8 Production Decline Analysis

Contents

8.1 Introduction

Production decline analysis is a traditional means of identifying well production problems and predicting well performance and life based on real production data. It uses empirical decline models that have little fundamental justifications. These models include the following:

- Exponential decline (constant fractional decline)
- Harmonic decline
- Hyperbolic decline

Although the hyperbolic decline model is more general, the other two models are degenerations of the hyperbolic decline model. These three models are related through the following relative decline rate equation (Arps, 1945):

$$\frac{1}{q}\frac{dq}{dt} = -bq^d, \tag{8.1}$$

where b and d are empirical constants to be determined based on production data. When $d = 0$, Eq. (8.1) degenerates to an exponential decline model, and when $d = 1$, Eq. (8.1) yields a harmonic decline model. When $0 < d < 1$, Eq. (8.1) derives a hyperbolic decline model. The decline models are applicable to both oil and gas wells.

8.2 Exponential Decline

The relative decline rate and production rate decline equations for the exponential decline model can be derived from volumetric reservoir model. Cumulative production expression is obtained by integrating the production rate decline equation.

8.2.1 Relative Decline Rate

Consider an oil well drilled in a volumetric oil reservoir. Suppose the well's production rate starts to decline when a critical (lowest permissible) bottom-hole pressure is reached. Under the pseudo–steady-state flow condition, the production rate at a given decline time t can be expressed as

$$q = \frac{kh(\bar{p}_t - p_{wf}^c)}{141.2B_0\mu\left[\ln\left(\frac{0.472r_e}{r_w}\right) + s\right]}, \tag{8.2}$$

where

$\bar{p}_t =$ average reservoir pressure at decline time t,
$p_{wf}^c =$ the critical bottom-hole pressure maintained during the production decline.

The cumulative oil production of the well after the production decline time t can be expressed as

$$N_p = \int_0^t \frac{kh(\bar{p}_t - p_{wf}^c)}{141.2B_o\mu\left[\ln\left(\frac{0.472r_e}{r_w}\right) + s\right]}dt. \tag{8.3}$$

The cumulative oil production after the production decline upon decline time t can also be evaluated based on the total reservoir compressibility:

$$N_p = \frac{c_tN_i}{B_o}(\bar{p}_0 - \bar{p}_t), \tag{8.4}$$

where

$c_t =$ total reservoir compressibility,
$N_i =$ initial oil in place in the well drainage area,
$\bar{p}_0 =$ average reservoir pressure at decline time zero.

Substituting Eq. (8.3) into Eq. (8.4) yields

$$\int_0^t \frac{kh(\bar{p}_t - p_{wf}^c)}{141.2B_o\mu\left[\ln\left(\frac{0.472r_e}{r_w}\right) + s\right]}dt = \frac{c_tN_i}{B_o}(\bar{p}_0 - \bar{p}_t). \tag{8.5}$$

Taking derivative on both sides of this equation with respect to time t gives the differential equation for reservoir pressure:

$$\frac{kh(\bar{p}_t - p_{wf}^c)}{141.2\mu\left[\ln\left(\frac{0.472r_e}{r_w}\right) + s\right]} = -c_tN_i\frac{d\bar{p}_t}{dt} \tag{8.6}$$

Because the left-hand side of this equation is q and Eq. (8.2) gives

$$\frac{dq}{dt} = \frac{kh}{141.2B_0\mu\left[\ln\left(\frac{0.472r_e}{r_w}\right) + s\right]}\frac{d\bar{p}_t}{dt}, \tag{8.7}$$

Eq. (8.6) becomes

$$q = \frac{-141.2c_tN_i\mu\left[\ln\left(\frac{0.472r_e}{r_w}\right) + s\right]}{kh}\frac{dq}{dt} \tag{8.8}$$

or the relative decline rate equation of

$$\frac{1}{q}\frac{dq}{dt} = -b, \tag{8.9}$$

where

$$b = \frac{kh}{141.2\mu c_tN_i\left[\ln\left(\frac{0.472r_e}{r_w}\right) + s\right]}. \tag{8.10}$$

8.2.2 Production rate decline

Equation (8.6) can be expressed as

$$-b(\bar{p}_t - p_{wf}^c) = \frac{d\bar{p}_t}{dt}. \tag{8.11}$$

By separation of variables, Eq. (8.11) can be integrated,

$$-\int_0^t bdt = \int_{\bar{p}_0}^{\bar{p}_t} \frac{d\bar{p}_t}{(\bar{p}_t - p_{wf}^c)}, \tag{8.12}$$

to yield an equation for reservoir pressure decline:

$$\bar{p}_t = p_{wf}^c + \left(\bar{p}_0 - p_{wf}^c\right)e^{-bt} \tag{8.13}$$

Substituting Eq. (8.13) into Eq. (8.2) gives the well production rate decline equation:

$$q = \frac{kh(\bar{p}_0 - p_{wf}^c)}{141.2B_o\mu\left[\ln\left(\frac{0.472r_e}{r_w}\right) + s\right]}e^{-bt} \tag{8.14}$$

or

$$q = \frac{bc_tN_i}{B_o}(\bar{p}_0 - p_{wf}^c)\,e^{-bt}, \tag{8.15}$$

which is the exponential decline model commonly used for production decline analysis of solution-gas-drive reservoirs. In practice, the following form of Eq. (8.15) is used:

$$q = q_ie^{-bt}, \tag{8.16}$$

where q_i is the production rate at $t = 0$.

It can be shown that $\frac{q_2}{q_1} = \frac{q_3}{q_2} = \ldots\ldots = \frac{q_n}{q_{n-1}} = e^{-b}$. That is, the fractional decline is constant for exponential decline. As an exercise, this is left to the reader to prove.

8.2.3 Cumulative production

Integration of Eq. (8.16) over time gives an expression for the cumulative oil production since decline of

$$N_p = \int_0^t qdt = \int_0^t q_ie^{-bt}dt, \tag{8.17}$$

that is,

$$N_p = \frac{q_i}{b}\left(1 - e^{-bt}\right). \quad (8.18)$$

Since $q = q_i e^{-bt}$, Eq. (8.18) becomes

$$N_p = \frac{1}{b}(q_i - q). \quad (8.19)$$

8.2.4 Determination of decline rate

The constant b is called the *continuous decline rate*. Its value can be determined from production history data. If production rate and time data are available, the b value can be obtained based on the slope of the straight line on a semi-log plot. In fact, taking logarithm of Eq. (8.16) gives

$$\ln(q) = \ln(q_i) - bt, \quad (8.20)$$

which implies that the data should form a straight line with a slope of $-b$ on the log(q) versus t plot, if exponential decline is the right model. Picking up any two points, (t_1, q_1) and (t_2, q_2), on the straight line will allow analytical determination of b value because

$$\ln(q_1) = \ln(q_i) - bt_1 \quad (8.21)$$

and

$$\ln(q_2) = \ln(q_i) - bt_2 \quad (8.22)$$

give

$$b = \frac{1}{(t_2 - t_1)}\ln\left(\frac{q_1}{q_2}\right). \quad (8.23)$$

If production rate and cumulative production data are available, the b value can be obtained based on the slope of the straight line on an N_p versus q plot. In fact, rearranging Eq. (8.19) yields

$$q = q_i - bN_p. \quad (8.24)$$

Picking up any two points, (N_{p1}, q_1) and (N_{p2}, q_2), on the straight line will allow analytical determination of the b value because

$$q_1 = q_i - bN_{p1} \quad (8.25)$$

and

$$q_2 = q_i - bN_{p2} \quad (8.26)$$

give

$$b = \frac{q_1 - q_2}{N_{p2} - N_{p1}}. \quad (8.27)$$

Depending on the unit of time t, the b can have different units such as month^{-1} and year^{-1}. The following relation can be derived:

$$b_a = 12b_m = 365b_d, \quad (8.28)$$

where b_a, b_m, and b_d are annual, monthly, and daily decline rates, respectively.

8.2.5 Effective decline rate

Because the exponential function is not easy to use in hand calculations, traditionally the effective decline rate has been used. Since $e^{-x} \approx 1 - x$ for small x-values based on Taylor's expansion, $e^{-b} \approx 1 - b$ holds true for small values of b. The b is substituted by b', the effective decline rate, in field applications. Thus, Eq. (8.16) becomes

$$q = q_i(1 - b')^t. \quad (8.29)$$

Again, it can be shown that $\frac{q_2}{q_1} = \frac{q_3}{q_2} = \ldots\ldots = \frac{q_n}{q_{n-1}} = 1 - b'$.

Depending on the unit of time t, the b' can have different units such as month^{-1} and year^{-1}. The following relation can be derived:

$$(1 - b'_a) = (1 - b'_m)^{12} = (1 - b'_d)^{365}, \quad (8.30)$$

where b'_a, b'_m, and b'_d are annual, monthly, and daily effective decline rates, respectively.

Example Problem 8.1 Given that a well has declined from 100 stb/day to 96 stb/day during a 1-month period, use the exponential decline model to perform the following tasks:

1. Predict the production rate after 11 more months
2. Calculate the amount of oil produced during the first year
3. Project the yearly production for the well for the next 5 years

Solution

1. Production rate after 11 more months:

$$b_m = \frac{1}{(t_{1m} - t_{0m})}\ln\left(\frac{q_{0m}}{q_{1m}}\right) = \left(\frac{1}{1}\right)\ln\left(\frac{100}{96}\right) = 0.04082/\text{month}$$

Rate at end of 1 year:

$$q_{1m} = q_{0m}e^{-b_m t} = 100e^{-0.04082(12)} = 61.27\,\text{stb/day}$$

If the effective decline rate b' is used,

$$b'_m = \frac{q_{0m} - q_{1m}}{q_{0m}} = \frac{100 - 96}{100} = 0.04/\text{month}.$$

From

$$1 - b'_y = (1 - b'_m)^{12} = (1 - 0.04)^{12},$$

one gets

$$b'_y = 0.3875/\text{yr}$$

Rate at end of 1 year:

$$q_1 = q_0(1 - b'_y) = 100(1 - 0.3875) = 61.27\,\text{stb/day}$$

2. The amount of oil produced during the first year:

$$b_y = 0.04082(12) = 0.48986/\text{year}$$

$$N_{p,1} = \frac{q_0 - q_1}{b_y} = \left(\frac{100 - 61.27}{0.48986}\right)365 = 28{,}858\,\text{stb}$$

or

$$b_d = \left[\ln\left(\frac{100}{96}\right)\right]\left(\frac{1}{30.42}\right) = 0.001342\frac{1}{\text{day}}$$

$$N_{p,1} = \frac{100}{0.001342}(1 - e^{-0.001342(365)}) = 28{,}858\,\text{stb}$$

3. Yearly production for the next 5 years:

$$N_{p,2} = \frac{61.27}{0.001342}(1 - e^{-0.001342(365)}) = 17{,}681\,\text{stb}$$

$$q_2 = q_i e^{-bt} = 100e^{-0.04082(12)(2)} = 37.54\,\text{stb/day}$$

$$N_{p,3} = \frac{37.54}{0.001342}(1 - e^{-0.001342(365)}) = 10{,}834\,\text{stb}$$

$$q_3 = q_i e^{-bt} = 100e^{-0.04082(12)(3)} = 23.00\,\text{stb/day}$$

$$N_{p,4} = \frac{23.00}{0.001342}(1 - e^{-0.001342(365)}) = 6639\,\text{stb}$$

$$q_4 = q_i e^{-bt} = 100e^{-0.04082(12)(4)} = 14.09\,\text{stb/day}$$

$$N_{p,5} = \frac{14.09}{0.001342}(1 - e^{-0.001342(365)}) = 4061\,\text{stb}$$

In summary,

Year	Rate at End of Year (stb/day)	Yearly Production (stb)
0	100.00	—
1	61.27	28,858
2	37.54	17,681
3	23.00	10,834
4	14.09	6,639
5	8.64	4,061
		68,073

8.3 Harmonic Decline

When $d = 1$, Eq. (8.1) yields differential equation for a harmonic decline model:

$$\frac{1}{q}\frac{dq}{dt} = -bq, \tag{8.31}$$

which can be integrated as

$$q = \frac{q_0}{1 + bt}, \tag{8.32}$$

where q_0 is the production rate at $t = 0$.

Expression for the cumulative production is obtained by integration:

$$N_p = \int_0^t q dt,$$

which gives

$$N_p = \frac{q_0}{b}\ln(1 + bt). \tag{8.33}$$

Combining Eqs. (8.32) and (8.33) gives

$$N_p = \frac{q_0}{b}[\ln(q_0) - \ln(q)]. \tag{8.34}$$

8.4 Hyperbolic Decline

When $0 < d < 1$, integration of Eq. (8.1) gives

$$\int_{q_0}^{q} \frac{dq}{q^{1+d}} = -\int_0^t b dt, \tag{8.35}$$

which results in

$$q = \frac{q_0}{(1 + dbt)^{1/d}} \tag{8.36}$$

or

$$q = \frac{q_0}{\left(1 + \frac{b}{a}t\right)^a}, \tag{8.37}$$

where $a = 1/d$.

Expression for the cumulative production is obtained by integration:

$$N_p = \int_0^t q dt,$$

which gives

$$N_p = \frac{aq_0}{b(a-1)}\left[1 - \left(1 + \frac{b}{a}t\right)^{1-a}\right]. \tag{8.38}$$

Combining Eqs. (8.37) and (8.38) gives

$$N_p = \frac{a}{b(a-1)}\left[q_0 - q\left(1 + \frac{b}{a}t\right)\right]. \tag{8.39}$$

8.5 Model Identification

Production data can be plotted in different ways to identify a representative decline model. If the plot of log(q) versus t shows a straight line (Fig. 8.1), according to Eq. (8.20), the decline data follow an exponential decline model. If the plot of q versus N_p shows a straight line (Fig. 8.2), according to Eq. (8.24), an exponential decline model should be adopted. If the plot of log(q) versus log(t) shows a straight line (Fig. 8.3), according to Eq. (8.32), the

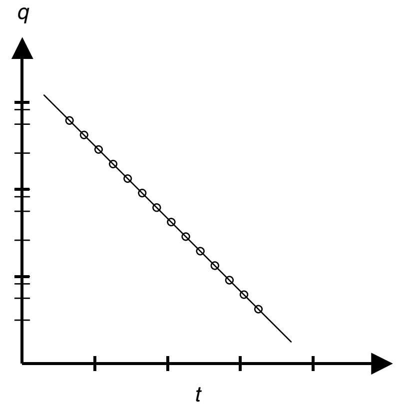

Figure 8.1 *A semilog plot of* q *versus* t *indicating an exponential decline.*

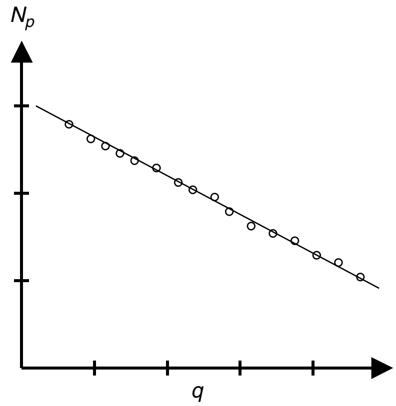

Figure 8.2 *A plot of* N_p *versus* q *indicating an exponential decline.*

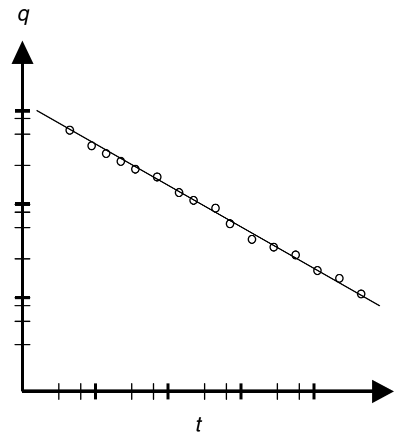

Figure 8.3 *A plot of log(*q*) versus log(*t*) indicating a harmonic decline.*

decline data follow a harmonic decline model. If the plot of N_p versus log(q) shows a straight line (Fig. 8.4), according to Eq. (8.34), the harmonic decline model should be used. If no straight line is seen in these plots, the hyperbolic decline model may be verified by plotting the relative decline rate defined by Eq. (8.1). Figure 8.5 shows such a plot. This work can be easily performed with computer program *UcomS.exe*.

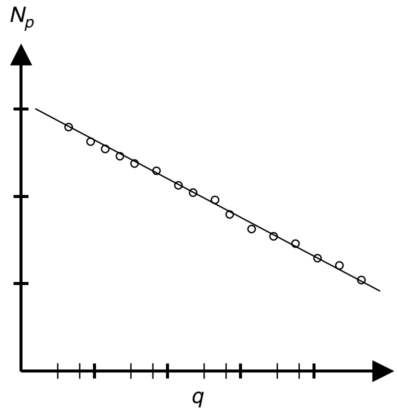

Figure 8.4 *A plot of* N_p *versus log(*q*) indicating a harmonic decline.*

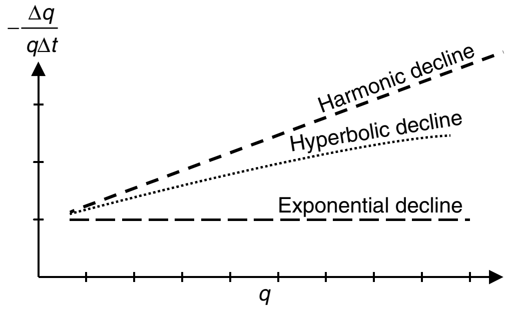

Figure 8.5 *A plot of relative decline rate versus production rate.*

8.6 Determination of Model Parameters

Once a decline model is identified, the model parameters a and b can be determined by fitting the data to the selected model. For the exponential decline model, the b value can be estimated on the basis of the slope of the straight line in the plot of log(q) versus t (Eq. [8.23]). The b value can also be determined based on the slope of the straight line in the plot of q versus N_p (Eq. [8.27]).

For the harmonic decline model, the b value can be estimated on the basis of the slope of the straight line in the plot of log(q) versus log(t) or Eq. (8.32):

$$b = \frac{\frac{q_0}{q_1} - 1}{t_1}. \tag{8.40}$$

The b value can also be estimated based on the slope of the straight line in the plot of N_p versus log(q) (Eq. [8.34]).

For the hyperbolic decline model, determination of a and b values is somewhat tedious. The procedure is shown in Fig. 8.6.

Computer program *UcomS.exe* can be used for both model identification and model parameter determination, as well as production rate prediction.

8.7 Illustrative Examples

Example Problem 8.2 For the data given in Table 8.1, identify a suitable decline model, determine model parameters, and project production rate until a marginal rate of 25 stb/day is reached.

Solution A plot of log(q) versus t is presented in Fig. 8.7, which shows a straight line. According to Eq. (8.20), the exponential decline model is applicable. This is further evidenced by the relative decline rate shown in Fig. 8.8.

Select points on the trend line:

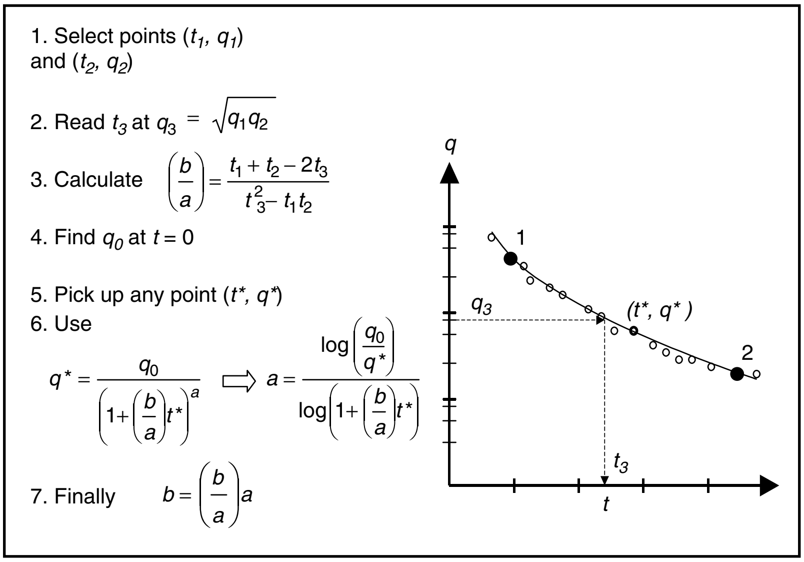

Figure 8.6 *Procedure for determining* a- *and* b-*values.*

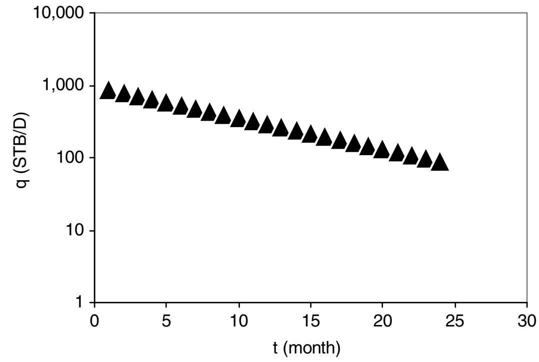

Figure 8.7 *A plot of log(q) versus t showing an exponential decline.*

Table 8.1 *Production Data for Example Problem 8.2*

t (mo)	q (stb/day)	t (mo)	q (stb/day)
1.00	904.84	13.00	272.53
2.00	818.73	14.00	246.60
3.00	740.82	15.00	223.13
4.00	670.32	16.00	201.90
5.00	606.53	17.00	182.68
6.00	548.81	18.00	165.30
7.00	496.59	19.00	149.57
8.00	449.33	20.00	135.34
9.00	406.57	21.00	122.46
10.00	367.88	22.00	110.80
11.00	332.87	23.00	100.26
12.00	301.19	24.00	90.720

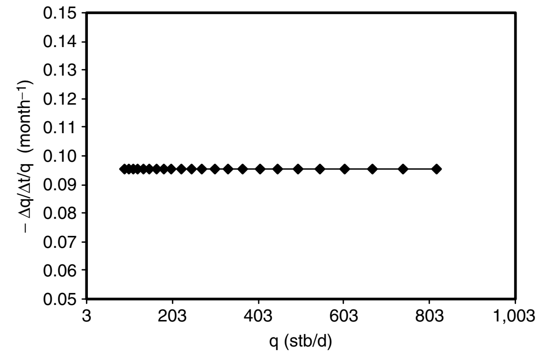

Figure 8.8 *Relative decline rate plot showing exponential decline.*

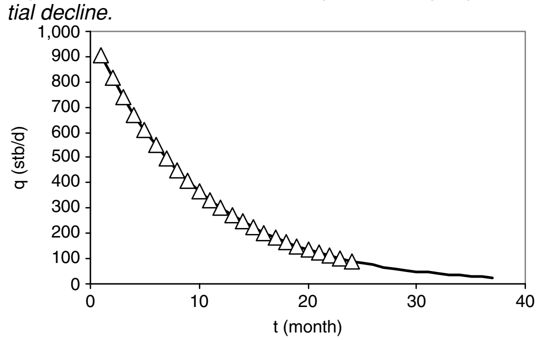

Figure 8.9 *Projected production rate by a exponential decline model.*

$t_1 = 5$ months, $q_1 = 607\,\text{stb/day}$
$t_2 = 20$ months, $q_2 = 135\,\text{stb/day}$

Decline rate is calculated with Eq. (8.23):

$$b = \frac{1}{(5-20)} \ln\left(\frac{135}{607}\right) = 0.11/\text{month}$$

Projected production rate profile is shown in Fig. 8.9.

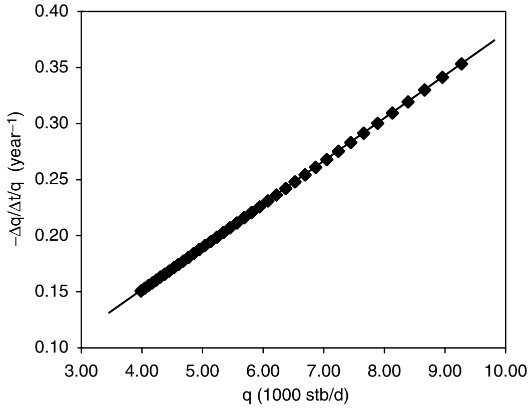

Figure 8.10 *Relative decline rate plot showing harmonic decline.*

Example Problem 8.3 For the data given in Table 8.2, identify a suitable decline model, determine model parameters, and project production rate until the end of the fifth year.

Solution A plot of relative decline rate is shown in Fig. 8.10, which clearly indicates a harmonic decline model.

On the trend line, select

$$q_0 = 10{,}000\,\text{stb/day at } t = 0$$

$$q_1 = 5{,}680\,\text{stb/day at } t = 2 \text{ years.}$$

Therefore, Eq. (8.40) gives

Table 8.2 *Production Data for Example Problem 8.3*

t (yr)	q (1,000 stb/day)	t (yr)	q (1,000 stb/day)
0.20	9.29	2.10	5.56
0.30	8.98	2.20	5.45
0.40	8.68	2.30	5.34
0.50	8.40	2.40	5.23
0.60	8.14	2.50	5.13
0.70	7.90	2.60	5.03
0.80	7.67	2.70	4.94
0.90	7.45	2.80	4.84
1.00	7.25	2.90	4.76
1.10	7.05	3.00	4.67
1.20	6.87	3.10	4.59
1.30	6.69	3.20	4.51
1.40	6.53	3.30	4.44
1.50	6.37	3.40	4.36
1.60	6.22	3.50	4.29
1.70	6.08	3.60	4.22
1.80	5.94	3.70	4.16
1.90	5.81	3.80	4.09
2.00	5.68	3.90	4.03

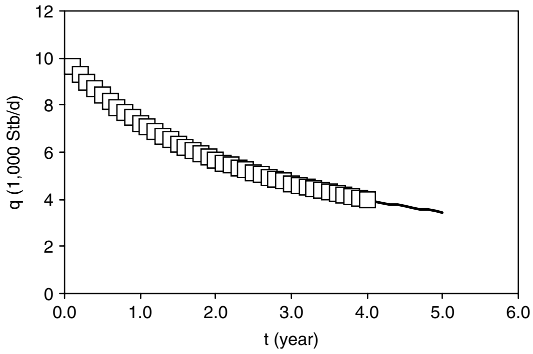

Figure 8.11 *Projected production rate by a harmonic decline model.*

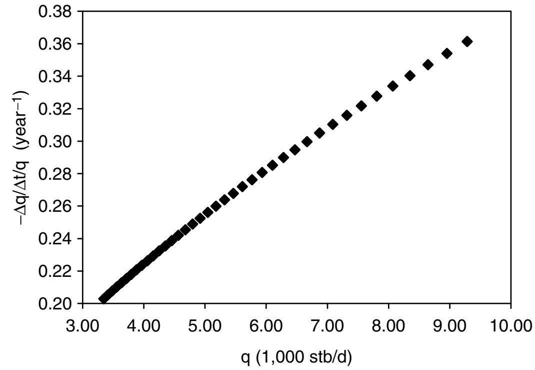

Figure 8.12 *Relative decline rate plot showing hyperbolic decline.*

$$b = \frac{\frac{10,000}{5,680} - 1}{2} = 0.38 \ 1/\text{yr}.$$

Projected production rate profile is shown in Fig. 8.11.

Example Problem 8.4 For the data given in Table 8.3, identify a suitable decline model, determine model parameters, and project production rate until the end of the fifth year.

Solution A plot of relative decline rate is shown in Fig. 8.12, which clearly indicates a hyperbolic decline model.

Select points

$$t_1 = 0.2 \text{ year}, \ q_1 = 9,280 \text{ stb/day}$$

$$t_2 = 3.8 \text{ years}, \ q_2 = 3,490 \text{ stb/day}$$

$$q_3 = \sqrt{(9,280)(3,490)} = 5,670 \text{ stb/day}$$

$$\left(\frac{b}{a}\right) = \frac{0.2 + 3.8 - 2(1.75)}{(1.75)^2 - (0.2)(3.8)} = 0.217$$

Read from decline curve (Fig. 8.13) $t_3 = 1.75$ years at $q_3 = 5,670$ stb/day.

Read from decline curve (Fig. 8.13) $q_0 = 10,000$ stb/day at $t_0 = 0$.

Pick up point ($t^* = 1.4$ years, $q^* = 6,280$ stb/day).

Table 8.3 *Production Data for Example Problem 8.4*

t (yr)	q (1,000 stb/day)	t (yr)	q (1,000 stb/day)
0.10	9.63	2.10	5.18
0.20	9.28	2.20	5.05
0.30	8.95	2.30	4.92
0.40	8.64	2.40	4.80
0.50	8.35	2.50	4.68
0.60	8.07	2.60	4.57
0.70	7.81	2.70	4.46
0.80	7.55	2.80	4.35
0.90	7.32	2.90	4.25
1.00	7.09	3.00	4.15
1.10	6.87	3.10	4.06
1.20	6.67	3.20	3.97
1.30	6.47	3.30	3.88
1.40	6.28	3.40	3.80
1.50	6.10	3.50	3.71
1.60	5.93	3.60	3.64
1.70	5.77	3.70	3.56
1.80	5.61	3.80	3.49
1.90	5.46	3.90	3.41
2.00	5.32	4.00	3.34

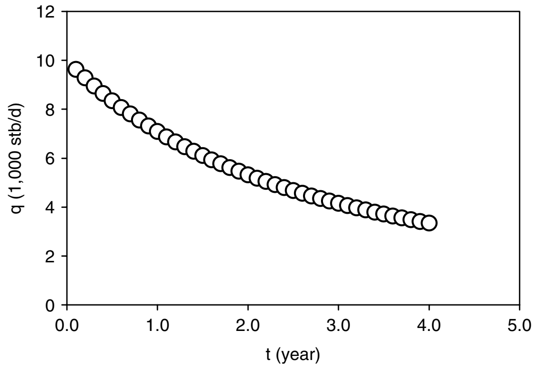

Figure 8.13 *Relative decline rate shot showing hyperbolic decline.*

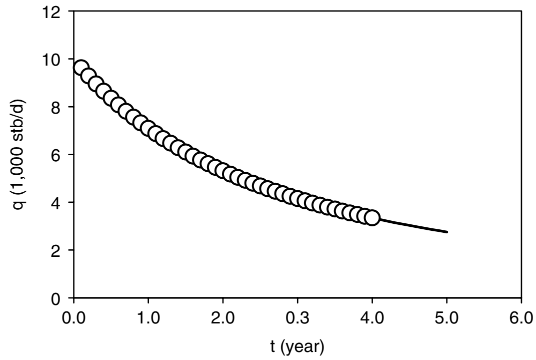

Figure 8.14 *Projected production rate by a hyperbolic decline model.*

$$a = \frac{\log\left(\frac{10,000}{6,280}\right)}{\log\left(1 + (0.217)(1.4)\right)} = 1.75$$

$$b = (0.217)(1.758) = 0.38$$

Projected production rate profile is shown in Fig. 8.14.

Summary

This chapter presents empirical models and procedure for using the models to perform production decline data analyses. Computer program *UcomS.exe* can be used for model identification, model parameter determination, and production rate prediction.

References

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Problems

8.1 For the data given in the following table, identify a suitable decline model, determine model parameters, and project production rate until the end of the tenth year. Predict yearly oil productions:

8.2 For the data given in the following table, identify a suitable decline model, determine model parameters, predict the time when the production rate will decline to a marginal value of 500 stb/day, and the reverses to be recovered before the marginal production rate is reached:

Time (yr)	*Production Rate* (1,000 stb/day)
0.1	9.63
0.2	9.29
0.3	8.98
0.4	8.68
0.5	8.4
0.6	8.14
0.7	7.9
0.8	7.67
0.9	7.45
1	7.25
1.1	7.05
1.2	6.87
1.3	6.69
1.4	6.53
1.5	6.37
1.6	6.22
1.7	6.08
1.8	5.94
1.9	5.81
2	5.68
2.1	5.56
2.2	5.45
2.3	5.34
2.4	5.23
2.5	5.13
2.6	5.03
2.7	4.94
2.8	4.84
2.9	4.76
3	4.67
3.1	4.59
3.2	4.51
3.3	4.44
3.4	4.36

Time (yr)	*Production Rate* (stb/day)
0.1	9.63
0.2	9.28
0.3	8.95
0.4	8.64
0.5	8.35
0.6	8.07
0.7	7.81
0.8	7.55
0.9	7.32
1	7.09
1.1	6.87
1.2	6.67
1.3	6.47
1.4	6.28
1.5	6.1
1.6	5.93
1.7	5.77
1.8	5.61
1.9	5.46
2	5.32
2.1	5.18
2.2	5.05
2.3	4.92
2.4	4.8
2.5	4.68
2.6	4.57
2.7	4.46
2.8	4.35
2.9	4.25
3	4.15
3.1	4.06
3.2	3.97
3.3	3.88
3.4	3.8

8.3 For the data given in the following table, identify a suitable decline model, determine model parameters, predict the time when the production rate will decline to a marginal value of 50 Mscf/day, and the reverses to be recovered before the marginal production rate is reached:

Time (mo)	*Production Rate* (Mscf/day)
1	904.84
2	818.73
3	740.82
4	670.32
5	606.53
6	548.81
7	496.59
8	449.33
9	406.57
10	367.88
11	332.87
12	301.19
13	272.53
14	246.6
15	223.13
16	201.9
17	182.68
18	165.3
19	149.57
20	135.34
21	122.46
22	110.8
23	100.26
24	90.72