AI1110 Assignment 6

DEEPSHIKHA-CS21BTECH11016

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Outline

Abstract

- This document contains the solution to Question of Chapter 5 of Papoulis book.

LaTeX

Question

Exercise 5.19

If X is an exponential random variable with parameter λ. Show that $Y = X^{\frac{1}{\beta}}$ has a Weibull distribution.

Theory

Weibull Distribution

1. Probability density function

$$f(x) = \kappa\lambda^{\kappa}x^{\kappa-1}e^{(-\lambda x)^{\kappa}} \tag{1}$$

where, λ=Positive scale parameter
and κ=Positive shape parameter

2. Exponential Distribution is a special case when $\kappa = 1$.
3. Rayleigh Distribution is a special case when $\kappa = 2$.

Solution

Let the random variable X have the exponential distribution with probability density function,

$$f_X(x) = \lambda e^{\lambda x} u(x) \tag{2}$$

$$Y = X^{\frac{1}{\beta}} \implies x_1 = y^{\beta} \tag{3}$$

$$|\frac{dy}{dx}| = \frac{1}{\beta} x^{\frac{1}{\beta - 1}} \tag{4}$$

$$\implies |\frac{dy}{dx}| = \frac{1}{\beta} y^{1-\beta} \tag{5}$$

$$f_Y(y) = \frac{1}{|\frac{dy}{dx}|} f_X(x_1) \tag{6}$$

$$= \lambda \beta y^{\beta - 1} e^{-\lambda y^{\beta}} U(y) \tag{7}$$

and it represents a Weibull distribution.