Al1110 Assignment 8

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May 29, 2022



Outline

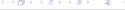
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Abstract

 This document contains the solution to Question of Chapter 6 of Papoulis book.





Question

Exercise 6.23

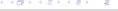
The random variables x and y are independent with respective densities $\mathcal{X}^2(m)$ and $\mathcal{X}^2(n)$. Show that if

$$z = \frac{\frac{x}{m}}{\frac{y}{n}}$$

then

$$f_z(z) = \gamma \frac{z^{\frac{m}{2}-1}}{\sqrt{(1+\frac{mz}{n})^{m+n}}} U(z)$$





Theory

Densities

• The density of x/m is given by

$$f_1(x) = \frac{m(mx)^{(\frac{m}{2}-1)}e^{\frac{-mx}{2}}}{\Gamma(\frac{m}{2})2^{\frac{m}{2}}}$$
(1)

2 The density of y/n is given by

$$f_2(y) = \frac{n(nx)^{(\frac{n}{2}-1)}e^{\frac{-nx}{2}}}{\Gamma(\frac{n}{2})2^{\frac{n}{2}}}$$
(2)

The density of z is given by

$$f_z(z) = \int_0^\infty y f_{xy}(yz, y) \, dy \tag{3}$$

Solution

$$f_z(z) = \int_0^\infty y \left(\frac{m(mzy)^{(\frac{m}{2}-1)} e^{\frac{-mzy}{2}}}{\Gamma(\frac{m}{2})} \right) \left(\frac{n(nx)^{(\frac{n}{2}-1)} e^{\frac{-nx}{2}}}{\Gamma(\frac{n}{2}) 2^{\frac{n}{2}}} \right) dy \qquad (4)$$

$$=\frac{\left(\frac{m}{2}\right)^{\frac{m}{2}}\left(\frac{n}{2}\right)^{\frac{n}{2}}}{\Gamma\left(\frac{m}{2}\right)}z^{\left(\frac{m}{2}-1\right)}\int_{0}^{\infty}y^{\left(\frac{m+n}{2}-1\right)}e^{y\left(\frac{n+mz}{2}\right)}dy\tag{5}$$

$$=\frac{\left(\frac{m}{2}\right)^{\frac{m}{2}}\left(\frac{n}{2}\right)^{\frac{n}{2}}}{\Gamma\left(\frac{m}{2}\right)}z^{\left(\frac{m}{2}-1\right)}\Gamma\left(\frac{m+n}{2}\right)\left(\frac{2}{n+mz}\right)^{\left(\frac{m+n}{2}\right)} \tag{6}$$





Solution

$$=\frac{\Gamma(\frac{m+n}{2})(m)^{\frac{m}{2}}(n)^{\frac{n}{2}}}{\Gamma(\frac{m}{2})\Gamma(\frac{n}{2})}\frac{z^{(\frac{m}{2}-1)}}{(n+mz)^{\frac{m+n}{2}}}$$
(7)

$$=\frac{\Gamma(\frac{m+n}{2})(\frac{m}{n})^{\frac{m}{2}}}{\Gamma(\frac{m}{2})\Gamma(\frac{n}{2})}\frac{z^{(\frac{m}{2}-1)}}{\sqrt{(1+\frac{mz}{n})^{m+n}}}$$
(8)

$$= \gamma \frac{z^{\frac{m}{2}-1}}{\sqrt{\left(1 + \frac{mz}{n}\right)^{m+n}}} U(z) \tag{9}$$

Proved.



