Week 2: Risk and Reward: Modeling High Uncertainty Settings

Session 1

- High-Uncertainty Settings: Stock Price Example
- Probability Distributions: Scenario Approach
- Parameters of the Probability Distributions: Expected Value, Variance, Standard Deviation
- Uncertainty and Risk

Week 2: Risk and Reward: Modeling High Uncertainty Settings

- Common Scenarios for Multiple Random Variables
- Risk Reduction Example: Investing in a Pair of Stocks
- Calculating and Interpreting Correlation Values

Session 2

- Using Scenarios for Optimizing Under High Uncertainty: Portfolio Selection Problem
- Sensitivity Analysis and Efficient Frontier

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Session 3

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Low-Uncertainty vs. High Uncertainty Settings

- In our first example in Week 1, we have looked at a company (Hudson Readers Inc.) faced with a decision of how to allocate its advertising budget for a new product

Low-Uncertainty vs. High Uncertainty Settings

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- All of the parameters in that example were assumed to take **deterministic** values

- For example, the sales response to advertising the Standard version in India is assumed to be 0.05, rather than, say, having 50%-50% chance of being either 0.03 or 0.07

Low-Uncertainty vs. High Uncertainty Settings

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- All of the parameters in that example were assumed to take **deterministic** values

- For example, the sales response to advertising the Standard version in India is assumed to be 0.05, rather than, say, having 50%-50% chance of being either 0.03 or 0.07

- Ignoring randomness in the data (for example, by replacing random quantities by their expected values) dramatically simplifies the process of finding the best solution

High-Uncertainty Setting: A Stock Price Example

- Consider a set of daily closing prices for a hypothetical stock A for a period of 40 consecutive trading days (Stock A.xlsx)
- "Closing price" is the last price at which a stock was traded on a particular day

Trading Day	Closing Price for Stock A (in $)
1	35.79
2	36.96
3	36.15
…	…
37	43.37
38	43.43
39	43.21
40	43.70

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Closing price on Day 1 = $35.79

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Closing price on Day 1 = $35.79

Closing price on Day 2 = $36.96

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Closing price on Day 1 = $35.79

Closing price on Day 2 = $36.96

Closing price on Day 40 = $43.70

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- Historical values for closing stock prices are available, for example, at Yahoo Finance (http://finance.yahoo.com/q/hp?s=YHOO)

High-Uncertainty Setting: A Stock Price Example

- Analysis of randomness is often focused on stock “returns”
- The “return” on a particular trading day is the relative (percentage) change between the closing price on that trading day and the closing price on the previous trading day

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Return on Day 2 = ($36.96-$35.79)/$35.79 ≈ 0.03269

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Return on Day 3 = ($36.15-$36.96)/$36.96 ≈ - 0.02191

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Return on Day 40 = ($43.70-$43.21)/$43.21 ≈ 0.01134

Investing in Stock A: Modeling Future Value

- Consider an investor that purchases a number of shares of stock A at the closing price on day 40

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- What value will this investment have at the closing of trading on the next day?

- This value depends on the return on stock A on the next day, R

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- This value depends on the return on stock A on the next day, R

- How do we model the value of R?

Modeling Future Values

- Modeling future values is a complex task that can combine statistical analysis of historical data and subjective inputs, such as expert opinions

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Modeling Future Values

- Modeling future values is a complex task that can combine statistical analysis of historical data and subjective inputs, such as expert opinions

- Experience with making decisions in a particular business context can be a major factor in determining how historical data are to be used and how to combine historical data with subjective inputs

- Testing alternative plausible models of the future may be necessary to increase confidence in the recommended decisions

Scenario Approach to Modeling Future Realizations of A Random Quantity

- We are going to base our analysis of the future price of stock A on the following **modeling assumption**: the daily return on stock A is a random value that can take each of 20 values observed in the past 20 trading days, with equal probability (1/20)

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- In other words, we are making an assumption that the last 20 values of the return on stock A completely describe all the possible values of tomorrow's return, and that each of those 20 values is equally likely to be repeated tomorrow

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- The term **"scenario"** is used to describe each of the past realizations of the random quantity – and modeling the future using a number of scenarios is called **"scenario approach"**

Scenario Approach to Modeling Future Realizations of A Random Quantity

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  - 1) historical return values observed beyond the last 20 days are not likely to be relevant for predicting tomorrow's return, and
  - 2) each of the values observed in the past 20 days is equally likely to be observed tomorrow

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- Stock A.xlsx

Scenario Approach to Modeling the Value of R: Complete Probability Distribution

Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
-0.01178	0.05
-0.01515	0.05
-0.00353	0.05
-0.01772	0.05
-0.02345	0.05
0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Scenario 1: Return R_1 = -0.00024, occurring with probability p_1 = 0.05

- Stock A.xlsx

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0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Scenario 20: Return R_{20} = 0.01134, occurring with probability p_1 = 0.05

- 40 parameters provide ***complete description*** of this distribution

Scenario Approach to Modeling the Value of R: Complete Probability Distribution

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-0.00024	0.05
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0.01214	0.05
0.00138	0.05
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Expected Value of R

- ***Expected value*** tells you what you will get if you average the values of the infinite number of independent random “draws” from a distribution

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Expected Value of R:
$E(R) = p_1 * R_1 +$

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Expected Value of R:

$E(R) = p_1{}^*R_1 + p_2{}^*R_2 +$

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0.01276	0.05
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Expected Value of R:

$$E(R) = p_1{}^*R_1 + p_2{}^*R_2 + \ldots + p_{19}{}^*R_{19} + p_{20}{}^*R_{20}$$

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0.01276	0.05
0.01214	0.05
0.00138	0.05
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0.01134	0.05

Expected Value of R:

$E(R) = p_1*R_1 + p_2*R_2 + \ldots + p_{19}*R_{19} + p_{20}*R_{20}$

- In Excel, you can use the =SUMPRODUCT() function to calculate the expected value of R

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Expected Value of R:

$E(R) = p_1{*}R_1 + p_2{*}R_2 + \ldots + p_{19}{*}R_{19} + p_{20}{*}R_{20} \approx \boxed{0.003467}$

- While, on average, R's value is 0.003467, on any particular random "draw", the actual value of R can be as low as **-0.02345** or as high as **0.03562**

Parameters Summarizing the Properties of a Distribution: Variance and Standard Deviation

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0.01276	0.05
0.01214	0.05
0.00138	0.05
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0.01134	0.05

- ***Variance*** and ***standard deviation*** indicate how "far away", on average, a random value of R is from its expected value E(R) = 0.003467

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0.01134	0.05

Variance of R:

$$Var(R) = p_1*(R_1 - E(R))^2 +$$

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Variance of R:

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Variance of R:

$$Var(R) = p_1 * (R_1 - E(R))^2 + p_2 * (R_2 - E(R))^2 + \ldots + p_{19} * (R_{19} - E(R))^2 + p_{20} * (R_{20} - E(R))^2$$

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Variance of R:

$$Var(R) = p_1*(R_1 - E(R))^2 + p_2*(R_2 - E(R))^2 + \ldots + p_{19}*(R_{19} - E(R))^2 + p_{20}*(R_{20} - E(R))^2$$

Standard Deviation of R:

$$SD(R) = \sqrt{Var(R)}$$

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0.01134	0.05

Variance of R:

$$\mathrm{Var}(R) = p_1 * (R_1 - E(R))^2 + p_2 * (R_2 - E(R))^2 + \ldots + p_{19} * (R_{19} - E(R))^2 + p_{20} * (R_{20} - E(R))^2$$

Standard Deviation of R:

$$SD(R) = \sqrt{\mathrm{Var}(R)}$$

- In Excel, variance can be computed by first evaluating, for each scenario, the squared deviation from the expected value, and then using =SUMPRODUCT()

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Variance of R:

$$\mathrm{Var}(R) = p_1{*}(R_1 - E(R))^2 + p_2{*}(R_2 - E(R))^2 + \ldots + p_{19}{*}(R_{19} - E(R))^2 + p_{20}{*}(R_{20} - E(R))^2 \approx \boxed{0.000327}$$

Standard Deviation of R:

$$\mathrm{SD}(R) = \sqrt{\mathrm{Var}(R)} = \boxed{0.01808}$$

- In Excel, variance can be computed by first evaluating, for each scenario, the squared deviation from the expected value, and then using =SUMPRODUCT()

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- At the same time, a standard deviation of 1.808% indicates that the actual value of R can be that far away, on average, from 0.3467%

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- Standard deviation can serve as an indicator of the degree of ***uncertainty*** in the actual value of R
- Some decision makers may be averse to uncertainty, and, therefore, would prefer smaller values of standard deviation if they have a choice

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- ***Risk*** can be defined as the likelihood and/or magnitude of ***undesirable*** outcome(s)

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- Risk measures can come in different forms, and the same probability distribution can be used to evaluate multiple different risk measures

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- Others may prefer to focus on risk measures they associate with specific undesirable scenarios

Measures of Risk: Loss Probability

Scenario	Probability
-0.00024	0.05
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-0.02114	0.05
-0.01178	0.05
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- For example, some decision makers may choose to focus on the likelihood of a loss

- In the distribution of R we use, the negative returns occur in 9 scenarios out of 20, with the total probability of a loss being 9*0.05=0.45

Measures of Risk: Probability of a “Substandard” Return

Scenario	Probability
-0.00024	0.05
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-0.01178	0.05
-0.01515	0.05
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- Others would like to know the likelihood of generating a return that is below some threshold they consider acceptable, for example, a threshold of 1.5%
- In the distribution of R we use, the returns below 1.5% occur in 14 scenarios out of 20, with the total probability being 14*0.05=0.70

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- The best alternative in high-uncertainty settings can then be identified by maximizing the reward while imposing constraints on the values of risk measures