债券交易系统讲义

债券基础知识

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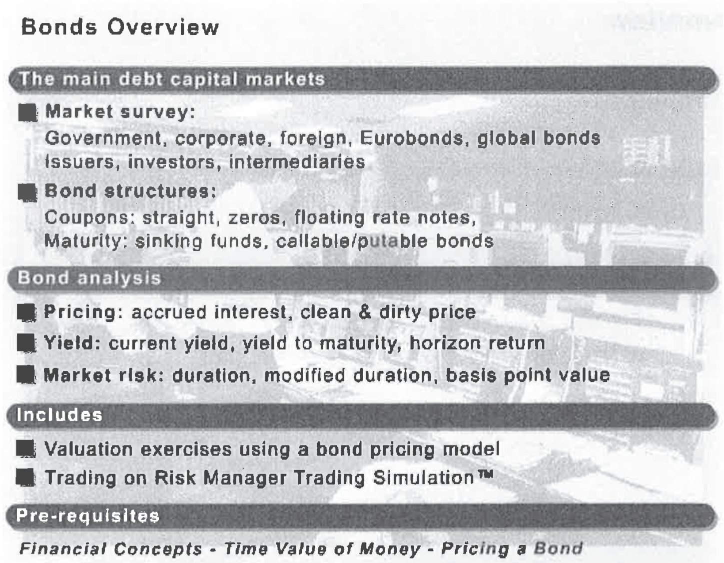
1. Overview



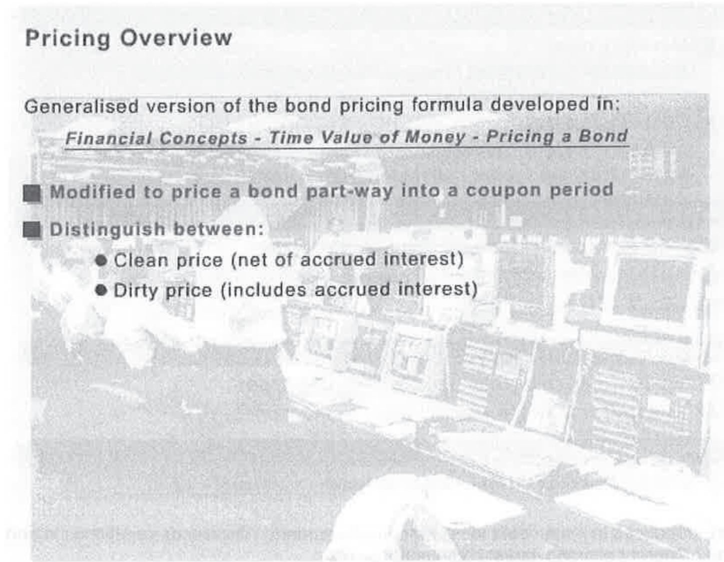
This course introduces the main debt capital market instruments. We discuss variations in bond structure and different accrued interest conventions.

We also explain:

- The difference between clean and dirty prices
- Various measures of bond yield
- How to estimate the market risk on fixed income instruments

2. Bond Pricing

2.1. Overview



In this module we take a deeper look at the bond pricing formula that we developed in *Time Value of Money - Pricing a Bond*.

- We modify the formula to price a bond part-way into a coupon period
- We distinguish between a bond's:
  - Clean price, which is net of accrued interest, and
  - Dirty price, which includes accrued interest.

Learning Objectives

By the end of this module, you will be able to:

- Explain the difference between **clean** and **dirty price**
- Calculate **accrued interest** on bonds using the appropriate **day-count conventions** that apply in different markets

2.2. Valuation Formula

Pricing on a Coupon Date

The bond pricing formula developed in Time Value of Money - Pricing a Bond was:

$$\textbf{Bond price} = \frac{C/t}{(1 + R/t)} + \frac{C/t}{(1 + R/t)^2} + \frac{C/t}{(1 + R/t)^3} + \ldots + \frac{\text{Principal} + C/t}{(1 + R/t)^{nxt}}$$

Where:
C = Coupon rate
R = Return on the investment (the yield)
t = Number of coupon payments per year (= compounding period)
n = Number of years

This formula takes the following factors into account:

- The bond's principal amount
- The coupon rate
- The timing of the coupon payments - e.g. annual, semi-annual, etc.
- The required return (or yield) on the bond - the discount rate applied to discount the bond's future cash flows.

However, this formula assumes that we are pricing the bond at the start of a coupon period, so the next coupon is exactly one coupon period away from the settlement date.

Pricing off a Coupon Date

? What happens if we trade the bond part-way through a coupon period?

The formula is modified so that:

- The first discount factor is raised to a power of **a**, where **a** is the fraction of the coupon period to the next coupon payment
- The second discount factor is raised to the power of **a + 1**
- The third discount factor is raised to the power of **a + 2**
- And so on.

Below is the industry-standard formula for calculating the present value of a bond.

$$\text{Present value} = \frac{C/t}{(1 + R/t)^a} + \frac{C/t}{(1 + R/t)^{a+1}} + \frac{C/t}{(1 + R/t)^{a+2}} + \ldots + \frac{\text{Principal} + C/t}{(1 + R/t)^{a+m}}$$

Where:
C = Coupon rate
R = Return on the investment (the yield)
t = Number of coupon payments per year (= compounding period)
m = Number of complete coupon periods to maturity

$$a = \frac{\text{Number of days to next coupon}}{\text{Number of days in current coupon period}}$$

Equivalent formulations:

$$\text{Present value} = C/t \times \Sigma \frac{1}{(1 + R/t)^{a+i}} + \frac{\text{Principal}}{(1 + R/t)^{a+m}}$$

for i = 0 … m

$$= C/t \times \frac{(D_a - D_{a+m+1})}{(1 - D_1)} + \text{Principal} \times D_{a+m}$$

Where D_i = Discount factor = $1/(1 + R/t)^i$

Different bond market sectors use different day-count conventions for calculating the **fractional coupon period** - the proportion of the current coupon period already elapsed - hence the variable **a** in the pricing formula, as we shall see in section *Accrued Interest*.

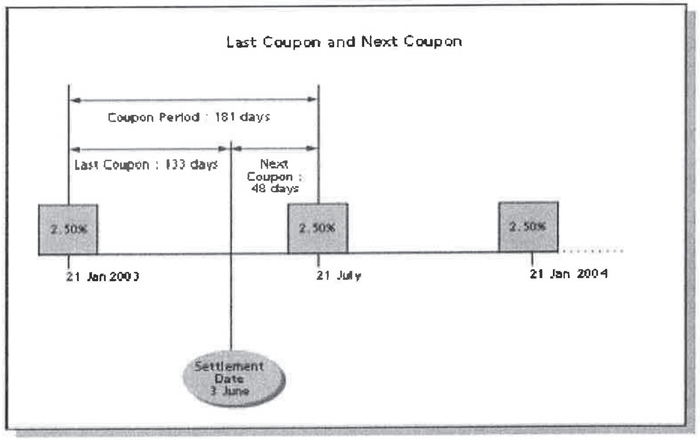
Pricing Example

Security: 5% US Treasury note maturing 21 January 2005
Type: Semi-annual
Settlement date: 3 June 2003

? What should be the price of this bond to give the investor a return (or yield) 8.00%?

Analysis

With a coupon rate of 5%, the bond will pay a coupon amount equal to 2.50% of its face value on 21 July and 21 January each year.[1] The figure below shows where we are in the current coupon period (21 January - 21 July 2003).



[1] Or on the next business day, if the coupon date falls on a week-end or public holiday (the so-called **next business day convention**).

To find the present value of these future cash flows, the convention in this market is to proceed as follows:

1. Calculate the number of days from the settlement date (3 June) to the next coupon date (21 July) as a fraction of the current coupon period:

   48/181 = 0.2652

2. The next coupon is therefore 0.2652 of a coupon period away, so it is discounted by a factor $(1 + 0.08/2)^{0.2652}$.

   The second cash flow is then discounted by a factor $(1 + 0.08/2)^{1.2652}$, the third cash flow by $(1 + 0.08/2)^{2.2652}$, and so on.

The bond's present value is the sum of the present values of these cash flows:

Coupon Date	Interest Period	Cashflow	Discounted at 8%	Present Value
21 Jul 2003	0.2652	2.50	$\frac{2.50}{(1 + 0.08/2)^{0.2652}}$	= 2.47
21 Jan 2004	1.2652	2.50	$\frac{2.50}{(1 + 0.08/2)^{1.2652}}$	= 2.38
21 Jul 2004	2.2652	2.50	$\frac{2.50}{(1 + 0.08/2)^{2.2652}}$	= 2.29
21 Jan 2005	3.2652	102.50	$\frac{102.50}{(1 + 0.08/2)^{3.2652}}$	= 90.18
Total				**= 97.32**

Discounted at 8% the bond's present value comes to 97.32% of its face value, so a certificate which repays USD 1,000 at maturity would cost USD 973.20.

2.3. Clean and Dirty Prices

Effect of a Coupon Payment

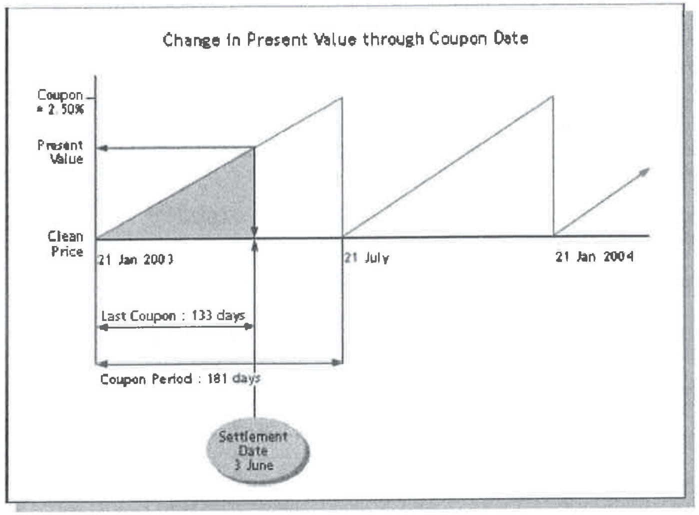
In the example in *Valuation Formula* we priced the following bond:

Security: 5% US Treasury note maturing 21 January 2005
Type: Semi-annual
Settlement date: 3 June 2003
Required yield: 8.00%
Present value: 97.32

Imagine that we hold this bond through the next coupon date, while market conditions remain the same. As the coupon date approaches all the future cash flows get closer, so the present value of the bond rises steadily.

! Then, on or before 21 July, the bond goes ex-coupon and, suddenly, the bond loses 2.5% of its market value!

Similar price drops would occur every time a coupon is paid, as the figure below illustrates.



To eliminate such 'technical' fluctuations the markets quote bond prices on a **clean** (or **flat**) basis: they subtract from the bond's present value the interest amount accrued since the start of the current coupon period (in the figure, the vertical distance between the bond's present value and the horizontal timeline). Thanks to this technique, other things being equal there should be no perceptible change in the bond's quoted price as it goes through a coupon date.

The clean price gives investors a measure of market value that is not affected by the payment of a coupon.

Accrued interest can be calculated directly from the bond's details, so in practice market makers quote bonds prices on a clean basis and leave it to their respective settlement departments to work out the accrued interest, and therefore the dirty price payable by the buyer (i.e. the bond's present value).

Accrued interest and clean price calculation

To calculate the clean price, the procedure is as follows:

Actual number of days since last coupon (21 Jan - 3 June): 133
Actual number of days in current coupon period (21 Jan - 21 July): 181

Fractional coupon period = 133/181
= 0.7348

Accrued interest = 0.7348 x 2.50
= 1.837 or **1.84**% rounded.

Subtracting this accrued interest from the bond's present value gives:

Clean price = 97.32 - 1.84
= 95.48[2]

[2] Bond prices in the US Treasury market are quoted in 32nds, rather than in decimal, and are typically rounded to the nearest 64th of a percent. Thus, the 0.48 in the price of 95.48 is equivalent to:

48 x 32 / 100

See Money Market Cash Instruments - Accrued Interest for a description of the day-count conventions used in the money markets.

The examples illustrate the differences that can arise, depending on the day-count convention used.

Example 1
Accrued interest and settlement amount calculation with Actual/Actual convention.

Security: 5% British government bond maturing 21 January 2005
Type: semi-annual
Settlement date: 3 June 2003
Quoted price: 97.32
Trade amount: GBP 5 million.

Actual number of days since last coupon (21 Jan - 3 June): 133
Actual number of days in current coupon period (21 Jan - 21 July) : 181

The number of days in a coupon period could be anything between 181 and 184 days, depending on the dates and whether it's a leap year or not.

Fractional coupon period = 133/181
= 0.734806629834.

Accrued interest = 0.734806629834 x 2.50
= 1.83701657458 or **1.84%** rounded.

= **4,957,850.83**

Example 2
Accrued interest and settlement amount calculation with Actual/365 convention.

Security: 5% Japanese Government Bond maturing 21 January 2005
Type: Semi-annual
Settlement date: 3 June 2003
Quoted price: 97.32
Trade amount: JPY 5 million.

Actual number of days Since last coupon (21 Jan - 3 June): 133
Number of days in Current coupon period: 365/2 = 182.5

It is assumed that every year has 365 days - even leap years! - and therefore on a semi-annual coupon bond each coupon period is exactly 182.5 days.

Fractional coupon period = 133/182.5
= 0.72876712329

Accrued interest = 0.72876712329 x 2.50
= 1.82191780822 or **1.82**% rounded.

Settlement amount $= 5{,}000{,}000 \times \left(\frac{97.32 + 1.82191780822}{100} \right)$

= **4,957,095.89**

General Formula

Clean price = Dirty price - Accrued interest

Dirty price = Settlement price
= Present value (as per pricing formula)

Accrued interest = C/t x Fractional coupon period

Where:
C = Coupon rate (%)
t = Number of coupon payments per year (= compounding period)

$$\text{Fractional coupon period} = \frac{\text{Number of days since last coupon}}{\text{Number of days in current coupon period}}$$

The fractional coupon period is calculated differently in different bond market sectors, depending on the day-count convention used:

- Actual/actual - as in the example here
- Actual/365
- 30/360 (for Eurobonds issued prior to January 1st 1999)

We shall examine these conventions in section *Accrued Interest*.

2.4. Accrued Interest

Principal Conventions

Not all bond markets calculate accrued interest in the same way. Below are the main day-count conventions used in the debt capital markets.

Debt Capital Market Accruals

$$\text{Accrued interest} = \text{Principal} \times \frac{\text{Coupon rate}}{\text{Nr. Coupons per year}} \times \text{Fractional coupon period}$$

Where Fractional coupon period is calculated in different ways, depending on local market conventions:

- Actual/Actual (e.g. US Treasuries; Eurobonds issued after 31 Dec 1998)
= Actual number of days since last coupon was paid divided by the actual number of days in the current coupon period

- Actual/365 (e.g. Japanese Government bonds)
= Actual number of days since last coupon was paid divided by 182.5 (if semi-annual coupons) or 365 (if annual coupons)

- 30/360 (e.g. US corporate bonds; Eurobonds issued before 1 Jan 1999)
= Number of days since last coupon, assuming every month has 30 days, divided by 180 (semi-annual coupons) or 360 (annual coupons)

= 15.36 or 15½ / 32, rounded.

A price like 95.48 would therefore be quoted as **95-15+** or **95*15+** (the "+" signifies the extra ½ tick over 15).

Example 3
Accrued interest and settlement amount calculation with 30/360 convention.

Security: 5% Coca Cola maturing 21 January 2005
Type: semi-annual (domestic US corporate)
Settlement date: 3 June 2003
Quoted price: 97.32
Trade amount: USD 5 million.

Every month is assumed to have 30 days - even February! - and therefore every year has 360 days.

Number of days since last coupon:

21 January - 1 February:		10
1 February - 1 June:	4 x 30 =	120
1 June - 3 June:		2
Total		**132**

Number of days in current coupon period: 360/2 = 180

Fractional coupon period = 132/180
= 0.733...

Accrued interest = 0.733... x 2.50
= 1.833 or **1.83%** rounded.

Settlement amount $= 5{,}000{,}000 \times \left(\frac{97.32 + 1.833}{100} \right)$

= 4,957,666.67

Selected Markets

The table below summarises the main compounding and accrued interest conventions used in the major bond market sectors.

	Coupon Frequency	Accrued Interest
Government Bonds		
USA	Semi-annual	Actual/Actual
Japan	Semi-annual	Actual/365
UK	Semi-annual	Actual/Actual
France	Annual	Actual/Actual
Germany	Annual	Actual/Actual
Netherlands	Annual	Actual/Actual
Canada	Semi-annual	Actual/365
Australia	Semi-annual	Actual/Actual
Italy	Semi-annual	Actual/Actual
Corporate Bonds		
USA	Annual or Semi-annual	30/360
UK	Semi-annual	Actual/365 or Actual/Actual[3]
Eurobonds		
Issued before 1/1/99	Annual (some Semi-annual)	30E/360[4]
Issued after 31/12/98	Annual (some Semi-annual)	Actual/Actual[5]

[3] For bonds issued after January 1999, accrued interest is calculated on an Actual/Actual basis, instead of the traditional Actual/365.
[4] The 'E' in the 30E/360 (or ISMA) basis is to distinguish this convention from the one that applies in the domestic US corporate bond market, which is also 30/360 but does not include the **end-month rule**. The end-month rule means that the number of days from the 1st to the end of a 31-day month (e.g. 1 May to 31 May) is also counted as 29, rather than 30, which is how it would be counted under the US 30/360 convention. In all other respects the two conventions are identical.
[5] The ISMA accrued interest convention has been changed to Actual/Actual for Eurobonds issued after 1 January 1999, *unless the bond is denominated in US dollars*, in which case accrued interest will continue to be calculated on a 30/360 basis (ISMA Rule 251).

2.5. Exercise

Question 1

How many days of accrued interest are there on a bond maturing on 15 September 2006, for settlement on 2 November 2004, if the bond is:

a) A US Treasury (Actual/Actual)? Write your answer in the box below, then validate.

b) A US domestic corporate bond (30/360)?

Question 2

A 10% sterling corporate bond pays coupons on 21 January and 21 July and is bought for settlement on 6 June 2002. Calculate the amount of accrued interest payable on a GBP 1 million deal under the following day count conventions. Write your answer rounded to the nearest pence.

a) Accrued interest, Actual/365 (GBP):

b) Accrued interest, Actual/Actual (GBP):

Question 3

Security:	7.50% Coca-Cola maturing 15 December 2005
Type:	Eurobond, annual, 30E/360
Settlement date:	12 August 2004
Amount dealt:	USD 10 million
Yield:	6.75%

a) What is total accrued interest payable on this trade?

b) What is the bond's dirty price, rounded to the nearest 2 decimal places?

c) What is the clean price on this bond, rounded to the nearest 1/8%? (Write in decimal).

d) Assuming you bought the bond at the price calculated in (c), what is the total settlement amount of the transaction?

e) Would you need to pay more or less for this bond if you required a yield of 7%?

- Less
- More

Question 4

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model, which should have been provided for you to go with this exercise.

Security:	7½% US Treasury bond maturing 21 October 2009
Type:	semi-annual, actual/actual
Settlement date:	20 April 2004
Yield:	7.00%.

a) What is the bond's clean price, rounded to the nearest 1/32%? (Enter in decimal).

b) What is its dirty price, rounded to 2 decimal places?

c) Given the same yield, what would be the bond's clean price for value 22 April 2004, rounded to the nearest 1/32%?

Enter the result in decimal, to 2 decimal places.

d) What would be its dirty price, rounded to 2 decimal places?

e) Explain the differences between (b) and (d), above.

- The clean price changed
- A coupon was paid on 21 April
- The bond has been 'sold down'
- The bond's yield rose

Question 5

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model.

Security:	7½ FNMA bond maturing 21 October 2009
Type:	USD Eurobond, annual 30/360
Settlement date:	20 April 2004
Yield:	7.00%.

a) What is the bond's clean price, rounded to 2 decimal places?

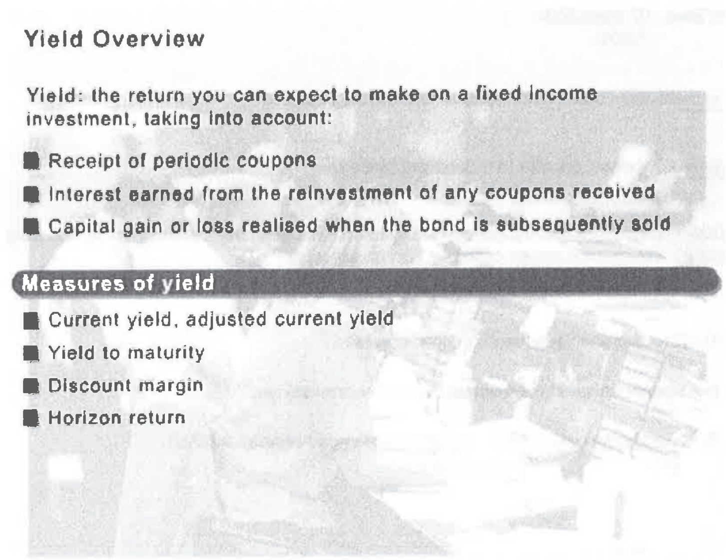
b) What is its dirty price, rounded to 2 decimal places?

c) Since the settlement, maturity, coupon and yield on this bond are the same as for the bond in *Question 4*, why are the prices of the two bonds different?

- ☐ This bond has a lower credit rating
- ☐ The two bonds were issued on different rates
- ☐ A semi-annual yield is equivalent to a lower annual yield
- ☐ This bond is annual 30/360; the other one is semi-annual Actl/Actl

3. Bond Yield

3.1. Overview



Someone buying a bond wants to know the return, or **yield**, he or she can expect to make on the investment. This yield may arise from:

- The receipt of periodic coupons
- Interest earned from the reinvestment of any coupons received
- A capital gain or loss realised when the bond is subsequently sold in the secondary market, or redeemed at maturity

The different measures of yield that we define here either take only some of these factors into account or make simplifying assumptions about them, and we shall highlight the strengths and weaknesses of each measure in turn.

Learning Objectives

By the end of this module, you will be able to:

- Define different measures of bond yield and explain their limitations:
  - **Current yield**
  - **Adjusted current yield**
  - **Yield to maturity**
  - **Yield to call / yield to put**
- Calculate the **yield to worst** on a callable bond and the **yield to best** on a putable bond

- Convert between **annual** and **semi-annual** yields
- Calculate the **horizon return** on a bond investment
- Infer the interest rate expectations implied in a **positive**, **flat** or **inverted yield curve**
- Formulate various basic yield curve strategies, including **riding the curve**
- Estimate the **net carry** on trading positions
- Calculate the **breakeven** on a bond position

3.2. Current Yield

Definition

$$\text{Current yield} = \frac{\text{Coupon rate}}{\text{Clean price}} \times 100$$

Also known as: **Running yield**; **Income yield**.

Example

Coupon rate: 6%
Clean Price: 95.00

? What is the bond's current yield?

Current yield = coupon rate / clean price x 100
= 6 / 95 x 100
= **6.32%**

Bond Price and Current Yield

If a bond trades:	Its CY will be:
At par	Equal to the coupon rate
At a premium to par	Less than the coupon rate
At a discount to par	Greater than the coupon rate

Limitations of Current Yield

Current yield is conceptually similar to the dividend yield on equities: it measures the income-generating capacity of the investment. But it ignores:

- Any interest earned on the reinvestment of the coupons received
- Any capital gain or loss realised when the bond is subsequently sold in the secondary market, or is redeemed at maturity.

If the bond in the example was held to maturity and redeemed at par the investor would also make a guaranteed capital gain of 5% on the asset.

Indeed, current yield is totally unsuitable for calculating the return on a zero coupon bond, since zeros do not pay any coupons. Zeros always trade at a discount to par and the return to the investor is implied in the capital gain made if the bond is held to maturity.

Despite its obvious shortcomings, current yield is often used in the context of equity convertible bonds where investors compare the current yield on the convertible with the dividend yield on the underlying equity. On this measure, convertibles typically yield more than the underlying equities, and this is one of their attractions.

Adjusted Current Yield

Adjusted Current Yield

$$= \frac{\text{[Coupon rate + (100 - Clean price) / Yrs. to maturity]}}{\text{Clean price}} \text{ x100}$$

$$= \frac{\text{Current yield + [(100 - Clean price) / Yrs. to maturity]}}{\text{Clean price}} \text{ x100}$$

Where:
Yrs. to maturity = Full years + Fraction of a year

$$\text{Fraction of a year} = \frac{\text{Number of days to next coupon}^{6}}{\text{Number of days in year}^{7}}$$

Also known as: **Japanese simple yield**.

Example

Coupon rate: 6%
Clean Price: 95.00
Maturity: 4.75 years

? What is the bond's adjusted current yield?

Current yield = 6 / 95 x 100
= 6.32%

$$\text{Adjusted current yield (ACY)} = 6.32 + \frac{[\,(100 - 95) / 4.75\,]}{95} \text{ x 100}$$

= 6.32 + 1.11
= **7.43%**

As we saw earlier, current yield ignores the fact that a bond bought at 95 will pay 100 at maturity, representing a substantial capital gain. ACY amortises this gain over the life of the bond (in this example at a rate of 1.11% per annum) and adds it to the current yield.

Bond Price and ACY

If a bond trades:	Its ACY will be:
At at par	Equal to the coupon rate
At a premium to par	Less than the coupon rate
At a discount to par	Greater than the coupon rate

[6] Using appropriate day-count convention.
[7] For bonds issued after January 1999, accrued interest is calculated on an Actual/Actual basis, instead of the traditional Actual/365.

Limitations of ACY

ACY is a relatively simple formula which used to be popular in the days before electronic calculators. However, it does have two flaws:

- Like current yield, it ignores any interest earned from the reinvestment of coupons
- It ignores the timing of the bond's cash flows - there is no allowance for the time value of money.

Nevertheless, ACY is useful as a quick reckoner of a bond's return and is still commonly quoted in some markets, notably Japanese government bonds.

More examples
Practise calculating the current yield and adjusted current yields.

Question

Security: 2.50% Japanese Government Bond maturity 10 March 2010
Type: Semi-annual, Actual/365
Settlement date: 5 December 2002
Clean Price: 95.00

Calculate the current and adjusted current yields for this bond. Enter your answers in the boxes below, in percent to 2 decimal places, then validate.

a) Current yield:

b) Adjusted current yield (ACY):

3.3. Yield to Maturity

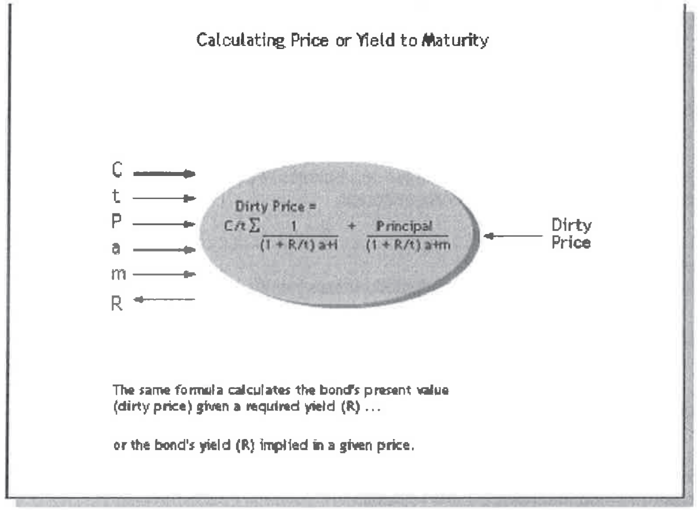
Definition

Yield to maturity:

- The uniform discount rate which makes the present value of a bond's future cash flows equal to its quoted dirty price
- The return that would be achieved on the bond if:
  - it is bought at the quoted price
  - *and* it is held until maturity
  - *and* any coupons received are reinvested at the same rate
- The internal rate of return on all of the bond's cash flows, including the initial outlay

This is the most widely used measure of return in the bond markets, and in fact when market participants speak of yield they typically mean yield to maturity (YTM).

To calculate the YTM on a bond we use the same pricing model that we developed in Bond Pricing - Valuation Formula, except that this time we use it 'in reverse', as the figure below indicates:



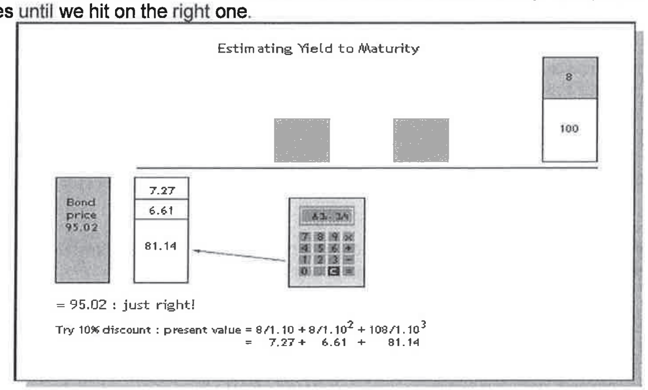
The formula is the same, only the direction of the calculation is different. The calculation is difficult to perform because the formula cannot be re-arranged into an expression for **R**. So **R** has to be found by trial-and-error (**iteratively**) by computing present values at different discount rates until we find the one that is equal to the bond's dirty price. Fortunately, dedicated bond calculators can do this in an instant.

Example

Security: 8% bond maturing in exactly 3 years
Type: annual
Price: 95.03 (no accrued)

? What is the yield to maturity?

The figure below illustrates how the YTM on this bond is arrived at iteratively, by trying different discount rates until we hit on the right one.



Bond Price and Yield to Maturity

If a bond trades:	Its YTM will be:
At a par	Equal to the coupon rate
At a premium to par	Less than the coupon rate
At a discount to par	Greater than the coupon rate

Limitations of YTM

YTM takes into account all the three components of return:

- The periodic coupon payments
- Interest earned on the reinvestment of the coupons received
- A capital gain or loss realised when the bond is redeemed

But it makes two fundamental assumptions:

- That the investor actually holds the bond until maturity (there is no guarantee that the bond can be sold at par before maturity)

- That the coupons received will all be reinvested at the bond's YTM!

YTM is therefore a theoretical calculation: it does not compute the actual return that an investor will make on the bond, even if it was held to maturity. The actual return will depend on future reinvestment rates.

Nevertheless, YTM is useful as a means of comparing the return on bonds with similar maturity and credit quality: a bond may be considered cheap if it yields more than a comparable issue. In this context, assumptions about future reinvestment rates may be less critical, since the same reinvestment rates will apply to all the bonds being compared.

3.4. Yield to Call/Put

Yield to Worst

So far we have considered the yield to maturity on bullet bonds, but how do you calculate the yield on bonds with uncertain maturity dates, such as callable bonds?

The traditional approach is to calculate the yield to each call date, as well as the yield to maturity, and then take the worst of these yields as the basis for the investment decision.

A yield to call is similar to a yield to maturity, except that in the pricing model we use:

- The call date instead of the maturity date
- The call price, rather than par, as the principal repayment amount.

As with yield to maturity, yield to call assumes that any coupons received will be reinvested at the same rate.

Example

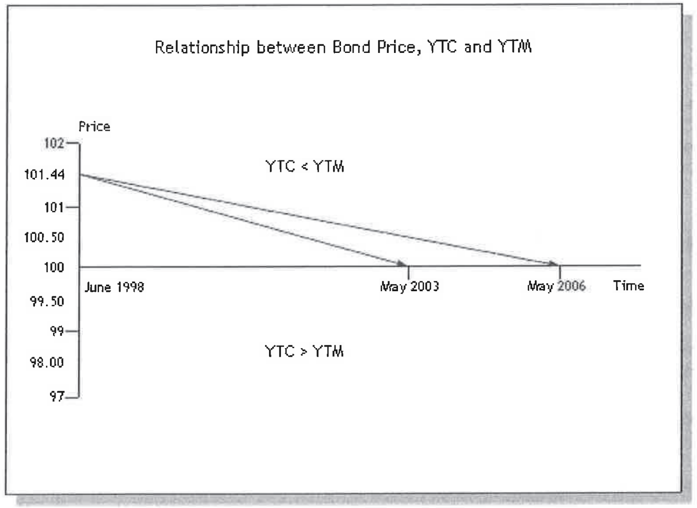
Security: 8% UK government maturing 5 May 2006
Type: domestic **double-dated gilt**,
semi-annual, actual/actual

Call features: callable at par on 5 May 2003
Settlement: 18 June 2002
Price: 101.44 (decimal)

Analysis
Yield to maturity (YTM on 5 May 2006): 7.55%
Yield to call (YTC on 5 May 2003): 6.22%

In the current market, yields are lower than the 8% coupon rate, so the bond trades at a premium to par and there is a built-in capital loss of 1.44% if the bond is redeemed at par, either in 2003 or in 2006. If the bond is called in 2003, the capital loss would have to be amortised over a shorter period, and that is why in this case YTC is lower than YTM.

In a different scenario, if yields were higher than 8% the bond would trade at a discount to par and its YTC would be higher than its YTM.



If the bond trades at a premium, the worst-case scenario is that it is called in 2003. In this case the investor earns 6.22%. If the investor finds this yield attractive enough, then clearly the bond is worth buying because in any other scenario (e.g. the bond is not called) the return would be higher.

Valuing the Embedded Option

Focusing on the worst-case scenario makes sense, because with a callable bond that is also the most likely scenario:

- If the bond trades at a premium (i.e. market yields are low), it is quite probable that the issuer will call the bond, financing the repurchase through the issue of a new bond carrying a lower coupon. The investor is therefore likely to end up earning the YTC, which in this scenario is lower than the YTM.

- On the other hand, if the bond trades at a discount it is unlikely that the issuer will call the bond at or above par. The investor is more likely to end up earning the YTM, which in this scenario is lower than the YTC.

But focusing exclusively on the yield to worst is an ultra-conservative approach to investment, because we are always assuming that the worst *will* happen, without considering the possibility that it may not.

A more sophisticated approach is to value the embedded option in the callable bond and then calculate its **option-adjusted yield** as a weighted average of YTM and YTC, where the weights are proportional to the probability of the embedded option being exercised (see Callable Bonds - Pricing). However, this approach is significantly more complex than a simple yield to worst calculation; many investors therefore still use yield to worst as their benchmark.

Yield to Best

A similar approach may be used for valuing putable bonds. Here it is the investor who has the option to obtain early repayment and therefore can be assured the most favourable yield. The method is as follows:

- Calculate the yield(s) to put:
  - using the put date(s) instead of the maturity date
  - and the corresponding put price(s) instead of par

- Calculate the YTM

- Take the best of these as the basis for the investment decision.

3.5. Yield Conversions

Annual & Semi-annual Yields

Yields on fixed income securities are not always comparable because of the different compounding and accrued day-count conventions used in various markets.

The formulas for converting between money market rates and bond equivalent yields are discussed in *Money Market Cash Instruments - Yield Conversions*. In this section we look at the conversions necessary to compare yields on bonds with different coupon periods.

We saw in *Time Value of Money - Simple and Compound Interest* that the effective - as opposed to the nominal - interest rate depends on the compounding frequency: the shorter the compounding period, the higher is the effective rate. The example below shows that quoted yields on annual and semi-annual coupons are not directly comparable.

The example below shows that quoted yields on annual and semi-annual coupons are not directly comparable.

Example

Security 1:	8% US Treasury bond maturing 5 May 2010
Type	Domestic, semi-annual, actual/actual
Settlement	18 June 2002
Quoted yield	7.75% (semi-annual)
Price:	101.44 (decimal)

Security 2:	8% AAA-rated bond maturing 5 May 2010
Type	Eurodollar bond, annual, 30/360
Settlement	18 June 2002
Quoted yield	7.75% (annual)
Price	101.40 (decimal)

? Why does the US Treasury bond have a higher price than the USD Eurobond, if both have the same credit rating and have identical coupon rates, maturities and quoted yields?

Because the US Treasury bond pays coupons semi-annually, so the investor receives one half of the next coupon rate earlier (on 5 November) and the balance on 5 May 2003, whereas the Eurobond does not pay anything until 5 May 2003. The Treasury bond's cash flows have a higher present value than the Eurobond's.

Put a different way, for the same price the Treasury bond's effective (annual) yield would be higher than the Eurobond's.

The General Formula

We can use the interest rate conversion formula developed in Time Value of Money - Simple and Compound Interest to convert between semi-annual and annual-equivalent yields.

Annual and Semi-annual Yield Equivalences

Annual return = Effective semi-annual return

$(1 + \text{Annual Rate}) = (1 + \text{Semi-annual Rate}/2)^2$

Therefore:

$\text{Annual Rate} = (1 + \text{Semi-annual Rate}/2)^2 - 1$

$\text{Semi-annual Rate} = [\sqrt{(1 + \text{Annual Rate})} - 1] \times 2$

In our example, the annual yield equivalent on the US Treasury bond would be:

$= (1 + 0.0775 / 2)^2 - 1$
$= 0.0790$ or **7.90%**

The 15 basis points difference with the yield on the Eurobond is clearly significant: the unwary investor might interpret the gilt as being overpriced, whereas in fact it is not.

The higher the yield level, the wider is the gap between annual and semi-annual yields.

Selected Markets

The table below summarises the accrued interest and yield conventions used in the major fixed income markets.

	Coupon Frequency	Accrued Interest	Yield Convention
Government Bonds			
USA	Semi-annual	Actual/Actual	YTM, Semi-annual
Japan	Semi-annual	Actual/365	ACY, Semi-annual
UK	Semi-annual	Actual/Actual	YTM, Semi-annual
France	Annual	Actual/Actual	YTM, Annual
Germany	Annual	Actual/Actual	YTM, Annual
Netherlands	Annual	Actual/Actual	YTM, Annual
Canada	Semi-annual	Actual/365	YTM, Semi-annual
Australia	Semi-annual	Actual/Actual	YTM, Semi-annual
Italy	Semi-annual	Actual/Actual	YTM, Annual
Corporate Bonds			
USA	Annual or Semi-annual	30/360	YTM, Semi-annual
UK	Semi-annual	Actual/365 or Actual/Actual[8]	YTM, Semi-annual
Eurobonds			
Issued before 1/1/99	Annual or Semi-annual	30E/360[9]	YTM, Annual
Issued after 31/12/98	Annual or Semi-annual	Actual/Actual[10]	YTM, Annual
Money Markets			
Fixed deposits, CDs	Bullet, but periodic if longer than 12 mths.	Actual/360 or Actual/365	Money market yield
T-bills, BAs, CP	No coupons	Actual/360 or Actual/365	Discount rate

[8] Using the day-count convention appropriate to the bond being analysed.
[9] The 'E' in the 30E/360 (or ISMA) basis is to distinguish this convention from the one that applies in the domestic US corporate bond market, which is also 30/360 but does not include the **end-month rule**. The end-month rule means that the number of days from the 1st to the end of a 31-day month (e.g. 1 May to 31 May) is also counted as 29, rather than 30, which is how it would be counted under the US 30/360 convention. In all other respects the two conventions are identical.
[10] *Unless the bond is denominated in US dollars*, in which case accrued interest will continue to be calculated on a 30/360 basis (ISMA Rule 251).

3.6. Discount Margin

Yield on an FRN

How do you measure the yield on a floating rate note (FRN)?

So far in this module we have focused exclusively on straight bonds. The FRN is a hybrid between a debt capital market and a money market security:

- Its original maturity typically exceeds 12 months (indeed most FRNs have longer maturities than straight corporate bonds), and its price is quoted as a percentage of face value, like a bond
- Its coupon rate is reset at each coupon date in line with a money market reference rate such as LIBOR, plus a fixed spread.

Unlike a straight bond, the price of an FRN is not very sensitive to changes in market rates, because its coupons are reset periodically in line with the market. However, its price is sensitive to changes in the credit quality of the issuer: a note rated single-A paying LIBOR + 45 basis points and issued at par will trade at a discount to par in the secondary market if the debt of the issuer was to be downgraded to BBB/Baa.

Definition

Dealers and investors assess the investment value of the FRN by reference to its discount margin.

Discount Margin:

- The risk premium which, when added to the risk-free rate, makes the PV of the FRN equal to its market price
- The spread over LIBOR which should be paid on the FRN in order to make its market price equal to par.

Also known as: **Effective LIBOR spread**.

The discount margin is the spread over LIBOR at which the note effectively trades, given its market price. It is calculated using a variant of the bond valuation formula:

$$\mathbf{PV} = \frac{(L_0 + S) \times d_0}{(1 + R + M)^a} + \frac{(L_1 + S) \times d_1}{(1 + R + M)^{a+1}} + \dots + \frac{(L_n + S) \times d_n + Par}{(1 + R + M)^{a+n}}$$

Where:

L_i = The LIBOR fixing for coupon period i

d_i = Number of days in coupon period i divided by 360 or 365, depending on the money market accrued interest day-count convention used
(see Money Market Cash Instruments - Accrued Interest)

S = The note's spread over LIBOR

R = The periodic risk-free discount rate (e.g. annual rate / 2 if the note is semi-annual)

M = The discount margin

a = Fractional period to the next coupon, calculated using the same day count convention as used for R

n = Number of complete coupon periods to maturity

Like a yield to maturity, M is calculated by a process of **iteration** - trying different values of M until you arrive at the one that equates the note's PV with its market price.

The LIBOR for the current coupon period may have already been reset, but in order to calculate M we also need to somehow 'fix' the future LIBORs. And we also need to consider what risk-free discount rate we should use. There are two common approaches:

1. Assume all future LIBORs will be the same as the current LIBOR and let R = LIBOR
2. Let the future LIBORs be equal to the forward rates implied in the LIBOR yield curve and let the risk-free rate corresponding to each coupon period be equal to the zero-coupon rate derived from the same curve (forward and zero-coupon rates are explained in detail in Spot Yields and Forward Yields).

Strictly speaking, the second approach gives a more accurate valuation of the note's future cash flows, but unless the yield curve is very steep the pricing differences between the two will not be significant. So the first method, which you can apply using a simple financial calculator, is frequently used. Whichever method you use:

If the note trades at par:	**Discount margin = LIBOR spread**
If the note trades at a discount to par:	**Discount margin > LIBOR spread**
If the note trades at a premium to par:	**Discount margin < LIBOR spread**

Example

Security:	Asian Development Bank GBP FRN maturing 29 October 2010
Rating:	Single A
Coupon rate:	6 month LIBOR + 0.15%
Day count:	Actual / 365
Settlement:	15 April 2002
Current LIBOR fix:	5.25%
Clean price:	98.75

? What is the discount margin on this security?

Analysis

Using method 1, first we 'fix' the coupon rate on the note at the current LIBOR:

Coupon = 5.25 + 0.15
= 5.40%

Then, using a financial calculator or the bond pricing model , we compute the yield to maturity on this 'bond', given its current market price.

Calculated yield (semi-annual, actual/365) = 5.583%

Finally, we calculate the discount margin as the difference between this yield and LIBOR:

Discount margin = 5.583% - 5.250%
= **0.333%**

In other words, if the note paid LIBOR + 33 basis points, instead of LIBOR + 15, then it would trade at par. An investor expecting to earn LIBOR + 40 on single-A rated paper would consider this note to be trading rich.

Yield Conversions

This approach is very straightforward, but you must be careful to use the appropriate day-count conventions when moving between the bond markets and the money markets. Thus, if the coupons on this note were calculated on an Actual/360 basis:

- The bond-equivalent coupon rate would be closer to:
  365/360 x 5.40% = 5.475%
  (see Money Market Cash Instruments -Yield Conversions)
- The calculated bond 'yield' would be 5.658%, which is a money market equivalent yield of:
  360/365 x 5.658% = 5.580%
- And the discount margin would be:
  5.580 - 5.250 = **0.330%**

In this example the difference is quite small, but in different conditions it could be significant.

3.7. Horizon Return

YTM and Horizon Yield Compared

We have seen how yield to maturity (YTM) is limited by the assumptions that:

- The bond will be held until maturity
- And any coupons received will be reinvested at the same rate (the YTM).

Horizon return or yield (HY) is a method of calculating the actual return that will be earned on the bond, taking into account:

- A given holding period (or horizon date), which may be earlier than maturity
- *And* any desired reinvestment rate assumption.

Horizon return, or yield (HY), takes into account:

- A given **holding period** (or **horizon date**), which may be earlier than maturity
- And any desired reinvestment rate assumption.

Example 1

Settlement date:	12 March 2002
Security:	8% Eurobond maturing 12 March 2012
Type:	annual, 30/360
Price:	90.00

Calculated yield to maturity: **9.60%**

What is the horizon yield if the investor expects to:

? - Hold the bond for 3 years, at which point it is expected to trade at 93.00 (for a YTM of 9.41%)

- And reinvest all the coupons received at 7% (annual 30/360)?

Analysis
The procedure is as follows:

Step 1: calculate the **future value** of all the coupons plus interest on these at 7%.

There are 3 coupon payments payable during the holding period: the first one next year which will be reinvested for 2 years, so its future value will be 8 x $(1 + 0.07)^2$.

The next coupon is payable in 2 years and will be reinvested for one year, so its future value will be 8 x (1 + 0.07). The final coupon is payable at the end of year 3 and will not be reinvested.

The future value of the coupons, including reinvestment income, will therefore be:

Future value of coupons
$= 8 \times (1 + 0.07)^2 + 8 \times (1 + 0.07)^1 + 8$
$= 8 \times [(1 + 0.07)^2 + (1 + 0.07)^1 + 1]$
$= 8 \times [1.1449 + 1.07 + 1]$
= 25.7192%

Step 2: add the future value of the coupons to the estimated future price of the bond, to arrive at the expected **horizon cash flow** - i.e. its total future value:

Horizon cash flow
= 25.7192 + 93.00
= 118.7192

Step 3: use the present value formula developed in Time Value of Money - Present & Future Value to calculate the yield implied in the two cash flows: payment of 90.00 today for a return of 118.7192 in 3 years.

$$\text{Present value} = \frac{\text{Horizon cash flow}}{(1 + \text{Horizon yield} / t)^{nxt}}$$

Where:
t = Compounding frequency (in this case 1)
n = Number of years

In this case:

$$90.00 = \frac{118.7192}{(1 + \text{Horizon yield})^3}$$

$$\text{Horizon yield} = \left\{ \frac{118.7192}{90.00} \right\}^{1/3} - 1$$

$$= 0.09671 \text{ or } \mathbf{9.67\%}$$

Conclusion
Calculated yield to maturity: **9.60%**
Calculated horizon yield: **9.67%**

By making his own assumptions, the investor has arrived at a yield estimate that is different from the bond's yield to maturity. Not surprisingly, this is higher than the yield to maturity. Of course, there is no guarantee that the assumptions made will be borne out.

You can verify these results using the bond pricing model, which should have been provided for you to go with this exercise.

The General Formula

Calculating horizon yield is relatively straightforward when the settlement and the horizon dates fall on coupon dates. The general formula, below, can handle situations when this is not the case.

$$\text{Horizon return} = \left[\left\{ \frac{\text{Horizon cash flow}}{\text{Cash dirty price}} \right\}^{1/(a+m+b)} - 1 \right] \times t$$

Where:
Horizon cash flow = Horizon dirty price + Future value of coupons

$$\text{FV of coupons} = C/t \times [(1 + r/t)^{m+b} + (1 + r/t)^{m+b-1} + \ldots + (1 + r/t)^b]$$

$$= C/t \times (1 + r/t)^b \times [(1 + r/t)^m + (1 + r/t)^{m-1} + \ldots + 1]$$

$$\text{Which reduces to} = \frac{C/t \times (1 + r/t)^b \times [(1 + r/t)^{m+1} - 1]}{r/t}$$

Where:
C = Coupon rate
t = Number of coupon payments per year (= compounding period)
r = Reinvestment rate on the coupons
m = Number of complete coupon periods to horizon date
a = (1 - Current fractional coupon period)
b = Fractional coupon period at horizon date

$$\text{Fractional coupon period} = \frac{\text{Number of days since previous coupon}^{11}}{\text{Number of days in coupon period}^{12}}$$

Example 2

Practise calculating the horizon yield on another investment.

Security:	8% Eurobond maturing 10 October 2001
Type:	Annual, 30/360
Settlement date:	5 January 1998
Price:	93.516 (clean)
Yield to maturity:	10.134%
Horizon date:	10 October 2001 (the maturity date)
Horizon price:	100.00

? What is the horizon yield, assuming a reinvestment rate of 10.13% (annual, 30/360)?

Number of Eurobond days since last coupon (10 October 1997 - 5 January1998)		= 85
Fractional coupon period	= 85/360	= 0.23611
Accrued interest	= 8 x 0.23611	= 1.889
Dirty price	= 93.516 + 1.889	= 95.405

[11] The formula represents the first two terms of a **Taylor's expansion** of the bond pricing formula.
[12] Take care to work with your calculator's full precision – i.e. do not to round your result until you reach the final calculation.

Terms used in the horizon return formula:

a = 1 – 0.23611 = 0.76389
b = 0
m = 3 (there are 3 whole coupon periods to the horizon date)

$$\text{Future value of coupons} = \frac{8 \times [\,(1 + 0.10134)^4 - 1\,]}{0.10134}$$

$$= 37.2013$$

Horizon cash flow = 100.00 + 37.2013
= 137.2013

$$\text{Horizon return} = \left\{ \frac{137.2013}{95.405} \right\}^{1/3.76389} - 1$$

$$= 0.10134 \text{ or } \mathbf{10.134\%}$$

The horizon yield in this case is the same as the bond's yield to maturity. The example demonstrates the crucial assumption behind yield to maturity: that the coupons are all reinvested at the same yield, which of course is not always the case!

3.8. Exercise

Question 1

Security: 4½% Japanese government bond maturing 23 Sep 2005
Type: domestic, semi-annual actual/365
Price: 108.55
Settlement date: 21 June 2002

Calculate:

a) The bond's current yield (CY) in percent. (Enter your answer, to 2 decimal places, in the box below, then validate.)

b) The bond's adjusted current yield (ACY) rounded to 2 decimal places.

Question 2

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model, which should have been provided for you to go with this exercise.

Security: 4½% Japanese government bond maturing 22 Sep 2005
Type: Domestic, semi-annual actual/365
Settlement date: 20 June 2002
Price: 108.55

a) Calculate the bond's yield to maturity, rounded to the nearest 2 decimal places.

Question 3

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model

Security:	4½% Asian Investment Bank maturing 22 Sep 2005
Type:	JPY Eurobond, annual 30E/360
Settlement date:	20 June 2002
Price:	108.55

a) Calculate the bond's yield to maturity, rounded to the nearest 2 decimal places.

b) Why is the yield on this bond different from the yield on the bond calculated in *Question 2*, if the two bonds have the same coupon, maturity and price?

- ☐ The two bonds were issued on different dates.
- ☐ A semi-annual yield is equivalent to a lower annual yield.
- ☐ This bond is annual 30/360; the other one is semi-annual Actl/Actl.
- ☐ This bond has a lower credit rating.

Question 4

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model.

Security:	7½% Ford Motor Finance, maturing 22 October 2009	
Type:	Eurobond, annual 30E/360	
Call features:	Callable at:	101.00 on 22 October 2007
		100.50 on 22 October 2008
Settlement date:	19 April 2002	
Price:	102¾	

a) What is the bond's yield to maturity, rounded to the nearest 2 decimal places?

b) What is the bond's yield to worst, rounded to 2 decimal places?

c) Without doing the calculations, could you have predicted which was the worst call date?

- ☐ No
- ☐ Yes

Question 5

This exercise is too complex to perform with a simple calculator. You should use a financial calculator such as the HP 17 (or later model) or the bond pricing model.

Security	6¼% British government bond (UK Gilt) maturing 20 October 2010
Type:	Semi-annual, Actual/actual
Price:	102.34375
Settlement:	19 April 2002

What is the bond's yield to maturity:

a) On a semi-annual basis, in percent, rounded to the nearest 3 decimal places?

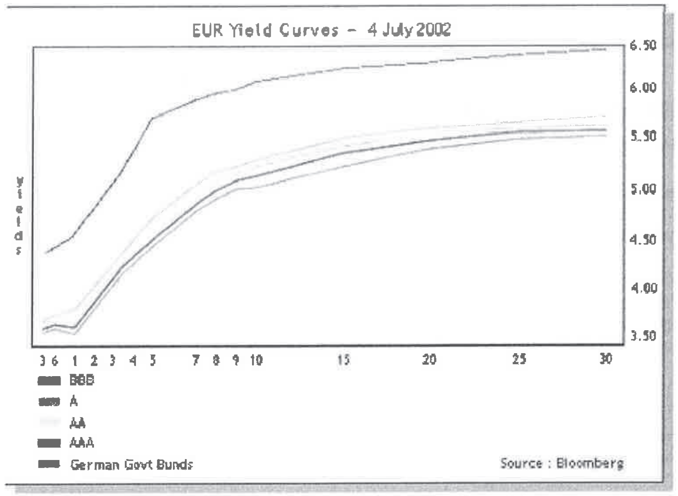
b) On an annualised basis (to 2 decimal places)?

3.9. The Yield Curve

Definition

The yield curve displays graphically the relationship between interest rates (or yields) and term to maturity - the term structure of interest rates - for borrowers with comparable credit rating.

The figure below plots a separate yield curve for each class of borrowers with the same credit risk. We can see the pure 'price of waiting' most clearly in the yield curve for government securities, which in most developed markets is assumed to be free of credit risk.



The Shape of the Curve

It is clear from the shape of these curves that lenders require a different price for locking their money into investments of different maturities. The cost of three or six month money is lower than the cost of, say, two or ten year money. Why?

- *Not* because long term debt has more credit risk than short term debt: the government yield curve has no credit risk at all

- *Not* because long term bonds have higher market risk than short term bonds: as we explain in Bond Market Risk - Using Duration, short bonds are actually riskier for the balance sheets of long term funds: pension funds and insurance companies need the security of long-term fixed income securities in order to service their long-term liabilities. One investor's medicine is another's poison...

The answer has to do with market expectations:

- A positive curve (upward sloping) indicates the market expects short term rates to rise

  If an investor expects interest rates to rise within the next 12 months he will demand a higher rate to lock into 1 or 2 year investments because by committing his funds for a long period he foregoes the opportunity of benefiting from higher rates in the near future.

- An inverted curve (downward sloping) would imply that the market anticipates rates to fall

  If the investor expects future rates to fall, he can afford to lock into long-term lending at a lower rate.

Analysis of the shape of the yield curve is central to some trading strategies which rely on the yield curve changing slope (see Bond Market Risk - Trading Applications).

Factors Affecting the Curve

Different economic forces tend to be behind the short and the long end of the yield curve:

- **At the short end**: central banks have a powerful influence on money market rates. They can alter the amount of liquidity available in the banking system through **open market operations**:

  - They can **add liquidity** to the system and drive rates down by purchasing securities from the market or through repo transactions (see Repurchase Agreements - Applications). If the market expects the authorities to tighten liquidity conditions in the future the curve will tend to become more positive.

  - They can **drain liquidity** from the system and drive short term rates up by selling some of their holdings of government or other securities to the private sector, or through matched funding. If the market expects tight liquidity conditions to be temporary the curve will tend to invert.

- **At the long end**: the most important factor here is investors' inflation expectations. Investors locking into fixed nominal yields look for real yields - i.e. inflation-adjusted - on their portfolios. Thus, the market adjusts nominal yields up if it perceives higher inflation and vice-versa.

  Therefore any economic information that has a potential bearing on future inflation also has an impact on long term yields, and therefore prices of long term securities:

  - **Economic activity indicators** - e.g. unemployment, employment and payroll data; surveys of manufacturing orders, etc.

  - **Financial indicators** - e.g. consumer credit granted, money supply growth, the exchange rate, commodity prices, etc.

3.10. Yield Curve Strategies

Bull & Bear Positions

The shape and slope of the yield curve gives traders important information about market sentiment. In this section we focus on directional yield curve strategies and in section *Carry & Breakeven* we examine their funding implications. The examples below focus on cash securities, although the same strategies may also be implemented using fixed income derivatives.

Scenario 1 – Bull Market

Situation:
The inflation threat is receding and the authorities are expected to begin easing monetary conditions, so you anticipate the curve will shift down.

Strategy: go long the curve

- Switch out of cash and into long-dated bonds
- Receive fixed on swaps
- Buy bond futures

Evolution:
The curve may already be inverted and may invert further before it shifts down as investors position themselves at the long end for maximum leverage.

Scenario 2 – Bear Market

Situation:
Inflation may be accelerating or the currency is weak. You expect interest rates to rise, especially at the long end.

Strategy: go short the curve

- Switch out of bonds and into cash
- Pay fixed on a swap
- Sell bond futures

Evolution:
The curve may already be positive and may steepen further before it shifts up, as investors leave the long end for the relative safety of the short end.

Riding the Curve

Scenario 3 – Steady Market

Market yields	**(Semi-annual, Actual/Actual):**
6 MTHS	5.75 - 5.70
12 MTHS	6.05 - 6.00

Situation:
Rates are expected to remain on hold and the curve is mildly positive. You wish to place $1 million for 6 months (183 days).

Strategy:
Roll down the curve (or **ride the curve**): buy the 12 month paper and resell it after 6 months.

Analysis:
The horizon return on this strategy should exceed the yield on the 6 or the 12 month paper. Assuming the curve remains static, then after 6 months we should be able to sell the securities, at which point they only have 6 months to maturity so they should yield 5.75% bid. Using the formula developed in section *Horizon Return*:

$$\textbf{Horizon return} = \left[\left\{\frac{\text{Horizon cash flow}}{\text{Cash dirty price}}\right\}^{1/(a+m+b)} - 1\right] \times t$$

Where:
t = Number of coupon payments per year; in this case 2
m = Number of complete coupon periods to horizon date; in this case 1
a = (1 - Current fractional coupon period); in this case 0
b = Fractional coupon period at horizon date; in this case 0

We shall assume for simplicity that the bond is bought at par, so the cash flow at spot settlement is $1 million.

Horizon cash flow = Dirty price of bond at @5.75% yield
= Current coupon + PV of final cash flow

$$= 60{,}000/2 + \frac{1{,}000{,}000 \times (1 + 0.06/2)}{(1 + 0.0575/2)}$$

= $1,031,215.07

Horizon return = [(1,031,215.07 / 1,000,000) - 1] x 2
= **0.0624 or 6.24%**

We achieved a return which is higher than the original yield on the security because we were able to resell it for a lower yield. You can think of this return as consisting of two elements:

Gross profit = Yield on underlying bond (6.00%)
+ Capital gain (0.24%)

This is a gross profit because we have not yet considered the cost of funding the strategy.

3.11. Carry & Breakeven

Net Carry = Accrued interest - Funding cost

Positive carry: Accrued interest > Funding cost
Negative carry: Accrued interest < Funding cost

Trading positions seldom pay off immediately. They may have to be carried for days, weeks or even months. In this section we consider funding costs and we highlight some of the funding risks.

Scenario 1 –Positive Curve

A positive curve allows you to fund a trading book more cheaply by borrowing on a short term basis at lower rates. The positive carry builds up a useful cushion of profit, but the funding may have to be rolled over perhaps many times before the position is unwound. If funding rates rise the cost of carry may negate any trading profits.

Example

Market yields:
O/N repo 5.70 - 5.65 (annual, Actl/360)
12 MTHS 6.05 - 6.00 (semi annual, Actl/Actl)

Strategy:
Expecting a stable market, we buy 12 month paper at 6.00% and fund it for 6 months (182 days) in the overnight repo market at 5.70%.

Analysis:
Over the 6 months we make a net 30 basis points (bps) profit on the carry. This raises the forward breakeven rate on the position.

Forward breakeven: the price (or yield) at which you need to unwind a position in order to cover net carry costs.

A seat-of-the pants calculation of the forward breakeven yield is:

Net carry = Current yield - Funding rate
= 6.00 - 5.70
= 0.30%

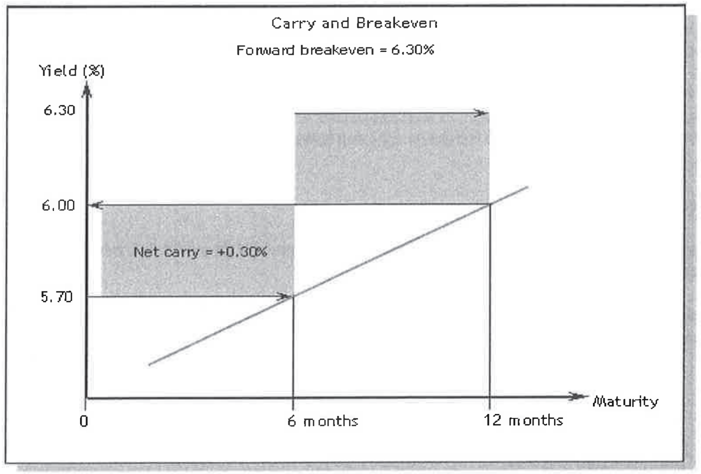
Breakeven yield = Cash yield + Net carry
= 6.00 + 0.30
= **6.30%**

To make a profit we need to sell the **tail** of the bond position in 6 months for a yield of less than 6.30%. We have in fact created a synthetic futures position in the bond, at a yield of 6.30%. In effect, we are betting on the future yield on the bond being lower than its implied forward yield.

In a positive yield curve:

- **Being long the curve requires the view that yields will not rise by more than the forward rates**
- **Being short the curve requires the view that yields will rise by more than the forward rates**

The figure below highlights the risks involved from a funding perspective. The funding rate after 6 months has not yet been fixed: if the rate rises beyond the breakeven then we shall lose money overall.



The Compounding Effect

The seat-of-the-pants calculation above does not allow for the different compounding or day-count conventions that may apply in the bond and the repo markets. The bond is funded with a series of overnight repos, so interest compounds daily and this will reduce the net carry benefit. As we show in Bond Futures - Pricing, the breakeven forward clean price (FP) is calculated from the clean price for cash or spot settlement (SP) as follows:

FP + Forward accrued = SP + Spot accrued + Funding cost

If we assume for simplicity that the bond was purchased at par with no accrued interest:

$$FP + 6.00/2 = 100 \times (1 + 0.057 / 360)^{182}$$
$$FP = 102.92 - 3.0$$
$$= 99.92$$

Now we can calculate the implied yield on this forward price. On the forward date, the bond pays a coupon, so on that date its present value = clean price:

$$\text{Present Value} = \frac{\text{Maturity value}}{(1 + \text{Yield}/2)}$$

$$99.92 = \frac{103.00}{(1 + \text{Yield}/2)}$$

Yield = 0.0617 or **6.17%**

Having allowed for the different day-count conventions and compounding speeds, we discover that the carry is in fact much less favourable!

Scenario 2 –Inverted Curve

Carrying a bull position in an inverted yield curve can become prohibitively expensive:

- The compounding effect is against you
- The carry is negative

If yields fail to fall far enough or fast enough the windfall gain may not be sufficient to cover the funding costs incurred.

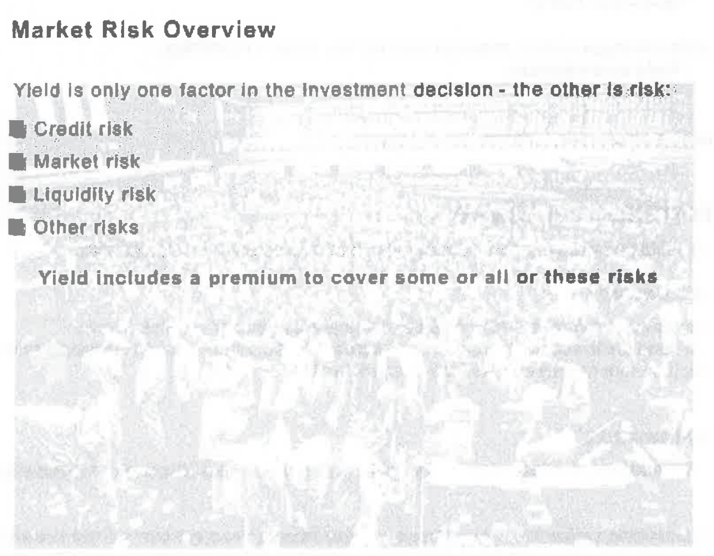
On the other hand, if we are short the bond in an inverted yield curve, then both the carry and the compounding effect work in our favour. In this case, there is an additional risk: the securities borrowed may go on special and the repo rate for them may fall below the required breakeven (see Repurchase Agreements).

In an inverted yield curve:

- **Being long the curve requires the view that interest rates will fall by more than the breakeven**
- **Being short the curve requires the view that interest rates will *not* fall by more than the breakeven**

4. Bond Market Risk

4.1. Overview



Yield is only one factor in the investment decision - the other factor is risk:

- **Credit risk** - the risk of default or delays in payments of coupons or principal on fixed income security
- **Market risk** - the potential loss that could arise from an adverse change in the price of a security
- **Liquidity risk** - the ability to trade in large size without significantly affecting the market price
- **And other risks**

The yield that an investor requires on a security must include a risk premium to reflect some or all of these risks. In this module we focus on market risk. We discuss various ways of quantifying this risk, and we explore how investors use them to manage fixed income positions.

Learning Objectives

By the end of this module, you will be able to:

- Define three measures of bond market risk and highlight their limitations:
  - **Macaulay duration**
  - **Modified duration**
  - **Basis point value**
- Design strategies which hedge or alter the risk profile of a portfolio:
  - **Yield curve spreads**
  - **Credit spreads**
- Identify any residual risks on spread positions

4.2. Fixed Income Laws

By now you should be aware of some fundamental laws of the fixed income markets:

Bond Market Law No 1

Bond price varies inversely with yield: it falls as market yield goes up, and vice-versa
The price of a bond is the sum of its discounted future cash flows: the higher the discount rate (or yield) applied, the more heavily are the future cash flows discounted.

Bond Market Law No 2

The longer the maturity of a fixed income security, the more sensitive is its price to a given change in yield.

An investor committing capital to the purchase of a fixed income security earns a fixed stream of cash flows over the term. If, during this time, interest rates were to rise the investor misses the opportunity to earn the higher rate. The fall in the price of the security reflects the present value of the interest rate difference.

- If you buy a security maturing a day later, and in the meantime interest rates rise, then you only lose one day's interest at the higher rate
- But if you invest in a 20-year bond the opportunity loss is correspondingly larger - and so is the change in the bond's price

Bond Market Law No 3

The lower the coupon rate on a bond, the more sensitive is its price to a given change in yield.
This law is not so intuitive. If interest rates rise, any coupon paid on the bond unlocks funds which can be reinvested at the higher rates. The higher reinvestment income offsets some of the opportunity loss on the principal tied up, so the higher the coupon rate on the bond the more stable is its price.

A zero-coupon bond pays no coupons which could be reinvested at the higher rates, so zeros are more price-sensitive to yield changes than equivalent coupon bonds.

4.3. Macaulay Duration

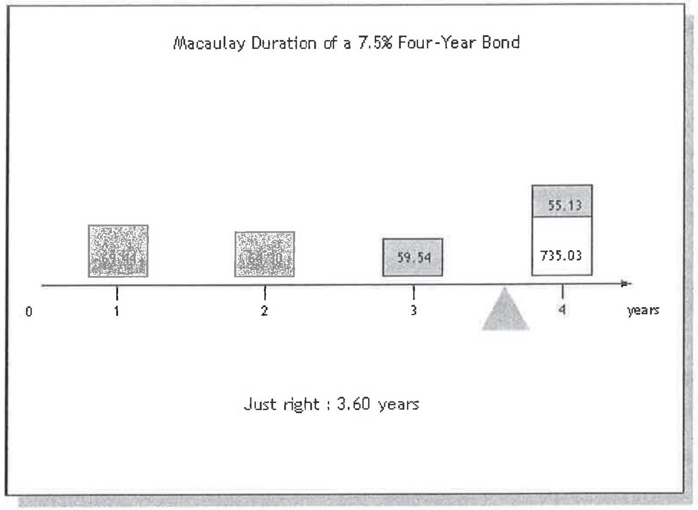
Definition

Macaulay duration is a weighted average of a bond's life, taking into account:

- The size of each cash flow
- Its timing.

Duration is a measure of market risk: the higher the bond's duration, the greater is its risk.

The concept was developed in the 1930s by the Scottish actuary Frederick R. Macaulay. The figure below shows duration as the 'centre of gravity' of a bond, taking into account the 'weight' and position of each cash flow along the timeline.



The weight of each coupon in the picture becomes progressively smaller as you look towards the right because the more distant cash flows are more heavily discounted.

Mathematically, duration is a weighted average of the time to each cash flow. The 'weight' given to each time period is the present value of the cash flow payable at that point in time, as a proportion of the present value of the whole bond (i.e. its dirty price).

Properties

Duration is useful because it brings all the factors that affect a bond's market risk 'under one roof'. Here are the bond market laws introduced earlier, restated in terms of duration, plus a new one:

- **Bond Market Law No 2** (restated): the longer the maturity of a bond, the higher is its duration - the pivot point in the see-saw above must be moved to the right.
- **Bond Market Law No 3** (restated): the lower the coupon rate on the bond the higher is its duration - again the pivot must be shifted to the right to preserve the balance. The duration of a zero coupon bond is equal to its maturity: a zero has only one cash flow, so the pivot point lies directly underneath it.
- **Bond Market Law No 4** (new): the duration of a bond increases as its yield falls, and vice versa. At higher yields the more distant cash flows are discounted proportionately more heavily, so the pivot point has to be moved to the left.

Properties of Macaulay Duration

Laws 2 and 4 indicate that duration is not a constant: it drifts over time as the bond approaches maturity and market yields change. Here are a couple of other interesting observations:

- The maximum duration of any coupon bond is about 14 years. Beyond a certain maturity, the present value of the more distant cash flows - even the principal - are so small as to be irrelevant! So the market risk on Walt Disney's 100-year Eurobond is virtually the same as the risk on British Telecom's 50-year bond. Only zero coupon bonds are able to breach this 14-year 'sound barrier'.
- The duration of a floating rate note is equal to the length of its current coupon period. The coupons on a floater are reset periodically in line with current rates, so its market risk is limited to the time left to the next coupon fixing.

The General Formula

Macaulay Duration (D)

$$= \frac{1}{PV} \times \left[a \times \frac{C/t}{(1 + R/t)^{a}} + (a+1) \times \frac{C/t}{(1 + R/t)^{a+1}} + \ldots + (a+m) \times \frac{Principal + C/t}{(1 + R/t)^{a+m}} \right]$$

Where:
PV = Dirty price
C = Coupon rate (%)
R = Yield to maturity
t = Number of coupon payments per year (compounding period)
m = Number of complete coupon periods to maturity
a = (1 - Fractional coupon period)

$$\text{Fractional coupon period} = \frac{\text{Number of days since last coupon}}{\text{Number of days in current coupon period}}$$

Equivalent formulation:

$$D = \frac{C/t}{PV} \times \Sigma \frac{a + i}{(1 + R/t)^{a+i}} + \frac{(a + m)}{PV} \times \frac{Principal}{(1 + R/t)^{a+m}}$$

for i = 0 ... m

Example
Macaulay Duration calculation.

Security: 5% US Treasury note maturing 21 January 2005
Type: Semi-annual, actual/actual
Settlement date: 3 June 2003

Price: 95.48
Accrued: 1.84
Yield: 8.00%

? What is the bond's Macaulay duration?

We priced this bond in *Bond Pricing - Valuation Formula*!

Nr. days from settlement to next coupon	(3 June - 21 July)	=	48
Nr. days in current coupon period	(21 January - 21 July)	=	181
Fraction of period to next coupon	= 48/181	=	0.2652

In the table below we proceed in three stages:

1. Calculate the present value of each cash flow (column 2)
2. Multiply each PV by its corresponding time period (column 3)
3. Divide the sum of column 3 by the sum of column 2 (the bond's dirty price).

Coupon Date	(1) Time Period	Cashflow	Discounted at 8%	(2) Present Value	(3) = (1) x (2)
21 Jul 1998	0.2652	2.50	$\frac{2.50}{(1 + 0.08/2)^{0.2652}}$	2.47	0.655
21 Jan 1999	1.2652	2.50	$\frac{2.50}{(1 + 0.08/2)^{1.2652}}$	2.38	3.011
21 Jul 1999	2.2652	2.50	$\frac{2.50}{(1 + 0.08/2)^{2.2652}}$	2.29	5.187
21 Jan 2000	3.2652	102.50	$\frac{102.50}{(1 + 0.08/2)^{3.2652}}$	90.18	294.456
Sum				**97.32**	**303.309**

$$\text{Macaulay duration} = \frac{303.309}{97.32}$$

$$= 3.12 \text{ coupon periods}$$
$$= \mathbf{1.56} \text{ years}$$

4.4. Using Duration

Example – Asset & Liability Management

An insurance company sold a single-premium savings plan giving the policy holder title to a fixed lump-sum cash payment when the plan matures in 9 years.

? How should the fund invest the premium proceeds to ensure it can meet this known future liability?

The Alternatives

- **Equities**: in the long run equities tend to outperform bonds but their return is uncertain. There is no guarantee that the value of the equities in 9 years will be sufficient to meet the company's obligations.

- **Money market investments**: although the market risk on short term paper may be small, the reinvestment risk is high: the interest and principal on the instruments would have to be rolled many times over the 9 years at varying yields, so the terminal value of the fund would be uncertain.

- **Coupon bonds**:
  - A bond with less than 9 years to maturity would expose the fund to reinvestment risk on the principal, as well as on the coupons.
  - One with more than 9 years to maturity would have to be liquidated before maturity, so it would expose the fund to market risk, as well as to reinvestment risk on the coupons.
  - One with exactly 9 years to maturity would have reinvestment risk on its coupons (though not on the principal), so its horizon yield would also be uncertain.

- **Zero coupon bonds**: only zeros repay a fixed cash flow at maturity with no reinvestment risk (there are no coupons to reinvest!). Ideally, the fund should purchase a zero maturing on the same date as the underlying savings plan.

  In practice, it is unlikely that there will a zero that matches exactly each of the fund's future liabilities on thousands of different savings plans.

Duration-matching

The solution in this case is to apply a technique known as **duration-matching**. This involves building a portfolio of assets that has the same Macaulay duration as that of the fund's liabilities.

The duration of a fixed income asset portfolio is the weighted average of the durations of its constituent securities.

The weight of each security in the portfolio is the present value (PV) of that security relative to the PV of the total portfolio.

Duration of a Portfolio

$$D_{portfolio} = \frac{1}{PV} \times \Sigma (PV_i \times D_i)$$

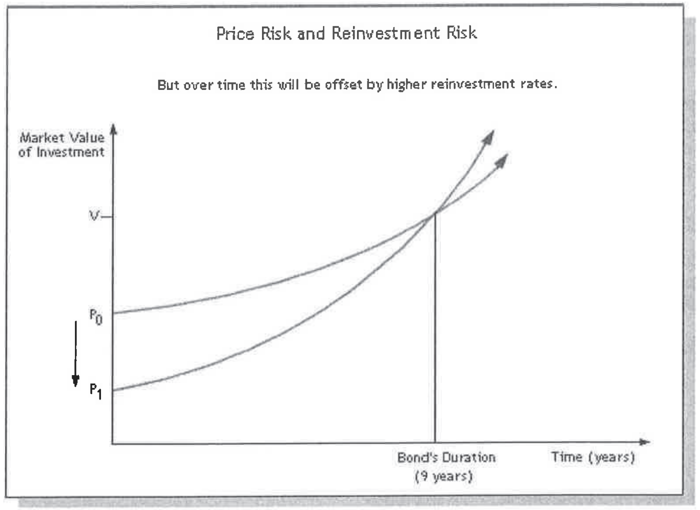
for i = 1 ... n

Where:
PV_i = Present value of asset i
D_i = Duration of asset i
n = Number of assets in the portfolio

Similarly, the duration of a liability portfolio is the weighted average of the durations of its constituent future liabilities.

Analysis

A fixed income portfolio that is duration-matched is said to be **immunised**: its future value will not be affected by interest rate movements. The figure below illustrates how this powerful technique works in the case of our 9-year forward liability.



Explanation

Suppose today the fund purchases a bond with a duration of 9 years at a price P_0. Over time the value of the fund will rise to V; the rise is exponential because interest compounds as the coupons are reinvested.

Now suppose that immediately after the bond is purchased market yields rise, resulting in a fall in the value of the fund to P1. Although on a mark to market the fund might now be technically insolvent, future coupons on this bond can be reinvested at higher rates. Other things being equal, by year 9 the higher reinvestment income should have fully offset the price loss today!

Managing Duration

A balance sheet is:

- **Short-funded** (or **net short duration**) if the duration of its assets exceeds that of its liabilities: a rise in interest rates will hurt its profit & loss, and vice versa.
- **Long-funded** (or **net long duration**) if the duration of its assets is less than that of its liabilities: a rise in rates will improve its profit & loss, and vice versa.
- **Match-funded** (or **duration matched**) if the duration of its assets is the same as that of its liabilities: a rise or a fall in rates will not impact on the profit & loss.

The technique of duration-matching is commonly used by banks, as well as fixed income funds, to manage the net interest rate risk on their balance sheets. The asset & liability managers calculate the net duration of the company as a whole and create interest rate hedging positions, using cash or derivative instruments, to ensure it is reasonably matched.

Of course, the duration of the assets (and the liabilities) will change daily, requiring the company to rebalance its hedging portfolio at regular intervals. As long as the company remains duration-matched, its balance sheet will be immune to fluctuations in market yields. This was Macaulay's contribution to interest rate risk management.

4.5. Modified Duration

Macaulay duration allows us to rank fixed income securities in terms of their market risk, but it does not tell us how risky the securities are in terms of profit or loss. However, Macaulay duration can be easily modified to quantify risk in those terms.

$$\text{Modified duration} = \frac{\text{Percentage change in a bond's dirty price}}{\text{1 percentage point change in yield}}$$

$$= \frac{\text{Macaulay duration (in years)}}{(1 + R/t)}$$

Where:
R = Yield to maturity, in decimal (i.e. 5% is 0.05)
t = Number of coupons per year (1= annual; 2= semi-annual, etc.)

Also known as: **Adjusted duration**.

We shall not prove the relationship between Macaulay and modified duration here, but we can illustrate its use with a simple example .

Example
Modified Duration calculation

Security:	5% US Treasury note maturing 21 January 2005
Type:	Semi-annual, actual/actual
Settlement date:	3 June 2003
Price:	95.48
Accrued:	1.84
Yield:	8.00%
Macaulay duration:	1.56 years
Amount held:	USD 1 million.

? What is the potential loss on this investment if yields rose to 9%?

We calculated the Macaulay duration of this bond in section *Macaulay Duration*!

$$\text{Modified duration} = \frac{1.56}{(1 + 0.08/2)}$$

$$= \mathbf{1.50\%}$$

A rise in yield from 8% to 9% would result in a capital loss on this investment of approximately 1½%.

$$\text{Risk in cash terms} = \frac{1.50}{100} \times \frac{(95.48 + 1.84)}{100} \times 1{,}000{,}000$$

= USD **14,598.00**

The capital loss is only approximate. As we said in section *Macaulay Duration*, Bond Market Law No. 4 means that Macaulay duration depends on the yield level at which we discount the bond's future cash flows. Therefore a yield change of this magnitude will also affect the numerator in the modified duration formula. We shall explore this phenomenon in section *Convexity*.

In any case, a 1% change in yield is a very large move in the US Treasury markets. Traders and risk managers typically estimate risk based on smaller yield changes, as we shall see in section *Basis Point Value*.

4.6. Basis Point Value

Basis point value (BPV): the change in the price of a bond, per 100 nominal,
for a 1 basis point (0.01%) change in its yield.

$$\text{BPV} = \frac{\text{Modified duration(\%)}}{100} \times \frac{\text{Dirty Price}}{100}$$

Also known as: **Present value of a basis point** (PVBP), **Value of an 01** (Val-01),
Dollar modified duration, **Risk factor**, **Delta**.

In the BPV formula we first divide modified duration by 100 to convert it from a percentage into decimal (i.e. 5% is 0.05). The second divisor of 100 reduces the scale of risk from a 100 basis point change in yield (modified duration) to just 1 basis point.

BPV is the unit of risk in the fixed income markets.

It is an essential building block in the design of a variety of trading strategies, as we shall explore in the next sections.

Example
Basis Point Value

Security:	5% US Treasury note maturing 21 January 2005
Type:	Semi-annual, actual/actual
Settlement date:	3 June 2002
Price:	95.48
Accrued:	1.84
Yield:	8.00%
Macaulay duration:	1.56 years
Modified duration:	1.50%
Amount held:	USD 1 million.

We calculated the modified duration of this bond in section *Modified Duration*!

$$\text{BPV} = \frac{1.50}{100} \times \left(\frac{95.48 + 1.84}{100}\right)$$

$$= \mathbf{0.014598}$$

Thus, a 1 basis point rise in the bond's yield will result in:

- A fall in the price from 95.4800 to 95.4654
- A loss of 1.46 cents per USD 100 nominal (remember the bond price is expressed in percent)
- A loss of USD 145.98 on a USD 1 million position

BPV tends to come out as a very small figure with many decimal places. For convenience many bond analysis systems scale the BPV figure by a factor of 100, so in our example the **risk factor** would be 1.4598. Thus, a 100-point change in the bond's yield would result in:

- A fall in the price from 95.48 to 94.02 (minus 1.46)
- A loss of USD 1.46 per 100 nominal
- A loss of USD 14,598 on a USD 1 million position

4.7. Trading Applications

Example – Yield Curve Spreads

Settlement date: 14 October 2002

Situation

We observe the following:

- The 8½% German government bond (Bund) maturing 8 Sep 2004 yields 4.65%
- The 6% maturing 3 April 2006 yields 4.70%

The 5 basis point yield spread between 2 and 4 years appears too narrow. We expect the yield curve to steepen (i.e. pivot anticlockwise) and the spread between the two bonds to widen. That is, we expect the yield of the 2006 to rise relative to the yield on the 2004.

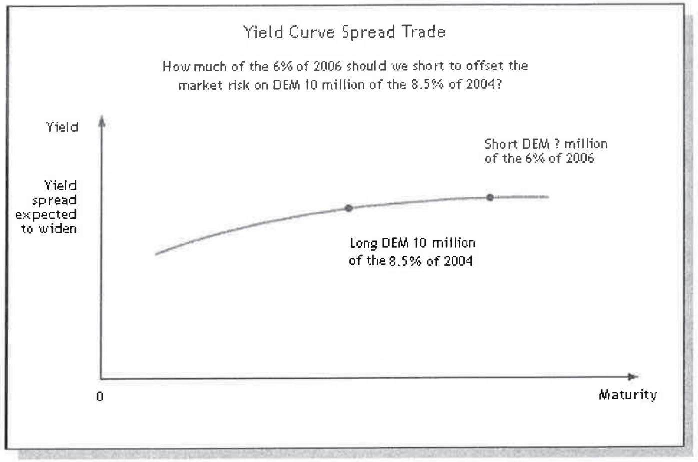
We want to profit from an expected widening of the yield spread between the two bonds, but we don't want to take an outright market exposure: risking a loss if the entire yield curve were to shift up or down.

Analysis

The technique is to create a **risk-weighted spread** position as follows:

- Buy the 8½% of 2004
- Sell short the 6% of 2006

The idea is that if the whole curve were to shift up in parallel, then any losses on the 2004 would be offset by trading gains on the short position in the 2006s; and vice versa if the curve were to shift down. But if the curve were to pivot anticlockwise, then the gain on the 2004 position should more than offset any losses on the 2006 position.



The Solution

? How many of the 2006 should we short if we were to buy EUR 10 million of the 2004?

The solution involves calculating the BPV of each bond:

BPV of 8½% of 2004 = EUR 0.01877
BPV of 6% of 2006 = EUR 0.03227

The price of the 2004 changes by 1.88 cents, per EUR 100, for every basis point change in yield. The 2006 changes by 3.23 cents, so it is riskier. Therefore if we bought EUR 10 million of the 2004 we should short:

$$\frac{0.01877}{0.03227} \times 10{,}000{,}000 = \textbf{EUR 6,057,079} \text{ of the 2006.}$$

In practice we may have to round this amount to the nearest EUR 50,000 - 100,000, although most market makers nowadays are well-used to trading odd amounts with their hedge fund clients.

This is the idea behind the risk-weighted yield curve spread trade: shifts in the yield curve should have no net effect on the profit/loss. Only a change in the yield spread between the two bonds will make an impact:

- A steepening of the curve would be profitable
- A flattening of the curve would be unprofitable

Yield curve pivot risk: the risk of loss as a result of a steepening or flattening of the yield curve.

Also known as: **Turning point risk**.

Credit Spreads

The technique of risk-weighting is also used when designing a credit spread trade, for example:

- Short a corporate bond whose credit rating you expect to weaken
- Buy a government bond with comparable maturity to hedge the outright market risk

The net position should make a profit if the yield spread of the corporate bond widens over the yield on the government bond. Again, the amounts dealt should be BPV-weighted so that there is no net profit/loss if both the government and the corporate yield curves were to shift up or down together.

4.8. Convexity

Definition

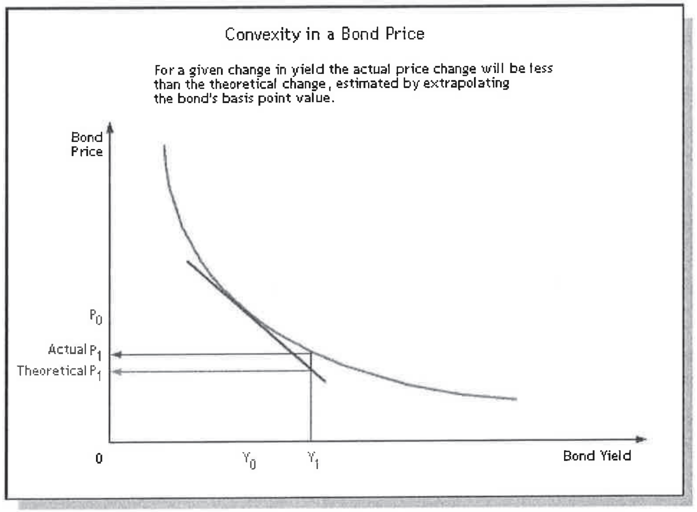
The higher the yield on a straight bond the lower is its price risk. This is true whichever measure of bond price sensitivity you look at. As yield rises:

- The distant cash flows on the bond are discounted proportionately more heavily, so Macaulay duration falls

- The denominator in the modified duration formula is higher, so modified duration also falls

- The price of the bond falls, so BPV falls as well.

This phenomenon is depicted in the figure below which plots the relationship between bond price and yield. The shape of the curve indicates why the bond is said to have **positive convexity**.



Measurement

Convexity (%) = $\dfrac{\text{Change in modified duration}}{\text{100 basis points change in yield}}$

Convexity may also be defined as the change in BPV for a given change in yield, but it is more frequently defined as above. Defining convexity in this way allows us to use a formula that gives a better approximation to what actually happens to a bond's price for a given change in yield (ΔYTM).

Percentage change in bond's dirty price:
= - Modified Duration x ΔYTM + Convexity x 1/2 x ΔYTM^2

The proof of this approximation formula[13] lies outside the scope of this training programme, but the example shows how it is used.

Implications

For a bond holder convexity is a Good Thing:

- As yields rise the loss on a bond portfolio becomes progressively less steep - as if the bond had a built-in 'parachute'
- As yields fall the gains on the bond portfolio become progressively larger - as if the bond had a turbo-booster!

All straight bonds have positive convexity but there are some bonds, for example callable bonds, where the issuer has the option to repay the principal ahead of its scheduled maturity, which display **negative convexity**. Callable bonds tend to trade with higher yields than equivalent straights, because for a bond holder negative convexity is a Bad Thing. The convexity behaviour of callable bonds is discussed in more detail in *Callable Bonds - Convexity*.
For straight bonds, convexity adjustments tend to be small and in most bond trading contexts they are usually ignored. However, for large institutional investors holding portfolios worth tens of billions of dollars, the convexity error becomes significant.

Example
Convexity

Security:	5% US Treasury note maturing 21 January 2005
Type:	Semi-annual, actual/actual
Settlement date:	3 June 2002
Price:	95.48
Accrued:	1.84
Yield:	8.00%
Modified duration:	1.50%
Convexity:	0.01%
Amount held:	USD 1 million.

? What is the potential loss on this investment if yields rose to 8.50%?

[13] US Treasuries are quoted to the nearest 1/32% of par value, so 124-11 means 124 + 11/32%. Half a tick (i.e. 1/64%) is represented with a '+', so 124-11+ means 124 + 11.5/32%, or 124 + 23/64%.

We calculated the modified duration of this bond in section *Modified Duration*! For a 50 basis point rise in yield, the percentage loss on the bond's dirty price will approximate:

$$= (-1.50 \times 0.5) + \left[\frac{0.01 \times (0.5)^2}{2}\right]$$

$= -0.75 \quad + 0.00125$
= - 0.74875%

We now use this convexity-adjusted figure in the same way as we used modified duration before:

Risk in cash terms = $\dfrac{0.74875}{100} \times \dfrac{(95.48 + 1.84)}{100} \times 1{,}000{,}000$

= **USD 7,287**

4.9. Exercise

Question 1

Try to estimate the Macaulay duration of each of the securities below just by visualising it on the Macaulay 'see-saw' (see section *Macaulay Duration*).
Enter your answers in the boxes below, then validate.

a) Non-interest bearing demand deposit:

b) Six months Eurodeposit (in months):

c) Ten-year Floating Rate Note where the coupon rate is reset semi-annually, in line with the 6 month LIBOR (in months):

d) Ten-year zero coupon bond (in years):

e) A 10-year 10% annual coupon bond (approximate duration in years?):

Question 2

You currently hold the following:

Security:	10% bond maturing in 10 years
Type:	Annual, E30/360
Price:	103.00
Portfolio held:	USD 10 million.

a) If the yield on this bond were to rise by 100 basis points, what percentage of your capital would you lose? In other words, calculate the bond's modified duration.
Using a financial calculator or the bond pricing model supplied:

1. Calculate the yield on the bond at its current price
2. Add 1% to the calculated yield and re-price the bond[14]
3. Take the difference between the two prices
4. Divide this difference by the bond's original dirty price

Enter your answer in percent to 3 decimal places.

b) Using the analytical formula developed in section *Modified Duration*, calculate the bond's modified duration, given its price of 103.00 and Macaulay duration of 6.809 years.

$$\textbf{Modified duration} = \frac{\textbf{Macaulay duration (in years)}}{\textbf{(1 + Yield/Nr Coupons per year)}}$$

Enter your answer in percent to 3 decimal places.

c)
- Because of rounding errors
- Because the bond has convexity
- Because the bond has credit risk, as well as market risk
- Because of specific supply and demand factors

Question 3

Consider the following securities:

- 12 month T-bill
- 14 year 8% US Treasury bond yielding 12%
- 6 month CD
- 16 year 15% US Treasury bond yielding 12%

a) Rank these securities in order of price risk (basis point value) by entering the appropriate number (1 = lowest risk, 4 = highest risk) in each box below.

12 month T-bill
14 year 8% US Treasury bond yielding 12%
6 month CD
16 year 15% US Treasury bond yielding 12%

Question 4

Settlement date:	16 April 2002
Security:	12¼% US T-Bond maturing on 4 January 2008
Type:	Semi-annual, Actl/Actl
Price:	124-11+[15]
Accrued:	3.4517%
Yield:	7.00%
Duration:	4.32 years

a) Using the bond's Macaulay duration, calculate its modified duration to 4 decimal places.

b) Using the result in (a), calculate the bond's BPV, per USD 100 nominal, to 5 decimal places.

c) Estimate the expected change in the bond's price if its yield were to rise by 10 basis points, from 7.00% to 7.10% (your answer to 2 decimal places).

4.11. Portfolio Construction

Question 1

In this exercise you will design a fixed income portfolio that meets certain stated duration, yield and credit objectives using the portfolio model which should have been provided for you to go with this exercise.

Below is the policy statement of well-known investment fund.

UK Corporate Bond Fund

The Fund invests in sterling-denominated fixed and variable rate securities, including corporate bonds and debentures with the aim of achieving a higher return from investment than would be obtainable from a pre-defined benchmark of UK Government fixed interest securities (gilts) for the same duration.

The Fund may hold gilts where it is deemed appropriate. If the Managers deem it appropriate, up to 35% of the Fund's assets may consist of Government and other public sector securities issued by any of the entities specified in Schedule A.

The quality of corporate securities in which the fund invests is of investment grade, with a credit rating of not less than Baa/BBB. In addition, to spread the risk of default, not more than 7% of the fund may be invested in any single security.

Schedule A
Includes Governments of : Australia, Austria, Belgium, Canada, France, Germany, Japan, Switzerland, UK European Bank for Reconstruction and Development, European Investment Bank, IBRD, International Financial Corporation.

Fund Statistics - 28 June 2002
Value of Fund: £898.5 million
Weighted yield: 5.80%
Duration: 6.98 years

Three largest investments:
Treasury 8% 2003
Hyder 9.5% bonds 2016
Lloyds TSB 8.5% bonds 2049

a) In the Bond Universe worksheet you will see the composition of the fund's benchmark portfolio. Copy this data into the appropriate cells of the Analysis worksheet. Both worksheets should have been provided for you to go with this exercise.

What is the weighted yield of the fund's benchmark portfolio?

b) In the **Bond Universe** worksheet you will see the composition of the fund's benchmark portfolio. Copy this data into the appropriate cells of the **Analysis** worksheet.

What is the duration of the fund's benchmark portfolio?

c) Since the fund's risk-weighted yield was 5.80% as at 28 June 2002, what is your assessment of the average credit risk in its portfolio?

- A/A: yield spread = 1.00 – 1.40%
- Baa/BBB: yield spread = 1.80 – 2.30%
- Aaa/AAA: yield spread = 0.00 – 0.04%
- Aa/AA: yield spread = 0.10 – 0.25%

Question 2

Copy the details of the bonds you select from the Bond Universe worksheet into the appropriate cells of the **Analysis** worksheet. You will then have to allocate amounts to each bond so as to:

- Match the duration of the benchmark
- Match the amount of capital available for investment

Using the list of available bonds in the **Bond Universe** worksheet, construct a portfolio of not more than 15 securities that would yield a higher return than the fund's 5.80%, while at the same time matching the duration of the benchmark to within 3 months. Remember that you may not allocate more than 7% of the fund's capital to any single security.

a) Is it possible to achieve a yield of 6.50% or higher?

- Yes
- No

b) Suppose you had to make a cash payment of £50 million out of your portfolio on 15 June 2009. If your fund matches the duration of the benchmark, what would be the current market value of the assets that you would need to set aside today in order to fulfill this commitment?

- £36.57 million
- The PV of £50 million, discounted at the fund's risk-weighted yield
- £50 million less accrued interest
- £50 million.