Blending Problems

Dealing with Proportional Constraints
The Words and the Algebra

Section 9.4 Blending Models

- Blending relationships are common across many industries
- Two types of blending constraints
  - **Proportional constraints** (individual elements must be a constrained proportion of the total)
  - **Property constraints** (the output blend has to meet a particular constraint on a specific property)

Proportional Constraint

- Consider the furniture production problem in the beginning of section 9.2

Veerman makes three kinds of office furniture: chairs, desks, and tables. Each requires labor in the parts, assembly, and shipping depts. Maximum potential sales are estimated for the coming quarter. Profit contribution for each item is known. Determine the product mix that maximizes profit.

Solving the Basic Problem

	Chairs	Desks	Tables			
	0	275	100			
Profit	15	24	18	$8400		
Constraints						
Labor Constraint		Hours per Unit				
Fabrication	4	6	2	1850	<=	1850 labor hours
Assembly	3	5	7	2075	<=	2400 labor hours
Shipping	3	2	4	950	<=	1500 labor hours
Demand Constraint						
Chairs	1			0	<=	360
Desks		1		275	<=	300
Tables			1	100	<=	100

Upon seeing this solution, the marketing department balks. We need to make some chairs, too! New constraint: no less than 25% of total pieces

Proportional Constraints

- Such constraints are based on a fraction of each decision variable to the total of the DVs
- Although this is a valid constraint, it is not a *linear* constraint (it divides DVs by DVs)
- We can convert it to a linear constraint by multiplying both sides to get
- Then we collect the terms and move them all to the left-hand side to get

Proportional Constraints Continued

- For all three furniture types, we will have three new constraint inequalities:

Reformulated Furniture Production

General Procedure: Proportional Constraints

1. Write the fraction that expresses the constrained proportion.
2. Write the inequality implied by the lower bound or upper bound
3. Multiply both sides of the inequality by the denominator and collect terms
4. The new inequality should be ready to incorporate into the model

Mixing Problems

- A different type of blending problem
- Materials are mixed in various amounts
  - Each input material has a particular property or properties
  - The final product has a value of that property that is a weighted average of the input material properties
- Note: a mixing problem assumes the decision includes how much of each component to include in the mix
  - If the proportions are fixed, you don't need these methods

Example, Page 250:
The Diaz Coffee Company

- Three types of beans (Brazilian, Columbian, and Peruvian) available for input into final product
- Each bean type has different values of Aroma Rating and Strength Rating
- There are limits on the amount of each type of bean available
- The beans cost different amounts per pound

Diaz Coffee Data

- For the three beans, here is the data

- The company wants its final blend to have an aroma rating of at least 78 and a strength of at least 16
- The company wants to make 4,000,000 at the lowest cost possible

Solving the Problem

- What must be decided?
  - How many pounds of each kind of bean to include in the mix
- What is the objective?
  - Cost, a function of the pounds of each times the cost per pound
- What are the constraints?
  - Pounds available
  - Aroma and strength

The Aroma Mixing Constraint

- Aroma of the final product is a weighted average of the aromas of the beans used

- Rearranging and collecting terms this becomes

The Strength Mixing Constraint

- Strength must be greater than 16, so:

- And this becomes:

Diaz Coffee: Solved

Gas Blending (Problem 9-9)

- An oil company makes three brands of oils: Regular, Midgrade, and Supreme
- Each brand is produced from four crude stocks
- Each brand must meet minimum viscosity standard and sells for a different price
- The supply of each crude stock is limited, and the cost is different for each
- Determine the optimal production plan for a single day, assuming that oil produced can be sold, either now or in the future

Gas Blending Problem Data

Crude Stock	Viscosity	Cost per Barrel	Supply per Day
1	20	$ 7.10	1000
2	40	$ 8.50	1100
3	30	$ 7.70	1200
4	55	$ 9.00	1100

Production Data

Brand	Min Viscosity Index	Selling Price	Daily Demand
Regular	25	$ 8.50	2000
Multigrade	35	$ 9.00	1500
Supreme	50	$ 10.00	750

Structure of the Problem

- What are the decision variables?
  - The amounts of each CS used in each of the 3 blends
  - Twelve variables:

    $x_{1R}, x_{2R}, x_{3R}, x_{4R}, x_{1M}, x_{2M}, \ldots, x_{4S}$
- What is the objective function?
  - Profit = revenue minus cost
  - Revenue = amount of each product time its price
  - Cost = amount of each CS times its cost

The Viscosity Constraint

- This is an example of a *proportional* constraint
- We first write the constraint in terms of the actual formula
- In this case, the mean viscosity must be larger than a limiting value. For Regular this becomes:

Proportional Constraints

- This is a nonlinear constraint, which causes difficulties for solver, but it can be converted to a linear constraint:
- Multiply both sides by the denominator to get:

- We can regroup to get

Proportional Constraints – cont'd

- Using the same procedure for the other two products (MG and SUP), we have three new constraints:

- These ensure we meet the viscosity mix

Part of Your Challenge:
What Problem Do I Have?

- The original, straightforward LP problem mixes input proportions as well, but the mix coefficients are *fixed*
  - Consider the palette problem
- In a *blending* problem you can choose how much you of each component you should include in each output product
- Homework and tests could draw from either one

Move to solver for blending…

- State the three viscosity constraints as weighted averages
- Multiply through by the denominator and recollect terms
- Insert the three new constraints into solver
- Solve