Week 4: Balancing Risk and Reward Using Simulation

- Modeling Uncertainty: From Scenarios to Continuous Distributions
- Example: Designing a New Apartment Building
- Connecting Random Inputs and Random Outputs in a Simulation
- Setting up and Running a Simulation in Excel
- Analyzing and Interpreting Simulation Output
- Evaluating Alternative Decisions using Simulation Results

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- Modeling Uncertainty: From Scenarios to Continuous Distributions
- Example: Designing a New Apartment Building **Session 1**
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- Modeling Uncertainty: From Scenarios to Continuous Distributions
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- Connecting Random Inputs and Random Outputs in a Simulation
- Setting up and Running a Simulation in Excel **Session 2**
- Analyzing and Interpreting Simulation Output
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- Example: Designing a New Apartment Building
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- Analyzing and Interpreting Simulation Output **Session 3**
- Evaluating Alternative Decisions using Simulation Results

Simulation: Results for n=10 Simulation Runs

- The profit (in $):

  Π = 500,000*min(D_R,R) + 900,000*min(D_L,L) + 100,000*(R-min(D_R,R)) + 150,000*(L-min(D_L,L))

- Stargrove.xlsx

Stargrove.xlsx
Modeling Risk and Realities MOOC

Apartments	Regular	Luxury
Profit during the year	$ 500,000.00	$ 900,000.00
Salvage Profit	$ 100,000.00	$ 150,000.00
Profit Threshold	$ 45,000,000.00	

	Reward	$ 50,778,195.32
	Risk	0.2

	Regular	Luxury
Numbers of Apartments	96	12
Demand Parameters		
Expected	90	10
St. Dev.	25	3

Demand Realizations	RV for Regular Demand	RV for Luxury Demand	DR	DL	Profit from Regular apartments	Profit from Luxury apartments	Total Profit	Profit below Threshold?
1	34.65997588	6.536523668	34	6	$ 23,463,990.35	$ 6,702,392.75	$ 30,166,383.10	1
2	95.14134513	2.548728187	95	2	$ 47,656,538.05	$ 3,711,546.14	$ 51,368,084.19	0
3	103.4140237	9.157716957	103	9	$ 48,000,000.00	$ 8,668,287.72	$ 56,668,287.72	0
4	83.66079351	8.136827344	83	8	$ 43,064,317.40	$ 7,902,620.51	$ 50,966,937.91	0
5	146.1676643	11.60464879	146	11	$ 48,000,000.00	$ 10,503,486.59	$ 58,503,486.59	0
6	57.91029844	7.837553565	57	7	$ 32,764,119.37	$ 7,678,165.17	$ 40,442,284.55	1
7	128.539838	9.745067498	128	9	$ 48,000,000.00	$ 9,108,800.62	$ 57,108,800.62	0
8	159.5054041	12.00413297	159	12	$ 48,000,000.00	$ 10,800,000.00	$ 58,800,000.00	0
9	76.73592927	6.531155375	76	6	$ 40,294,371.71	$ 6,698,366.53	$ 46,992,738.24	0
10	116.6149982	9.286600314	116	9	$ 48,000,000.00	$ 8,764,950.24	$ 56,764,950.24	0

Simulation: Results for n=10 Simulation Runs

- Sample of random variables from the normal distribution with mean 90 and standard deviation of 25 and its descriptive statistics

Demand Realizations	RV for Regular Demand
1	34.65997588
2	95.14134513
3	103.4140237
4	83.66079351
5	146.1676643
6	57.91029844
7	128.539838
8	159.5054041
9	76.73592927
10	116.6149982

RV for Regular Demand	
Mean	100.235027
Standard Error	12.31380659
Median	99.2776844
Mode	#N/A
Standard Deviation	38.9396755
Sample Variance	1516.298328
Kurtosis	-0.581372514
Skewness	-0.101693197
Range	124.8454282
Minimum	34.65997588
Maximum	159.5054041
Sum	1002.35027
Count	10
Confidence Level(95.0%)	27.85576579

- Sample mean is based on a small sample of n=10 instances of the underlying random variable. It is just an approximation to the true expected value of the random variable being simulated

Simulation: Results for n=10 Simulation Runs

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- **95% confidence level** identifies the “95% confidence interval” for the true expected value of the simulated random variable: based on the results of this simulation, we are 95% confident that the true expected value is in the interval = sample mean +/- 95% confidence level ≈ 110.24 +/- 27.86 = [82.38, 138.10]

Simulation: Results for n=10 Simulation Runs

- Sample of random variables from the normal distribution with mean 90 and standard deviation of 25 and its descriptive statistics

Demand Realizations	RV for Regular Demand
1	34.65997588
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Simulation: Results for n=10 Simulation Runs

- With a simulation that samples the random input variables only 10 times, the reliability of the estimates for the mean and the standard deviation for any random quantity involved may be limited

Total Profit
$ 30,166,383.10
$ 51,368,084.19
$ 56,668,287.72
$ 50,966,937.91
$ 58,503,486.59
$ 40,442,284.55
$ 57,108,800.62
$ 58,800,000.00
$ 46,992,738.24
$ 56,764,950.24

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Reward	$	50,778,195.32
Risk		0.2

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Reward	**$ 50,778,195.32**
Risk	**0.2**

Profit	
Mean	50778195.32
Standard Error	2945866.396
Median	54018185.95
Mode	#N/A
Standard Deviation	9315647.495
Sample Variance	8.67813E+13
Kurtosis	1.570184843
Skewness	-1.419981677
Range	28633616.9
Minimum	30166383.1
Maximum	58800000
Sum	507781953.2
Count	10
Confidence Level(95.0%)	6664012.769

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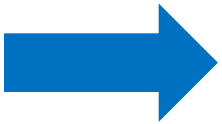

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- Our estimate for the reward measure, ≈ **$50,778,195** may also be quite removed from the true expected profit value associated with the decision we consider

Simulation: Results for n=10 Simulation Runs

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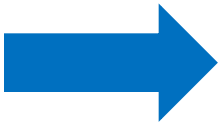

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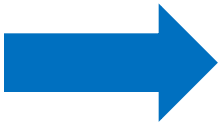

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Range	28633616.9
Minimum	30166383.1
Maximum	58800000
Sum	507781953.2
Count	10
Confidence Level(95.0%)	6664012.769

- Based on the results of this simulation, we can be 95% confident that the true expected profit under the decision we consider lies in the interval ≈ $50,778,195 +/- $6,664,013 = **[$44,114,182, $57,442,208]**

Simulation: Results for n=1000 Simulation Runs

- Stargrove_1000.xlsx, seed 123 for the B column and seed 1234 for the C column

Total Profit
$ 30,166,383.10
$ 51,368,084.19
$ 56,668,287.72
$ 50,966,937.91
$ 58,503,486.59
$ 40,442,284.55
$ 57,108,800.62
$ 58,800,000.00
$ 46,992,738.24
$ 56,764,950.24
$ 57,920,800.05
$ 35,017,702.31
$ 45,448,387.76
$ 40,417,837.90
$ 56,460,689.30
$ 56,113,851.70
$ 55,434,315.43
$ 58,302,165.39
$ 53,191,844.26
$ 56,471,197.37

Reward	**$ 52,131,111.07**
Risk	**0.156**

Profit	
Mean	52131111.07
Standard Error	205518.4113
Median	54261174.8
Mode	58800000
Standard Deviation	6499062.808
Sample Variance	4.22378E+13
Kurtosis	0.783172165
Skewness	-1.151004166
Range	32537281.28
Minimum	26262718.72
Maximum	58800000
Sum	52131111066
Count	1000
Confidence Level(95.0%)	403297.2995

- Based on the results of this longer simulation, we can now be 95% confident that the true expected profit under the decision we consider lies in the interval ≈ $52,131,111 +/- $403,297 = **[$51,727,814, $52,534,408]**

Simulation: Results for n=1000 Simulation Runs

- Stargrove_1000.xlsx

Total Profit	
$	30,166,383.10
$	51,368,084.19
$	56,668,287.72
$	50,966,937.91
$	58,503,486.59
$	40,442,284.55
$	57,108,800.62
$	58,800,000.00
$	46,992,738.24
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$	57,920,800.05
$	35,017,702.31
$	45,448,387.76
$	40,417,837.90
$	56,460,689.30
$	56,113,851.70
$	55,434,315.43
$	58,302,165.39
$	53,191,844.26
$	56,471,197.37

Reward	**$ 52,131,111.07**
Risk	**0.156**

below Threshold?	
Mean	0.156
Standard Error	0.011480235
Median	0
Mode	0
Standard Deviation	0.363036907
Sample Variance	0.131795796
Kurtosis	1.609118878
Skewness	1.898921967
Range	1
Minimum	0
Maximum	1
Sum	156
Count	1000
Confidence Level(95.0%)	0.022528141

- Based on the results of this longer simulation, we can be 95% confident that the true value of the risk measure under the decision we consider lies in the interval ≈ 0.156+/- 0.023 = **[0.133, 0.179]**

Simulation: Comparing Two Alternatives

- Suppose that Stargrove would like to compare the decision of building 12 regular floors and 3 luxury floors (R=96 and L=12) with the decision of building 11 regular floors and 4 luxury floors (R=88 and L=16)

- We can use 1000 random values we have already generated for the demand for regular apartments and 1000 random values we have already generated for the demand for luxury apartments to estimate the reward and the risk associated with the decision of R=88 and L=16

- We can then compare reward and risk estimates for the two decisions

Simulation: Comparing Two Alternatives

- Stargrove_1000_TwoDecisions.xlsx

Profit for R=96, L=12		*Profit for R=96, L=12 below Threshold?*	
Mean	52131111	Mean	0.156
Standard Error	205518.4	Standard Error	0.011480235
Median	54261175	Median	0
Mode	58800000	Mode	0
Standard Deviation	6499063	Standard Deviation	0.363036907
Sample Variance	4.22E+13	Sample Variance	0.131795796
Kurtosis	0.783172	Kurtosis	1.609118878
Skewness	-1.151	Skewness	1.898921967
Range	32537281	Range	1
Minimum	26262719	Minimum	0
Maximum	58800000	Maximum	1
Sum	5.21E+10	Sum	156
Count	1000	Count	1000
Confidence Level(95.0%)	403297.3	Confidence Level(95.0%)	0.022528141

Profit for R=88, L=16		*Profit for R=88, L=16 below Threshold?*	
Mean	50660188.5	Mean	0.159
Standard Error	176801.7579	Standard Error	0.011569
Median	52266311.62	Median	0
Mode	58400000	Mode	0
Standard Deviation	5590962.492	Standard Deviation	0.365859
Sample Variance	3.12589E+13	Sample Variance	0.133853
Kurtosis	1.760500397	Kurtosis	1.491811
Skewness	-1.349726248	Skewness	1.867841
Range	32337281.28	Range	1
Minimum	26062718.72	Minimum	0
Maximum	58400000	Maximum	1
Sum	50660188497	Sum	159
Count	1000	Count	1000
Confidence Level(95.0%)	346945.4199	Confidence Level(95.0%)	0.022703

Simulation: Comparing Two Alternatives

- Stargrove_1000_TwoDecisions.xlsx

Profit for R=96, L=12		*Profit for R=96, L=12 below Threshold?*	
Mean	52131111	Mean	0.156
Standard Error	205518.4	Standard Error	0.011480235
Median	54261175	Median	0
Mode	58800000	Mode	0
Standard Deviation	6499063	Standard Deviation	0.363036907
Sample Variance	4.22E+13	Sample Variance	0.131795796
Kurtosis	0.783172	Kurtosis	1.609118878
Skewness	-1.151	Skewness	1.898921967
Range	32537281	Range	1
Minimum	26262719	Minimum	0
Maximum	58800000	Maximum	1
Sum	5.21E+10	Sum	156
Count	1000	Count	1000
Confidence Level(95.0%)	403297.3	Confidence Level(95.0%)	0.022528141

Profit for R=88, L=16		*Profit for R=88, L=16 below Threshold?*	
Mean	50660188.5	Mean	0.159
Standard Error	176801.7579	Standard Error	0.011569
Median	52266311.62	Median	0
Mode	58400000	Mode	0
Standard Deviation	5590962.492	Standard Deviation	0.365859
Sample Variance	3.12589E+13	Sample Variance	0.133853
Kurtosis	1.760500397	Kurtosis	1.491811
Skewness	-1.349726248	Skewness	1.867841
Range	32337281.28	Range	1
Minimum	26062718.72	Minimum	0
Maximum	58400000	Maximum	1
Sum	50660188497	Sum	159
Count	1000	Count	1000
Confidence Level(95.0%)	346945.4199	Confidence Level(95.0%)	0.022703

Reward and risk measures for two policies

Simulation: Comparing Two Alternatives

- Stargrove_1000_TwoDecisions.xlsx

Decision	**R=96, L=12**	**R=88, L=16**
95% Confidence Interval for Reward, in $ millions	[51.73, 52.53]	[50.31, 51.01]
95% Confidence Interval for Risk	[0.133, 0.179]	[0.136, 0.182]

- Based on the results of the simulation with n=1000 runs, we are 95% confident that the expected profit under the decision R=96, L=12 is higher than the expected profit under the decision R=88, L=16

Simulation: Comparing Two Alternatives

- Stargrove_1000_TwoPolicies.xlsx

Decision	**R=96, L=12**	**R=88, L=16**
95% Confidence Interval for Reward, in $ millions	[51.73, 52.53]	[50.31, 51.01]
95% Confidence Interval for Risk	[0.133, 0.179]	[0.136, 0.182]

- Based on the results of the simulation with n=1000 runs, we are 95% confident that the expected profit under the decision R=96, L=12 is higher than the expected profit under the decision R=88, L=16

- The results of this simulation do not allow us to distinguish between the levels of risk associated with those two decisions at the same level of confidence

Simulation: Comparing Two Alternatives

- We can add other reasonable decisions to our comparison set
- If two decisions cannot be distinguished on the basis of the results of a particular simulation, we can also run longer simulations to obtain more narrow confidence intervals for reward and risk measures

Simulation: Comparing Two Alternatives

- We can add other reasonable decisions to our comparison set
- If two decisions cannot be distinguished on the basis of the results of a particular simulation, we can also run longer simulations to obtain more narrow confidence intervals for reward and risk measures
- Ultimately, the goals are to 1) limit the consideration set to decisions that result in risk measures limited by the tolerance level of a decision maker, and 2) among the decisions that satisfy constraint(s) on acceptable risk level(s), choose one that generates highest reward, at the selected confidence level
- Simulation provides “imperfect” estimates of reward and risk, but the notion of confidence intervals enables a decision maker to compare alternatives even using those imperfect estimates