magnetic field at the point P requires integrating over the current source,

  $$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2}$$

Biot-Savart law to derive the expression for the magnetic field on the axis of a current-carrying circular loop

Magnetic field lines due to a circular current loop :





$$d\vec{B}_x = \frac{\mu_0}{4\pi}\frac{I\delta l \sin\alpha}{r^2}$$

The direction of $d\vec{B}$ is perpendicular to the plane containing, $d\vec{l}$ and $\vec{r}$ and is given by right hand screw rule. As the angle between $I\,d\vec{l}$ and r is 90° the magnitude of the magnetic induction $d\vec{B}$ is given by,

$$\vec{dB} = \frac{\mu_0 I}{4\pi}\frac{dl \sin 90^\circ}{r^2}$$

$$= \frac{\mu_0 I dl}{4\pi r^2}$$

The component of $d\vec{B}$ along the axis,

$$\vec{dBc} = \frac{\mu_0 I dl}{4\pi r^2}\sin\alpha$$

But $\sin\alpha = \frac{R}{r}$ and $r = \left(R^2 + x^2\right)^{1/2}$

$$\therefore\ d\vec{B}_x = \frac{\mu_0 I dl}{4\pi r^2}\cdot\frac{R}{r} = \frac{\mu_0 IR}{4\pi r^3}dl$$

$$= \frac{\mu_0 IR}{4\pi\left(R^2 + x^2\right)^{3/2}}dl$$

Total field $\vec{B}$

$$= \oint \frac{\mu_0 IR}{4\pi\left(R^2 + x^2\right)^{3/2}}dl$$

$$= \frac{\mu_0 IR}{4\pi\left(R^2 + x^2\right)^{3/2}}\oint dl$$

But, $\oint dl$ = length of the loop = $2\pi R$

Therefore, $B = \frac{\mu_0 IR}{2\pi\left(R^2 + x^2\right)^{3/2}}(2\pi R)$

$$\Rightarrow \vec{B} = B_x\hat{i} = \frac{\mu_0 IR^2}{2\left(R^2 + x^2\right)^{3/2}}\hat{i}$$

If the coil contains N turns, then

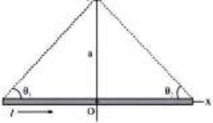
$$B = \frac{\mu_0 NIR^2}{2\left(R^2 + x^2\right)^{3/2}}\text{ tesla.}$$

— *KEY NOTE* —

The integral is a vector integral, which means that the expression for B is really three integrals, one for each component of $\vec{B}$. The vector nature of this integral appears in the cross product. Understanding how to evaluate this cross product and then perform the integral will be the key to learning how to use the Biot-Savart law.

Magnetic Field Due To A Finite Straight Wire

- A thin, straight wire carrying a current I is placed along the x-axis, as shown in Figure. Evaluate the magnetic field at point P due to the segment shown in figure.



$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_2 + \cos\theta_1)$$

- The first term involving θ_2 accounts for the contribution from the portion along the +x axis, while the second term involving θ_1 contains the contribution from the portion along the −x axis. The two terms add.

Special cases :

(i) **Magnetic field on the perpendicular bisector of a finite straight wire of length 2L**

In this case where $\theta_1 = \theta_2 = \theta$, the field point P is located along the perpendicular bisector. If the length of the rod is 2L, then $\cos\theta = L/\sqrt{L^2 + a^2}$ and the magnetic field is

$$B = \frac{\mu_0 I}{2\pi a}\cos\theta = \frac{\mu_0 I}{2\pi a}\frac{L}{\sqrt{L^2 + a^2}}$$

(ii) **Magnetic field due to semi-infinite straight wire**

Here $\theta_1 = 90^\circ, \theta_2 = 0^\circ$ or $\theta_1 = 0^\circ, \theta_2 = 90^\circ$

$$B = \frac{\mu_0 I}{4\pi a}$$

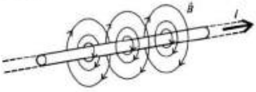
(iii) **Magnetic field due to infinite straight wire**

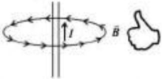
Here $\theta_1 = \theta_2 = 0$

$$B = \frac{\mu_0 I}{2\pi a}$$

Direction of magnetic field of a straight wire

- Note that in this limit, the system possesses cylindrical symmetry, and the magnetic field lines are circular, as shown in Figure.

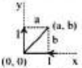




- In fact, the direction of the magnetic field due to a long straight wire can be determined by the right-hand rule (Figure). If you direct your right thumb along the direction of the current in the wire, then the fingers of your right hand curl in the direction of the magnetic field.

Train Your Brain

Q. Two semi-infinitely long straight current carrying conductors are in form of an 'L' shape as shown in the figure. The common end is at the origin. What is the value of magnetic field at a point (a, b), if both the conductors carry the same current I?



Ans. For the conductor along the X axis, the magnetic field

$$B_1 = \frac{\mu_0 I}{4\pi b}[\cos\theta_2 + \cos 0] \text{ along the negative Z-axis}$$

$$= \frac{\mu_0 I}{4\pi b}\left[1 + \frac{a}{\sqrt{a^2+b^2}}\right]$$

For the conductor along Y-axis, the magnetic field is

$$B_2 = \frac{\mu_0 I}{4\pi a}\left[1 + \frac{b}{\sqrt{a^2+b^2}}\right] \text{ along the negative z-axis}$$

$\therefore$ The net magnetic field is,

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$= \frac{\mu_0 I}{4\pi}\left[\left(\frac{1}{a}+\frac{1}{b}\right) + \frac{1}{\sqrt{a^2+b^2}}\left(\frac{a}{b}+\frac{b}{a}\right)\right]$$

$$= \frac{\mu_0 I}{4\pi}\left[\frac{(a+b)}{ab} + \frac{\sqrt{a^2+b^2}}{ab}\right]$$

$$= \frac{\mu_0 I}{4\pi ab}\left[(a+b) + \sqrt{a^2+b^2}\right]$$

Q. The magnetic field at a point 50 mm from a long straight line carrying a current of 3A will be-

a. 0.12 G b. 1.2 G
c. 12 G d. 0.012 G

Ans. (a) For the conductor along the X axis, the magnetic field

$$B = \frac{\mu_0 i}{2\pi r} = \frac{4\pi \times 10^{-7} \times 3}{2\pi \times 50 \times 10^{-3}} = 1.20 \times 10^{-5}\text{ T} = 0.12\text{ G}.$$

$[1\text{G} = 10^{-4}\text{T}]$

Magnetic Field Due To A Current Carrying Arc At Its Centre

$$dl = a\,d\theta$$

$$dB = \frac{\mu_0}{4\pi}\frac{I(a\,d\theta)\sin 90^\circ}{a^2} = \frac{\mu_0 I}{4\pi a}d\theta$$

$$B = \int dB = \frac{\mu_0 I}{4\pi a}\int_0^\beta d\theta$$

$$B = \frac{\mu_0 I}{4\pi a}(\beta)$$

Special Case:

Case : 1

For current carrying circular coil with N no. of turns

$\beta = 2\pi$

$$\boxed{B_{centre} = N\frac{\mu_0 I}{2R}}$$

Case : 2

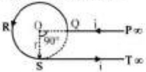
For current carrying semicircular coil with N no. to turns

$\beta = \pi$

$$\boxed{B_{centre} = N\frac{\mu_0 I}{4R}}$$

Train Your Brain

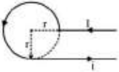
Q. A current i is flowing in a conductor shaped as shown in the figure. The radius of curved part is r and length of straight portions is very large. The value of magnetic field at the centre will be



a. $\frac{\mu_0 i}{4\pi r}\left[\frac{3\pi}{2}+1\right]$ b. $\frac{\mu_0 i}{4\pi r}\left[\frac{3\pi}{2}-1\right]$

c. $\frac{\mu_0 i}{4\pi r}\left[\frac{\pi}{2}+1\right]$ d. $\frac{\mu_0 i}{4\pi r}\left[\frac{\pi}{2}-1\right]$

Ans. (a)



$$B_0 = B_{PQ} + B_{QR} + B_{ST},\ B_{ST} = \frac{1}{2}\left[\frac{\mu_0 i}{2\pi r}\right]$$

$$B_{PQ} = 0,\ B_{QRS} = \frac{\mu_0 i}{4\pi r^2} \times \frac{3}{4} \times 2\pi r$$

$$B_0 = \frac{\mu_0 i}{4\pi r} + \frac{3}{4}\frac{\mu_0 i}{2r} = \frac{\mu_0 i}{4r}\left[\frac{3}{2}+\frac{1}{\pi}\right] = \frac{\mu_0 i}{4\pi r}\left[\frac{3\pi}{2}+1\right]$$

Q. A 6.28 m long wire is turned into a coil of diameter 0.2 m and a current of 1 amp. is passed in it. The magnetic induction at its centre will be

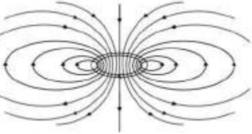
a. 6.28×10^{-5} Tesla b. 0
c. 6.28 Tesla d. 6.28×10^{-3} Tesla

Ans. (a) $l = (2\pi r)n$ or $n = \frac{l}{2\pi r}$

$$B = \frac{\mu_0 n i}{2r} = \frac{\mu_0 i l}{4\pi r^2} \text{ or } B = \frac{4\pi \times 10^{-7} \times 6.28 \times 1}{2 \times 2 \times \pi \times (0.10)^2}$$

$= 6.28 \times 10^{-5}$ Tesla

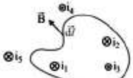
Magnetic field lines due to a circular current



AMPERE'S LAW

- Like Gauss's law in electrostatics, this law provides us a simple method to find magnetic fields in cases of symmetry
- Ampere's law gives another method to calculate the magnetic field due to a given current distribution.
- Statement: The circulation $\oint \vec{B}\cdot d\vec{l}$ of the resultant magnetic field (of a closed circuit or an infinite wire containing steady current) along a closed path (called amperian path) is equal to μ_0 times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant. Thus,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{en}$$



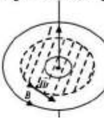
- In figure, the positive side is going into the plane of the diagram so that i_1 and i_2 are positive and i_3 is negative. Thus, the total current crossing the area is $i_1 + i_2 - i_3$. Any current outside the area is not included in writing the right - hand side of equation. The magnetic field on the left -hand side is the resultant field due to all the currents existing anywhere.

— *KEY NOTE* —

Ampere's law may be derived from the Biot-Savart law and Bio-Savart law may be derived from the Ampere's law. Thus, the two are equivalent in scientific content. However, Ampere's law is useful under certain symmetrical conditions.

Calculation Of Magnetic Field Due To Long Straight Wire

- Figure shows a long, straight current I. We have to calculate the magnetic field at a point P which is at a distance r from the wire. Figure shows the situation in the plane perpendicular to the wire and passing through P. The current is perpendicular to the plane of the diagram and is coming out of it.



- Let us draw a circle passing through the point and with the axis as wire. We put an arrow to show the positive sense of the circle. The radius of the circle is r. The magnetic field due to the long, straight current at any point on the circle is along the tangent as shown in the figure. Same is the direction of the length-element dl there. By symmetry, all points of the circle are equivalent and hence the magnitude of the magnetic field should be the same at all these points. The circulation of magnetic field along the circle is

$$\oint \vec{B}\cdot d\vec{l} = \oint Bdl = B\oint dl = B(2\pi r)$$

- The current crossing the area bounded by the circle is $\sum I_{en} = +I$
- Thus, from Ampere's law,

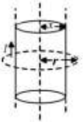
$$B(2\pi r) = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

Magnetic Field Due To A Current Carrying Thin Long Pipe

Case I : $r > R$

$$\oint \vec{B}\cdot d\vec{l} = \oint B\cdot dl = B\oint dl = B(2\pi r)$$



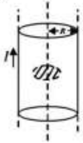
$$\sum I_{en} = +I$$

$$\Rightarrow B(2\pi r) = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

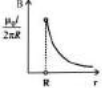
Case II : r < R

$$\oint \vec{B}\cdot d\vec{l} = \oint Bdl = B\oint dl = B(2\pi r)$$



$$\sum I_{en} = 0$$

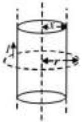
$$\Rightarrow B(2\pi r) = \mu_0(0)$$



Magnetic Field Due To Current Carrying Rod Having Uniform Current Density

Case I : $r > R$

$$\oint \vec{B}\times d\vec{l} = \oint B\times dl = B\oint dl = B(2\pi r)$$

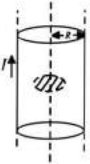


$$\sum I_{in} = +I$$

$$\Rightarrow B(2\pi r) = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

Case II : $r < R$

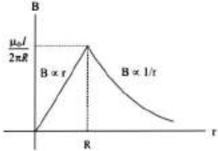


$$\oint \vec{B}\times d\vec{l} = \oint B dl = B\oint dl = B(2\pi r)$$

$$\sum I_{in} = \frac{I}{R^2}r^2$$

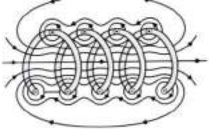
$$\Rightarrow B(2\pi r) = \mu_0 \frac{I}{R^2}r^2$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi R^2}r$$



Magnetic Field Due To A Long Solenoid

- A solenoid is a long coil of wire tightly wound in the helical form. Figure shows the magnetic field lines of a solenoid carrying a steady current I. We see that if the turns are closely spaced, the resulting magnetic field inside the solenoid becomes fairly uniform, provided that the length of the solenoid is much greater than its diameter. For an "ideal" solenoid, which is infinitely long with turns tightly packed, the magnetic field inside the solenoid is uniform and parallel to the axis, and vanishes outside the solenoid.



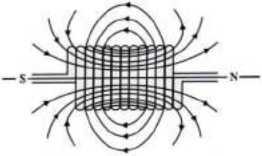


Fig.: Magnetic field lines of a solenoid

- We can use Ampere's law to calculate the magnetic field strength inside an ideal solenoid. The cross-sectional view of an ideal solenoid is shown in figure. To compute $\vec{B}$, we consider a rectangular path of length l and width w and traverse the path in a counterclockwise manner. The line integral of $\vec{B}$ along this loop is

$$\oint \vec{B}\cdot d\vec{s} = \oint_1 \vec{B}\cdot d\vec{s} + \oint_2 \vec{B}\cdot d\vec{s} + \oint_3 \vec{B}\cdot d\vec{s} + \oint_4 \vec{B}\cdot d\vec{s}$$

$$= 0 + 0 + Bl + 0$$

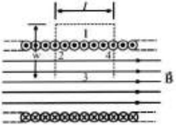


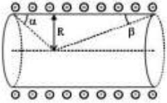
Fig.: Amperian loop for calculating the magnetic field of an ideal solenoid.

- In the above, the contributions along sides 2 and 4 are zero because $\vec{B}$ is perpendicular to $d\vec{s}$. In addition, $\vec{B} = \vec{0}$ along side 1 because the magnetic field is non-zero only inside the solenoid. On the other hand, the total current enclosed by the Amperian loop is $I_{enc} = nlI$, where n is the total number of turns per unit length. Applying Ampere's law yields

$$\oint \vec{B}\cdot d\vec{s} = Bl = \mu_0 nlI \qquad B = \mu_0 nI$$

The general formula for magnetic field inside solenoid.

$$\Rightarrow B = \frac{\mu_0 ni}{2}[\cos\alpha + \cos\beta]$$



Train Your Brain

Q. A solenoid of length 0.2 m has 500 turns on it. If $8.71 \times 10^{-6}\ W/m^2$ be the magnetic field at an end of the solenoid, then the current flowing in the solenoid will be

a. $\frac{0.174}{\pi}$ A b. $\frac{174}{\pi}$ A

c. $\frac{17.4}{\pi}$ A d. $\frac{174}{\pi}$ A

Ans. (a) We know, $B_{end} = \frac{\mu_0 ni}{2}$

Here $n = \frac{500}{0.2} = 2500/\text{metre}$, $i = ?$, $B_{end} = 8.71 \times 10^{-6}T$

$\therefore\ i = \frac{2B_{end}}{\mu_0 n} = \frac{2 \times 8.71 \times 10^{-6}}{4\pi \times 10^{-7} \times 2500}$

$= \frac{17.42 \times 10^{-2}}{\pi} = \frac{0.1742}{\pi}$ A

Magnetic Field Due To A Toroid

- Consider a toroid which consists of N turns, as shown in Figure. Find the magnetic field everywhere.

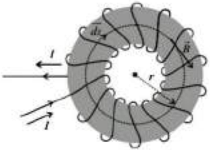


Fig.: A toroid with N turns

- One can think of a toroid as a solenoid wrapped around with its ends connected. Thus, the magnetic field is completely confined inside the toroid and the field points in the azimuthal direction (clockwise due to the way the current flows, as shown in Figure.)
- Applying Ampere's law, we obtain

$$\oint \vec{B}.d\vec{s} = \oint B ds = B\oint ds = B(2\pi r) = \mu_0 NI \text{ or } B = \frac{\mu_0 NI}{2\pi r}$$

where r is the distance measured from the center of the toroid. Unlike the magnetic field of a solenoid, the magnetic field inside the toroid is non-uniform and decreases as $1/r$.

Force On A Moving Charge In Magnetic Field

- Consider a particle of charge q and moving at a velocity $\vec{v}$. Experimentally we have the following observations:
  (a) The magnitude of the magnetic force $\vec{F}_B$ exerted on the charged particle is proportional to both v and q.
  (b) The magnitude and direction of $\vec{F}_B$ depends on $\vec{v}$ and $\vec{B}$.
  (c) The magnetic $\vec{F}_B$ force vanishes when $\vec{v}$ is parallel to $\vec{B}$. However, when $\vec{v}$ makes an angle θ with h $\vec{B}$, the direction of $\vec{F}_B$ is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$ and the magnitude of $\vec{F}_B$ is proportional to sin θ.
  (d) When the sign of the charge of the particle is switched from positive to negative (or vice versa), the direction of the magnetic force also reverses.

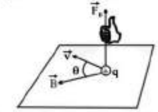


Fig.: The direction of the magnetic force

- The above observations can be summarized with the following equation:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

- The above expression can be taken as the working definition of the magnetic field at a point in space. The magnitude of $\vec{F}_B$ is given by

$$F_B = |q|\ vB \sin\theta$$

- The SI unit of magnetic field is the tesla (T):

$$1\text{ tesla} = 1T = 1\frac{\text{Newton}}{(\text{Coulomb})(\text{meter/second})} = 1\frac{N}{C.m/s} = 1\frac{N}{A.m}$$

- Another commonly used non-SI unit for $\vec{B}$ is the gauss (G), where $1T = 10^4 G$

— KEY NOTE —

- Note that $\vec{F}_B$ is always perpendicular to $\vec{v}$ and $\vec{B}$, and cannot change the particle's speed v (and thus the kinetic energy). In other words, magnetic force cannot speed up or slow down a charged particle. Consequently, **Magnetic field does no work on charged particle:**
  $dW = \vec{F}_B \cdot d\vec{s} = q(\vec{v} \times \vec{B}) \cdot \vec{v}dt = q(\vec{v} \times \vec{v}) \cdot \vec{B}dt = 0$
- The direction of $\vec{v}$ however, can be altered by the magnetic force

| Motion Of Charged Particle In Uniform Magnetic Field

- There are three possible paths in which a charged particle may move in presence of uniform magnetic field which is uniform in space.

(a) Straight line path (b) Circular path (c) Helical path

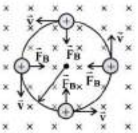
We shall see them one by one.

Straight line path :

- When the charged particle projected in the direction of or opposite to uniform magnetic field, magnetic field exerts no force hence it will travel along straight line with fixed speed.

Circular path :

- If a particle of mass m moves in a circle of radius r at a constant speed v, what acts on the particle is a radial force of magnitude $F = mv^2/r$ that always points toward the center and is perpendicular to the velocity of the particle.
- In previous section, we have also shown that the magnetic force $\vec{F}_B$ always points in the direction perpendicular to the velocity $\vec{v}$ of the charged particle and the magnetic field $\vec{B}$. Since $\vec{F}_B$ can do no work, it can only change the direction of $\vec{v}$ but not its magnitude. What would happen if a charged particle moves through a uniform magnetic field $\vec{B}$ with its initial velocity $\vec{v}$ at a right angle to $\vec{B}$? For simplicity, let the charge be +q and the direction of be into the page. It turns out that $\vec{F}_B$ will play the role of B a centripetal force and the charged particle will move in a circular path in a counterclockwise direction, as shown in figure.



$$qvB = \frac{mv^2}{r}$$

The radius of the circle is found to be $r = \frac{mv}{qB}$

(i) In this case path of charged particle is circular and magnetic force provides the necessary centripetal force, i.e., $qvB = \frac{mv^2}{r}$ $\Rightarrow$ radius of path

$$r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

where p = momentum of charged particle and K = kinetic energy of charged particle (gained by charged particle after accelerating through potential difference V) then p $= mv = \sqrt{2mK} = \sqrt{2mqV}$

(ii) If T is the time period of the particle then $T = \frac{2\pi m}{qB}$ (i.e., time period (or frequency) is independent of speed of particle).

Train Your Brain

Q. A uniform magnetic field of 30 mT exists in the + X direction. A particle of charge +e and mass 1.67×10^{-27} kg is projected into the field along the + Y direction with a speed of 4.8×10^6 m/s.

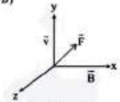
a. Find the force on the charged particle in magnitude and direction.

b. Find the force if the particle were negatively charged.

c. Describe the nature of path followed by the particle in the both the case.

Ans. (a) Force acting on a charge particle moving in the magnetic field

$\vec{F} = q(\vec{v} \times \vec{B})$



Magnetic field $\vec{B} = 30(\text{mT})\hat{j}$

Velocity of the charge particle $\vec{v} = 4.8 \times 10^6$ (m/s) $\hat{j}$

$\vec{F} = 1.6 \times 10^{-19}[(4.8 \times 10^6\,\hat{j}) \times (30 \times 10^{-3})(\hat{i})]$

Velocity of the charge particle (m/s)

$\vec{F} = 230.4 \times 10^{-16}(-\hat{k})$N

(b) If the particle were negatively charged, the magnitude of the force will be the same but the direction will be along (+z) direction.

(c) As $v \perp B$, the path described is a circle

$$R = \frac{mv}{qB}$$

$= (1.67 \times 10^{-27}).(4.8 \times 10^6)/(1.6 \times 10^{-19}).(30 \times 10^{-3})$

$= 1.67$m

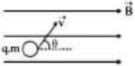
Helical Path:

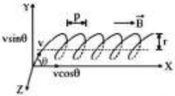
- In this situation velocity of the particle can be resolved into two components.

  (a) $v_{||}$ $\rightarrow$ projection of velocity parallel to the magnetic field.

  (b) $v_{\perp}$ $\rightarrow$ projection of velocity perpendicular to the magnetic field.

- Due to $v_{\perp}$ motion of the charged particle will be uniform circular in the plane perpendicular to the field. whereas $v_{||}$ remain unaffected as it is perpendicular to magnetic force.
- As a whole its motion is helical with constant pitch.





(i) The radius of this helical path is $r = \dfrac{m(v\sin\theta)}{qB}$.

(ii) Time period and frequency do not depend on velocity and so they are given by $T = \dfrac{2\pi m}{qB}$ and $\nu = \dfrac{qB}{2\pi m}$.

(iii) The pitch of the helix, (i.e., linear distance travelled in one rotation) will be given by

$p = T(v\cos\theta) = 2\pi\dfrac{m}{qB}(v\cos\theta) = \dfrac{2\pi r}{\tan\theta}$.

(iv) If pitch value is p, then number of pitches obtained in length l is given as

Number of pitches $= \dfrac{l}{p}$ and time required $t = \dfrac{l}{v\cos\theta}$.

Train Your Brain

Q. A charged particle P leaves the origin with speed $v = v_0$, at some inclination with the x-axis. There is uniform magnetic field B along the x-axis. P strikes a fixed target T on the x-axis for a minimum value of $B = B_0$. Find the condition so that P will also strike if you can change magnetic field and speed.

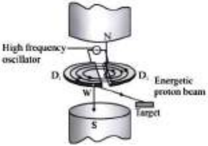
Ans. Let d = distance of the target T from the point of projection. P will strike T if d an integral multiple of the pitch.

$$\left(2\pi\frac{m}{qB_0}\right)v_0\cos\theta = N\left(2\pi\frac{m}{qB}\right)v\cos\theta$$

Here N is a natural number.

Cyclotron

- Cyclotron is a device used to accelerate positively charged particles (like, α-particles, deutrons etc.) to acquire enough energy to carry out nuclear disintegration etc.
- It is based on the fact that the electric field accelerates a charged particle and the magnetic field keeps it revolving in circular orbits of constant frequency.



- It consists of two hollow D-shaped metallic chambers D_1 and D_2 called dees. The two dees are placed horizontally with a small gap separating them. The dees are connected to the source of high frequency electric field. The dees are enclosed in a metal box containing a gas at a low pressure of the order of 10^{-3} mm mercury. The whole apparatus is placed between the two poles of a strong electromagnet NS as shown in figure. The magnetic field acts perpendicular to the plane of the dees.

(1) Cyclotron frequency: Time taken by ion to describe a semicircular path is given by $t = \dfrac{\pi r}{v} = \dfrac{\pi m}{qB}$.

If T = time period of oscillating electric field then $T = 2t = \dfrac{2\pi m}{qB}$ the cyclotron frequency $\nu = \dfrac{1}{T} = \dfrac{Bq}{2\pi m}$

(2) Maximum energy of particle : Maximum energy gained by the charged particle $E_{max} = \left(\dfrac{q^2B^2}{2m}\right)r_0^2$.

where r_0 = maximum radius of the circular path followed by the positive ion.

(3) Cyclotron cannot be used to accelerate small negative charges like electron because the relative mass increases with velocity and it becomes out of phase from cyclotron.

Lorentz Force

- In the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$, the total force on a charged particle is

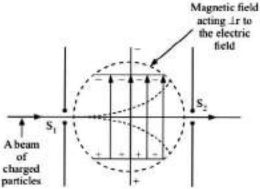
$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

- This is known as the Lorentz force.

Velocity Selector

Velocity filter is an arrangement of cross electric and magnetic fields in a region which helps as to select from a beam, charged particles of the given velocity irrespective of their charge and mass.

A velocity selector consists of two slits S_1 and S_2 held parallel to each other, with common axis, some distance apart. In the region between the slits, uniform electric and magnetic fields are applied, perpendicular to each other as well as to the axis of slits, as shown in Fig. When a beam of charged particles of different charges and masses after passing through slit S_1 enters the region of crossed electric field $\vec{E}$ and magnetic field $\vec{B}$, each particle experiences a force due to these fields. Those particles which are moving with the velocity v, irrespective of their mass and charge, the force on each such particle due to electric field (qE) is equal and opposite to the force due to magnetic field ($q\ v\ B$), then



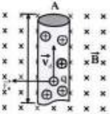
$qE = qvB$ or $v = E/B$

Such particles will go undeviated and filtered out of the region through the slit S_2. Therefore, the particles emerging form slit S_2 will have the same velocity even though their charge and mass may be different.

The velocity filter is used in mass spectrograph which helps to find the mass and specific charge (charge/mass) of the charged particle.

Magnetic Force on a Current-Carrying Wire

- We have just seen that a charged particle moving through a magnetic field experiences a magnetic force $\vec{F}_B$. Since electric current consists of a collection of charged particles in motion, when placed in a magnetic field, a current-carrying wire will also experience a magnetic force.
- Consider a long straight wire suspended in the region between the two magnetic poles. The magnetic field points out the page and is represented with dots (•). It can be readily demonstrated that when a downward current passes through, the wire is deflected to the left. However, when the current is upward, the deflection is rightward, as shown in figure.
- To calculate the force exerted on the wire, consider a segment of wire of length l and cross-sectional area A, as shown in figure. The magnetic field points into the page, and is represented with crosses (x).



- The charges move at an average drift velocity . Since the total amount of charge in this segment is $Q_{tot} = q(nAl)$, where n is the number of charges per unit volume, the total magnetic force on the segment is

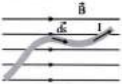
$$\vec{F}_B = Q_{tot}\vec{v}_d \times \vec{B} = qnAl(\vec{v}_d \times \vec{B}) = I(\vec{l} \times \vec{B})$$

where $I = nqv_dA$, and $\vec{l}$ is a length vector with a magnitude l and directed along the direction of the electric current.

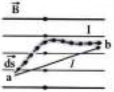
Special Case-1:

Wire of arbitrary shape placed in uniform magnetic field

- For a wire of arbitrary shape, the magnetic force can be obtained by summing over the forces acting on the small segments that make up the wire. Let the differential segment be denoted as $d\vec{s}$ (Figure).



- The magnetic force acting on the segment is : $d\vec{F}_B = Id\vec{s} \times \vec{B}$
- Thus, the total force is : $\vec{F}_B = I\int_a^b d\vec{s} \times \vec{B}$
- Where a and b represent the endpoints of the wire.
- As an example, consider a curved wire carrying a current I in a uniform magnetic field $\vec{B}$, as shown in Figure.



- Using the magnetic force on the wire is given by

$$\vec{F}_B = I\left(\int_a^b d\vec{s}\right) \times \vec{B} = I\vec{l} \times \vec{B}$$

where is the length $\vec{l}$ vector directed from a to b. However, if the wire forms a closed loop of arbitrary shape (Figure), then the force on the loop becomes

$$\vec{F}_B = I\left(\oint d\vec{s}\right) \times \vec{B}$$

Special Case-2:

Magnetic Force on a closed loop in uniform magnetic field

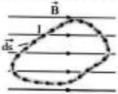


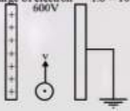
Fig.: A closed loop carrying a current I in a uniform magnetic field.

- Since the set of differential length elements $d\vec{s}$ form a closed polygon, and their vector sum is zero, i.e. $\oint d\vec{s} = 0$. The net magnetic force on a closed loop is $\vec{F}_B = \vec{0}$.

Train Your Brain

Q. A potential difference of 600 volt is applied across the plates of a parallel plate condenser placed in a magnetic field. The separation between the plates is 3 mm. An electron projected vertically upward parallel to the plates with a velocity of 2×10^6 m/s moves undeflected between the plates. The magnitude and direction of the magnetic field in the region between the condenser plates will be (in Wb/m²)

(Given charge of electron $= -1.6 \times 10^{-19}$ coulomb)



a. 0.1, vertically downward
b. 0.2 vertically downward
c. 0.3 vertically upward
d. 0.4 vertically downward

Ans. (a) The electron will pass undeviated if the electric force and magnetic force are equal and opposite.

Thus $\quad$ E.e. = Bev $\quad$ or $\quad$ B = E/v

but $\quad$ E = V/d

Therefore, $B = \frac{V}{v.d.} = \frac{600}{2 \times 10^6 \times 3 \times 10^{-3}}$

$\therefore$ B = 0.1 Wb/m²

The direction of field is perpendicular to the plane of paper vertically downward.

Force Between Two Parallel Wires

AB and CD are two straight very long parallel conductors placed in air at a distance a. They carry currents I_1 and I_2 respectively. (Fig) The magnetic induction due to current I_1 in AB at a distance a is

$$B_1 = \frac{\mu_0 I_1}{2\pi a} \quad ...(1)$$

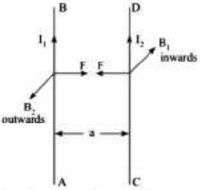


Fig.: Force between two long parallel current-carrying conductors

This magnetic field acts perpendicular to the plane of the paper and inwards. The conductor CD with current I_2 is situated in this magnetic field. Hence, force on a segment of length l of CD due to magnetic field B_1 is

$$F = B_1 I_2 l$$

Substituting equation (1)

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi a} \quad ...(2)$$

By Fleming's Left Hand Rule, F acts towards left. Similarly, the magnetic induction due to current I_2 flowing in CD at a distance a is

$$B_2 = \frac{\mu_0 I_2}{2\pi a} \quad ...(3)$$

This magnetic field acts perpendicular to the plane of the paper and outwards. The conductor AB with current I_1, is situated in this field. Hence force on a segment of length l of AB due to magnetic field B_2 is

$$F = B_2 I_1 l$$

Substituting equation (3)

$$\therefore F = \frac{\mu_0 I_1 I_2 l}{2\pi a} \quad ...(4)$$

By Fleming's left hand rule, this force acts towards right. These two forces given in equations (2) and (4) attract each other. Hence, two parallel wires carrying currents in the same direction attract each other and if they carry currents in the opposite direction, repel each other.

Definition of ampere

The force between two parallel wires carrying currents on a segment of length *l* is

$$F = \frac{\mu_0 I_1 I_2}{2\pi a} l$$

$\therefore$ Force per unit length of the conductor is

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

If $I_1 = I_2 = 1$A, a = 1m

$$\frac{F}{l} = \frac{\mu_0}{2\pi a} \frac{1 \times 1}{1} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} \text{ Nm}^{-1}$$

The above conditions lead the following definition of ampere.

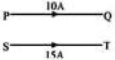
Ampere is defined as that constant current which when flowing through two parallel infinitely long straight conductors of negligible cross section and placed in air or vacuum at a distance of one meter apart, experience a force of 2×10^{-7} Newton per unit length of the conductor.

— KEY NOTE —

The conclusion we can draw from this simple calculation is that two parallel wires carrying currents in the same direction will attract each other. On the other hand, if the currents flow in opposite directions, the resultant force will be repulsive.

Train Your Brain

Q. In the adjoining figure the two parallel wires PQ and ST are at 30 cm apart. The currents flowing in the wires are according to figure. The force acting over a length of 5 m of the wires is



a. 5×10^{-4} N, (attraction)
b. 1×10^{-4} N, (attraction)
c. 5×10^{-4} N, (repulsion)
d. 1×10^{-4} N, (repulsion)

Ans. (a) When currents flow in two long, parallel wires in the same direction, the wires exert a force of attraction on each other. The magnitude of this force acting per meter length of the wires is given by

$$F = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{R} = 2\times10^{-7}\frac{i_1 i_2}{R} \text{ N/m}$$

Here $i_1 = 10$ A, $i_2 = 15$A,
$R = 30$ cm $= 0.3$ m

$$\therefore F = 2\times10^{-7}\frac{10\times15}{0.3}$$
$$= 1\times10^{-4} \text{ N/m}$$

$\therefore$ Force on 5 m length of the wire
$= 5 \times (1 \times 10^{-4}) = (5 \times 10^{-4})$
$= 5 \times 10^{-4}$ N (attraction)

Magnetic Moment

- Magnetic field (at large distances) due to current in a circular current loop is very similar in behavior to the electric field of an electric dipole. We know that the magnetic field on the axis of a circular loop, of a radius R, carrying a steady current I

$$B = \frac{\mu_0 I(2\pi a^2)}{4\pi(a^2+x^2)^{3/2}}$$

- Its direction is along the axis and given by the right-hand thumb rule Here, x is the distance along the axis from the centre of the loop.
- For x >> R, we may drop the R^2 term in the denominator. Thus

$$B = 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{IA}{x^3}\right)$$

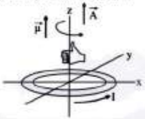
Where $A = \pi R^2$ = area of the loop

- The expression is very similar to an expression obtained earlier for the electric field of a dipole. The similarity may be seen if we we can define the magnetic dipole moment $\vec{\mu}$ as

$$\vec{\mu} = I\vec{A}$$

— KEY NOTE —

The direction of $\vec{\mu}$ is the same as the area vector $\vec{A}$ (perpendicular to the plane of the loop) and is determined by the right-hand rule (Figure). The SI unit for the magnetic dipole moment is ampere-meter^2 (A•m^2).

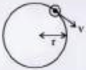


Train Your Brain

Q. Find the magnetic moment of an electron orbiting in a circular orbit of radius r with a speed v.

Ans. Magnetic moment $\mu = iA$
I = current; Since the orbiting electron behaves as current loop of current i,

we can write $i = \frac{e}{T} = \frac{e}{\frac{2\pi r}{v}} = \frac{ev}{2\pi r}$



A = area of the loop = πr^2

$$\Rightarrow \mu = (1)\left(\frac{ev}{2\pi r}\right)(\pi r^2) \Rightarrow \mu = \frac{evr}{2}$$

Torque on a Current Loop

- What happens when we place a rectangular loop carrying a current I in the xy plane and switch on a uniform magnetic field $\vec{B} = B\hat{i}$ which runs parallel to the plane of the loop, as shown in Figure (a)?

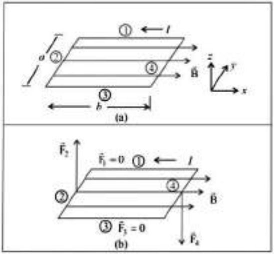


Fig.: (a) A rectangular current loop placed in a uniform magnetic field.
(b) The magnetic forces acting on sides 2 and 4.

- From Eq., we see the magnetic forces acting on sides 1 and 3 vanish because the length vectors $\vec{l}_1 = -b\hat{i}$ and $\vec{l}_3 = b\hat{i}$ are parallel and anti-parallel to $\vec{B}$ and their cross products vanish. On the other hand, the magnetic forces acting on segments 2 and 4 are non-vanishing:

$$\begin{cases} \vec{F}_2 = I(-a\hat{j})\times(B\hat{i}) = IaB\hat{k} \\ \vec{F}_4 = I(a\hat{j})\times(B\hat{i}) = -IaB\hat{k} \end{cases}$$

with $\vec{F}_2$ pointing out of the page and $\vec{F}_4$ into the page. Thus, the net force on the rectangular loop is

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{0}$$

as expected. Even though the net force on the loop vanishes, the forces $\vec{F}_2$ and $\vec{F}_4$ will produce a torque which causes the loop to rotate about the y-axis (Figure). The torque with respect to the center of the loop is

$$\vec{\tau} = \left(-\frac{b}{2}\hat{i}\right)\times\vec{F}_2\left(-\frac{b}{2}\hat{j}\right)\times F_4$$

$$= \left(-\frac{b}{2}\hat{i}\right)\times\left(IaB\hat{k}\right) + \left(\frac{b}{2}\hat{i}\right)\times\left(-IaB\hat{k}\right)$$

$$\left(\frac{IabB}{2} + \frac{IabB}{2}\right)\hat{j} = IabB\hat{j} = IAB\hat{j}$$

where A = ab represents the area of the loop and the positive sign indicates that the rotation is clockwise about the y-axis. It is convenient to introduce the area vector $\vec{A} = A\hat{n}$ where $\hat{n}$ is a unit vector in the direction normal to the plane of the loop. The direction of the positive sense of $\hat{n}$ is set by the conventional right-hand rule. In our case, we have $\hat{n} = +\hat{k}$. The above expression for torque can then be rewritten as

$$\vec{\tau} = I\vec{A}\times\vec{B}$$

- Notice that the magnitude of the torque is at a maximum when $\vec{B}$ is parallel to the plane of the loop (or perpendicular to $\vec{A}$).
- Consider now the more general situation where the loop (or the area vector $\vec{A}$) makes an angle θ with respect to the magnetic field.

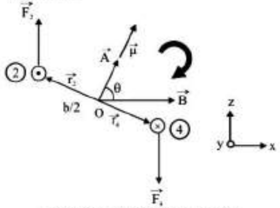


Fig.: Rotation of a rectangular current loop

From Figure, the lever arms and can be expressed as:

$$\vec{r}_2 = \frac{b}{2}\left(-\sin\theta\hat{i} + \cos\theta\hat{k}\right) = -\vec{r}_4$$

and the net torque becomes

$$\vec{\tau} = \vec{r}_2\times\vec{F}_2 + \vec{r}_4 = 2\vec{r}_2\times\vec{F}_2 = 2.\frac{b}{2}\left(-\sin\theta\hat{i} + \cos\theta\hat{k}\right)\times\left(IaB\hat{k}\right)$$

$$= \hat{j}IabB\sin\theta\,\hat{j} = I\vec{A}\times\vec{B}$$

- For a loop consisting of N turns, the magnitude of the toque is

$$\tau = \text{NIAB}\sin\theta$$

- The quantity $NI\vec{A}$ is called the magnetic dipole moment $\vec{\mu}$

$$\vec{\mu} = NI\vec{A}$$

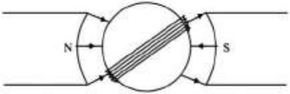
- Using the expression for $\vec{\mu}$, the torque exerted on a current-carrying loop can be rewritten as

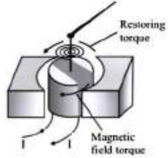
$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

- The above equation is analogous to $\vec{\tau} = \vec{p}\times\vec{E}$ in equation., the torque exerted on an electric dipole moment $\vec{P}$ in the presence of an electric field $\vec{E}$.

Moving Coil Galvanometer

- The main parts of a moving-coil galvanometer are shown in figure.





- The current to be measured is passed through the galvanometer. As the coil is in the magnetic field $\vec{B}$ of the permanent magnet, a torque $\vec{\tau} = ni\vec{A}\times\vec{B}$ acts on the coil. Here n = number of turns, i = current in the coil $\vec{A}$ = area-vector of the coil and $\vec{B}$ = magnetic field at the site of the coil. This torque deflects the coil from its equilibrium position.
- The pole pieces are made cylindrical. As a result, the magnetic field at the arms of the coil remains parallel to the plane of the coil everywhere even as the coil rotates. The deflecting torque is then $\tau = niAB$. As the upper end of the suspension strip

W is fixed, the strip gets twisted when the coil rotates. This produces a restoring torque acting on the coil. If the deflection of the coil is θ and the torsional constant of the suspension strip is k, the restoring torque is kθ. The coil will stay at a deflection θ where

$$niAB = k\theta \qquad \text{or,} \qquad i = \frac{k}{nAB}\theta$$

Hence, the current is proportional to the deflection. The constant $\frac{k}{nAB}$ is called the galvanometer constant.

We define the **current sensitivity** of the galvanometer as the deflection per unit current. From equation this current sensitivity is

$$\frac{\phi}{I} = \frac{NAB}{k}$$

A convenient way for the manufacturer to increase the sensitivity is to increase the number of turns N. We choose galvanometers having sensitivities of value, required by our experiment.

We define the **voltage sensitivity** as the deflection per unit volt of applied potential difference

$$\frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{1}{R}$$

An interesting point to note is that increasing the current sensitivity may not necessarily increase the voltage sensitivity. If N → 2N, i.e., we double the number of turns, then

$$\frac{\phi}{I} \rightarrow 2\frac{\phi}{I}$$

Thus, the current sensitivity doubles. However, the resistance of the galvanometer is also likely to double, since it is proportional to the length of the wire. In eq. N → 2N, and R → 2R, thus the voltage sensitivity,

$$\frac{\phi}{V} \rightarrow \frac{\phi}{V}$$

remains unchanged.

ILLUSTRATIONS

1. The horizontal component of earth's magnetic field at a certain place is 3.0×10^{-5}T and having a direction from the geographic south to geographic north. The force per unit length on a very long straight carrying a steady current of 1.2A in east to west direction is:

a. 3.0×10^{-5} N m^{-1} b. 3.2×10^{-5} N m^{-1}

c. 3.6×10^{-5} N m^{-1} d. 3.8×10^{-5} N m^{-1}

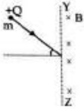
Ans. (c) Since $\vec{F} = I\vec{l} \times \vec{B} = IlB\sin\theta$

Now, force per unit length $f = \frac{F}{l} = IB\sin\theta$

When the current is flowing from east to west then $\theta = 90^0$, hence

$F = IB\sin 90^0 = 1.2 \times 3 \times 10^{-5} \times 1 = 3.6 \times 10^{-5}$ Nm^{-1}

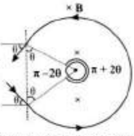
2. A particle with charge +Q and mass m enters a magnetic field of magnitude B, existing only to the right of the boundary YZ. The direction of the motion of the particle is perpendicular to the direction of B. Let the time spent by the particle in the field will be:



a. Tθ b. 2Tθ

c. $T\left(\frac{\pi+2\theta}{2\pi}\right)$ d. $T\left(\frac{\pi-2\theta}{2\pi}\right)$

Ans. (c)



A charged particle enters and leaves a uniform magnetic field symmetrically.

The time spent in the field is proportional to the angle (π + 2θ).

3. A particle with a specific charge s starts from rest in a region where the electric field has a constant direction, but whose magnitude increases linearly with time. The particle acquires a velocity v in time t.

a. $v \propto t$ b. $v \propto \sqrt{t}$

c. $v \propto t$ d. $v \propto t^2$

Ans. (a,d) E = αt (α = constant)

F = QE

a = F/m = QE/m = Es = αst

$\therefore \quad a = \frac{dv}{dt} = \alpha st$

or $\quad v = \frac{1}{2}\alpha st^2 \qquad \therefore v \propto t$ and t^2

4. A charged particle is fired at an angle θ to a uniform magnetic field directed along the x-axis. If the pitch of the helical path is equal to the maximum distance of the particle from the x-axis.

a. $\cos\theta = \frac{1}{\pi}$ b. $\sin\theta = \frac{1}{\pi}$

c. $\tan\theta = \frac{1}{\pi}$ d. $\tan\theta = \pi$

Ans. (d) Pitch $= \left(2\pi\frac{m}{QB}\right)v\cos\theta$, Also, $mv\sin\theta = QBr$ for motion perpendicular to the magnetic field $r = \frac{mv}{QB}\sin\theta$.

Maximum distance of the particle from the x-axis = 2r.

$\therefore \left(2\pi\frac{m}{QB}\right)v\cos\theta = 2.\frac{mv}{QB}\sin\theta$ or $\tan\theta = \pi$

5. Magnetic field at the centre of a circular loop of area A is B. The magnetic moment of the loop will be:

a. $\frac{BA^2}{\mu_0\pi}$ b. $\frac{BA^{3/2}}{\mu_0\pi}$

c. $\frac{BA^{3/2}}{\mu_0\pi^{1/2}}$ d. $\frac{2BA^{3/2}}{\mu_0\pi^{1/2}}$

Ans. (d) Magnetic field at the centre of a circular loop is given by, $B = \frac{\mu_0 I}{2R} \Rightarrow I = \frac{2BR}{\mu_0}$,

Magnetic moment,

$M = IA = \frac{2BR.\pi R^2}{\mu_0}$

$\Rightarrow M = \frac{2B\pi^{3/2}R^3}{\mu_0\pi^{1/2}} = \frac{2BA^{3/2}}{\mu_0\pi^{1/2}}$

6. A wire of length l carrying a current I ampere is bent into a circle. The magnitude of the magnetic moment is:

a. $\frac{Il^2}{2\pi}$ b. $\frac{Il^2}{4\pi}$

c. $\frac{l^2 I}{2\pi}$ d. $\frac{l^2 I}{4\pi}$

Ans. (d) The circumference of circle = length of wire

$\Rightarrow 2\pi r = l \Rightarrow r = \frac{l}{2\pi}$

$\therefore$ Magnetic moment, $M = IA = I\pi\frac{l^2}{4\pi^2} = \frac{Il^2}{4\pi}$

7. The cyclotron frequency v_c is given by:

a. $\frac{qB}{2\pi m}$ b. $\frac{mB}{2\pi q}$

c. $\frac{2\pi m}{qB}$ d. $\frac{2\pi B}{qm}$

Ans. (a) Time period, $T = \frac{2\pi m}{Bq}$

$\therefore$ Frequency, $v_c = \frac{Bq}{2\pi m}$

8. A circular loop of radius R carrying a current I is placed in a uniform magnetic field B perpendicular to the loop. The force on the loop is:

a. $2\pi RIB$ b. $2\pi RI^2B^3$

c. $\pi R^2 IB$ d. Zero

Ans. (d) $F_{net} = 0$. Also, torque, $\tau = MB\sin 0^\circ = 0$

9. A straight wire carrying a current of 13A is bent into a semi-circular arc of radius 2 cm as shown in figure. The magnetic field is 1.5×10^{-4} t at the centre of arc, then the magnetic field due to straight segment is:

a. 1.5×10^{4}T b. 2.5×10^{-4}T

c. Zero d. 3×10^{-4}T

Ans. (c) According to Biot savart law, $B = \frac{\mu_0}{4\pi}\frac{I|dl \times r|}{r^3}$

For each elements of straight segments, dl and r are parallel $\therefore dl \times r = 0 \therefore B = 0$

10. If a long straight wire carries a current of 40 A, then the magnitude of the field B at a point 15 cm away from the wire is:

a. 5.34×10^{-5}T b. 8.34×10^{-5}T

c. 9.6×10^{-5}T d. 10.2×10^{-5}T

Ans. (a) $B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{4\pi} \times \frac{2\times 40}{0.15} = 5.34 \times 10^{-5}$T

11. A steady electric current is flowing through a cylindrical conductor:

a. The magnetic field in the vicinity of the conductor is zero

b. The electric field in the vicinity of the conductor is zero

c. The magnetic field at the axis of the conductor is zero

d. The electric field at the axis of the conductor is zero

Ans. (c) Inside the conductor, magnetic field is given by,

$B = \frac{\mu_0 Ir}{2\pi R^2} \because$ at the axis $r = 0 \therefore B = 0$

12. If in moving coil galvanometer, a current i produces a deflection θ, then:

a. $i \propto \tan\theta$ b. $i \propto \theta$

c. $i \propto \theta^2$ d. $i \propto \sqrt{\theta}$

Ans. (b) For moving coil galvanometer

$i = \left(\frac{C}{NAB}\right)\theta$ or $i \propto \theta$

13. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity perpendicular to both $\vec{E}$ and $\vec{B}$ and comes out without any change in its magnitude or direction. Then:

a. $\vec{v} = \vec{B} \times \vec{E}/E^2$ b. $\vec{v} = \vec{E} \times \vec{B}/B^2$

c. $\vec{v} = \vec{B} \times \vec{E}/B^2$ d. $\vec{v} = \vec{E} \times \vec{B}/E^2$

Ans. (b) The particle comes out without any change in its magnitude & direction, $\therefore$ $F = qE - qvB = 0$

$\Rightarrow qE = qvB$

$\Rightarrow v = \frac{E}{B} = \frac{\vec{E}\times\vec{B}}{B^2}$

14. Which of the field patterns given below is valid for electric field as well as for magnetic field?

a. 

b. 

c. 

d. 

Ans. (c) The pattern of field lines shown in option (c) is correct because:

(i) a current carrying toroid produces magnetic field lines of such pattern.

(ii) a changing magnetic field with respect to time in a region perpendicular to the paper produces induced electric field lines of such pattern.

15. The magnetic field intensity due to a thin wire carrying current I in the figure shown is:

a. $\frac{\mu_0 I}{2\pi R}(\pi - \alpha + \tan\alpha)$ b. $\frac{\mu_0 I}{2\pi R}(\pi - \alpha)$

c. $\frac{\mu_0 I}{2\pi R}(\pi + \alpha)$ d. $\frac{\mu_0}{2\pi R}(\pi + \alpha - \tan\alpha)$

Ans. (a) Magnetic field due to circular arc is:

$$B_1 = \frac{\mu_0 I}{4\pi R}(2\pi - 2\alpha) = \frac{\mu_0 I}{2\pi R}(\pi - \alpha)$$

Magnetic field due to straight wire is:

$$B_2 = \frac{\mu_0 I}{4\pi R\cos\alpha}(\sin\alpha + \sin\alpha) = \frac{\mu_0 I}{4\pi R\cos\alpha}2\sin\alpha$$

$$B_2 = \frac{\mu_0 I}{2\pi R}\tan\alpha$$

$$\therefore B_{net} = B_1 + B_2 = \frac{\mu_0 I}{2\pi R}(\pi - \alpha + \tan\alpha)$$

16. If a particle is moving in a uniform magnetic field, then:

a. Both momentum and total energy will change

b. Its momentum changes but total energy remains the same

c. Both momentum and total energy remain the same

d. Its total energy changes but momentum remains the same

Ans. (b) In a uniform magnetic field, the direction of velocity changes but the magnitude remains constant.

$\therefore$ The momentum changes but the total energy remains constant.