In silico voting experiments

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Introduction

This work is devoted to computer modeling of voting. To create a voting model, two parameters must be set.

The first one is the economic environment in which voting takes place, that is, the number of voters, their preferences (options, opinions and utilities) and the number of alternatives. The second parameter is the voting rule and the decision-making behavior that guides voters if their preferences and other relevant information are known. The author of the article considers various combinations of these two parameters and conducts simulation results for each of the combinations.

Topic of the paper

The first thing that determines the author of the article is "the culture".

Def: The random generation of a profile of voter preferences is called a culture.

The article considers several types of cultures, many of which can be correlated with real-life examples, such juries or political elections. Here is the list of cultures for which the simulation was carried out:

1. Rousseauist cultures

   - *Individual preferences were modeled such that it may differ because of mistakes individuals make when forming their opinion about a pre-existing truth. If underlying true ranking of the alternatives is*

     $$1 \succ 2 \succ ... \succ K$$

     then each of n voters is correct with probability $p(k, k')$ when comparing k and k'.
   - $p(k, k')$ is defined by Truchon and Drissi-Bakhkhat (2004) and depends on two parameters (α, β).
   - This framework is called the Rousseauist culture of size (n, K) and parameters (α, β).

2. Impartial culture

   - *For n voters and K alternatives Impartial culture is obtained by chosing each individual preference at random uniformly among the $K!$ linear orderings of the alternatives, and independently of the preferences of the other voters.*
   - *There is a complete symmetry among alternatives.*

3. Distributive cultures

   - *Generated as follows: one unit of a divisible good (a "cake") has to be shared among n individuals. Each individuals wants her share to be as large as possible, and does not care about the other shares. The set of alternatives is here infinite, it is the n-simplex:*

     $$\triangle_n = \{x \in R^n : 0 < x_i, \sum_{i=0}^{n} x_i = 1\}$$
   - *Distributive cultures can represent redistributive politics with assumption, that there are not n individuals but n groups of individuals, the groups being of equal size. Each of the K candidates then choses to favor more or less the various groups.*
   - *Consensual and Inegalitarian cultures differ from each other by deffinition of x_i.*
   - Uses Gini index as measure of the inequality.

   3.1 Consensual redistributive culture

3.2 Inegalitarian distributive cultures

4. Spatial cultures

- *These cultures stem from the spatial theory of voting.*
- *In the Euclidean space R^d with d dimensions, each voter i has a bliss point ω_i and a utility function defined on R^d which is decreasing with the distance to ω_i:*

$$u_i(x) = -||x - \omega_i||$$

- *An alternative is a point in R^d and a culture is defined by the number of dimensions d and the probability distributions for n bliss points and K alternatives.*

4.1 Uni-dimensional spatial culture

4.2 Multi-dimensional cultures

Considering the various options for voting rules and summing up the voters, the author chose the following:

Plurality rule, the Borda rule, the Copeland rule and Approval Voting, when voters vote sincerely or strategically. Strategic behavior is introduced in a heuristic way as "responsive voting" without reference to equilibrium considerations.

1. Sincere voting

- *Three voting rules based on sincere behavior: the Plurality rule, the Borda rule and the Copeland rule, which is a familiar Condorcet-consistent agregation rule. For Approval Voting, sincere voting (in the usual definition) does not provide a well-specified behavior.*

2. Responsive voting

- *Here the voters respond to an annonced candidate score vector. The proposed reaction functions are derived from the theory of strategic voting: the voter holds some belief on the other voters' actions and rationally responds to this belief.*

4.1 Plurality voting

4.2 Approval voting

- *Author used the strategic best-response function introduced and justified in Laslier (2009).*

4.1 Borda voting

The main objective of the article was to confirm the fact that in addition to the voting rule itself, the behavior of voters is of paramount importance for predicting election results and, therefore, for assessing the quality of the voting rule.

Main results (one or two)

One of the main and general results of this work is the fact that exist voting rules that improve substancially on Plurality rule. In order to see that, we should take a look at the tables from different cultures.

For Consensual redistributive culture we can see, that Borda, Copeland and AV show do slightly better than random choice, whereas Plurality does slightly worse. And for Inegalitarian distributive culture we can see that Plurality rule behaves very badly, compare to other alternatives:

Consensual redistributive culture			
$n = 11$	$K = 3$	$K = 5$	$K = 15$
Rule	Gini		
random choice	.31	.31	.31
Plurality	.32	.36	.40
Copeland	.30	.30	.29
AV1	.31	.30	.31
Borda	.31	.30	.29

Inegalitarian distributive culture			
$n = 11$	$K = 3$	$K = 5$	$K = 15$
Rule	Gini		
random choice	.42	.42	.42
Plurality	.42	.47	.57
Copeland	.36	.32	.26
AV1	.36	.25	.25
Borda	.36	.32	.26

Also we can see, that Plurality doesn't obtain best results for Spatial cultures:

Uni-dimensional culture			
Pr. Condorcet :	1	1	1
$n = 11$	$K = 3$	$K = 5$	$K = 15$
Rule	Condorcet		
Plurality	.736	.528	.305
Copeland	1	1	1
AV1	1	1	1
Borda	.868	.777	.616
Borda 1	.850	.508	.158
Borda 2	.996	.557	.166
Borda 3	.983	.732	.223
Borda 4	.999	.739	.271
Borda 5	.998	.806	.340

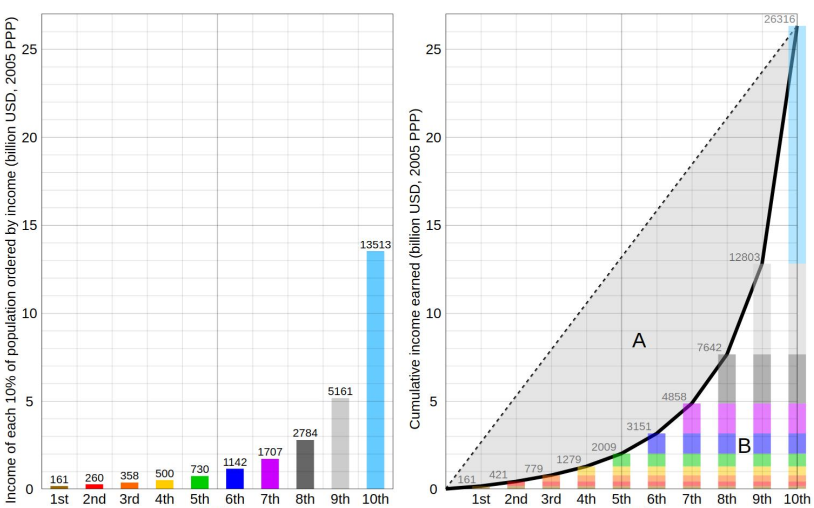
Bi-dimensional rectangular culture			
Pr. Condorcet :	.989	.966	.833
$n = 11$	$K = 3$	$K = 5$	$K = 15$
Rule	Copeland		
Plurality	.782	.538	.233
Copeland	1	1	1
AV1	.989	.966	.833
Borda	.874	.538	.624
Borda 1	.861	.479	.122
Borda 2	.988	.500	.133
Borda 3	.966	.661	.316
Borda 4	.990	.674	.393
Borda 5	.965	.753	.476

In case of unidimensional culture, the iterated strategic responses to the Borda ranking do very bad in the first iteration, as was already seen in previous sections but the situation here improves with successive iterations. And as you can see, in the table for Bi-dimensional rectangular culture the Borda rule does much better.

Clarifying figure/table

We were interested in better understanding of Gini index and wanted to show how it can be calculated in general case.

Usually Gini index is a statistical indicator of the degree of stratification of the society of a given country or region according to any studied characteristic. Sometimes the Gini coefficient is also used to identify the level of inequality in accumulated wealth and we would like to consider this interpretation in more detail. To define Gini index in this case we will start with an example:



On the diagram above (on the left side) you can see the income of each 10% of USA population in billions of dollars. On the right side of the same diagram you can see cumulative earned imcome in billions of dollars.

Black bold line connects values of cumulative income and plots the proportion of the total income of the population (y axis) that is cumulatively earned by the bottom x of the population (see diagram). The dotted line at 45 degrees thus represents perfect equality of incomes. The Gini coefficient can then be thought of as the ratio of the area that lies between the line of equality and the bold line (Lorenz curve in case of continues data) (marked A in the diagram) over the total area under the line of equality (marked A and B in the diagram); i.e., $G = \frac{A}{A+B}$.

Another paper citation

- Alessandro Lizzeri (1999) "Budget deficit and redistributive politics" Review of Economics Studies 66:909-928

In this paper there are theoretical models of redistributive politics that use economic environments which are identical or related to the set of alternatives n-simplex. More precisely it shows that the same forces that push candidates to redistribute resources across voters to pursue political advantage are forces that generate budget deficits.

Our paper citation

- Adrian Miriou (2012) "Experiments in Political Science: The Case of the Voting Rules" Probabilities,Laws, and Structures pp 403-416

As Laslier mentioned in the conclusion of his paper: "Apart the voting rule itself, the behavior of voters is of primary importance to predict the outcome of an election and therefore to assess the quality of a voting rule." And Miriou refers to that conclusion saying: "Domain rules help characterize voting procedures as complex institutions. They specify the way in which a collection of profiles is generated. As already mentioned, computer simulations have been used to investigate different "cultures", i.e. generations of collections of profiles. Different rules behave differently on such domains."