Summary of Godel's incompleteness theorems

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Gödel's two incompleteness theorems are among the most important results in modern science, and have deep implications for various issues.
axioms:- mathematical statement which we assume to be true without a proof is called an axiom. example:- 2+2=4
A system that does not include contradictions is called **consistent**
Any logical system (set of axioms) with unprovable statements is called **incomplete.**

1 first incompleteness theorem

Gödel's first incompleteness theorem says that if we have a consistent logical system (example, a set of axioms with no contradictions) in which we can do a certain amount of arithmetic, then there are statements in that system which are unprovable using just that system's axioms.

In other words, as long as our logical system is complicated enough to include addition and multiplication, then our logical system is incomplete. There are things we can't prove true or false!
example:- take the statement 'this statement cannot be proved'. Now if the statement is false, then it can be proved which would lead us to say that it is true. This is a contradiction, so cannot be correct. Therefore, the statement must be true, and therefore there are statements which are true that cannot be proved, which is a relatively simple version of the incompleteness theorem.

2 second incompleteness theorem

The second incompleteness theorem says that, within our mathematical system, we cannot prove that we can't have contradictions.Gödel's second incompleteness theorem gives a specific example of such an unprovable statement. And the example is quite a doozy.

The theorem says that inside of a similar consistent logical system (one without contradictions), the consistency of the system itself is unprovable!
The loss of certainty following the dissemination of Gödel's incompleteness theorems continues to have a profound effect on the **philosophy of mathematics.**