Al1110 Assignment 6

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Outline

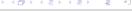
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Abstract

 This document contains the solution to Question of Chapter 2 of Papoulis book.





Question

Ex 2.13

A box contains white and black balls. When two balls are drawn without replacement, suppose the probability that both are white is 1/3. Find

- Find the smallest number of balls in the box.
- On the How small can the total number of balls be if black balls are even in number?





Theory

Let a = Number of white balls in the box.

Let b = Number of black balls in the box.

Let $W_k =$ "a white ball is drawn at the kth draw" .



Solution

We are given that $Pr(W_1W_2) = 1/3$.

$$Pr(W_1W_2) = Pr(W_2W_1) = Pr(W_2|W_1) Pr(W_1)$$
 (1)

$$\frac{1}{3} = \frac{a-1}{a+b-1} \cdot \frac{a}{a+b} \tag{2}$$

$$\frac{a}{a+b} < \frac{a-1}{a+b-1} \tag{3}$$

From equation (2) and (3), we can rewrite as,

$$\left(\frac{a-1}{a+b-1}\right)^2 < \frac{1}{3} < \left(\frac{a}{a+b}\right)^2 \tag{4}$$

This gives the inequalities,

$$(\sqrt{3}+1)b/2 < a < 1 + (\sqrt{3}+1)b/2$$





• For b = 1, this gives 1.36 < a < 2.36, or a = 2, and we get,

$$Pr(W_2W_1) = \frac{2}{3} \cdot \frac{1}{2}$$

$$= \frac{1}{3}$$
(6)

$$=\frac{1}{3}\tag{7}$$

Thus the smallest number of balls required is 3.





• For b=even number, we can use equation (4), with b = 2, 4, ... as shown in Table 1. From the table, 10 is the smallest number of balls (a = 6, b = 4) that gives the desired probability.





Table

b	а	$Pr(W_2W_1)$
2	3	$\frac{3}{4} \cdot \frac{2}{4} = \frac{3}{10} \neq \frac{1}{3}$
4	6	$\frac{6}{10} \cdot \frac{5}{4} = \frac{1}{3}$

Table 1: Probability



