200010021_Lab4

September 3, 2022

1 Lab 4 Clustering -> Part 1

In this lab I learned about two clustering algorithms they are :

1. K - means Clustering and
2. Gaussian Mixture Model

using mathematical theory and numpy, and data was visualised using matplotlib.

Then I learned how to use the above two algorithms in real life using scikit-learn and pandas.

New skills learned:

1. Pandas
2. Scikit-learn
3. Some scipy

2 K - means Clustering

K-Means Clustering is an Unsupervised Learning algorithm, which groups the unlabeled dataset into different clusters. Here K defines the number of pre-defined clusters that need to be created in the process, as if K=2, there will be two clusters, and for K=3, there will be three clusters, and so on.

Algorithm:

1. Select K random centroids.
2. Assign each data point to the closest centroid.
3. Calculate new centroids and assign points according to new centroids.
4. Repeat 2 & 3 until convergence.
5. The model is ready

Properties of Algorithm:

1. Hard partitioning occurs.
2. We set the condition for convergence (EG: $ $ < $ precision $)
3. Repeat training with various K to find optimal solutions.
4. Exploits mean of the data.

3 Gaussian Mixture Model

If we assume that there are K-clusters and each cluster follows guassian model with mean μ_k and variance Σ_k then we can group unlabled data into clusters.

Algorithm: 1. Initialise K centroids using K-means and find inital $\overline{\mu}$ *and* $\overline{\Sigma} = 1$. 2. Give each feature vector a probability $p(\frac{k}{X=x_i})$ for each cluster k. 3. Re-estimate model $\lambda(x, w, \mu, \Sigma)$ using EM algorithm until convergence. 4. The model is ready.

Properties of Algorithm:

1. Soft partitioning is achieved.
2. Each feature vector gets assign to each cluster with a probability.
3. Repeat with various K to find optimal solution.
4. Exploits both mean and variance of data.

4 Expectation Minimisation Algorithm

Definitions:

1. λ is the tuple (W, μ, Σ)
2. $g(X, \lambda)$ is the guassian ditribution for X $= x_i$ and λ_k
3. $p(\frac{k}{X=x_i}) = \frac{w_k * g(X=x_i, \lambda=k)}{\Sigma_k w_k * g(X=x_i, \lambda=k)}$
4. $p(X \mid \lambda) = \Sigma_k w_k * g(X = x_i, \lambda = k)$

re-estimations:

5. $\overline{w_k} = [\ \frac{\Sigma_M p(\frac{k}{X=x_i})}{M}\ ]$
6. $\overline{\mu_k} = [\ \frac{\Sigma_M p(\frac{k}{X=x_i}) * x_i}{\Sigma_M p(\frac{k}{X=x_i})}\ ]$
7. $\overline{\Sigma} = [\ \frac{\Sigma_M p(\frac{k}{X=x_i}) * (x_i - \mu) * (x_i - \mu)^T}{\Sigma_M p(\frac{k}{X=x_i})}\ ]$
8. Convergence condition -> $|p(x \mid \lambda) - p(x \mid \overline{\lambda})| < \delta$

Algorithm:

1. Get initial model λ from k-means or assume one.
2. Using given FV and λ re-estimate model parameters leading to $\overline{\lambda}$.
3. Repeat until convergence condition is met.
4. At the end we have our final model λ.

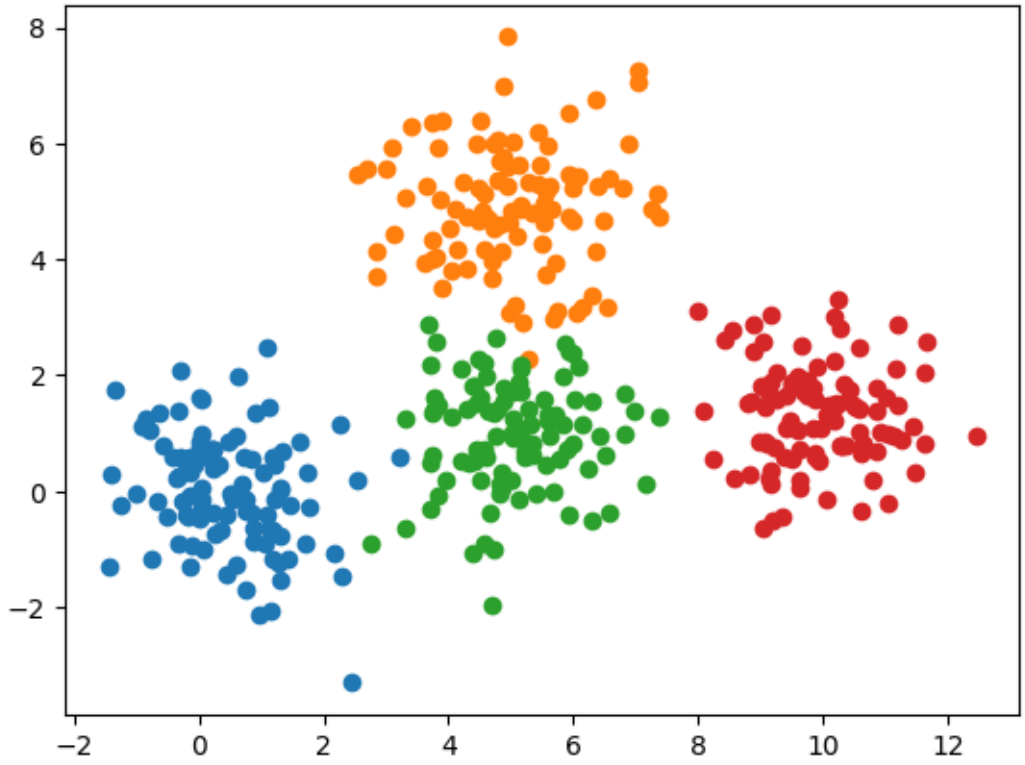
5 Q1) K-means

1. DATA GENERATION:

[10]:
```
import numpy as np
import matplotlib.pyplot as plt
```

```
mean_1 = np.array([0.5,0])
mean_2 = np.array([5,5])
mean_3 = np.array([5,1])
mean_4 = np.array([10,1.5])

Sigma  = np.array([[1,0],[0,1]])
distribution_1     = np.random.multivariate_normal(mean_1,Sigma,100)
distribution_2     = np.random.multivariate_normal(mean_2,Sigma,100)
distribution_3     = np.random.multivariate_normal(mean_3,Sigma,100)
distribution_4     = np.random.multivariate_normal(mean_4,Sigma,100)
Distribution       = np.
 ↪concatenate((distribution_1,distribution_2,distribution_3,distribution_4))

plt.figure()

plt.scatter(distribution_1[:,0], distribution_1[:,1])
plt.scatter(distribution_2[:,0], distribution_2[:,1])
plt.scatter(distribution_3[:,0], distribution_3[:,1])
plt.scatter(distribution_4[:,0], distribution_4[:,1])
```

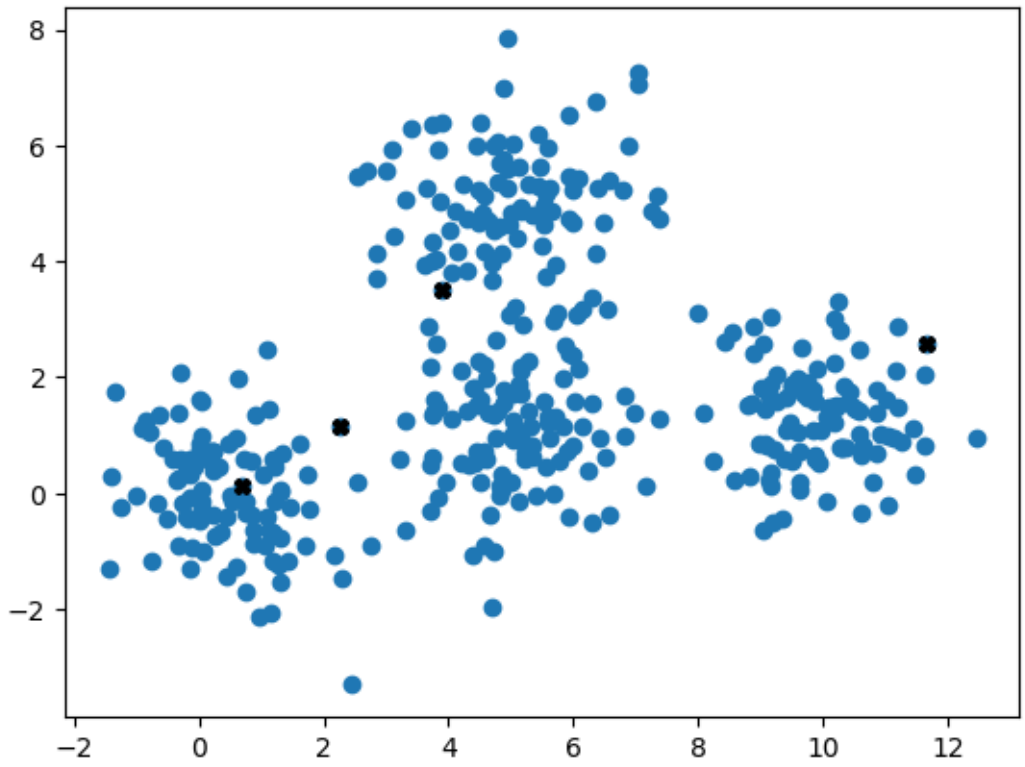
[10]: <matplotlib.collections.PathCollection at 0x7f04a4230d60>



Cluster Initialisation

[54]:
```
K = 4
Iterations = 10

rands = np.random.randint(0, Distribution.shape[0], K)
Centroids = Distribution[rands,:]


FV_tuple = np.zeros((Distribution.shape[0], K+2))

plt.scatter(Distribution[:,0],Distribution[:,1])
plt.plot(Centroids[:,0], Centroids[:,1], 'X', color='k')
```

[54]:
```
[<matplotlib.lines.Line2D at 0x7f049c62dd60>]
```

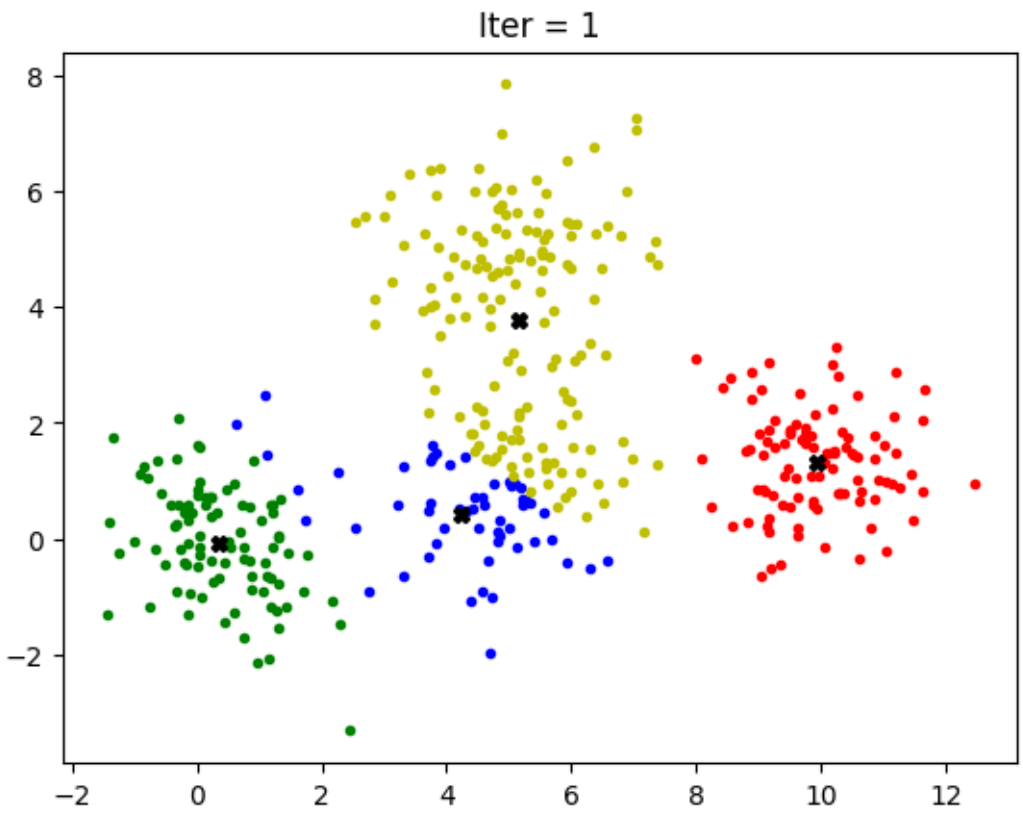


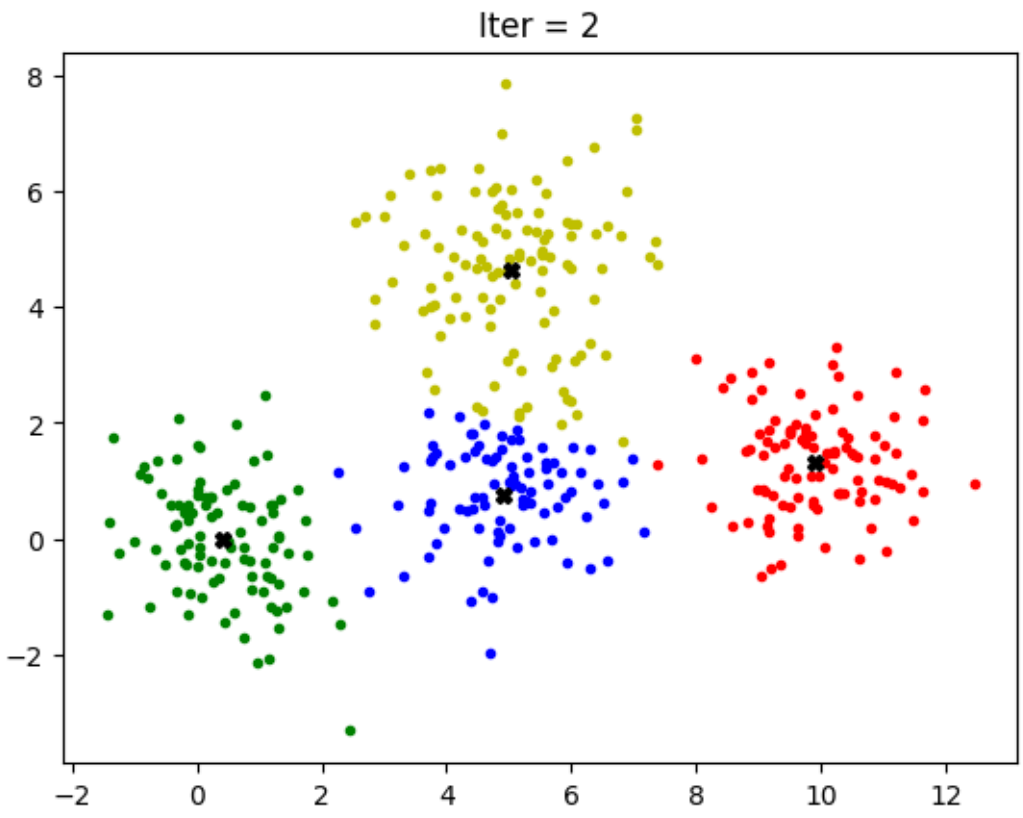
Cluster assignment and re-estimation

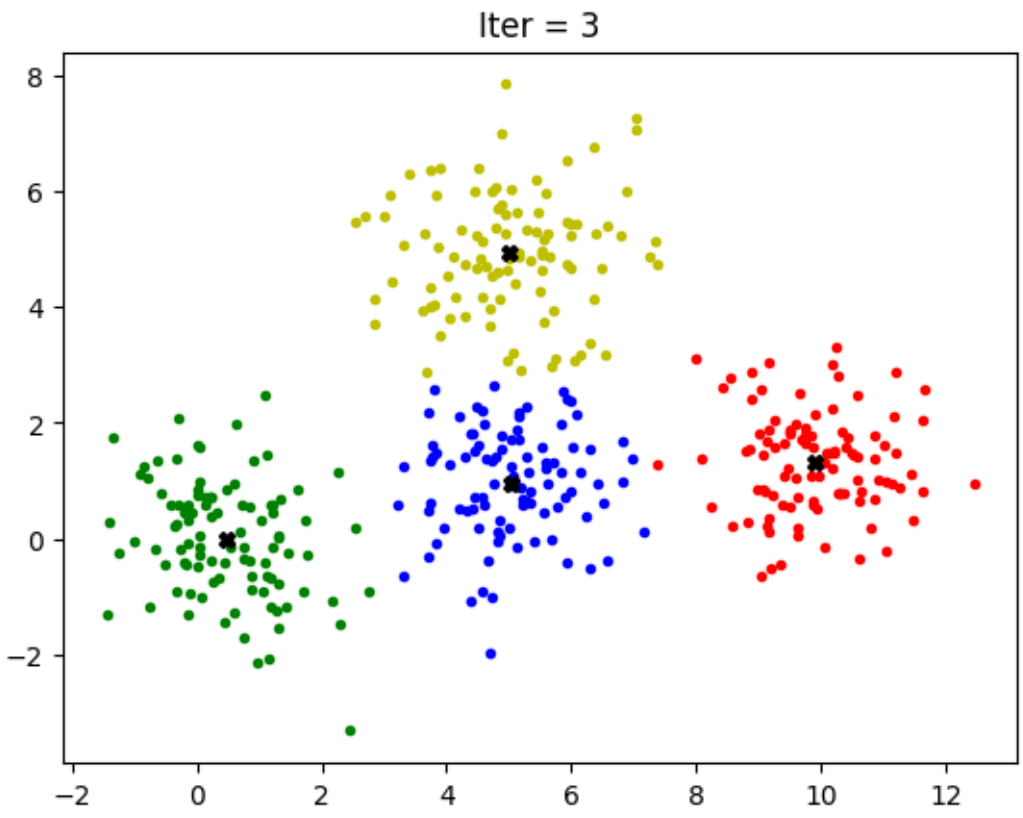
[55]:
```
error = []
precision = 1e-10
colour=['r','g','b','y','k','m','c']

for o in range(Iterations):
```

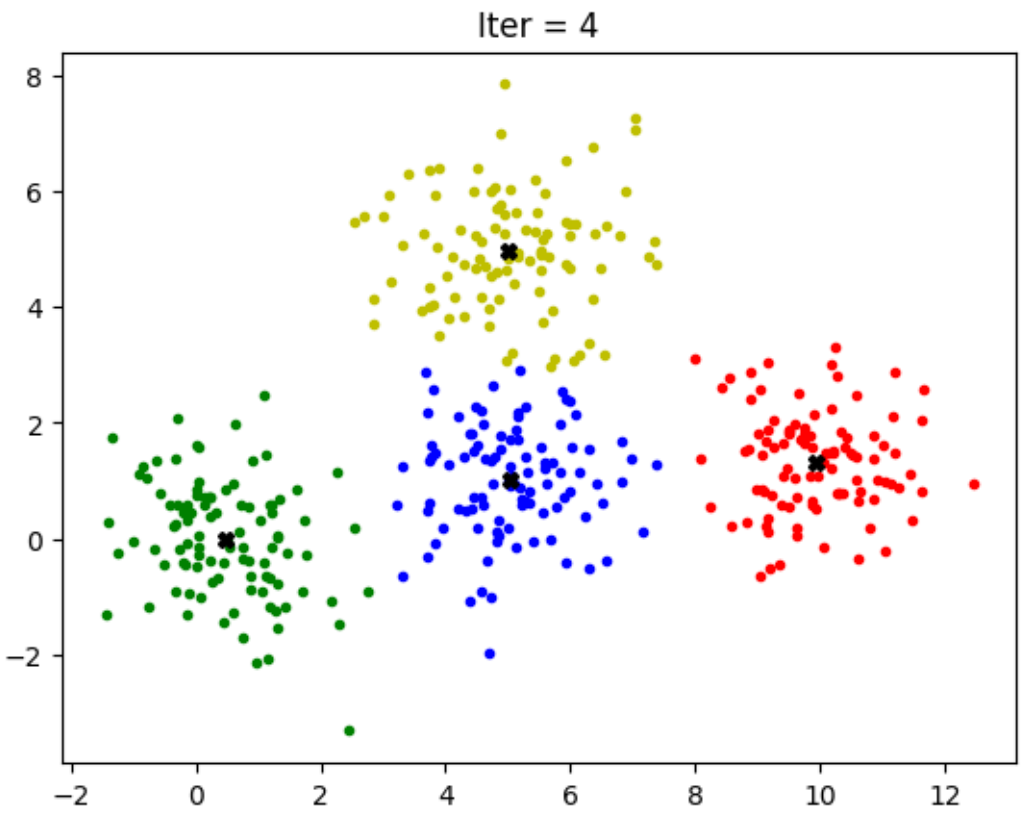
```
    for i in range(Distribution.shape[0]):
        for j in range(K):
            FV_tuple[i,j] = np.linalg.norm(Distribution[i,:] - Centroids[j,:])

        distance = np.min(FV_tuple[i,:K])
        index    = int((np.where(distance==FV_tuple[i,:K])[0]))
        FV_tuple[i,K] = index
        FV_tuple[i,K+1] = distance

    for i in range(K):
        index = np.where(FV_tuple[:,K]==i)
        Centroids[i,:] = np.mean(Distribution[index,:][0],axis=0)

    error.append(np.mean(FV_tuple[:,K+1]))

    plt.figure()

    for _ in range(K):
        ind = np.where(FV_tuple[:,K]==_)
        plt.plot(Distribution[ind,0],Distribution[ind,1],'.', color=colour[_] )


    plt.plot(Centroids[:,0], Centroids[:,1], 'X', color='k')
    plt.title(f"Iter = {o + 1}")

plt.figure()
plt.plot(error)
```

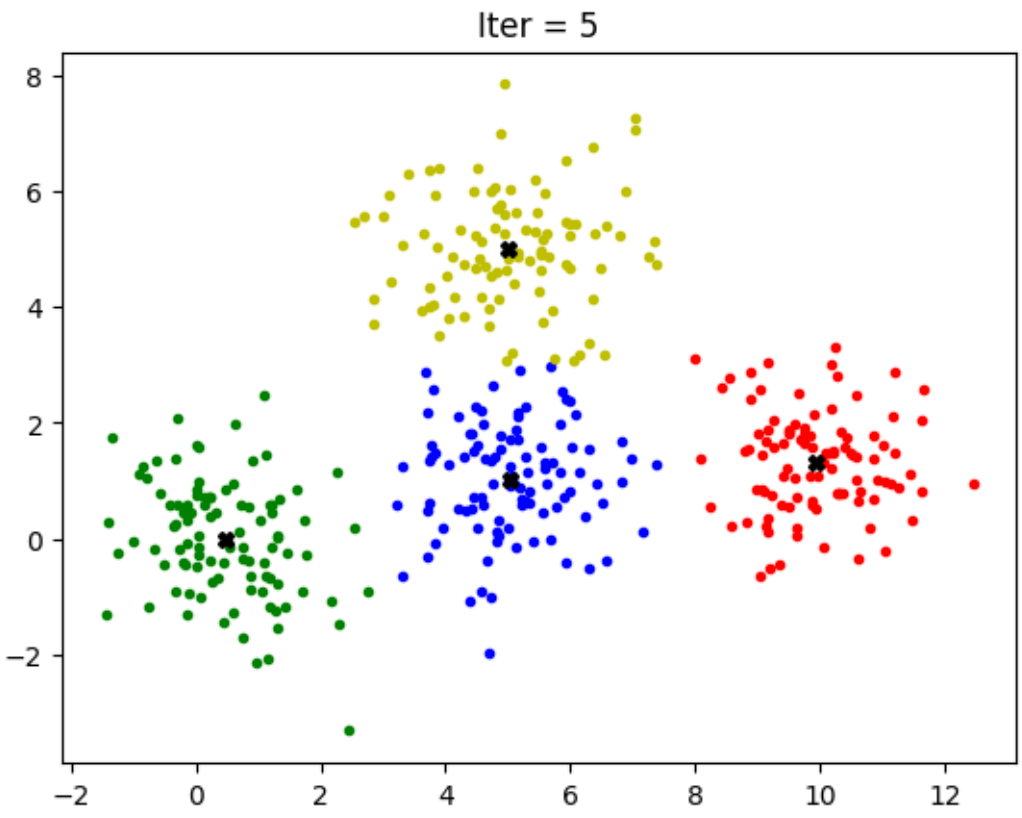
[55]: [<matplotlib.lines.Line2D at 0x7f049b6fa7f0>]

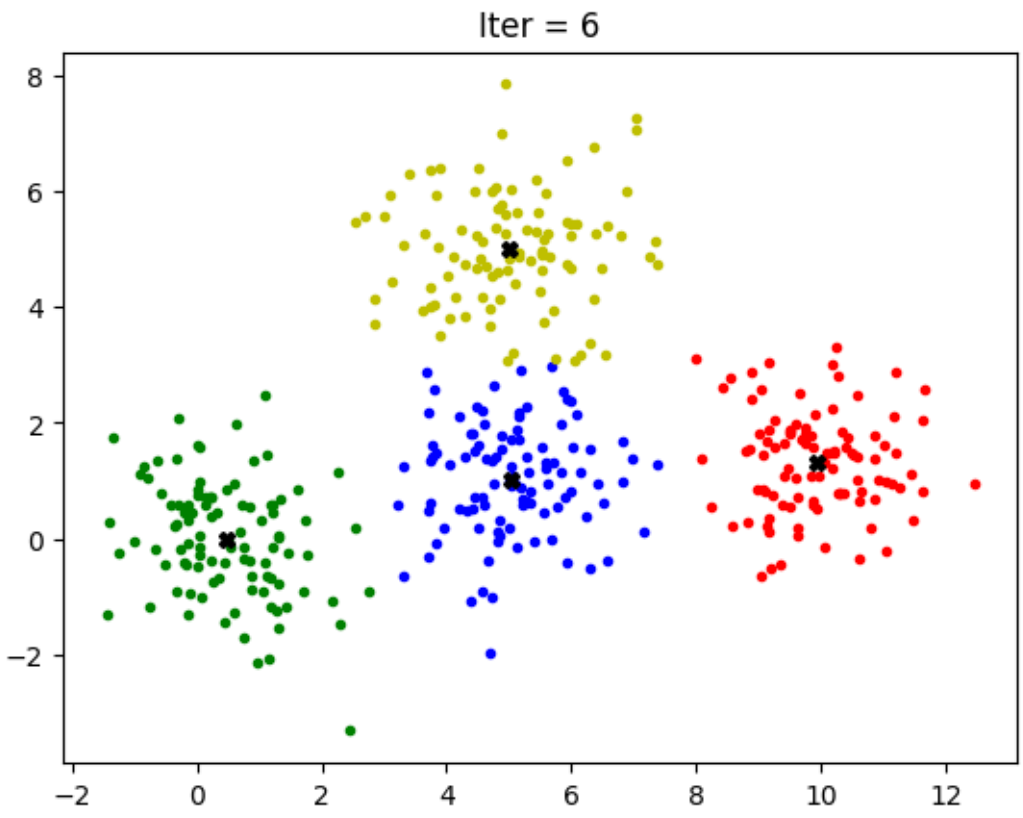

Iter = 1
8
6
4
2
0
−2
−2
0
2
4
6
8
10
12

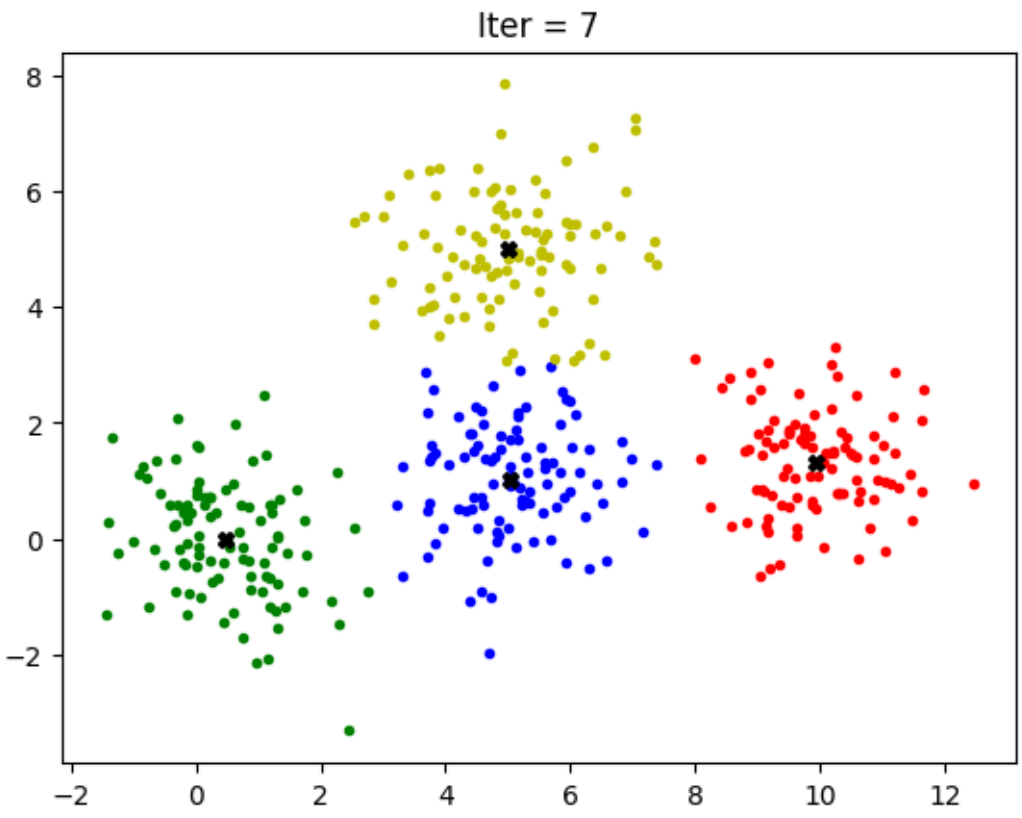

Iter = 2
8
6
4
2
0
−2
−2
0
2
4
6
8
10
12

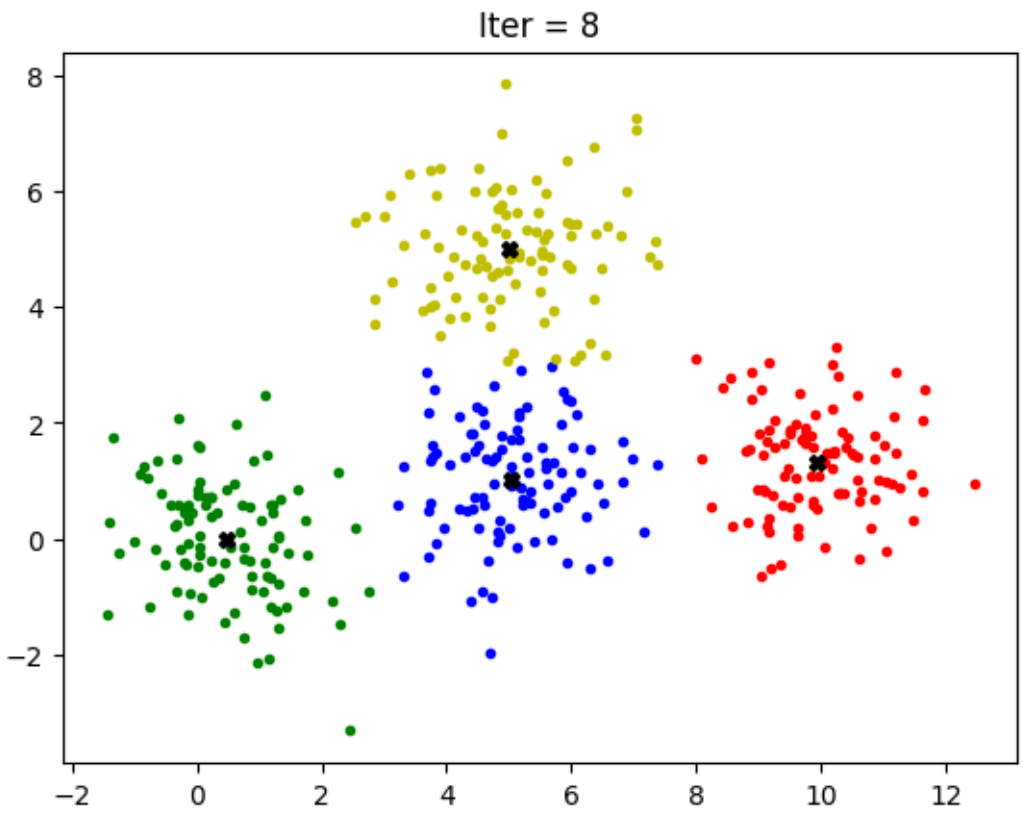

Iter = 3
8
6
4
2
0
−2
−2
0
2
4
6
8
10
12

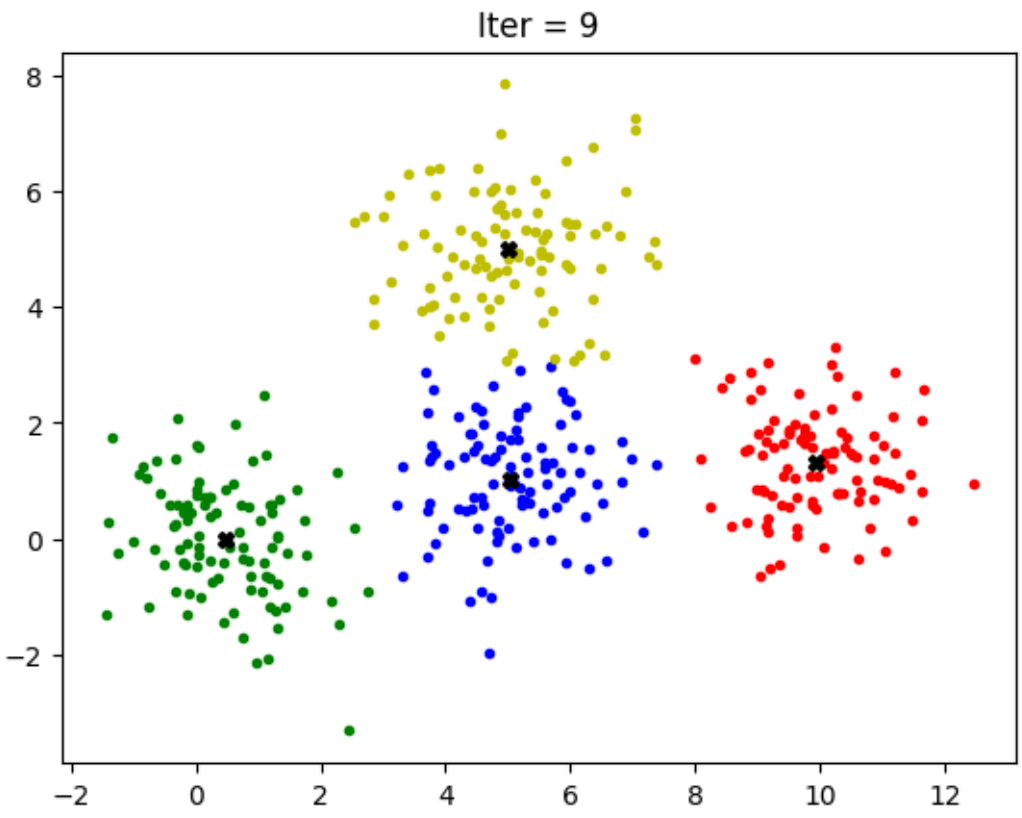

Iter = 4
8
6
4
2
0
−2
−2
0
2
4
6
8
10
12

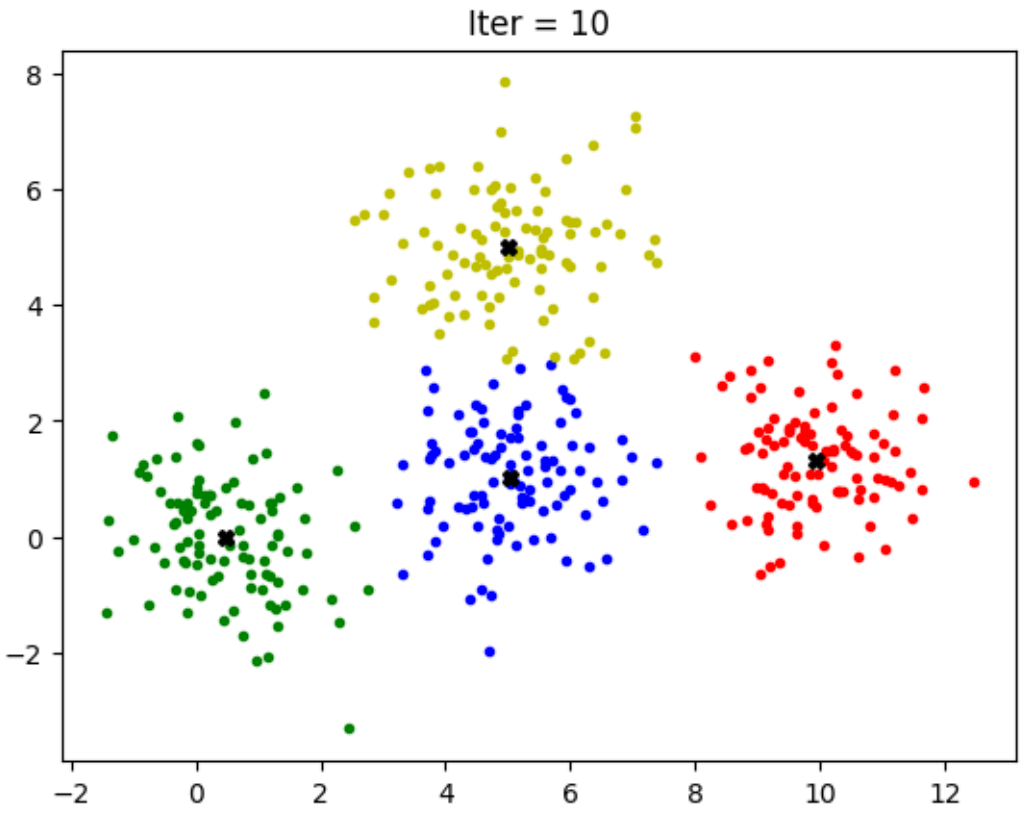

Iter = 5
8
6
4
2
0
−2
−2
0
2
4
6
8
10
12

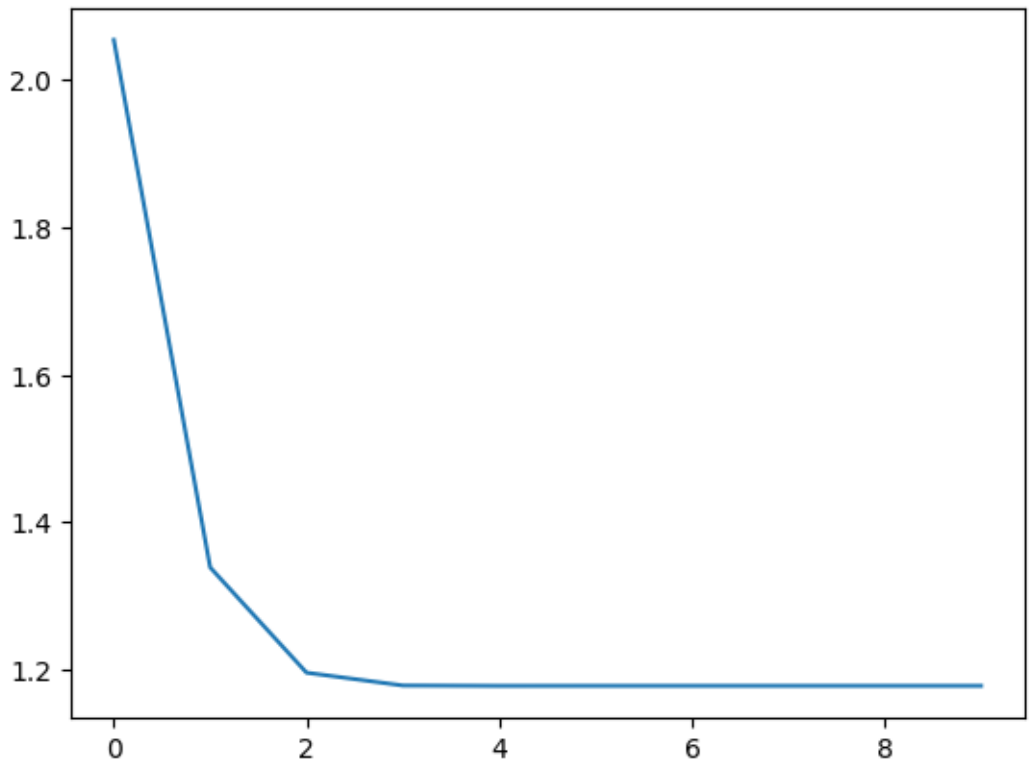

Iter = 6
8
6
4
2
0
−2
−2
0
2
4
6
8
10
12


Iter = 7
8
6
4
2
0
−2
−2
0
2
4
6
8
10
12


Iter = 8
8
6
4
2
0
−2
−2
0
2
4
6
8
10
12


Iter = 9
8
6
4
2
0
−2
−2
0
2
4
6
8
10
12


Iter = 10
8
6
4
2
0
−2
−2
0
2
4
6
8
10
12



[97]:
```
import sklearn
from sklearn import metrics

pred=FV_tuple[:,K]

cluster_1 = np.tile([1], distribution_1.shape[0])
cluster_2 = np.tile([2], distribution_2.shape[0])
cluster_3 = np.tile([3], distribution_3.shape[0])
cluster_4 = np.tile([4], distribution_4.shape[0])
cluster   = np.concatenate((cluster_1,cluster_2, cluster_3, cluster_4))


print('Performance=',metrics.homogeneity_score(pred,cluster))
```

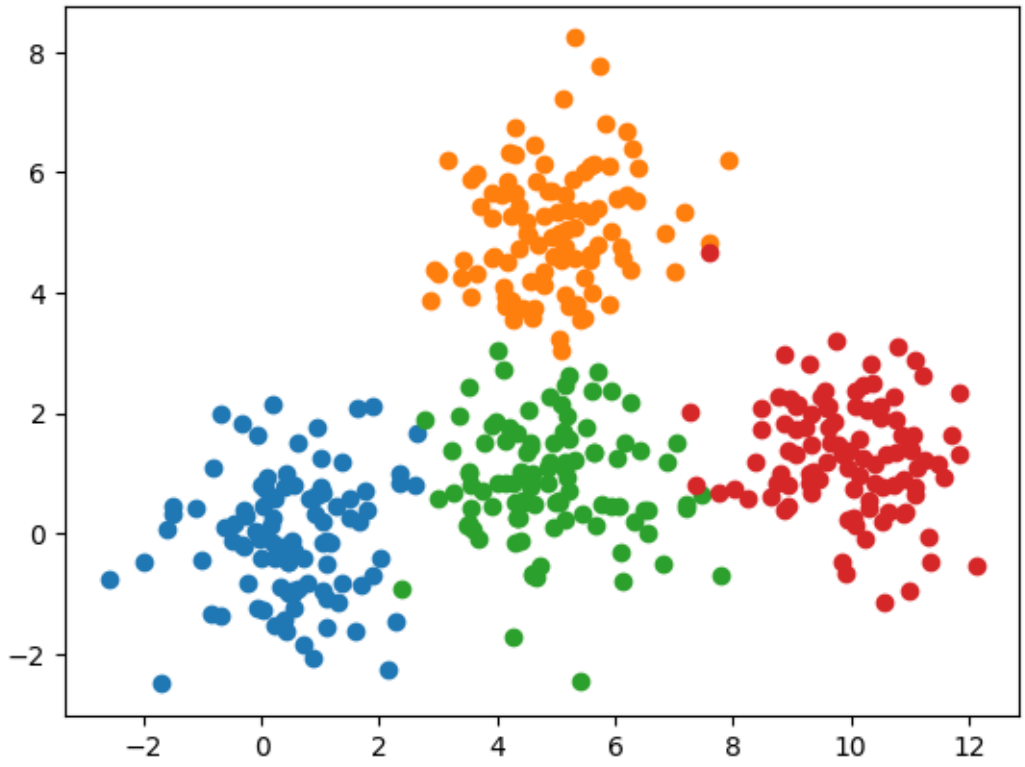
```
Performance= 0.9554955094923293
```

6 Gaussian Mixture Models Clustering

[111]:
```
import numpy as np
import matplotlib.pyplot as plt

mean_1 = np.array([0.5,0])
```

```
mean_2 = np.array([5,5])
mean_3 = np.array([5,1])
mean_4 = np.array([10,1.5])

Sigma  = np.array([[1,0],[0,1]])
distribution_1      = np.random.multivariate_normal(mean_1,Sigma,100)
distribution_2      = np.random.multivariate_normal(mean_2,Sigma,100)
distribution_3      = np.random.multivariate_normal(mean_3,Sigma,100)
distribution_4      = np.random.multivariate_normal(mean_4,Sigma,100)
Distribution        = np.
 ↪concatenate((distribution_1,distribution_2,distribution_3,distribution_4))

Distribution        = ((Distribution - (np.mean(Distribution,axis=0)))/(np.
 ↪std(Distribution, axis=0)))

plt.figure()

plt.scatter(distribution_1[:,0], distribution_1[:,1])
plt.scatter(distribution_2[:,0], distribution_2[:,1])
plt.scatter(distribution_3[:,0], distribution_3[:,1])
plt.scatter(distribution_4[:,0], distribution_4[:,1])
```

[111]: <matplotlib.collections.PathCollection at 0x7f049cc30d00>



[112]:
```
dimension = Distribution.shape[1]
mean = np.random.randint(0, Distribution.shape[0], K)
mean = Distribution[mean, :]
mean = mean.T

COV = np.zeros((dimension, dimension, K))
for i in range(K):
    COV[:,:,i] = np.eye(dimension)*np.max(Distribution, axis=None)

W  = np.ones((K,1)) / K
```

[113]:
```
from scipy.stats import multivariate_normal


def calculate_Expectation(mean, COV, W):
    ret_val = np.zeros((len(Distribution), K))

    for i in range(K):
        data_pt = 0

        for x in Distribution:

            denominator = 0

            N = multivariate_normal.pdf(x, mean=mean[:,i], cov=COV[:,:,i])

            ret_val[data_pt][i] = W[i]*N

            for j in range(K):
                denominator += W[j] * (multivariate_normal.pdf(x,mean=mean[:
 ↪,j], cov=COV[:,:,j])+1e-11)

            ret_val[data_pt][i] = ret_val[data_pt][i]/denominator
            data_pt += 1

    return ret_val
```

[114]:
```
def re_estimation(lamda):
    [data_pt, K] = lamda.shape

    dimension = Distribution.shape[1]

    N = np.sum(lamda,axis=0)

    W = N/data_pt
```

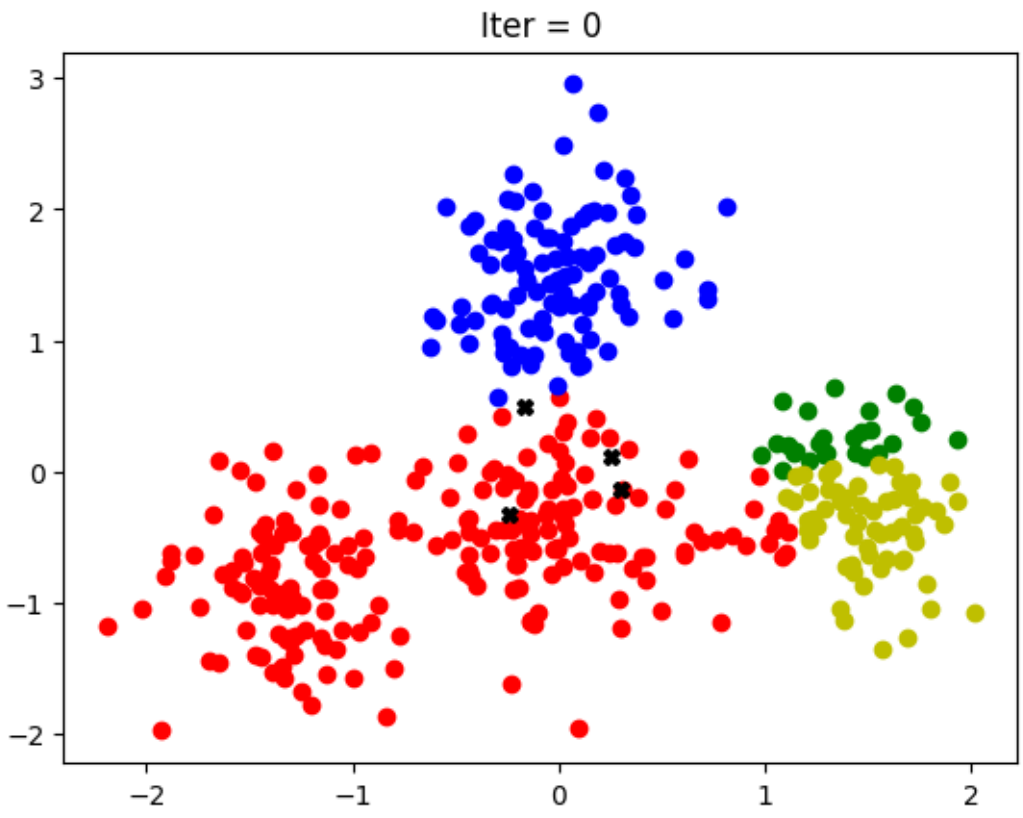
```
    mean = np.zeros((K, dimension))

    for k in range(K):

        t = lamda[:,k]
        temp = Distribution * t[:,None]
        mean[k] = (1/N[k])*np.sum(temp, axis=0)

    mean = mean.T

    COV = np.zeros((dimension, dimension, K))

    for k in range(K):
        for n in range(data_pt):
            a = Distribution[n,:] - mean[:,k]
            b = a @ a.T
            c = lamda[n,k] * b
            COV[:,:,k] += c

    ret = [mean, COV, W]

    err = 0
    log_error = 0
    """
    for pt in range(data_pt):
        for k in range(K):
            err += W[k]*(multivariate_normal.pdf(Distribution[pt,:],mean=mean[:
↪,k], cov=COV[:,:,k]))
        log_error += np.log(err)
    """
    return ret, log_error
```

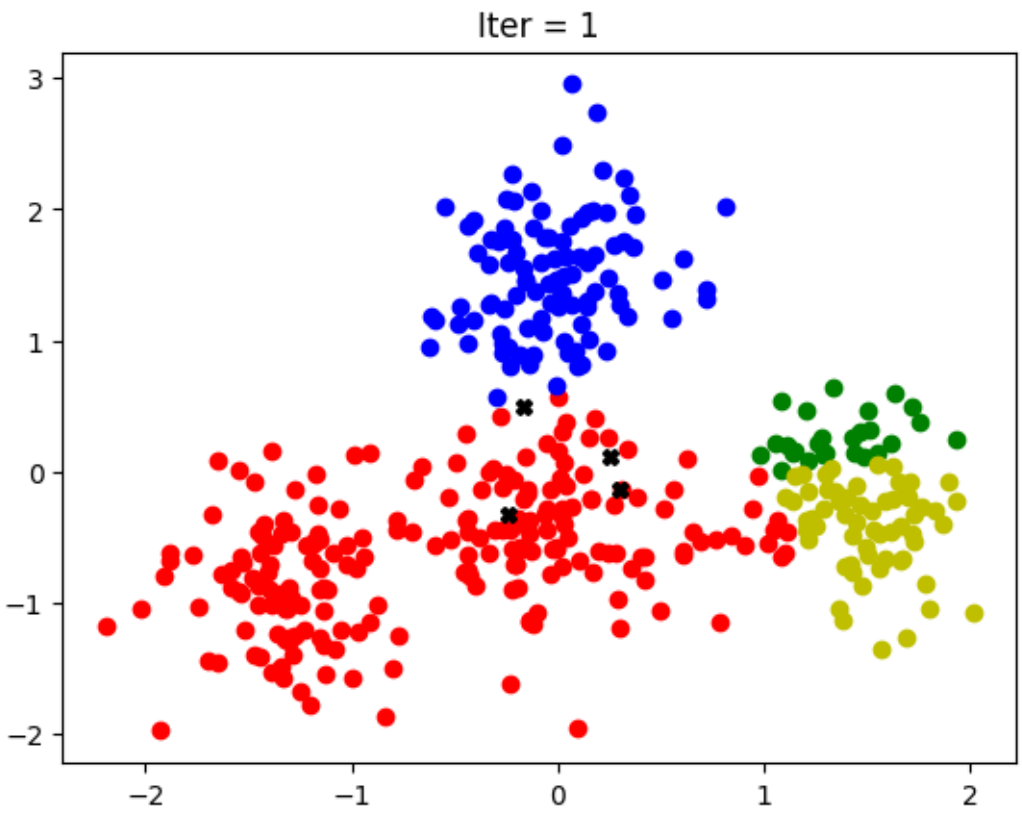
7 EM ALGORITHM

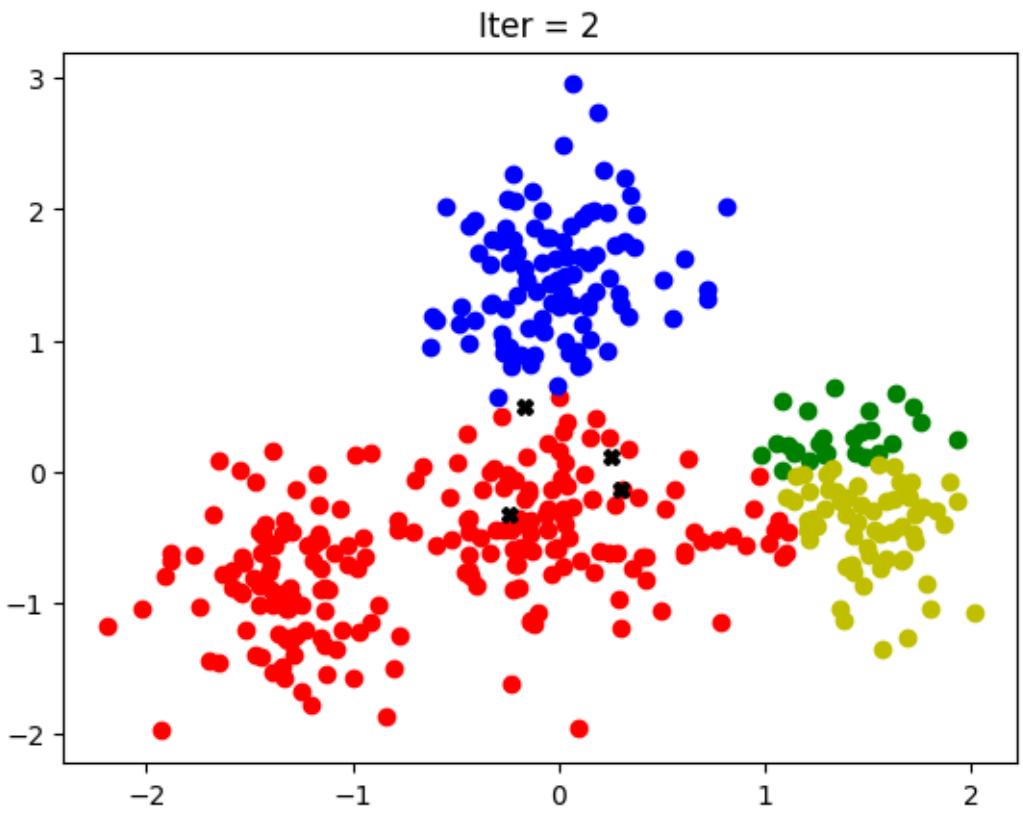
[116]:
```
log_err = []
Iterations = 30
precision = 1e-10
colour = ['r','g','b','y','k','m','c']

K = 4

lamda = [mean, COV, W]

for n in range(Iterations):
```

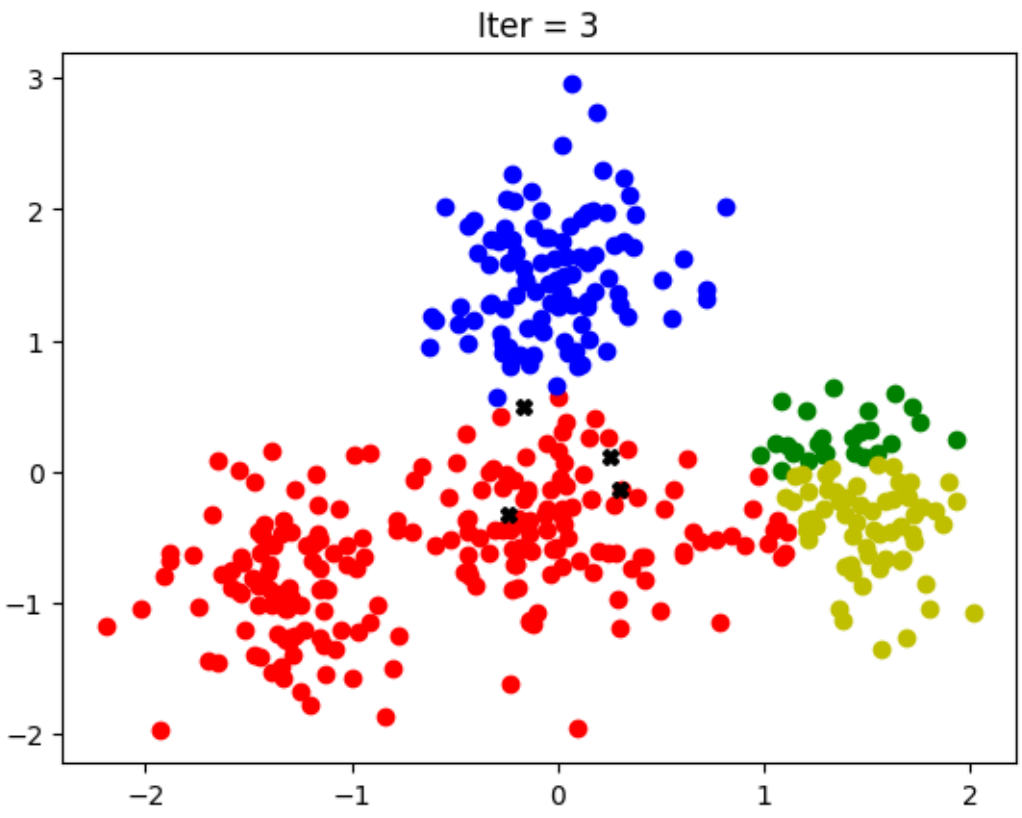
```
    expect = calculate_Expectation(mean,COV,W)

    lable = np.argmax(expect, axis=1)

    lamda, err = re_estimation(expect)

    log_err.append(err)

    plt.figure()

    for p in range(K):
        index = np.where(lable==p)
        plt.scatter(Distribution[index,0],Distribution[index,1],color=colour[p])

    Centroids = lamda[0].T

    plt.plot(Centroids[:,0], Centroids[:,1], "X", color='k')
    plt.title(f"Iter = {n}")

#plt.figure()
#plt.plot(log_err)
```

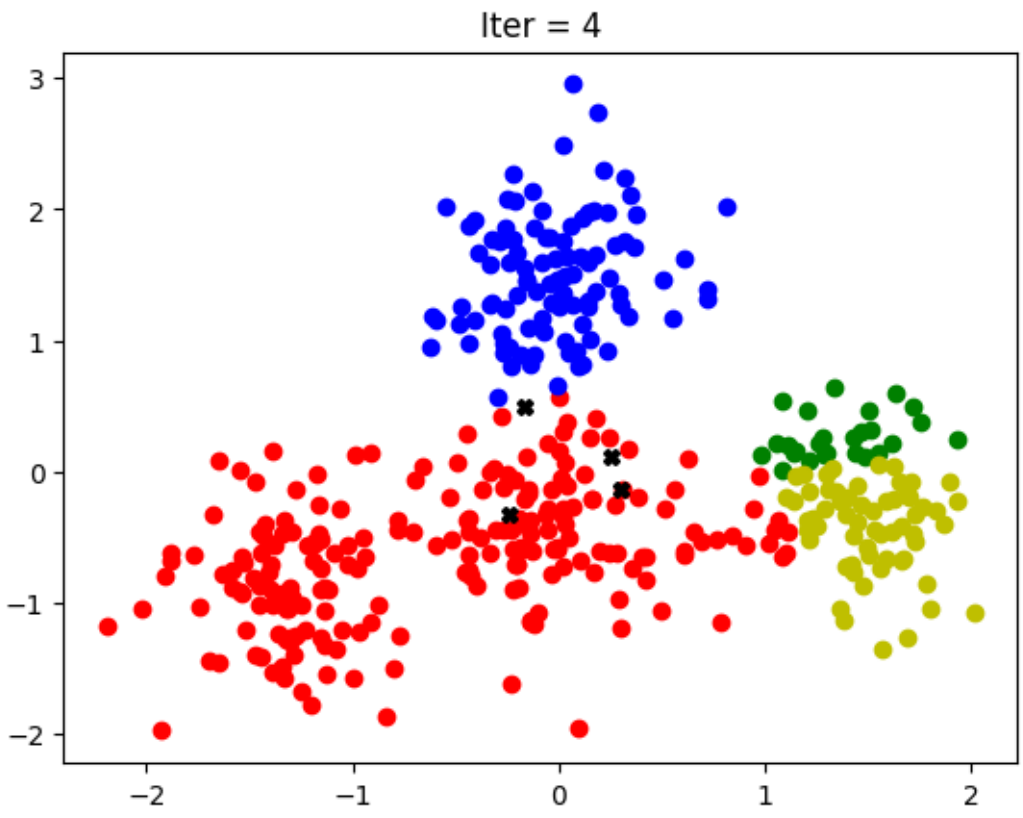
```
/tmp/ipykernel_656/2409286477.py:20: RuntimeWarning: More than 20 figures have
been opened. Figures created through the pyplot interface
(`matplotlib.pyplot.figure`) are retained until explicitly closed and may
consume too much memory. (To control this warning, see the rcParam
`figure.max_open_warning`).
  plt.figure()
```

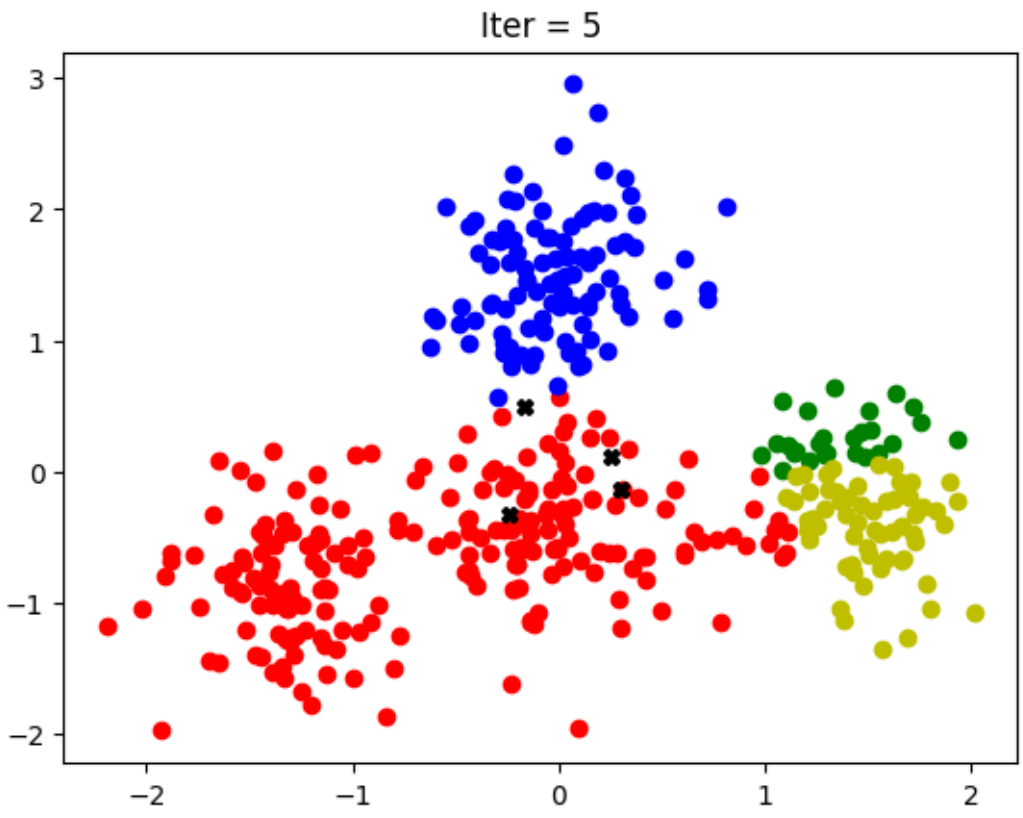

Iter = 0
3
2
1
0
−1
−2
−2
−1
0
1
2

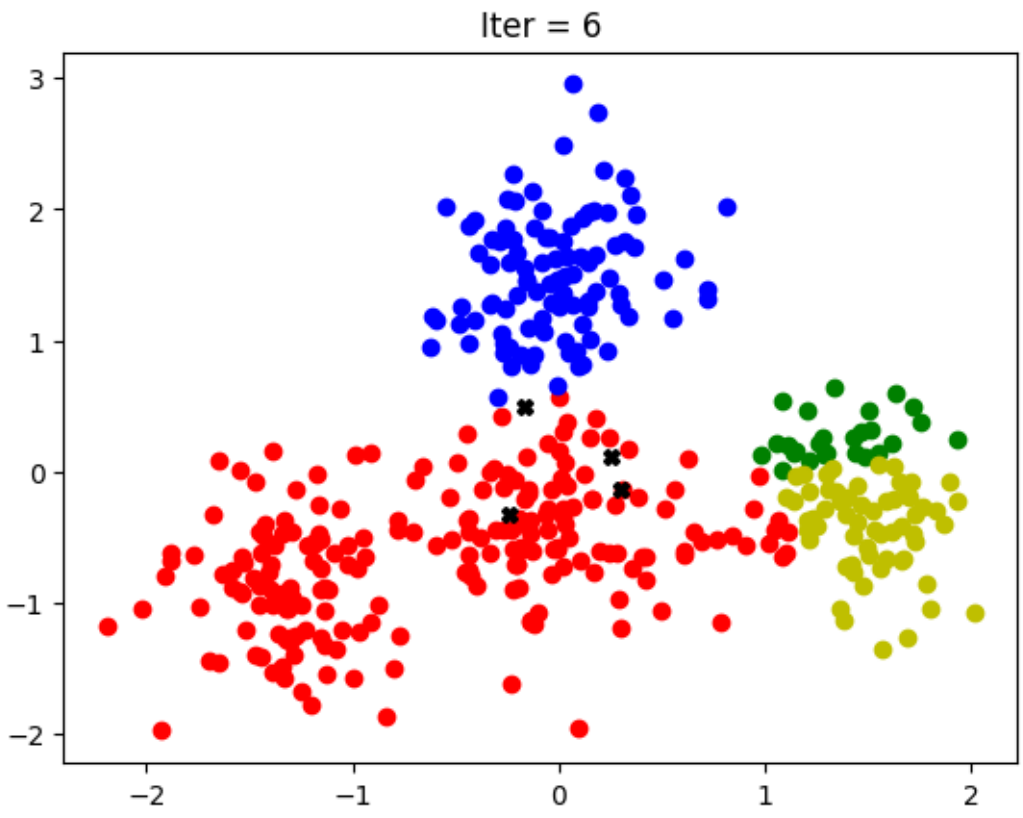

Iter = 1
3
2
1
0
−1
−2
−2
−1
0
1
2

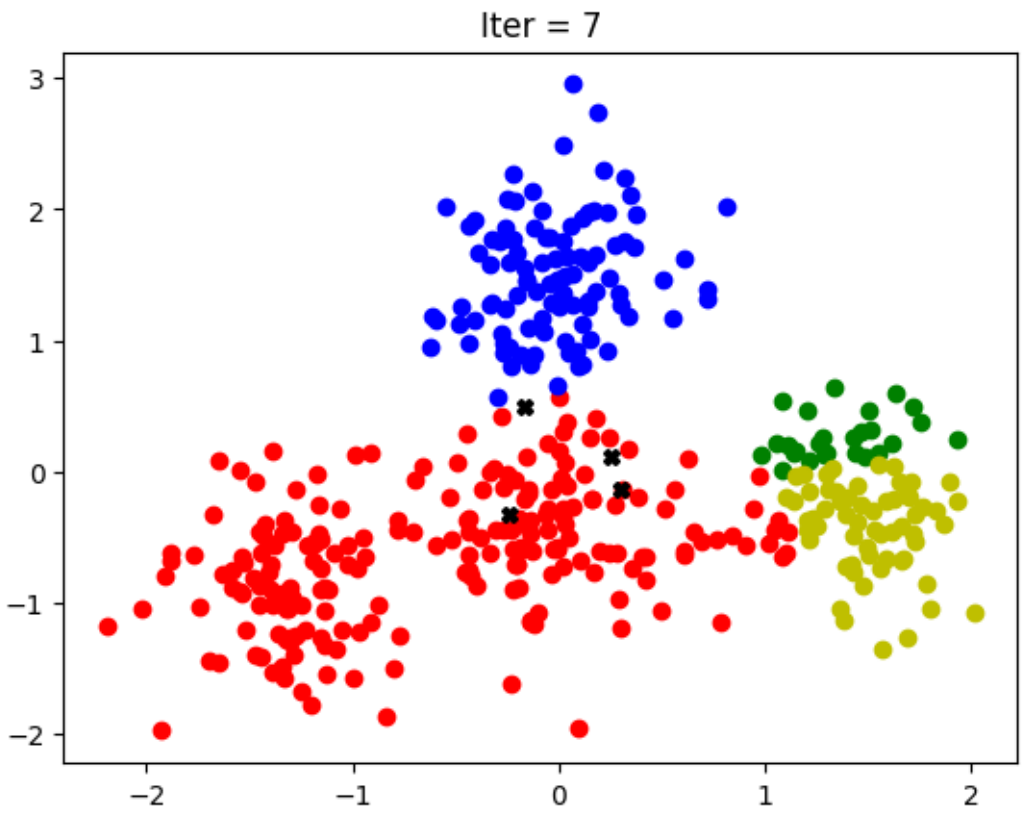

Iter = 2
3
2
1
0
−1
−2
−2
−1
0
1
2

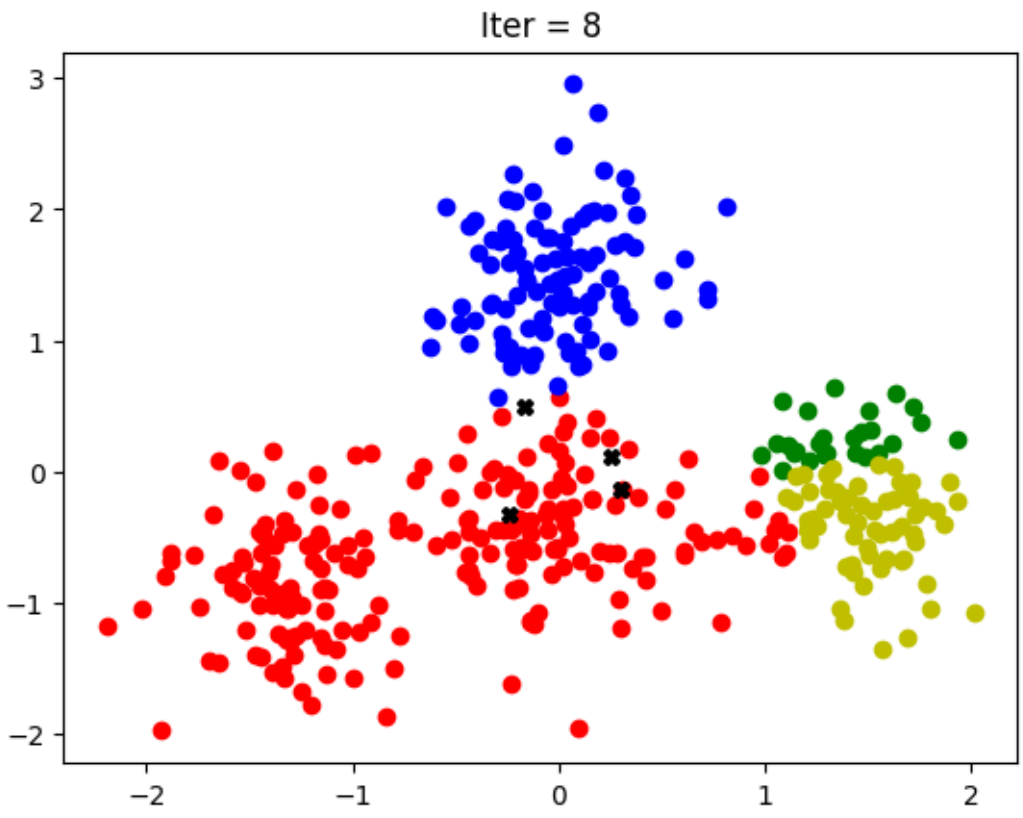

Iter = 3
3
2
1
0
−1
−2
−2
−1
0
1
2

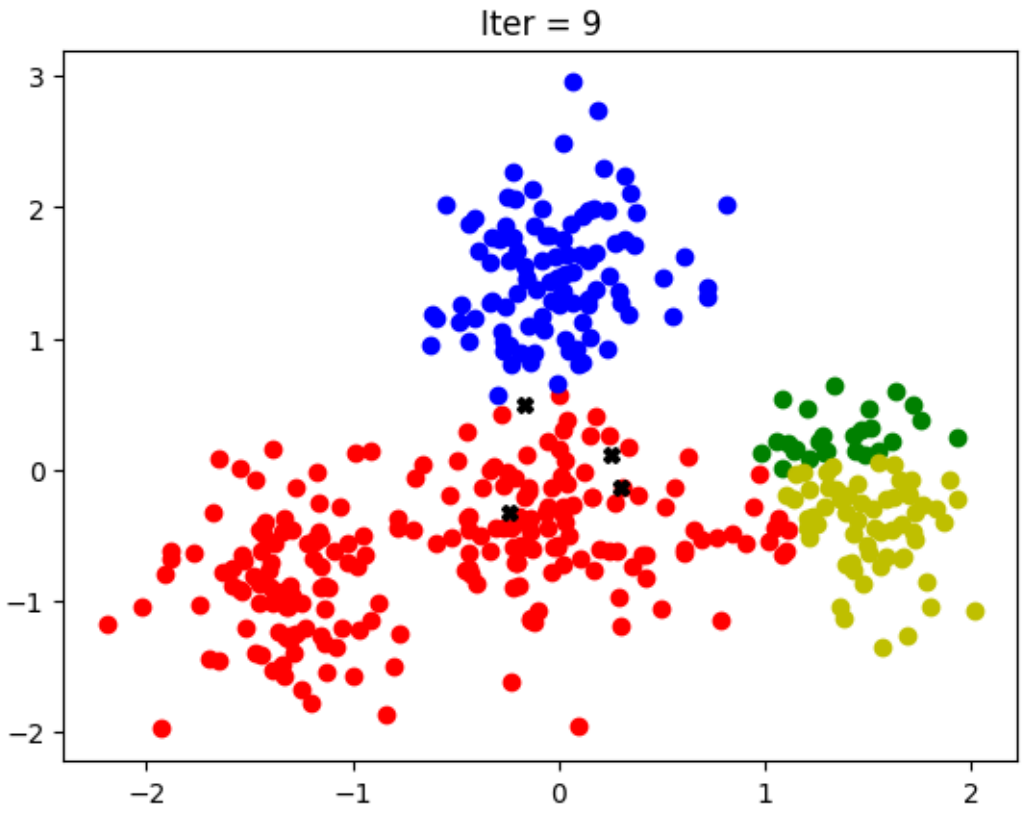

Iter = 4
3
2
1
0
−1
−2
−2
−1
0
1
2

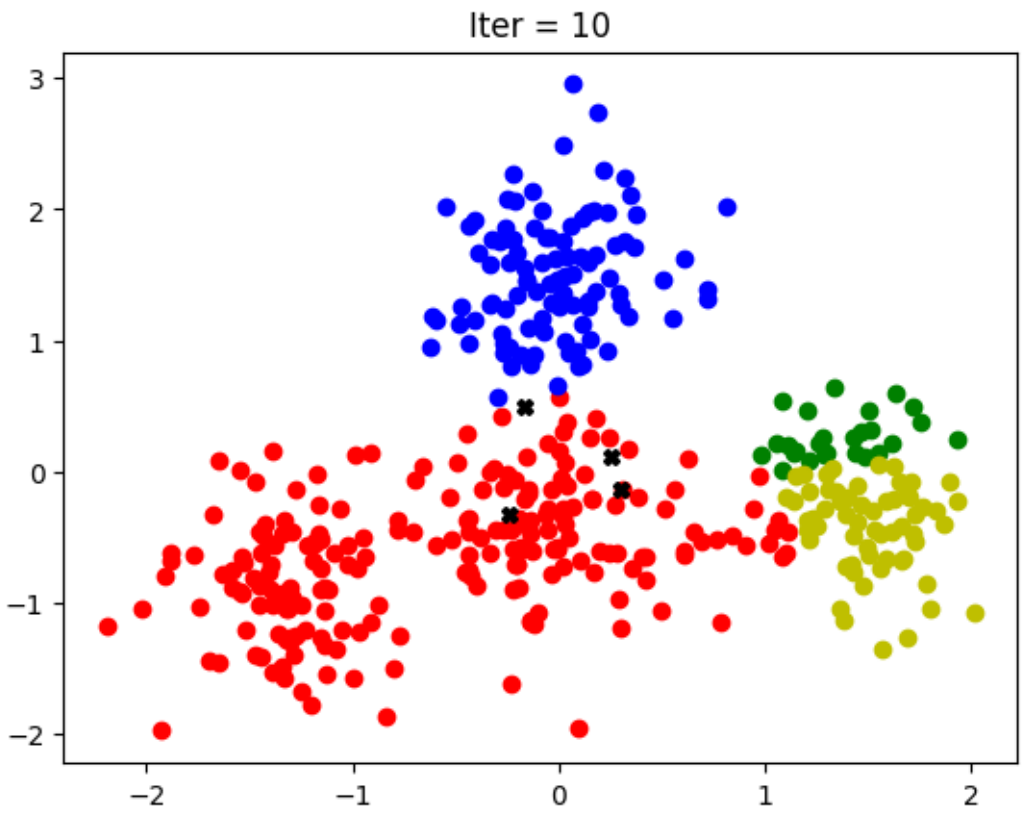

Iter = 5
3
2
1
0
−1
−2
−2
−1
0
1
2

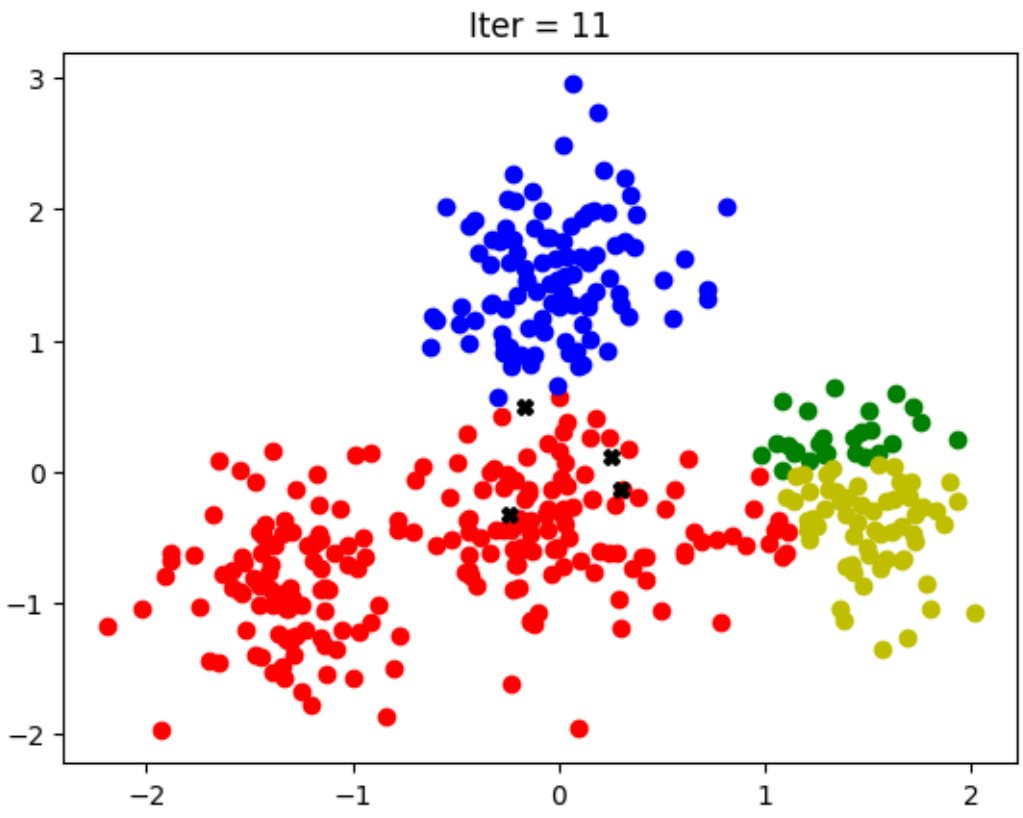

Iter = 6
3
2
1
0
−1
−2
−2
−1
0
1
2

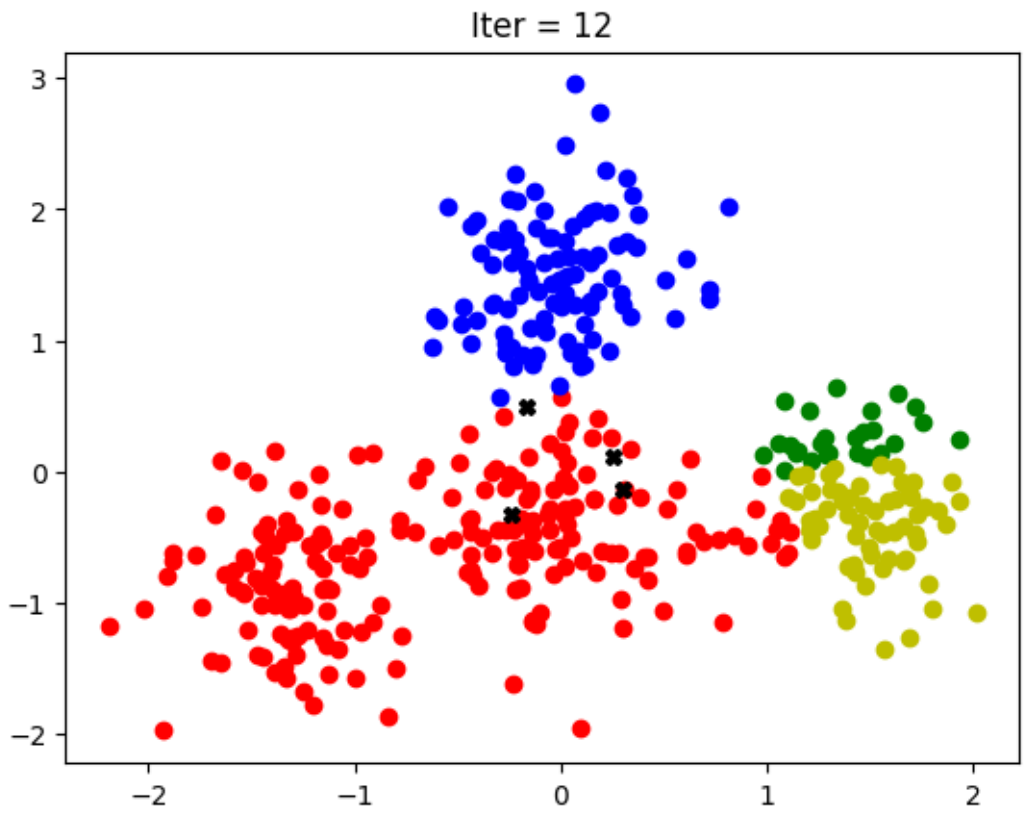

Iter = 7
3
2
1
0
−1
−2
−2
−1
0
1
2

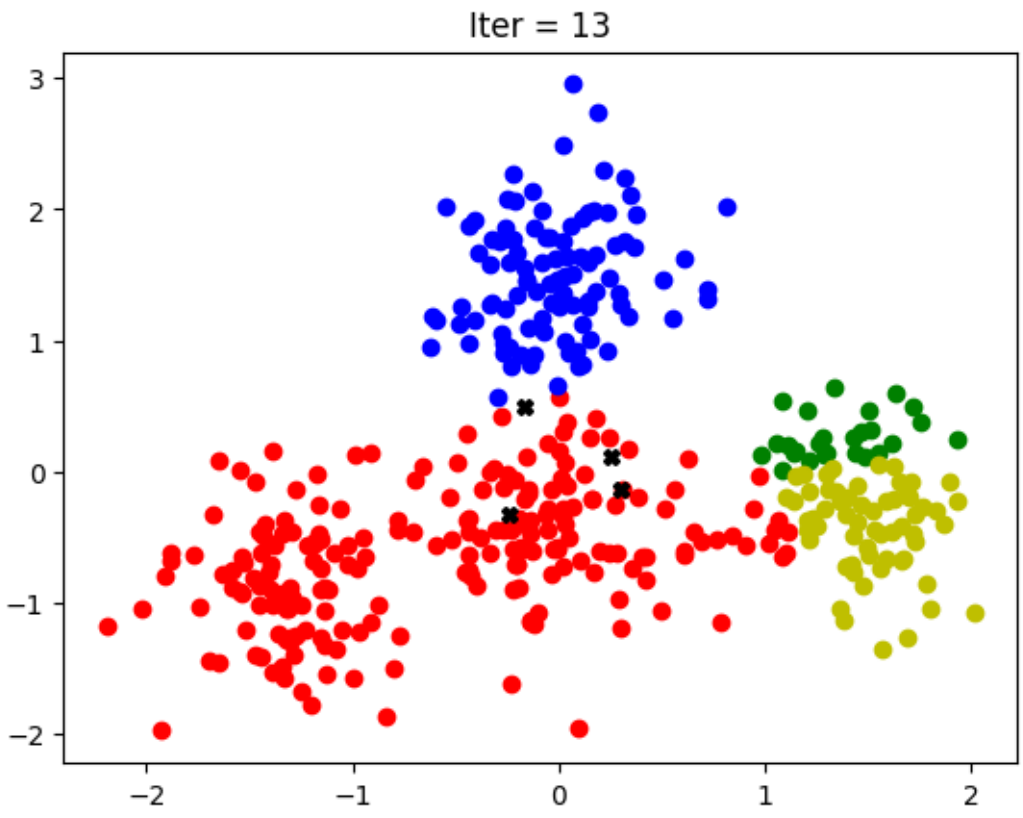

Iter = 8
3
2
1
0
−1
−2
−2
−1
0
1
2

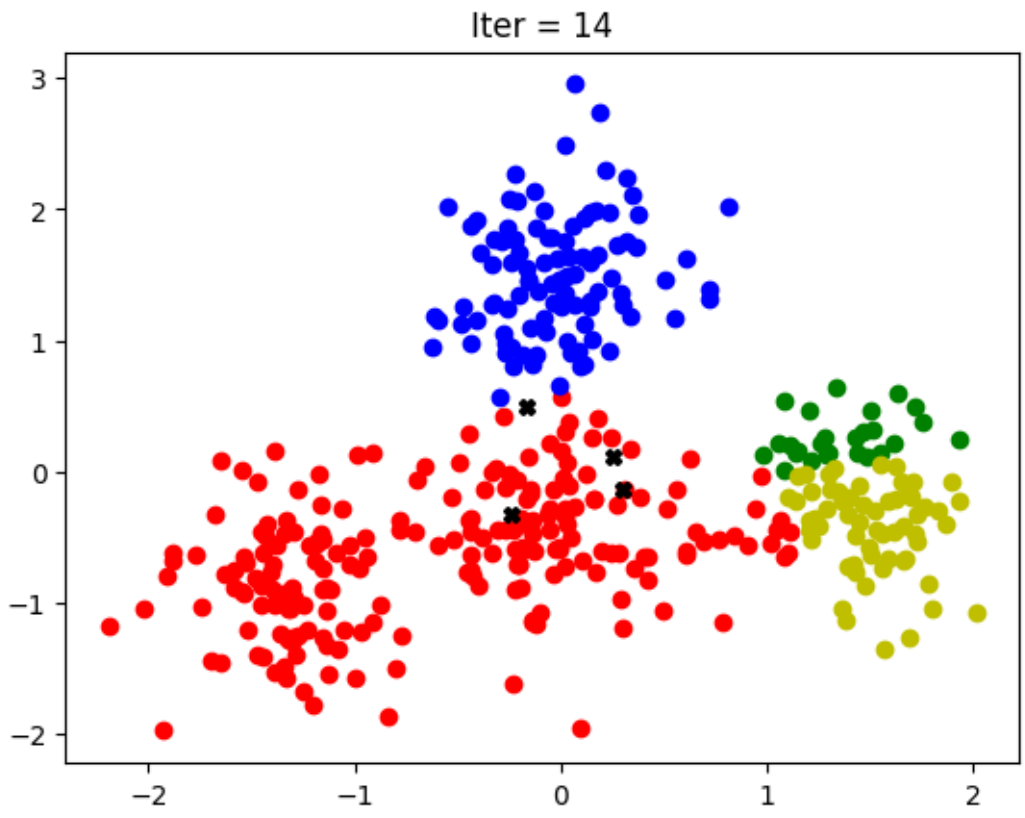

Iter = 9
3
2
1
0
−1
−2
−2
−1
0
1
2

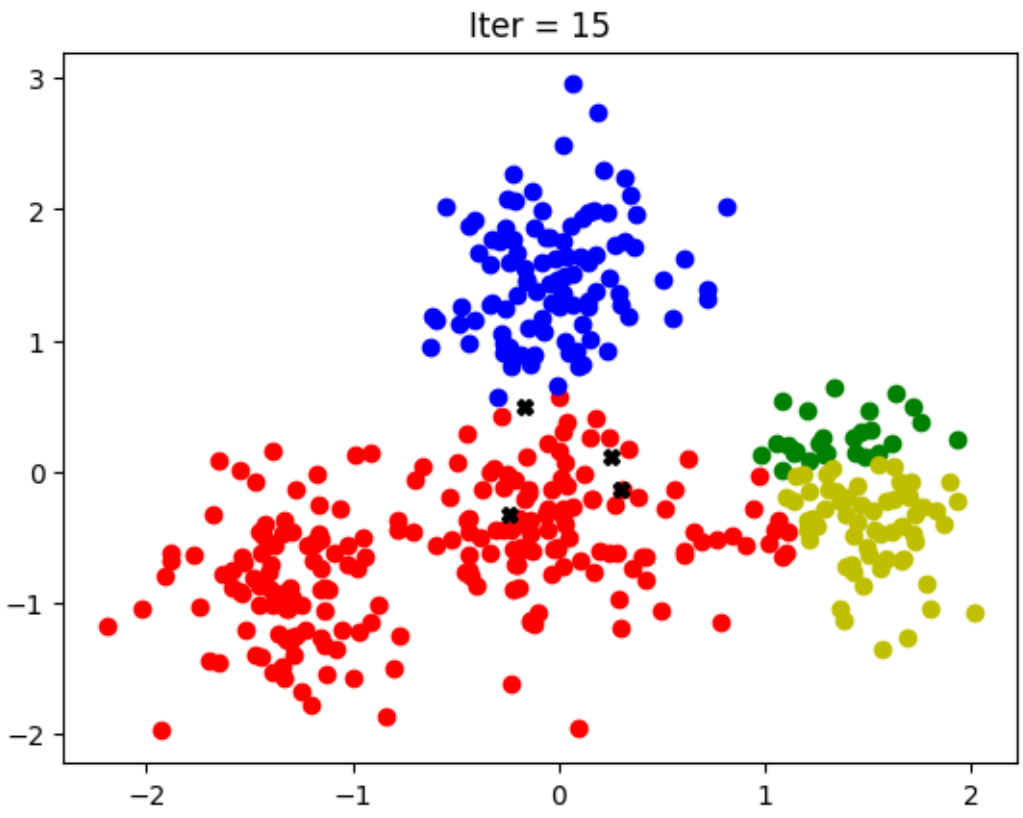

Iter = 10
3
2
1
0
−1
−2
−2
−1
0
1
2

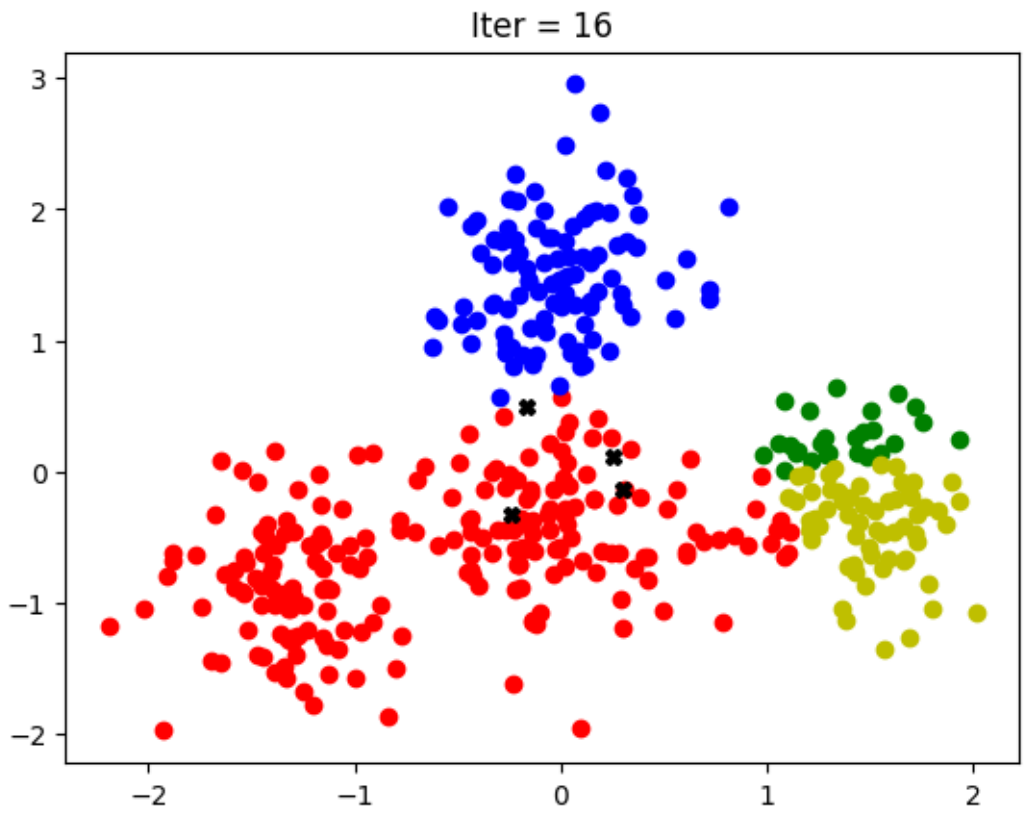

Iter = 11
3
2
1
0
−1
−2
−2
−1
0
1
2

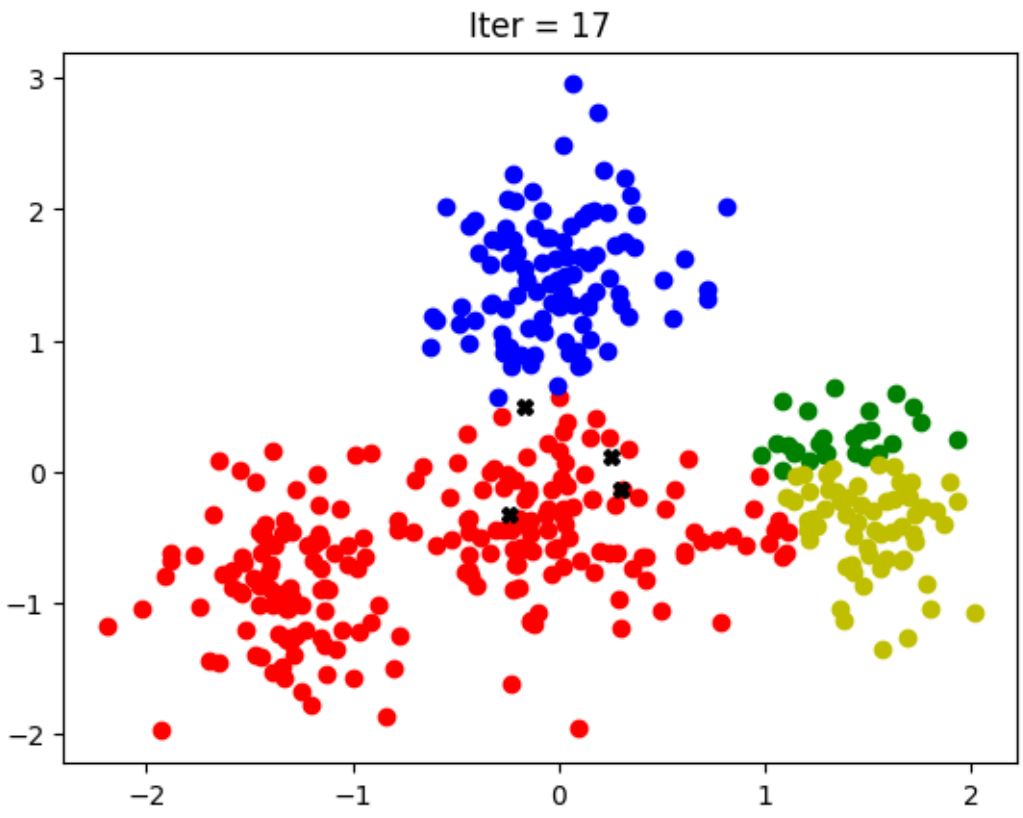

Iter = 12
3
2
1
0
−1
−2
−2
−1
0
1
2

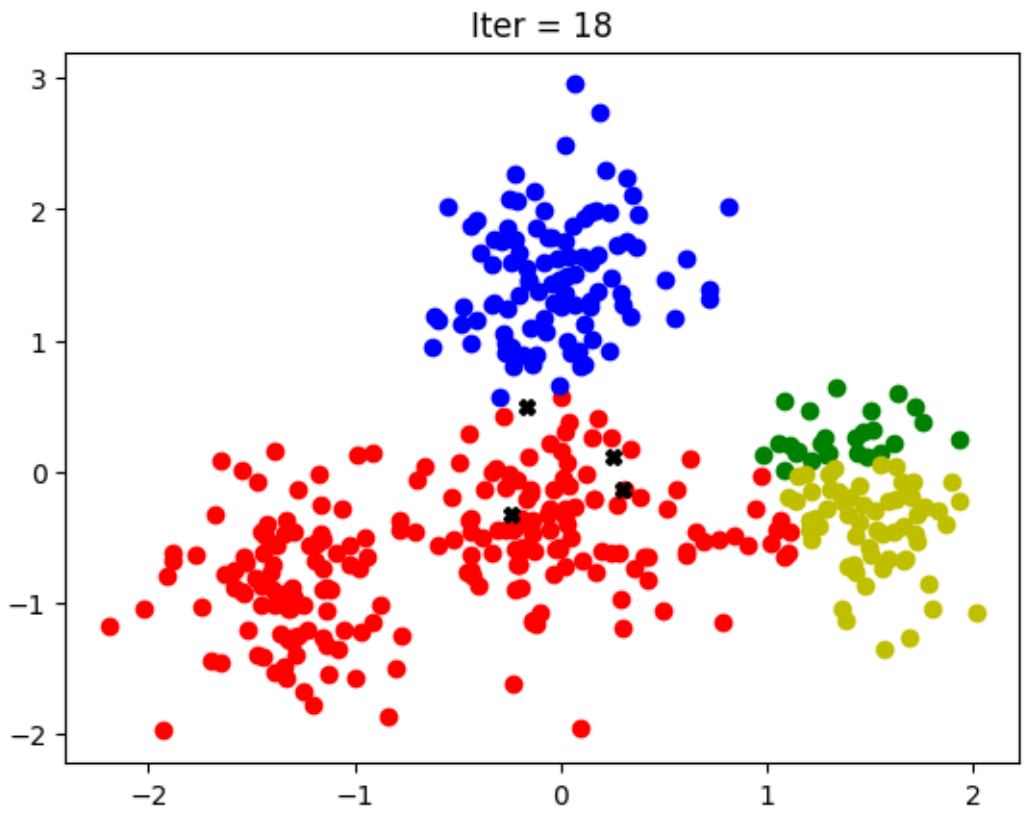

Iter = 13
3
2
1
0
−1
−2
−2
−1
0
1
2

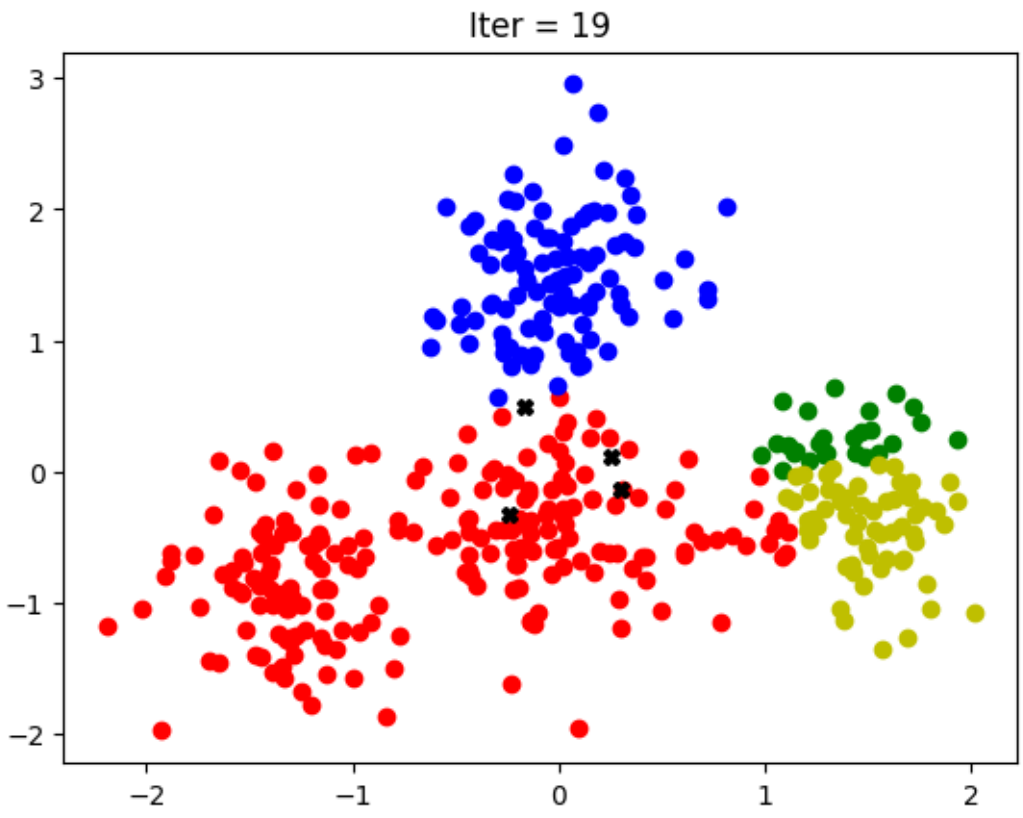

Iter = 14
3
2
1
0
−1
−2
−2
−1
0
1
2

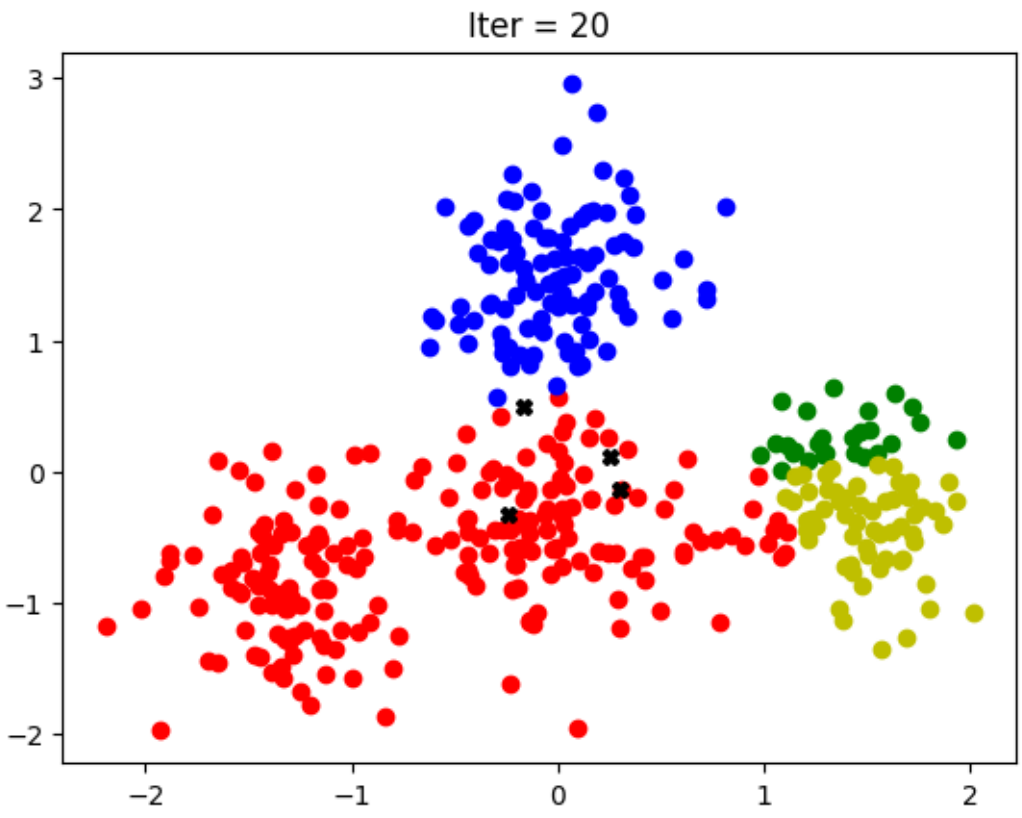

Iter = 15
3
2
1
0
−1
−2
−2
−1
0
1
2

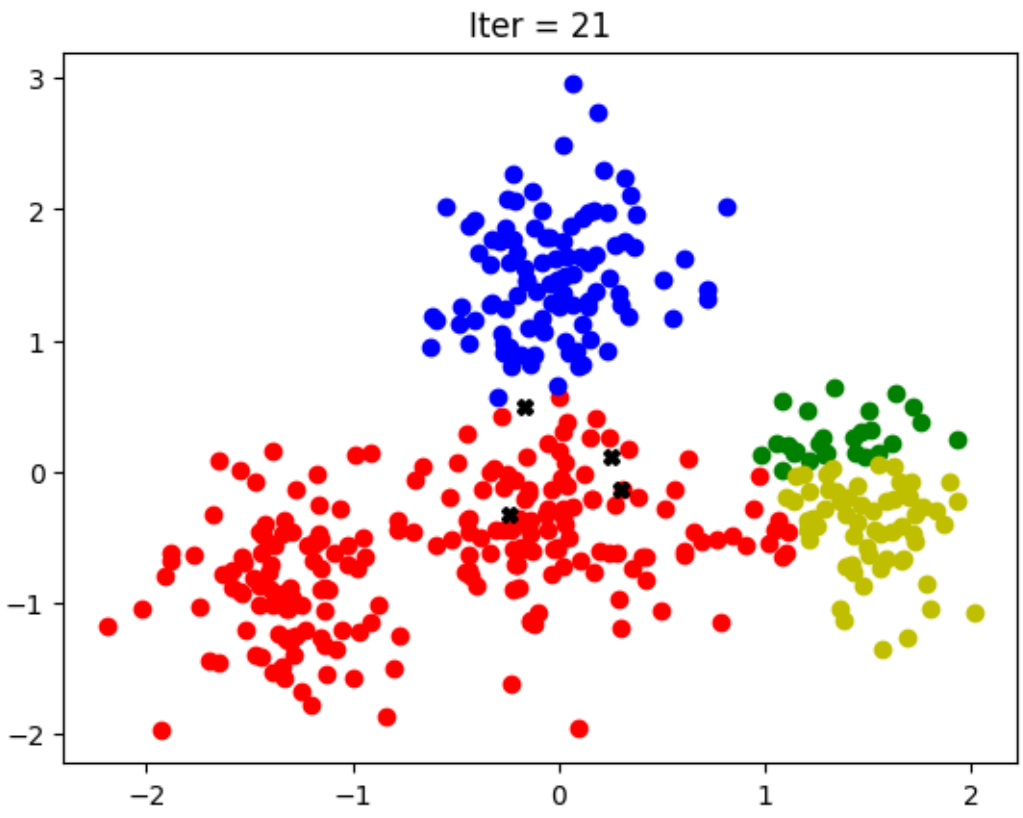

Iter = 16
3
2
1
0
−1
−2
−2
−1
0
1
2

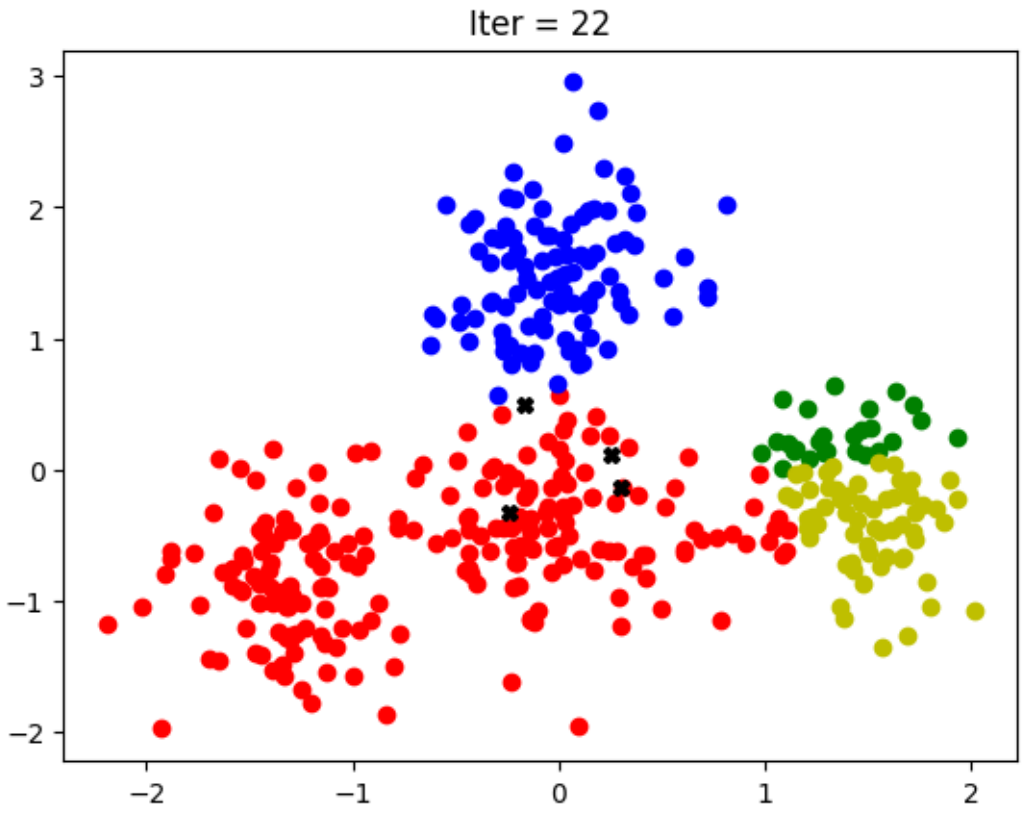

Iter = 17
3
2
1
0
−1
−2
−2
−1
0
1
2

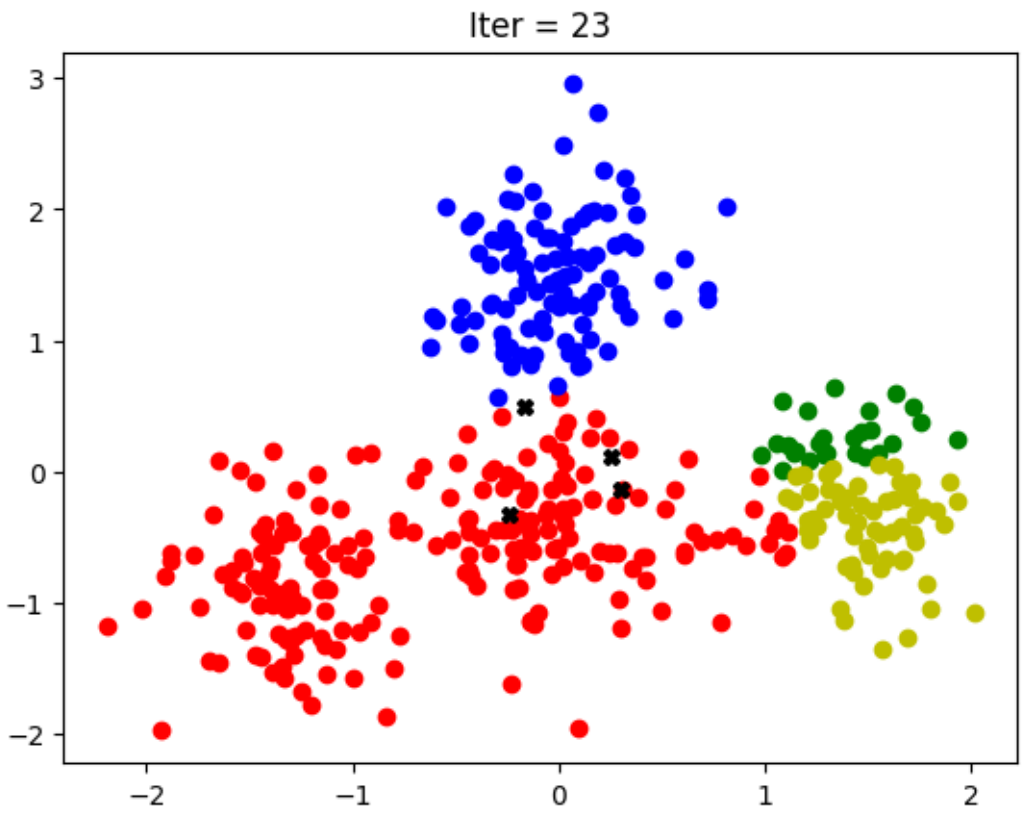

Iter = 18
3
2
1
0
−1
−2
−2
−1
0
1
2

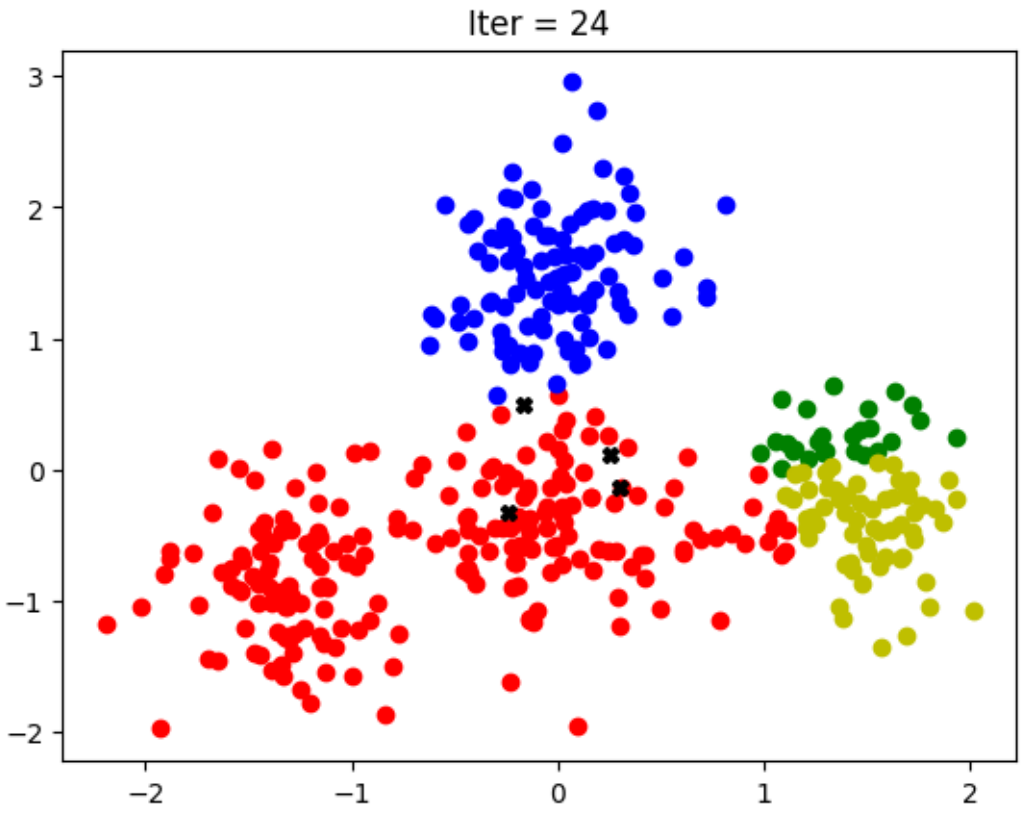

Iter = 19
3
2
1
0
−1
−2
−2
−1
0
1
2

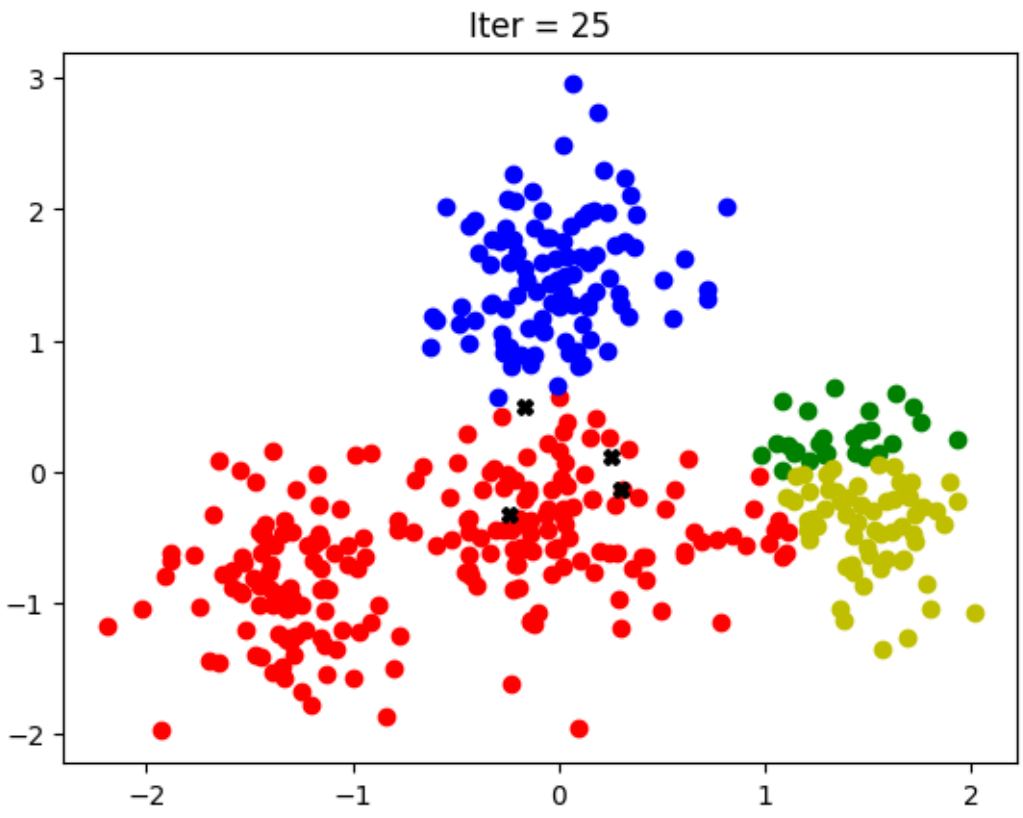

Iter = 20
3
2
1
0
−1
−2
−2
−1
0
1
2

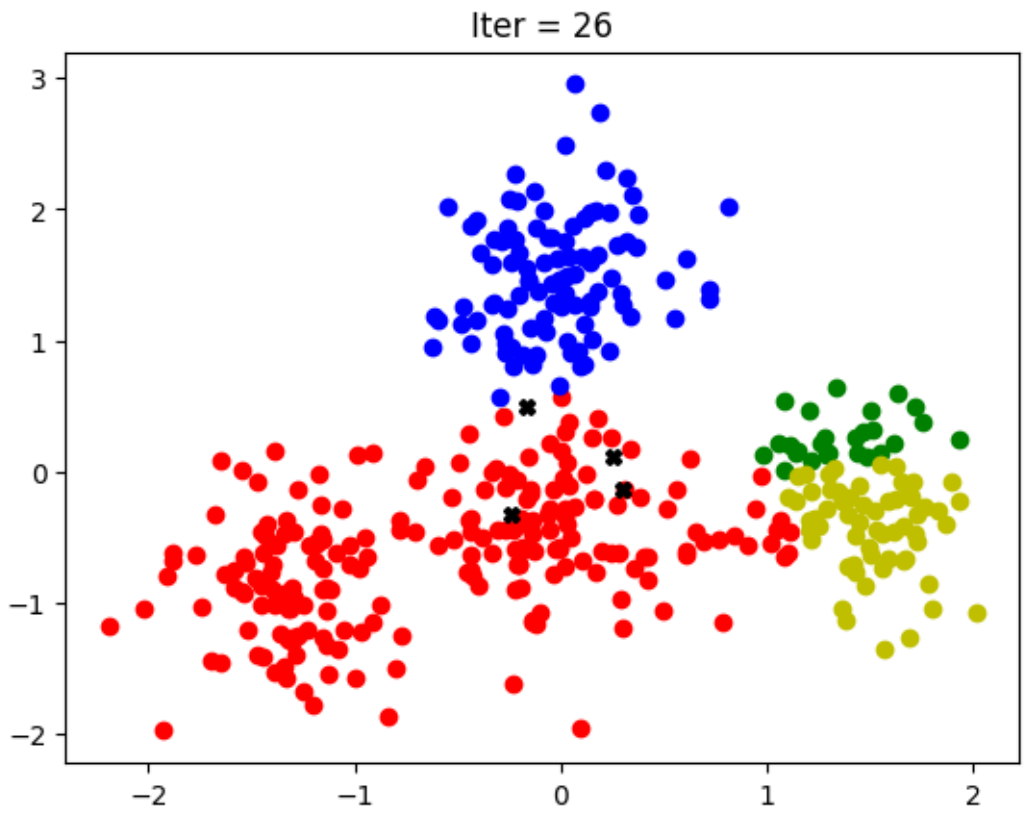

Iter = 21
3
2
1
0
−1
−2
−2
−1
0
1
2

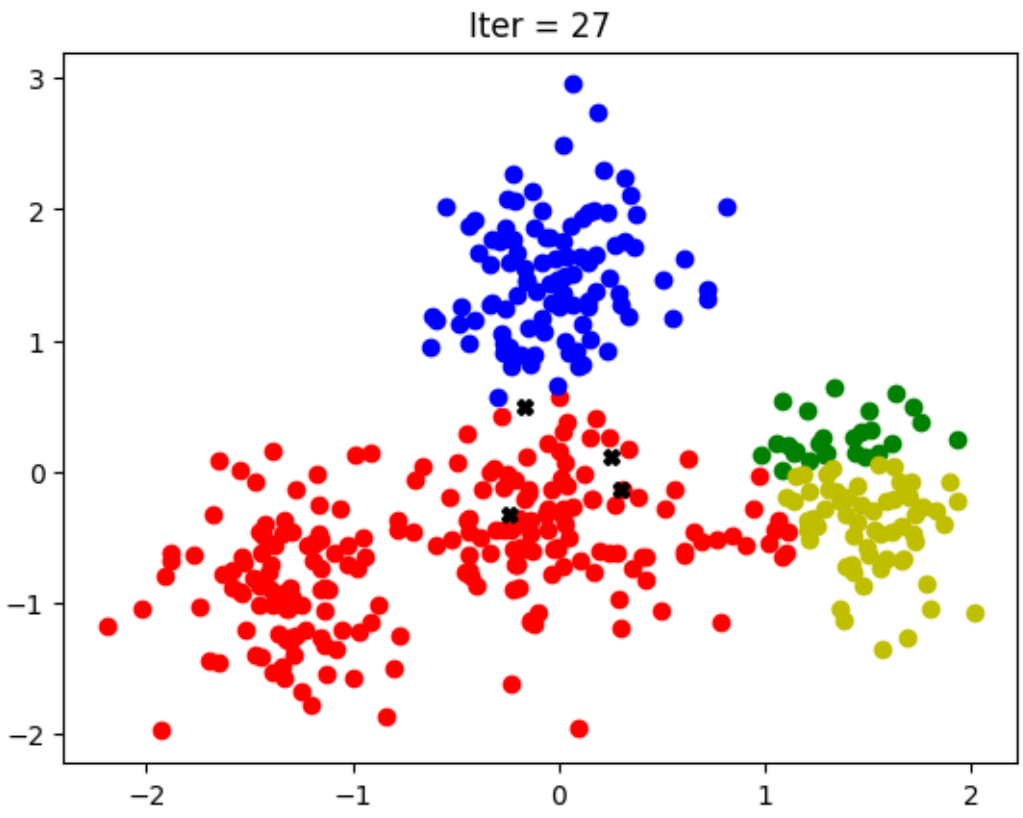

Iter = 22
3
2
1
0
−1
−2
−2
−1
0
1
2

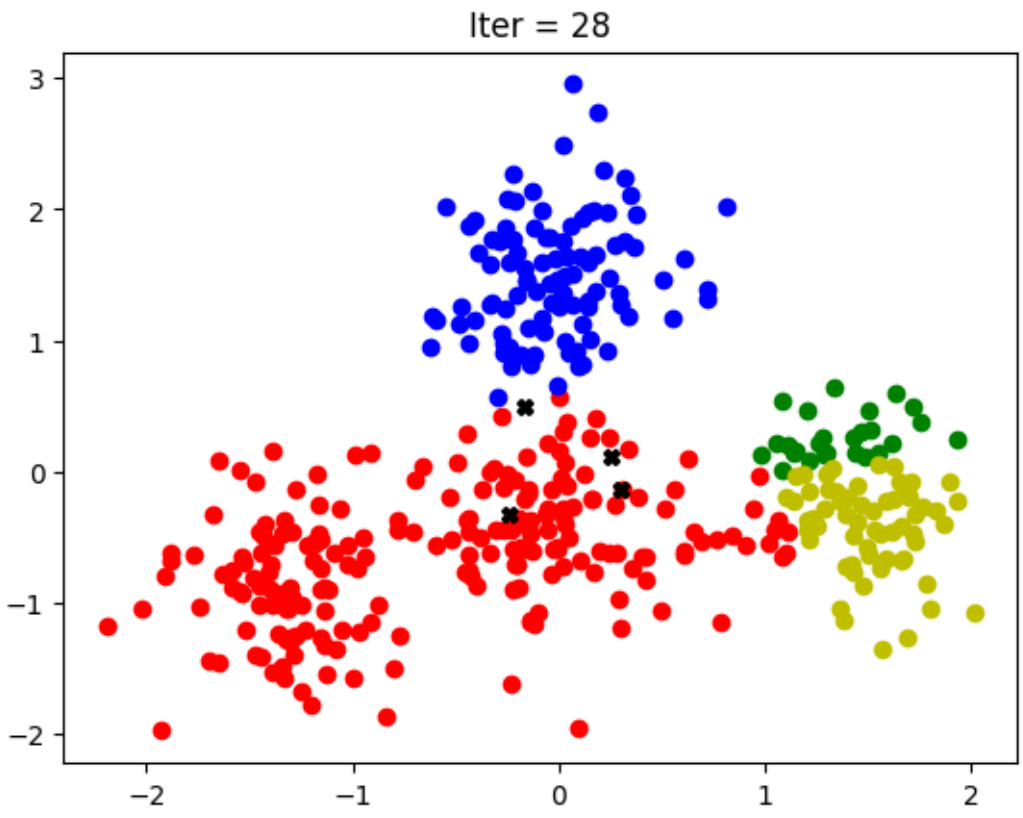

Iter = 23
3
2
1
0
−1
−2
−2
−1
0
1
2

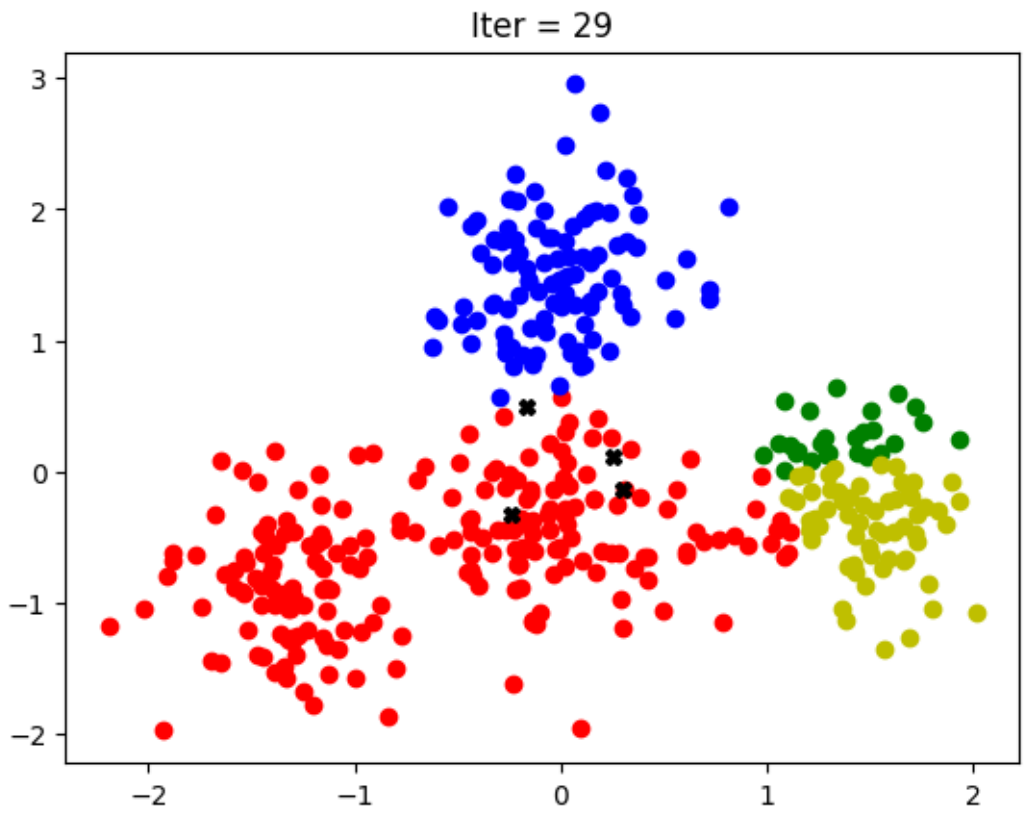

Iter = 24
3
2
1
0
−1
−2
−2
−1
0
1
2


Iter = 25
3
2
1
0
−1
−2
−2
−1
0
1
2


Iter = 26
3
2
1
0
−1
−2
−2
−1
0
1
2


Iter = 27
3
2
1
0
−1
−2
−2
−1
0
1
2


Iter = 28
3
2
1
0
−1
−2
−2
−1
0
1
2



8 Practical K - means Clustering

IRIS dataset

[8]:
```
import pandas as pd
import numpy as np
data = pd.read_csv('./IRIS.csv')
data[['target']] = data[['species']].apply(lambda col:pd.Categorical(col).codes)
data.head()
```

[8]:

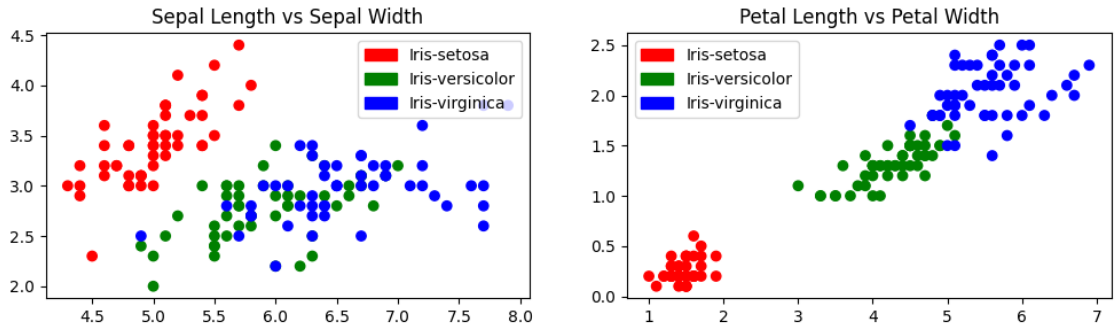
	sepal_length	sepal_width	petal_length	petal_width	species	target
0	5.1	3.5	1.4	0.2	Iris-setosa	0
1	4.9	3.0	1.4	0.2	Iris-setosa	0
2	4.7	3.2	1.3	0.2	Iris-setosa	0
3	4.6	3.1	1.5	0.2	Iris-setosa	0
4	5.0	3.6	1.4	0.2	Iris-setosa	0

[9]:
```
```

```
x = np.
 ↪asarray(data[['sepal_length','sepal_width','petal_length','petal_width']].
 ↪values.tolist())
y = (data[['species']].apply(lambda col:pd.Categorical(col).codes)).values.
 ↪tolist()
target = np.asarray([item for sublist in y for item in sublist])
```

[11]:
```
import matplotlib.pyplot as plt
import matplotlib.patches as mpatches
plt.figure(figsize=(12,3))
colors = np.array(['red', 'green', 'blue'])
iris_targets_legend = np.
 ↪array(['Iris-setosa','Iris-versicolor','Iris-virginica'])
red_patch = mpatches.Patch(color='red', label='Iris-setosa')
green_patch = mpatches.Patch(color='green', label='Iris-versicolor')
blue_patch = mpatches.Patch(color='blue', label='Iris-virginica')

plt.subplot(1, 2, 1)
plt.scatter(data['sepal_length'], data['sepal_width'], c=colors[data['target']])
plt.title('Sepal Length vs Sepal Width')
plt.legend(handles=[red_patch, green_patch, blue_patch])

plt.subplot(1,2,2)
plt.scatter(data['petal_length'], data['petal_width'], c=␣
 ↪colors[data['target']])
plt.title('Petal Length vs Petal Width')
plt.legend(handles=[red_patch, green_patch, blue_patch])
```

[11]:
```
<matplotlib.legend.Legend at 0x7f4f29a49c10>
```



[12]:
```
from sklearn.cluster import KMeans
kmeans = KMeans(n_clusters=3,n_init=100,max_iter=100,verbose=1)
kmeans.fit(x)
```

```
Initialization complete
```

```
Iteration 0, inertia 124.21000000000001.
Iteration 1, inertia 79.61434422799591.
Iteration 2, inertia 78.9450658259773.
Converged at iteration 2: strict convergence.
Initialization complete
Iteration 0, inertia 219.83.
Iteration 1, inertia 81.04960650613187.
Iteration 2, inertia 79.8179745332109.
Iteration 3, inertia 79.43376414532676.
Iteration 4, inertia 79.0107097222222.
Iteration 5, inertia 78.94506582597731.
Converged at iteration 5: strict convergence.
Initialization complete
Iteration 0, inertia 123.27000000000001.
Iteration 1, inertia 86.09778687119547.
Iteration 2, inertia 80.23054939305555.
Iteration 3, inertia 79.28654263977779.
Iteration 4, inertia 78.94084142614602.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 153.78.
Iteration 1, inertia 82.95893292565543.
Iteration 2, inertia 81.85008766287818.
Iteration 3, inertia 80.89577600000004.
Iteration 4, inertia 79.96297983461304.
Iteration 5, inertia 79.43376414532675.
Iteration 6, inertia 79.01070972222222.
Iteration 7, inertia 78.9450658259773.
Converged at iteration 7: strict convergence.
Initialization complete
Iteration 0, inertia 138.76000000000002.
Iteration 1, inertia 79.56835259340714.
Iteration 2, inertia 78.94084142614601.
Converged at iteration 2: strict convergence.
Initialization complete
Iteration 0, inertia 118.89999999999998.
Iteration 1, inertia 82.95893292565543.
Iteration 2, inertia 81.85008766287818.
Iteration 3, inertia 80.89577599999998.
Iteration 4, inertia 79.96297983461301.
Iteration 5, inertia 79.43376414532675.
Iteration 6, inertia 79.01070972222222.
Iteration 7, inertia 78.9450658259773.
Converged at iteration 7: strict convergence.
Initialization complete
Iteration 0, inertia 107.43999999999998.
Iteration 1, inertia 79.2390602412587.
Iteration 2, inertia 79.01070972222222.
```

```
Iteration 3, inertia 78.94506582597731.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 154.66000000000003.
Iteration 1, inertia 86.23349659863945.
Iteration 2, inertia 84.35467300347223.
Iteration 3, inertia 83.47963263888889.
Iteration 4, inertia 82.09358738904196.
Iteration 5, inertia 81.17033414092329.
Iteration 6, inertia 79.96297983461302.
Iteration 7, inertia 79.43376414532675.
Iteration 8, inertia 79.0107097222222.
Iteration 9, inertia 78.94506582597732.
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 128.04000000000002.
Iteration 1, inertia 88.35799644444445.
Iteration 2, inertia 85.04157943238866.
Iteration 3, inertia 84.10217888865148.
Iteration 4, inertia 83.13638186876975.
Iteration 5, inertia 81.83900206772621.
Iteration 6, inertia 80.895776.
Iteration 7, inertia 79.96297983461304.
Iteration 8, inertia 79.43376414532675.
Iteration 9, inertia 79.01070972222223.
Iteration 10, inertia 78.94506582597731.
Converged at iteration 10: strict convergence.
Initialization complete
Iteration 0, inertia 204.2.
Iteration 1, inertia 79.2865426397778.
Iteration 2, inertia 78.94084142614602.
Converged at iteration 2: strict convergence.
Initialization complete
Iteration 0, inertia 90.96.
Iteration 1, inertia 79.50383694891664.
Iteration 2, inertia 78.99945673645699.
Iteration 3, inertia 78.94084142614602.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 97.31999999999998.
Iteration 1, inertia 80.157420311637.
Iteration 2, inertia 79.43376414532675.
Iteration 3, inertia 79.01070972222222.
Iteration 4, inertia 78.94506582597732.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 158.32000000000002.
Iteration 1, inertia 145.5756810801416.
```

```
Iteration 2, inertia 144.42124818215095.
Iteration 3, inertia 143.9434757162304.
Iteration 4, inertia 143.5787278511598.
Iteration 5, inertia 143.4750399307518.
Iteration 6, inertia 143.45373548406212.
Converged at iteration 6: strict convergence.
Initialization complete
Iteration 0, inertia 126.8.
Iteration 1, inertia 86.38338573997092.
Iteration 2, inertia 79.95338439527465.
Iteration 3, inertia 79.28654263977779.
Iteration 4, inertia 78.94084142614602.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 95.32.
Iteration 1, inertia 79.34100415605006.
Iteration 2, inertia 79.01070972222222.
Iteration 3, inertia 78.94506582597731.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 150.44.
Iteration 1, inertia 83.64930208333334.
Iteration 2, inertia 82.5610868394502.
Iteration 3, inertia 81.63300278471787.
Iteration 4, inertia 80.89577600000001.
Iteration 5, inertia 79.96297983461304.
Iteration 6, inertia 79.43376414532673.
Iteration 7, inertia 79.01070972222222.
Iteration 8, inertia 78.9450658259773.
Converged at iteration 8: strict convergence.
Initialization complete
Iteration 0, inertia 91.96000000000002.
Iteration 1, inertia 79.31833613045693.
Iteration 2, inertia 78.94506582597732.
Converged at iteration 2: strict convergence.
Initialization complete
Iteration 0, inertia 155.13.
Iteration 1, inertia 88.33663409977032.
Iteration 2, inertia 80.91910465547278.
Iteration 3, inertia 79.2865426397778.
Iteration 4, inertia 78.94084142614601.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 112.54000000000002.
Iteration 1, inertia 85.33900550929934.
Iteration 2, inertia 84.1021788886515.
Iteration 3, inertia 83.13638186876975.
Iteration 4, inertia 81.83900206772621.
```

```
Iteration 5, inertia 80.89577599999998.
Iteration 6, inertia 79.96297983461304.
Iteration 7, inertia 79.43376414532675.
Iteration 8, inertia 79.0107097222222.
Iteration 9, inertia 78.9450658259773.
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 136.53000000000006.
Iteration 1, inertia 87.43310373227631.
Iteration 2, inertia 84.80172984452898.
Iteration 3, inertia 84.10217888865151.
Iteration 4, inertia 83.13638186876975.
Iteration 5, inertia 81.83900206772623.
Iteration 6, inertia 80.895776.
Iteration 7, inertia 79.96297983461304.
Iteration 8, inertia 79.43376414532675.
Iteration 9, inertia 79.01070972222222.
Iteration 10, inertia 78.94506582597731.
Converged at iteration 10: strict convergence.
Initialization complete
Iteration 0, inertia 99.25999999999999.
Iteration 1, inertia 82.44982528250972.
Iteration 2, inertia 81.41746489357331.
Iteration 3, inertia 80.1488266200554.
Iteration 4, inertia 79.66525726935403.
Iteration 5, inertia 79.08689895643232.
Iteration 6, inertia 78.9450658259773.
Converged at iteration 6: strict convergence.
Initialization complete
Iteration 0, inertia 140.08000000000004.
Iteration 1, inertia 82.11178341099625.
Iteration 2, inertia 80.89577600000001.
Iteration 3, inertia 79.96297983461301.
Iteration 4, inertia 79.43376414532673.
Iteration 5, inertia 79.01070972222222.
Iteration 6, inertia 78.9450658259773.
Converged at iteration 6: strict convergence.
Initialization complete
Iteration 0, inertia 109.17999999999999.
Iteration 1, inertia 86.06811700680274.
Iteration 2, inertia 84.35467300347224.
Iteration 3, inertia 83.4796326388889.
Iteration 4, inertia 82.09358738904196.
Iteration 5, inertia 81.17033414092327.
Iteration 6, inertia 79.96297983461304.
Iteration 7, inertia 79.43376414532675.
Iteration 8, inertia 79.01070972222223.
Iteration 9, inertia 78.94506582597731.
```

```
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 111.38.
Iteration 1, inertia 85.04157943238866.
Iteration 2, inertia 84.1021788886515.
Iteration 3, inertia 83.13638186876975.
Iteration 4, inertia 81.83900206772621.
Iteration 5, inertia 80.89577600000001.
Iteration 6, inertia 79.96297983461302.
Iteration 7, inertia 79.43376414532675.
Iteration 8, inertia 79.0107097222222.
Iteration 9, inertia 78.94506582597731.
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 95.56.
Iteration 1, inertia 85.04157943238866.
Iteration 2, inertia 84.10217888865147.
Iteration 3, inertia 83.13638186876973.
Iteration 4, inertia 81.83900206772623.
Iteration 5, inertia 80.89577599999998.
Iteration 6, inertia 79.96297983461302.
Iteration 7, inertia 79.43376414532675.
Iteration 8, inertia 79.0107097222222.
Iteration 9, inertia 78.94506582597732.
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 119.60000000000001.
Iteration 1, inertia 83.75124470912885.
Iteration 2, inertia 79.95338439527465.
Iteration 3, inertia 79.2865426397778.
Iteration 4, inertia 78.94084142614602.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 103.95000000000003.
Iteration 1, inertia 80.8232544842363.
Iteration 2, inertia 79.25791704724696.
Iteration 3, inertia 79.01070972222222.
Iteration 4, inertia 78.9450658259773.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 154.51000000000002.
Iteration 1, inertia 85.52354799506844.
Iteration 2, inertia 83.77629490216783.
Iteration 3, inertia 82.88459452600699.
Iteration 4, inertia 81.63300278471787.
Iteration 5, inertia 80.89577600000001.
Iteration 6, inertia 79.96297983461304.
Iteration 7, inertia 79.43376414532675.
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Iteration 8, inertia 79.01070972222222.
Iteration 9, inertia 78.94506582597732.
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 125.78999999999996.
Iteration 1, inertia 89.60853587119925.
Iteration 2, inertia 85.36986234996756.
Iteration 3, inertia 84.10217888865148.
Iteration 4, inertia 83.13638186876972.
Iteration 5, inertia 81.8390020677262.
Iteration 6, inertia 80.89577600000003.
Iteration 7, inertia 79.96297983461304.
Iteration 8, inertia 79.43376414532675.
Iteration 9, inertia 79.01070972222222.
Iteration 10, inertia 78.94506582597734.
Converged at iteration 10: strict convergence.
Initialization complete
Iteration 0, inertia 146.10999999999996.
Iteration 1, inertia 84.55382243690377.
Iteration 2, inertia 83.86059920961833.
Iteration 3, inertia 82.81641093072983.
Iteration 4, inertia 81.63300278471789.
Iteration 5, inertia 80.89577600000003.
Iteration 6, inertia 79.96297983461302.
Iteration 7, inertia 79.43376414532673.
Iteration 8, inertia 79.01070972222222.
Iteration 9, inertia 78.94506582597731.
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 126.60000000000001.
Iteration 1, inertia 89.60853587119925.
Iteration 2, inertia 85.36986234996758.
Iteration 3, inertia 84.1021788886515.
Iteration 4, inertia 83.13638186876973.
Iteration 5, inertia 81.83900206772621.
Iteration 6, inertia 80.895776.
Iteration 7, inertia 79.96297983461304.
Iteration 8, inertia 79.43376414532672.
Iteration 9, inertia 79.01070972222223.
Iteration 10, inertia 78.94506582597731.
Converged at iteration 10: strict convergence.
Initialization complete
Iteration 0, inertia 238.7400000000001.
Iteration 1, inertia 82.62569281080216.
Iteration 2, inertia 79.70512102298424.
Iteration 3, inertia 79.10144896074459.
Iteration 4, inertia 78.94084142614602.
Converged at iteration 4: strict convergence.
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Initialization complete
Iteration 0, inertia 120.36000000000003.
Iteration 1, inertia 81.88507343980544.
Iteration 2, inertia 80.89577600000001.
Iteration 3, inertia 79.96297983461304.
Iteration 4, inertia 79.43376414532673.
Iteration 5, inertia 79.01070972222222.
Iteration 6, inertia 78.94506582597731.
Converged at iteration 6: strict convergence.
Initialization complete
Iteration 0, inertia 135.06.
Iteration 1, inertia 86.38338573997089.
Iteration 2, inertia 79.95338439527465.
Iteration 3, inertia 79.2865426397778.
Iteration 4, inertia 78.94084142614601.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 105.66.
Iteration 1, inertia 79.99706873918686.
Iteration 2, inertia 79.10144896074462.
Iteration 3, inertia 78.94084142614601.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 109.07000000000001.
Iteration 1, inertia 80.23054939305554.
Iteration 2, inertia 79.28654263977782.
Iteration 3, inertia 78.94084142614604.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 131.85000000000005.
Iteration 1, inertia 83.46762438848647.
Iteration 2, inertia 80.77298622056264.
Iteration 3, inertia 79.96297983461304.
Iteration 4, inertia 79.43376414532675.
Iteration 5, inertia 79.01070972222223.
Iteration 6, inertia 78.94506582597731.
Converged at iteration 6: strict convergence.
Initialization complete
Iteration 0, inertia 179.85999999999996.
Iteration 1, inertia 91.18179124602467.
Iteration 2, inertia 85.7263582267561.
Iteration 3, inertia 84.35467300347223.
Iteration 4, inertia 83.4796326388889.
Iteration 5, inertia 82.09358738904194.
Iteration 6, inertia 81.17033414092329.
Iteration 7, inertia 79.96297983461302.
Iteration 8, inertia 79.43376414532673.
Iteration 9, inertia 79.01070972222222.
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Iteration 10, inertia 78.9450658259773.
Converged at iteration 10: strict convergence.
Initialization complete
Iteration 0, inertia 183.43.
Iteration 1, inertia 79.8179745332109.
Iteration 2, inertia 79.43376414532673.
Iteration 3, inertia 79.01070972222223.
Iteration 4, inertia 78.94506582597731.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 141.93.
Iteration 1, inertia 81.53982898086264.
Iteration 2, inertia 79.51834650631777.
Iteration 3, inertia 78.94506582597731.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 271.7899999999999.
Iteration 1, inertia 79.81797453321087.
Iteration 2, inertia 79.43376414532676.
Iteration 3, inertia 79.01070972222222.
Iteration 4, inertia 78.94506582597731.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 151.46000000000004.
Iteration 1, inertia 94.85373285549514.
Iteration 2, inertia 85.29856152165408.
Iteration 3, inertia 79.95338439527467.
Iteration 4, inertia 79.2865426397778.
Iteration 5, inertia 78.94084142614601.
Converged at iteration 5: strict convergence.
Initialization complete
Iteration 0, inertia 115.55.
Iteration 1, inertia 79.22285326604015.
Iteration 2, inertia 78.94084142614601.
Converged at iteration 2: strict convergence.
Initialization complete
Iteration 0, inertia 104.77999999999997.
Iteration 1, inertia 86.03620408163269.
Iteration 2, inertia 84.35467300347223.
Iteration 3, inertia 83.4796326388889.
Iteration 4, inertia 82.09358738904193.
Iteration 5, inertia 81.17033414092329.
Iteration 6, inertia 79.96297983461302.
Iteration 7, inertia 79.43376414532673.
Iteration 8, inertia 79.01070972222222.
Iteration 9, inertia 78.94506582597731.
Converged at iteration 9: strict convergence.
Initialization complete
```

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Iteration 0, inertia 114.55000000000001.
Iteration 1, inertia 79.17832654234827.
Iteration 2, inertia 78.94084142614602.
Converged at iteration 2: strict convergence.
Initialization complete
Iteration 0, inertia 164.30000000000013.
Iteration 1, inertia 82.3084080966332.
Iteration 2, inertia 79.54225115048625.
Iteration 3, inertia 78.99945673645699.
Iteration 4, inertia 78.94084142614602.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 148.30000000000004.
Iteration 1, inertia 87.38958022127547.
Iteration 2, inertia 84.80172984452896.
Iteration 3, inertia 84.10217888865148.
Iteration 4, inertia 83.13638186876973.
Iteration 5, inertia 81.83900206772621.
Iteration 6, inertia 80.89577600000003.
Iteration 7, inertia 79.96297983461302.
Iteration 8, inertia 79.43376414532673.
Iteration 9, inertia 79.01070972222222.
Iteration 10, inertia 78.94506582597731.
Converged at iteration 10: strict convergence.
Initialization complete
Iteration 0, inertia 142.56000000000003.
Iteration 1, inertia 93.60354359271405.
Iteration 2, inertia 85.29856152165408.
Iteration 3, inertia 79.95338439527467.
Iteration 4, inertia 79.28654263977782.
Iteration 5, inertia 78.94084142614601.
Converged at iteration 5: strict convergence.
Initialization complete
Iteration 0, inertia 140.94.
Iteration 1, inertia 87.85096547992099.
Iteration 2, inertia 85.04157943238864.
Iteration 3, inertia 84.10217888865148.
Iteration 4, inertia 83.13638186876973.
Iteration 5, inertia 81.8390020677262.
Iteration 6, inertia 80.89577600000001.
Iteration 7, inertia 79.96297983461304.
Iteration 8, inertia 79.43376414532675.
Iteration 9, inertia 79.01070972222222.
Iteration 10, inertia 78.9450658259773.
Converged at iteration 10: strict convergence.
Initialization complete
Iteration 0, inertia 99.48000000000002.
Iteration 1, inertia 79.26753115527623.
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Iteration 2, inertia 79.0107097222222.
Iteration 3, inertia 78.94506582597731.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 135.01999999999995.
Iteration 1, inertia 91.21921519396852.
Iteration 2, inertia 83.11004105257874.
Iteration 3, inertia 79.70512102298424.
Iteration 4, inertia 79.1014489607446.
Iteration 5, inertia 78.94084142614602.
Converged at iteration 5: strict convergence.
Initialization complete
Iteration 0, inertia 116.67999999999999.
Iteration 1, inertia 79.62832478329872.
Iteration 2, inertia 78.999456736457.
Iteration 3, inertia 78.94084142614604.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 95.46000000000001.
Iteration 1, inertia 82.23219169472503.
Iteration 2, inertia 81.17033414092327.
Iteration 3, inertia 79.96297983461304.
Iteration 4, inertia 79.43376414532675.
Iteration 5, inertia 79.01070972222222.
Iteration 6, inertia 78.9450658259773.
Converged at iteration 6: strict convergence.
Initialization complete
Iteration 0, inertia 122.65000000000002.
Iteration 1, inertia 89.60853587119927.
Iteration 2, inertia 85.36986234996756.
Iteration 3, inertia 84.10217888865148.
Iteration 4, inertia 83.13638186876975.
Iteration 5, inertia 81.8390020677262.
Iteration 6, inertia 80.89577600000001.
Iteration 7, inertia 79.96297983461304.
Iteration 8, inertia 79.43376414532675.
Iteration 9, inertia 79.01070972222222.
Iteration 10, inertia 78.94506582597732.
Converged at iteration 10: strict convergence.
Initialization complete
Iteration 0, inertia 159.99000000000007.
Iteration 1, inertia 95.72477783540306.
Iteration 2, inertia 86.3833857399709.
Iteration 3, inertia 79.95338439527465.
Iteration 4, inertia 79.2865426397778.
Iteration 5, inertia 78.94084142614602.
Converged at iteration 5: strict convergence.
Initialization complete
```

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Iteration 0, inertia 177.58.
Iteration 1, inertia 83.32918713118225.
Iteration 2, inertia 81.63300278471787.
Iteration 3, inertia 80.89577600000001.
Iteration 4, inertia 79.96297983461301.
Iteration 5, inertia 79.43376414532675.
Iteration 6, inertia 79.01070972222222.
Iteration 7, inertia 78.94506582597731.
Converged at iteration 7: strict convergence.
Initialization complete
Iteration 0, inertia 105.44000000000001.
Iteration 1, inertia 83.64930208333334.
Iteration 2, inertia 82.56108683945021.
Iteration 3, inertia 81.63300278471787.
Iteration 4, inertia 80.89577600000001.
Iteration 5, inertia 79.96297983461304.
Iteration 6, inertia 79.43376414532673.
Iteration 7, inertia 79.01070972222222.
Iteration 8, inertia 78.94506582597731.
Converged at iteration 8: strict convergence.
Initialization complete
Iteration 0, inertia 111.19999999999999.
Iteration 1, inertia 81.94304374947096.
Iteration 2, inertia 79.48372885317472.
Iteration 3, inertia 78.99945673645699.
Iteration 4, inertia 78.94084142614601.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 229.38000000000005.
Iteration 1, inertia 79.48372885317472.
Iteration 2, inertia 78.99945673645699.
Iteration 3, inertia 78.94084142614601.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 113.78999999999999.
Iteration 1, inertia 79.85961694083143.
Iteration 2, inertia 79.00734589639167.
Iteration 3, inertia 78.94084142614602.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 132.84000000000003.
Iteration 1, inertia 81.87067110706592.
Iteration 2, inertia 80.89577600000001.
Iteration 3, inertia 79.96297983461301.
Iteration 4, inertia 79.43376414532676.
Iteration 5, inertia 79.01070972222223.
Iteration 6, inertia 78.94506582597732.
Converged at iteration 6: strict convergence.
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Initialization complete
Iteration 0, inertia 116.44000000000003.
Iteration 1, inertia 85.04157943238866.
Iteration 2, inertia 84.1021788886515.
Iteration 3, inertia 83.13638186876975.
Iteration 4, inertia 81.83900206772621.
Iteration 5, inertia 80.895776.
Iteration 6, inertia 79.96297983461301.
Iteration 7, inertia 79.43376414532673.
Iteration 8, inertia 79.01070972222222.
Iteration 9, inertia 78.94506582597731.
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 154.74.
Iteration 1, inertia 81.52124049823632.
Iteration 2, inertia 79.2865426397778.
Iteration 3, inertia 78.94084142614602.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 103.95000000000002.
Iteration 1, inertia 80.82325448423632.
Iteration 2, inertia 79.25791704724696.
Iteration 3, inertia 79.01070972222223.
Iteration 4, inertia 78.94506582597732.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 135.45000000000005.
Iteration 1, inertia 87.43310373227632.
Iteration 2, inertia 84.80172984452898.
Iteration 3, inertia 84.1021788886515.
Iteration 4, inertia 83.13638186876973.
Iteration 5, inertia 81.83900206772621.
Iteration 6, inertia 80.89577600000001.
Iteration 7, inertia 79.96297983461302.
Iteration 8, inertia 79.43376414532675.
Iteration 9, inertia 79.0107097222222.
Iteration 10, inertia 78.94506582597731.
Converged at iteration 10: strict convergence.
Initialization complete
Iteration 0, inertia 122.69000000000003.
Iteration 1, inertia 79.124750487955.
Iteration 2, inertia 78.94084142614601.
Converged at iteration 2: strict convergence.
Initialization complete
Iteration 0, inertia 142.07999999999996.
Iteration 1, inertia 79.1340747486174.
Iteration 2, inertia 78.94506582597731.
Converged at iteration 2: strict convergence.
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Initialization complete
Iteration 0, inertia 111.55999999999997.
Iteration 1, inertia 85.72269753152831.
Iteration 2, inertia 79.95338439527465.
Iteration 3, inertia 79.2865426397778.
Iteration 4, inertia 78.94084142614602.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 97.52000000000001.
Iteration 1, inertia 79.00283107220376.
Iteration 2, inertia 78.94506582597731.
Converged at iteration 2: strict convergence.
Initialization complete
Iteration 0, inertia 114.38000000000001.
Iteration 1, inertia 79.95338439527465.
Iteration 2, inertia 79.2865426397778.
Iteration 3, inertia 78.94084142614602.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 99.46.
Iteration 1, inertia 82.23219169472503.
Iteration 2, inertia 81.17033414092327.
Iteration 3, inertia 79.96297983461304.
Iteration 4, inertia 79.43376414532675.
Iteration 5, inertia 79.01070972222222.
Iteration 6, inertia 78.94506582597731.
Converged at iteration 6: strict convergence.
Initialization complete
Iteration 0, inertia 90.66000000000001.
Iteration 1, inertia 84.80172984452898.
Iteration 2, inertia 84.10217888865148.
Iteration 3, inertia 83.13638186876973.
Iteration 4, inertia 81.8390020677262.
Iteration 5, inertia 80.895776.
Iteration 6, inertia 79.96297983461304.
Iteration 7, inertia 79.43376414532675.
Iteration 8, inertia 79.01070972222223.
Iteration 9, inertia 78.9450658259773.
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 153.2.
Iteration 1, inertia 80.66902766884532.
Iteration 2, inertia 79.13274230660844.
Iteration 3, inertia 78.94506582597731.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 122.27999999999999.
Iteration 1, inertia 79.94922003244893.
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Iteration 2, inertia 78.94084142614602.
Converged at iteration 2: strict convergence.
Initialization complete
Iteration 0, inertia 123.64999999999995.
Iteration 1, inertia 79.44547281875714.
Iteration 2, inertia 78.94084142614601.
Converged at iteration 2: strict convergence.
Initialization complete
Iteration 0, inertia 122.6.
Iteration 1, inertia 94.43252808127944.
Iteration 2, inertia 85.29856152165407.
Iteration 3, inertia 79.95338439527465.
Iteration 4, inertia 79.28654263977779.
Iteration 5, inertia 78.94084142614601.
Converged at iteration 5: strict convergence.
Initialization complete
Iteration 0, inertia 160.58999999999997.
Iteration 1, inertia 88.28736177777779.
Iteration 2, inertia 85.04157943238869.
Iteration 3, inertia 84.10217888865148.
Iteration 4, inertia 83.13638186876975.
Iteration 5, inertia 81.83900206772623.
Iteration 6, inertia 80.89577599999998.
Iteration 7, inertia 79.96297983461304.
Iteration 8, inertia 79.43376414532675.
Iteration 9, inertia 79.0107097222222.
Iteration 10, inertia 78.9450658259773.
Converged at iteration 10: strict convergence.
Initialization complete
Iteration 0, inertia 141.30999999999992.
Iteration 1, inertia 80.82250139949039.
Iteration 2, inertia 79.5183465063178.
Iteration 3, inertia 78.9450658259773.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 148.69999999999996.
Iteration 1, inertia 89.8982323280251.
Iteration 2, inertia 81.99626654702084.
Iteration 3, inertia 79.48372885317472.
Iteration 4, inertia 78.999456736457.
Iteration 5, inertia 78.94084142614602.
Converged at iteration 5: strict convergence.
Initialization complete
Iteration 0, inertia 217.45000000000005.
Iteration 1, inertia 81.81226653969546.
Iteration 2, inertia 80.3759032667653.
Iteration 3, inertia 79.66525726935403.
Iteration 4, inertia 79.08689895643232.
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Iteration 5, inertia 78.94506582597731.
Converged at iteration 5: strict convergence.
Initialization complete
Iteration 0, inertia 129.67999999999998.
Iteration 1, inertia 87.00356562736206.
Iteration 2, inertia 84.80172984452898.
Iteration 3, inertia 84.10217888865148.
Iteration 4, inertia 83.13638186876975.
Iteration 5, inertia 81.83900206772623.
Iteration 6, inertia 80.895776.
Iteration 7, inertia 79.96297983461304.
Iteration 8, inertia 79.43376414532673.
Iteration 9, inertia 79.01070972222222.
Iteration 10, inertia 78.94506582597731.
Converged at iteration 10: strict convergence.
Initialization complete
Iteration 0, inertia 146.48000000000002.
Iteration 1, inertia 86.64715183965589.
Iteration 2, inertia 79.95338439527464.
Iteration 3, inertia 79.2865426397778.
Iteration 4, inertia 78.94084142614601.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 129.93999999999997.
Iteration 1, inertia 80.95667365358243.
Iteration 2, inertia 79.28654263977782.
Iteration 3, inertia 78.94084142614602.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 137.31999999999996.
Iteration 1, inertia 85.4609644801579.
Iteration 2, inertia 84.10217888865148.
Iteration 3, inertia 83.13638186876975.
Iteration 4, inertia 81.83900206772621.
Iteration 5, inertia 80.89577600000001.
Iteration 6, inertia 79.96297983461304.
Iteration 7, inertia 79.43376414532673.
Iteration 8, inertia 79.01070972222223.
Iteration 9, inertia 78.94506582597731.
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 126.71.
Iteration 1, inertia 80.57862575295972.
Iteration 2, inertia 78.94084142614602.
Converged at iteration 2: strict convergence.
Initialization complete
Iteration 0, inertia 107.46000000000001.
Iteration 1, inertia 81.561961912123.
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Iteration 2, inertia 79.2865426397778.
Iteration 3, inertia 78.94084142614601.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 130.69000000000003.
Iteration 1, inertia 79.79657697653839.
Iteration 2, inertia 78.99945673645699.
Iteration 3, inertia 78.94084142614602.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 122.24.
Iteration 1, inertia 91.81167163435376.
Iteration 2, inertia 83.11004105257872.
Iteration 3, inertia 79.70512102298424.
Iteration 4, inertia 79.10144896074459.
Iteration 5, inertia 78.94084142614601.
Converged at iteration 5: strict convergence.
Initialization complete
Iteration 0, inertia 91.48.
Iteration 1, inertia 84.80172984452896.
Iteration 2, inertia 84.10217888865148.
Iteration 3, inertia 83.13638186876973.
Iteration 4, inertia 81.83900206772623.
Iteration 5, inertia 80.89577600000001.
Iteration 6, inertia 79.96297983461304.
Iteration 7, inertia 79.43376414532673.
Iteration 8, inertia 79.01070972222223.
Iteration 9, inertia 78.9450658259773.
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 130.12.
Iteration 1, inertia 90.87924759204469.
Iteration 2, inertia 82.54351817810456.
Iteration 3, inertia 79.70512102298424.
Iteration 4, inertia 79.1014489607446.
Iteration 5, inertia 78.94084142614602.
Converged at iteration 5: strict convergence.
Initialization complete
Iteration 0, inertia 134.18.
Iteration 1, inertia 84.58230506135827.
Iteration 2, inertia 80.70861016190007.
Iteration 3, inertia 79.63887734651712.
Iteration 4, inertia 79.13297335606687.
Iteration 5, inertia 78.94506582597731.
Converged at iteration 5: strict convergence.
Initialization complete
Iteration 0, inertia 137.61.
Iteration 1, inertia 83.14036626882982.
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Iteration 2, inertia 82.09154727068669.
Iteration 3, inertia 81.17033414092329.
Iteration 4, inertia 79.96297983461302.
Iteration 5, inertia 79.43376414532675.
Iteration 6, inertia 79.01070972222223.
Iteration 7, inertia 78.9450658259773.
Converged at iteration 7: strict convergence.
Initialization complete
Iteration 0, inertia 101.63000000000001.
Iteration 1, inertia 81.99626654702087.
Iteration 2, inertia 79.48372885317471.
Iteration 3, inertia 78.99945673645699.
Iteration 4, inertia 78.94084142614601.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 117.17000000000006.
Iteration 1, inertia 81.53739887253724.
Iteration 2, inertia 79.13199166666666.
Iteration 3, inertia 78.94084142614602.
Converged at iteration 3: strict convergence.
Initialization complete
Iteration 0, inertia 105.10000000000001.
Iteration 1, inertia 82.13983442505867.
Iteration 2, inertia 81.17033414092327.
Iteration 3, inertia 79.96297983461302.
Iteration 4, inertia 79.43376414532675.
Iteration 5, inertia 79.01070972222222.
Iteration 6, inertia 78.94506582597731.
Converged at iteration 6: strict convergence.
Initialization complete
Iteration 0, inertia 140.71.
Iteration 1, inertia 86.38338573997092.
Iteration 2, inertia 79.95338439527467.
Iteration 3, inertia 79.2865426397778.
Iteration 4, inertia 78.94084142614602.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 119.08999999999999.
Iteration 1, inertia 94.85373285549514.
Iteration 2, inertia 85.29856152165408.
Iteration 3, inertia 79.95338439527465.
Iteration 4, inertia 79.2865426397778.
Iteration 5, inertia 78.94084142614602.
Converged at iteration 5: strict convergence.
Initialization complete
Iteration 0, inertia 103.7.
Iteration 1, inertia 84.78260185497633.
Iteration 2, inertia 84.10217888865147.
```

```
Iteration 3, inertia 83.13638186876973.
Iteration 4, inertia 81.83900206772621.
Iteration 5, inertia 80.895776.
Iteration 6, inertia 79.96297983461304.
Iteration 7, inertia 79.43376414532676.
Iteration 8, inertia 79.01070972222223.
Iteration 9, inertia 78.94506582597732.
Converged at iteration 9: strict convergence.
Initialization complete
Iteration 0, inertia 107.98000000000002.
Iteration 1, inertia 83.11004105257874.
Iteration 2, inertia 79.70512102298426.
Iteration 3, inertia 79.10144896074462.
Iteration 4, inertia 78.94084142614602.
Converged at iteration 4: strict convergence.
Initialization complete
Iteration 0, inertia 181.74000000000004.
Iteration 1, inertia 97.45504277436234.
Iteration 2, inertia 87.38958022127548.
Iteration 3, inertia 84.80172984452896.
Iteration 4, inertia 84.10217888865148.
Iteration 5, inertia 83.13638186876973.
Iteration 6, inertia 81.83900206772623.
Iteration 7, inertia 80.895776.
Iteration 8, inertia 79.96297983461304.
Iteration 9, inertia 79.43376414532673.
Iteration 10, inertia 79.01070972222222.
Iteration 11, inertia 78.9450658259773.
Converged at iteration 11: strict convergence.
```

[12]:
```
KMeans(max_iter=100, n_clusters=3, n_init=100, verbose=1)
```

[13]:
```
from sklearn.mixture import GaussianMixture as Gmm
gmm=Gmm(n_components=3, init_params='kmeans',covariance_type='diag',verbose=1)
```

[14]:
```
gmm.fit(x)
```

```
Initialization 0
Initialization converged: True
```

[14]:
```
GaussianMixture(covariance_type='diag', n_components=3, verbose=1)
```