AI1110 Assignment 6

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Outline

Abstract

- This document contains the solution to Question of Chapter 2 of Papoulis book.

Question

Ex 2.13

A box contains white and black balls. When two balls are drawn without replacement, suppose the probability that both are white is $1/3$.
Find

1. Find the smallest number of balls in the box.
2. How small can the total number of balls be if black balls are even in number?

Theory

Let a = Number of white balls in the box.
Let b = Number of black balls in the box.
Let W_k = "a white ball is drawn at the kth draw" .

Solution

We are given that $\Pr(W_1 W_2) = 1/3$.

$$\Pr(W_1 W_2) = \Pr(W_2 W_1) = \Pr(W_2 | W_1) \Pr(W_1) \tag{1}$$

$$\frac{1}{3} = \frac{a-1}{a+b-1} . \frac{a}{a+b} \tag{2}$$

$$\frac{a}{a+b} < \frac{a-1}{a+b-1} \tag{3}$$

From equation (2) and (3), we can rewrite as,

$$\left(\frac{a-1}{a+b-1}\right)^2 < \frac{1}{3} < \left(\frac{a}{a+b}\right)^2 \tag{4}$$

This gives the inequalities,

$$(\sqrt{3}+1)b/2 < a < 1 + (\sqrt{3}+1)b/2 \tag{5}$$

1. For b = 1, this gives 1.36 < a < 2.36, or a = 2, and we get ,

$$\Pr\left(W_2 W_1\right) = \frac{2}{3}.\frac{1}{2} \tag{6}$$

$$= \frac{1}{3} \tag{7}$$

Thus the smallest number of balls required is 3.

1. For b=even number, we can use equation (4), with $b = 2, 4, \dots$ as shown in Table 1. From the table, 10 is the smallest number of balls ($a = 6$, $b = 4$) that gives the desired probability.

Table

b	a	$\Pr\left(W_2 W_1\right)$
2	3	$\frac{3}{4} \cdot \frac{2}{4} = \frac{3}{10} \neq \frac{1}{3}$
4	6	$\frac{6}{10} \cdot \frac{5}{4} = \frac{1}{3}$

Table 1: Probability