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Asymmetry between odd and even node weight in complex networks

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Abstract

Phenomenon of asymmetry between odd and even node weight distribution in some complex networks, such as public transportation networks and Internet, is presented. Reasons related to this phenomenon are summarized, i.e., (1) edges are solid connections between nodes which are embedded in real geometric background, and (2) preferential mechanism during the process of network formation. Incorporating these two points, complex networks are constructed by self-avoiding random walk on 2D grid and the simulation results are consistent well with that of empirical researches.

Keywords: Asymmetry; Node weight; Complex networks; Self-avoiding random walk

1. Introduction

It is a fact that many complex systems share similar network architecture. Examples pervade all of science, from neurobiology to statistical physics, including the Internet backbone and the World Wide Web, citation networks, electrical power grids, social networks, and biological and biochemical networks of various kinds [1]. The ubiquity of various networks spontaneously motivates the current intensive studies of complex networks, on both theory and applications [1–7]. Specially, small-world [8] and scale-free [9] properties have been repeatedly observed in various complex networks.

Recently, the rising interest in reproducing the features observed in real-world networks has fostered researches going beyond the simple topological structure. The empirical researches reveal that real-world networks usually display a large heterogeneity in the intensity or capacity of the connections (i.e., the weights of the links). For example, the amount of traffic in communication systems and large transport infrastructures [6,7,10–14], the closeness of any two scientists in scientific collaboration networks [14–18], the bandwidth or delay time on Internet [6], the energy transportation of predator–prey interactions in ecosystem [19,20], the reaction rates in metabolic networks [21], and the degree of acquaintances in social networks [22] are important to characterize these networks. Models of weighted complex networks in which the diversity of weights is taken into account have been formulated. Some models assign a statical weight to edges [23–30], i.e.,

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the weight on edges does not change with the evolution of the networks. Weight-driven models with evolving weights are also proposed [31–35] in which weights evolve according to different processes. All the above models are growing networks which are driven by the intensity of the weights along with a reinforcement mechanism (such as preferential mechanism or the survival of the fittest). Another kind of weight-driven models are built under mutual attraction between nodes [36,37].

Many of the complex networks mentioned above are embedded in the real space. For instance, transportation networks depend on distance, and many communication networks devices have short radio range [13,38–46]. An important example of such a ‘‘spatial’’ network is the Internet which is a set of routers linked by physical cables with different lengths and latency times [6].

Inspired by the researches discussed above, the public transportation systems and Internet on AS level are investigated as weighted complex networks taking the geometric constraints into account. Interestingly, an asymmetry between odd and even node weight is discovered, and the mechanism leading to the empirical results is analyzed in Section 2. Then, a simple model is developed to demonstrate the mechanism in Section 3. At last, concluding remarks are given in Section 4.

2. Empirical research

In this section, the empirical results about real-world networks are presented. The first is about bus system, and the second is about Internet on AS level.

Bus system is an important part of city transportation, which is of crucial role for the development of a city or a country [47]. Traffic dynamics have been extensively addressed in physicists' literature for many years [48]. But these works omit one basic fact that the movement of a bus is constrained by bus route whose geometric features heavily affect the dynamics of public transportation. According to recent researches, complex network is a proper tool to investigate the feature of bus routes. In this paper, we focus on node weight distribution of bus route networks. A weighted complex network is constructed based on the bus routes in a natural way. The nodes are bus stations, and the edges are links between the nearest neighboring nodes in the same bus route. One and only one edge will exist if there are bus routes going directly from node i to node j without any stop (i.e., more than one routes going between i and j will still be recorded as only one edge), and the edge's weight (w_{ij}) is characterized by the amount of bus routes flowing through it (here $w_{ij} = w_{ji}$). Node weight (w_i) is calculated by the sum of edge weights associated with it, i.e.,

$$w_i = \sum_{j \in S_i} w_{ij}. \tag{1}$$

Here, S_i is the set of edges connecting to node i.

According to the above definition, bus route networks in Beijing, Tianjin, and Shanghai are studied. When the count step is no less than 2, the power law distribution of node weight appears. However, if the count step is 1, the distribution does not obey the power law characterized by a single parameter. Both odd and even node weight obey power law distribution, but their slopes are different. This phenomenon is shown in Fig. 1.

As shown in Fig. 1(a), the distribution of node weight follows power law, it is consistent with Refs. [34,49]. Bus route network is a growing network. When a city grows bigger, more routes will be added into the bus route network to meet the demand of public transportation. A new route will prefer start at the place where more passengers are convenient to transfer or where more existing bus routes going through. This place is a node of bigger weight in bus route network. This positive feedback mechanism of bus route growth is similar to preferential attachment of complex network growth in Ref. [9].

In Fig. 1(b)–(d), asymmetry between odd and even node weight emerges. Exponent of odd node weight distribution is different from that of even node. Bus route networks are different from complex networks which are constructed by abstract links which represent abstract interactions between nodes, such as scientist collaborate networks, actors' networks, food webs, metabolic networks, and so on. Bus route networks are embedded in geometric background, their edges are solid connections linking nearest neighbors. And one route contributes the weight of its begin node and end node by 1, other nodes by 2 on condition that its begin

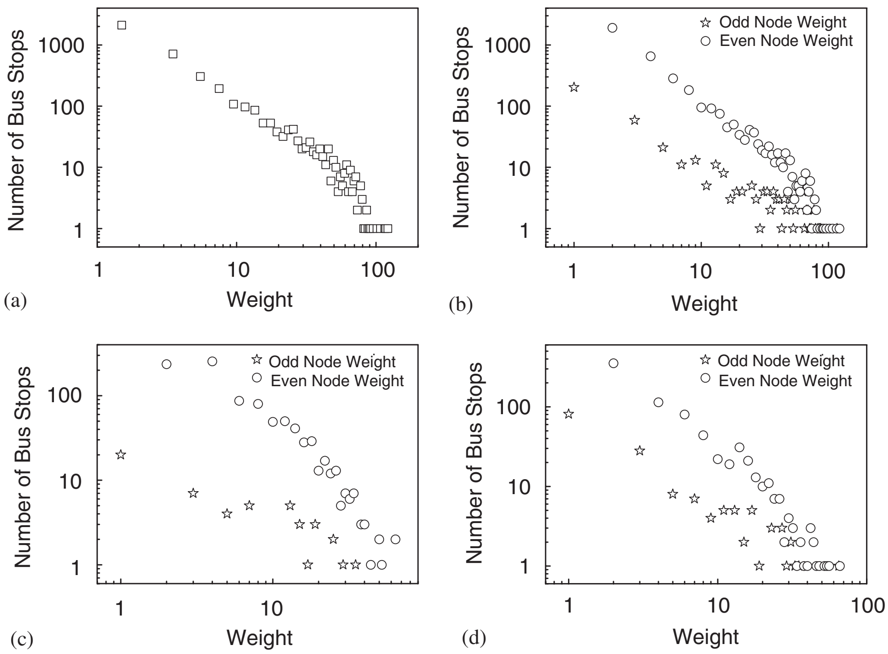

Fig. 1. Node weight distribution of bus route networks: (a) the count step is 2, and node weight distribution obeys power law with exponent −1.87062. It shows the result of Beijing, and the data of Shanghai and Tianjin have similar feature. In (b), (c), and (d) the count step is 1. Data are from Beijing, Shanghai, and Tianjin, respectively. The exponents of odd and even node weight distribution are −1.03764 and −1.95877 in (b) −0.77541 and −1.76105 in (c) −1.1164 and −1.91511 in (d).

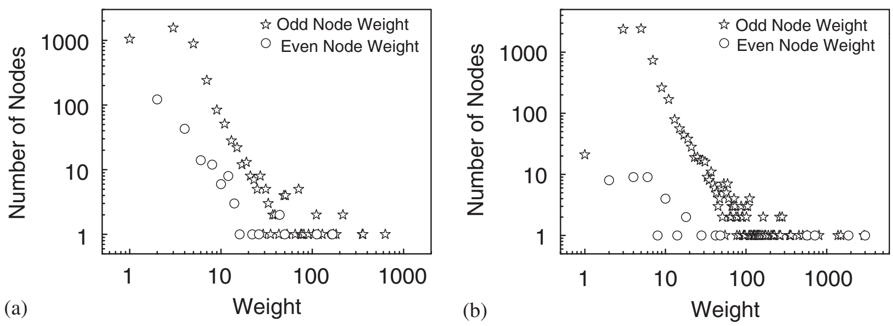

Fig. 2. Snapshots of Internet AS level: (a) is at November 8, 1997; (b) at January 2, 2000.

node differs from its end node. Despite of extreme situation that all route are closed circle, we can draw a conjecture: it is solid connection and geometric structure that cause asymmetry between odd and even node weight. To prove this conjecture, we explore the Internet on AS level, which is solidly connected by wire and embedded in three dimension space. Two complex networks of Internet on AS level are constructed according to the same way with bus route networks. The data are based on two snapshots of Internet on AS level. The results (see Fig. 2) prove our conjecture and the existence of asymmetry between odd and even node. Internet on AS level has similar mechanism with the bus route networks. When a new router is connected to the Internet, it prefers to record a new route which connects to an existing router who have recorded more routes, i.e., it prefers the connection with the existing router which can provide greater probability of transfer information to/from more other routers.

According to empirical researches and analyses, asymmetry between odd and even node weight could be explained by solid connection with geometric background; and the power law distribution is related to preferential mechanism.

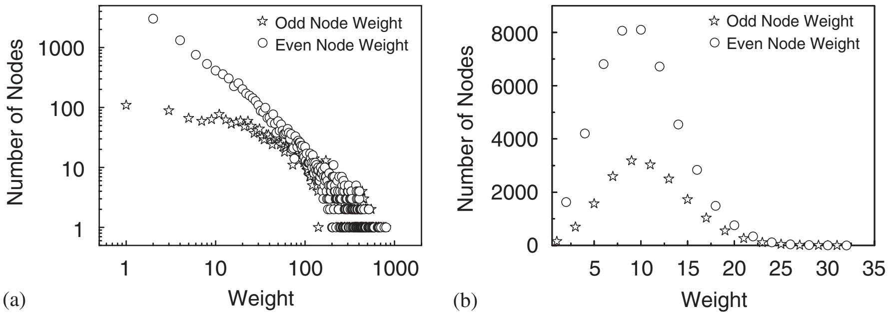

Fig. 3. Results of model. These results are averaged over 14 simulations. Each simulation is under the same parameters: 63×63 2D grid, $K = 600$ routes, and l is an uniform random variable ranging from 10 to 120 with an average of 65: (a) select start point according to Eq. (2) with $a = 0.0015$, i.e., preferential selection; (b) select start point by equal probability $1/N^2$.

3. A self-avoiding random walk model and its results

In the above section, empirical researches show that solid connection with geometric background and preferential mechanism lead to asymmetry between odd and even node and power law distribution. We build our complex network on the 2D grid $(x, y)(1 \leqslant x, y \leqslant N)$ as follows:

(1) Select one lattice (x, y) as the start point of one route according to the probability

$$p_{(x,y)} = \frac{w_{(x,y)} + a}{\sum_{1 \leqslant \tilde{x}, \tilde{y} \leqslant N} (w_{(\tilde{x},\tilde{y})} + a)}. \tag{2}$$

Parameter a reflects the initial attractiveness of any lattice and the strength of preference. The stronger a is, the weaker the preference is. If $a \gg 1$, we can get $p_{(x,y)} \sim 1/N^2$.

(2) Form one route by self-avoiding random walk (SARW). One route can be formed by l-step SARW along the lattices from the start point (x, y) selected in (1). The step size of SARW equals to 1, i.e., from one site to one of its four nearest neighbors randomly. After one route formed by SARW, some lattices of 2D grid may be included in our complex network and the node weight of complex network changes. Calculate the probability of $p_{(x,y)}$ according to new node weight. Return to (1) until there are K routes on 2D grid.

(3) Build the complex network in the same way as in Section 2.

Asymmetry between odd and even node weight emerges from above simulation as shown in Fig. 3. However, Fig. 3(a) is different from 3(b). In Fig. 3(a), the start point of route is selected with preferential mechanism. Odd and even node weight obeys the power law distribution respectively, which is consistent with empirical researches. In Fig. 3(b), the start point of route is selected without preferential mechanism and node weight does not follow the power law. The difference between these two figures indicates that preferential mechanism is one of the key factors of the power law behavior.

4. Concluding remarks

The phenomenon of asymmetry between odd and even node weight distribution is revealed by our empirical investigation on the bus route networks and Internet on AS level. This phenomenon typically indicates that connections between nodes of complex networks are solid and based on geometric background. It can be adopted as one standard in distinguishing one kind of complex networks based on solid connections with geometric background from other kinds.

This work can also give the inspiration that geometric background of networks is carrying more intrinsic features of real-world systems, and it is a reasonable way to go deep inside the reality. The methods presented in this paper are not intended to be regarded as a rigorous program for the study of real-world networks, but as an attempt towards the intuition when investigating these systems.

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