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Assessing fraction knowledge by a digital game

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ABSTRACT

Serious or educational games gain increasing research interest as tools to augment traditional instructional approaches on scholastic learning, especially in mathematics education. In this study, we investigated whether game-based approaches may not only be useful to foster numerical learning but may also be valid as an assessment tool. To measure their conceptual knowledge of fractions eleven-year-old students played a math game on tablet computers using tilt-control to navigate an avatar along a number line for a total of 30 min. Findings indicated that hallmark effects of fraction magnitude processing typically observed in basic research, such as the numerical distance effect, were successfully replicated using the game-based assessment. Moreover, fraction comparison performance as well as fraction estimation accuracy correlated significantly with students' math grades. Therefore, the results of the current study suggest that game-based learning environments for fraction education (even using tilt-control) may also allow for a valid assessment of students' fraction knowledge.

1. Introduction

Fractions are commonly considered as one of the most difficult relations to learn and even adults frequently fail to process them correctly (Gigerenzer, 2002; Siegler, Fazio, Bailey, & Zhou, 2013 for a review). Fraction understanding, however, seems crucial for math education (e.g. Booth & Newton, 2012). Not only that high school students' fraction knowledge correlates very highly with their actual mathematics achievement (up to $r = 0.8$), fifth graders' fraction knowledge also predicts future algebra and overall mathematics achievement in high school (e.g., Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012). Given the widespread difficulties many adults and children face with reasoning about fractions, traditional instructional methods may be reconsidered and complemented by new tools for fostering fraction knowledge. This seems specifically important because deficient numeracy skills are detrimental to individuals' job and life prospects (Parsons & Bynner, 2006).

Serious games or game-based applications have the potential to provide such new, engaging, and innovative ways of training children as well as adults in mathematics. Recently, the use of such games in cognitive training, learning and educational interventions increased considerably (for a systematic review see Boyle et al., 2016). Studies indicate that the use of game-based tasks can not only increase motivation and engagement of users but also their performance (e.g. Mekler, Brühlmann, Opwis, & Tuch, 2013; Ninaus et al., 2015; for a review see; Lumsden, Edwards, Lawrence, Coyle, & Munafò, 2016). Thus, in this study we used our game-based rational number research engine "Semideus" to tackle one of the major hurdles in mathematics education – students' fraction knowledge.

1.1. Number line estimation task and fractions

The processing and learning of fractions is one of the most challenging problems in mathematics education (National Mathematics Advisory Panel, 2008). A crucial part of fraction understanding and processing is the successful representation of fraction magnitude (reflecting the relation between denominator and numerator) and carrying out arithmetic operations on them. The concept of a mental number line is an often used metaphor to describe our mental representation of number magnitude. Accordingly, the number line estimation task, in which participants

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have to indicate the spatial position of a target number on a number line with only its start and endpoint specified (e.g., where goes 42 on a number line ranging from 0 to 100), is an often used approach to measure and train individuals' representation of number magnitude (e.g., Link, Moeller, Huber, Fischer, & Nuerk, 2013; Siegler & Opfer, 2003). Importantly, performance in the number line estimation task is associated with actual mathematical performance and can predict future mathematical achievement (e.g., Booth & Siegler, 2006). Therefore, recent research studies emphasized that children's mental representation of number magnitude can be fostered by training to map numbers (including fractions) onto space as in the number line estimation task (e.g. M. Schneider & Stern, 2010; Siegler & Ramani, 2008). Most of the time conventional computer tasks or paper-pencil versions are used to train and assess individuals' representation of number magnitude. However, in recent years new and innovative methods of training numerical skills were developed such as trainings with dance mats (Fischer, Dackermann, Cress, Nuerk, & Moeller, 2014), interactive whiteboards (Fischer, Moeller, Huber, Cress, & Nuerk, 2015), as well as game-like versions of the number line task (e.g. Kucian et al., 2011).

According to conceptual change theories, children form an initial conception of numbers as counting units before they encounter fractions, and later they draw heavily on this initial understanding to make sense of rational numbers (DeWolf & Vosniadou, 2015; Stafylidou & Vosniadou, 2004). As such, misconceptions and biases about rational numbers tend to originate in children's erroneous belief that properties of whole numbers can be applied to rational numbers. According to DeWolf and Vosniadou (2015) this is detrimental for children's understanding of fractions as it implies that they tend to treat denominators and numerators as two separate whole numbers instead of considering their relation to each other. From this conceptualization they often infer that the value of a fraction increases when either the denominator or the numerator increases. For example, 2/5 (0.4) is larger than 3/8 (0.375) although its numerator 2 is smaller than 3 and the denominator 5 is smaller than 8. This phenomenon is referred as a *whole number bias* (Ni & Zhou, 2005) or *natural number bias* (Alibali & Sidney, 2015).

The whole number bias has been found to cause difficulties in reasoning about the magnitude of fractions (e.g. Van Hoof, Lijnen, Verschaffel, & Van Dooren, 2013). Interestingly, even mathematics experts and educated adults have been shown to be slower to respond on tasks in which whole number features are incongruent with rational number features (Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013; Vamvakoussi, Van Dooren, & Verschaffel, 2012). Importantly, however, successful understanding of the magnitude of fractions was found to be an important precursor of later knowledge of rational numbers, such as the density of rational numbers (McMullen, Laakkonen, Hannula-Sormunen, & Lehtinen, 2014) and arithmetic operations with rational numbers (Van Hoof, Vandewalle, Verschaffel, & Van Dooren, 2015).

Other problems and misconceptions with fraction magnitude processing are of interest in examining how a digital game can assess students' conceptual knowledge of the numerical magnitude of fractions. Correct representation of fraction magnitude as the relation between denominator and numerator can be conceptualized as and assessed by being able to localize the magnitude of a fraction on a number line estimation task for the number range 0–1. Applying such fraction knowledge in a magnitude comparison task (e.g. which one of the numbers is larger: 3/8 or 4/5) would allow for further information on the representation of fraction magnitude. From basic research it is known that the so-called *distance effect* (i.e., longer and more error prone comparisons of fractions that are closer in magnitude than for fractions further apart in magnitude) indicates a successful representation of fraction magnitude (e.g., M. Schneider & Siegler, 2010).

1.2. Present study

The present study is a part of an ongoing research project in which we are developing a game-based rational number research engine called Semideus. The game is based on recent findings on fraction processing and numerical development (e.g. McMullen et al., 2014; Siegler, Thompson, & Schneider, 2011; Vamvakoussi, 2015) and theories that provide an account for the use of manipulatives in digital learning materials (Pouw, van Gog, & Paas, 2014). We implemented the assessment of children's conceptual fraction knowledge directly into the gameplay. In particular, students had to indicate the magnitude of a fraction as an analogue quantity by locating its position on a number line and to compare the magnitudes of two fractions by arranging them according to their magnitude on the number line. For the latter magnitude comparison task we were interested in replicate whole number bias and numerical distance effect in the current game. Furthermore, considering math grades as an external criterion allowed us to validate our game-based approach for assessing conceptual fraction knowledge.

The main purpose of the current study was to demonstrate the applicability of our game-based rational number research engine Semideus to assess fraction knowledge. More specifically, we investigated advantages and disadvantages of using a game as a research and assessment tool for investigating young students' biases in fraction processing and identifying factors from playing behaviour that may be used to define more exhaustive learning analytics for fraction knowledge.

In order to successfully apply the Semideus research engine for future research studies, one needs to demonstrate that our game-based approach provides similar effects (e.g. whole number bias, distance effect; see also 1.1) as reported with conventional (e.g. paper-pencil) and non-game-based assessment measures. Thus, in the current study we investigated 5th graders' performance of comparison and estimation of fraction magnitudes while playing the Semideus Exam game on a tablet with tilt control.

First, a general demonstration of participants' ability to master the user interface (including tilt-control) within the first few trials of assessment is necessary before any form of assessment can be performed. Following this, we assume that students' performance on the game will be defined by their understanding of fraction magnitude and not driven by general game mechanics. In order to evaluate this statement we set up six hypotheses that direct this study. All the hypotheses are deduced from previous research on numerical cognition and thus this approach provides important information about the applicability and usefulness of the Semideus game as an assessment and research tool.

Overall, whole number comparisons tasks should be relatively trivial at this age. Thus, we expected that children should perform worse in comparisons of fraction magnitudes (*Hypothesis 1*) than in comparisons of whole number magnitudes (DeWolf & Vosniadou, 2015; M.; Schneider & Siegler, 2010). Second, however, we expected that there will be a positive relation between students' whole number comparison speed and their fraction comparison speed (*Hypothesis 2*), as would be expected by the integrative theory of numerical development (e.g. Siegler et al., 2011).

Previous research has shown that fractions that are consistent with whole number ordering are mastered better than fractions that are inconsistent with whole number ordering due to the whole number bias (Vamvakoussi et al., 2012; Van Hoof et al., 2015). Therefore, we expect that students ability to solve consistent items is better (*Hypothesis 3a*) and faster (*Hypothesis 3b*) than for inconsistent items. Additionally, we expect to observe a standard distance effect for fraction comparisons with fraction pairs with a large distance being easier (*Hypothesis 4a*) and responded to faster as compared to pairs with smaller distances (*Hypothesis 4b*).

Both magnitude comparison and number line estimation tasks reflect fraction magnitude understanding and we expect that there is a reliable correlation between performance in these tasks as previous research has shown (*Hypothesis 5*; e.g. U.S. students in Torbeyns, Schneider, Xin, & Siegler, 2014). Moreover, it has been shown that fraction understanding is crucial for math education and overall mathematics achievement in high school (e.g., Bailey et al., 2012; Booth & Newton, 2012). Thus, we expect that performance in fraction comparison (*Hypothesis 6a*) and fraction estimation accuracy (*Hypothesis 6b*) as assessed by the Semideus game engine should be highly correlated with students' general math performance as indicated by their previous math grades in which fraction knowledge was not taken into account.

2. Method

2.1. Participants

Three Finnish fifth grade classes participated in the study. All classes were from the same public primary school in a middle-class district. The school was randomly selected from three schools in the City of Pori that were equipped with iPads. Fifty-six students were involved in the study, from which 54 followed the designed playing protocol and were included in the analysis. Of the participants 29 were female, 25 male. Mean age was 11.26 years ($SD = 0.48$ years). None of the participants had diagnosed math disabilities.

2.2. Description of the semideus research engine

Semideus is a rational number research engine that can be used to create games that aim at supporting the development of children's conceptual fraction knowledge. The engine was used to configure a game, Semideus Exam, which was used in the current study. The gameplay is founded on tasks that require working with number lines implemented as walkable platforms of a mountain. The game is set in ancient Greek times. In the game the player controls the character Semideus, who tries to find gold coins that Goblin Kobalos has stolen from Zeus. Kobalos has hidden the coins along the trails of Mount Olympos. Semideus has discovered the locations of the hidden coins, encrypted in mathematical symbols, and must race the goblin to retrieve the coins from trails of Mount Olympos.

Several different fraction processing tasks can be included in Semideus games, for instance number line estimation, magnitude comparison, magnitude ordering, and density awareness tasks. Fig. 1 shows examples of comparison and estimation tasks that were used in the present study. In the number line estimation tasks the player tries to locate a gold coin based on a given number (e.g., whole numbers or fractions). In magnitude comparison levels, the player has to compare two stones with values on them by arranging the stones in ascending order with regard to the numerical magnitudes depicted on them. The exact spot on the number line (ranging from smaller numbers on the left to larger numbers on the right) does not matter in magnitude comparison levels as long as the order of the stones is correct. The player can also pile the stones up when magnitudes are equivalent.

All tasks were embedded into the game in an engaging way and games created with the Semideus engine have been designed to work as non-invasive assessment instruments as well (Kiili, Devlin, Perttula, & Tuomi, 2015). In terms of embedded embodied interactions, the game character and other movable items can be considered as visual manipulatives that help the player to concretize the mental number line.

2.2.1. Level structure of the Semideus Exam game

The Semideus Exam game was a web based application played through an iPad browser (full screen mode). The game world consisted of seven levels. Fig. 2 shows the level structure of the game. The first two levels were designed for onboarding. The aim of onboarding levels was to familiarize players with the basic controlling mechanics and rules of the game as well as to set expectations for what the game exam is about. From an assessment perspective this phase of the game is crucial, because we cannot assume that all players master the user interface and controls right from the beginning. In the initial three items of the whole number estimation tasks (Level 1) the correct location/answer was indicated by a coin on the ground (see Fig. 2: Level 1). This allows players to immediately understand that the correct solution of the estimation task indicates the location of the coin on the number line. Additionally, performance in the first three items was used to evaluate whether players were able to use the tilting user interface. The actual exam of fraction understanding consisted of five levels (Levels 3–7). Level 3 included ten fraction estimation items and Levels 4–7 included seven fraction comparison items per level.

The game was configured in a way that each participant could play all seven levels once starting from level one. Although a player could run out of virtual energy in a level (100 units energy in the beginning), she or he was still able to complete the level. However, at the mountain top the player did not earn the bonus that is given based on remaining virtual energy. Within levels players were allowed only one answer to each item. Feedback was immediately shown after each answer. Fig. 3 shows the feedback that the player got for inaccurate and accurate estimations: For inaccurate estimations (i.e., estimations more than 8% away from the correct location) the avatar was struck by lightning and the player lost 15 units of virtual energy and the correct location of the coin was shown with a green marker on the walkable platform or number line, respectively. When the location was estimated accurately the player got 100–500 coins depending on estimation accuracy (over 98% correct = 500 coins; 95%–98% = 300 coins; 92%–94% = 100 coins) and the correct location of the coin was indicated by a green marker on the number line. Moreover, accuracy percentage was prompted on the display. After feedback presentation, players progressed to the next platform (i.e., faced the next item). In the comparison tasks the number of coins awarded to the player depended on the response time (under 9 s = 500 coins; 9–12 s = 400 coins; 12–16 s = 300 coins; 16–20 s = 200 coins; over 20 s = 100 coins; wrong answer = 0 coins and 20 units energy loss).

After completing a level (reaching the top of the mountain) a player got additional feedback according to their overall performance in the level: 1–3 stars and earned coins were shown (i.e., one star for completing the level and reaching the mountaintop, one star for collecting enough coins, and one star from accuracy reflecting that enough virtual energy was left). Additionally, a bonus was given based on remaining energy (energy % * 500 coins).

The user interface of the game also involved some onscreen buttons that allowed for additional touch interactions. In the top-left corner an answer button was placed (i.e., the shovel icon). Additionally, on comparison tasks, the top-right corner had a carry button for picking up and putting down a stone (i.e., to arrange them according to their numerical value by pressing the pick up and weighing scale icon on the screen, see right chart in Fig. 1). Touching the screen anywhere else, but the buttons, resulted in a jump of the avatar. Movement of the character Semideus was controlled by tilting the tablet to the right or left. Movement speed depended on the angle of tilting with a steeper angle resulting in

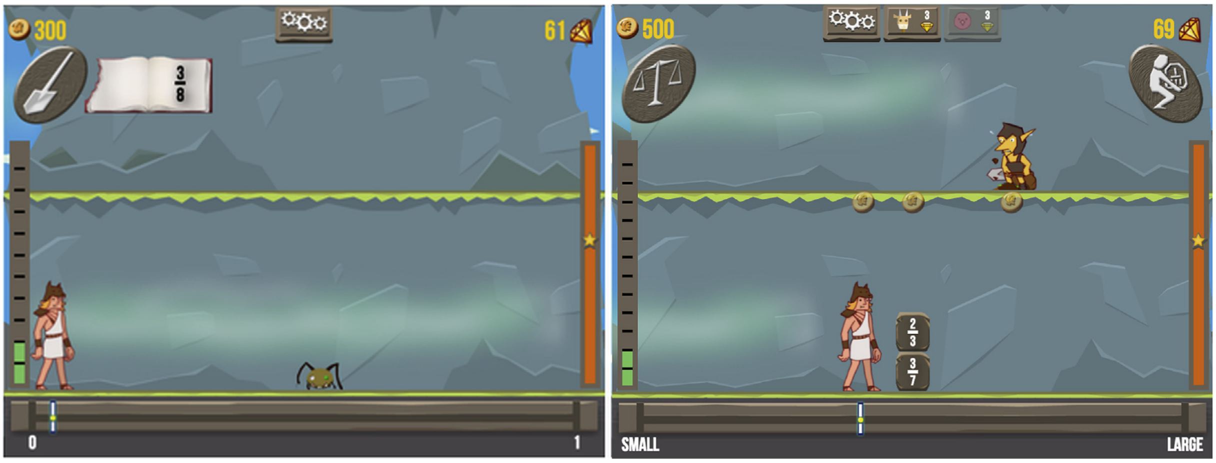

Fig. 1. Examples of number line estimation (left chart) and magnitude comparison tasks (right chart).

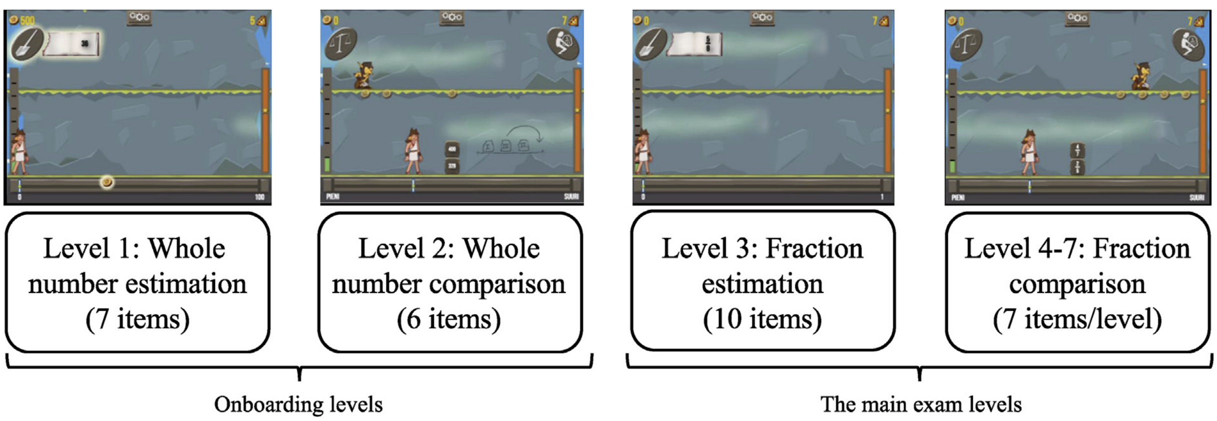

Fig. 2. The level structure of the Semideus Exam game world.

higher speed. Moreover, jumping length also depended on the angle of tilting; the steeper the tablet was tilted at the start of the jump the farther it was. Importantly, the possibility to alter walking speed on tablets was required as the tilting interface with appropriate static speed was not precise enough (based on earlier pilot studies) and self-paced slower movement allowed fine tuning of an estimation answer.

2.2.2. Tasks of the Semideus Exam game

The onboarding phase (cf. Fig. 2: Onboarding Levels) included a natural number estimation task involving seven items (Level 1: 85, 36, 69, 50, 75, 44, and 11) and a whole number comparison task with six items (Level 2: 328 vs. 400, 996 vs. 799, 202 vs. 501, 890 vs. 338, 167 vs. 266, and 621 vs. 468). The whole number tasks worked as onboarding phase of the assessment and were also used to

Fig. 3. Negative (left chart) and positive feedback (right chart) illustrated for a number line estimation task. A green bar is displayed during feedback to indicate the correct location. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

ensure that students got familiar with the user interface of the game.

The exam phase (cf. Fig. 2: Main Exam Levels) started with a fraction estimation task comprising ten items (Level 3: 1/8, 1/5, 1/3, 3/8, 4/9, 4/7, 5/8, 3/4, 5/6, and 8/9). The order of fraction estimation items was randomized across players. The second and major part of the exam phase (Levels 4–7) consisted of a fraction comparison task. This part comprised 28 fraction comparison items (see Table 1). The numerical magnitude of each of the fractions used in the comparison tasks was less than one. The fraction comparison task was conducted as a within-subject 2 × 2 design discerning the factors numerical distance (small vs. large) and consistency with whole number ordering (fraction type: consistent vs. inconsistent). Distance was considered as large if the difference between fractions was larger than 0.3. Fig. 1 (right chart) shows an example of the starting point of a fraction comparison task (distance: small & type: inconsistent).

The average numerical distance of small distance pairs of both consistent and inconsistent conditions was 0.14. The consistent large condition had an average distance of 0.36 and the inconsistent large condition had an average distance of 0.37. Overall the consistent condition had an average distance of 0.25 and the inconsistent condition had an average distance of 0.26 (for further details see Table 2). Moreover, the fraction comparison items were designed to have pairs that were on different sides of ½ (6 pairs/condition), the same side of ½ (6 pairs/condition), and include ½ (2 pairs/condition). These factors were equated across conditions and counterbalanced so that half of the time the larger fraction stone appeared on top of the smaller stone and the other half time the smaller fraction stone appeared on top of the larger stone.

2.3. Procedure

All students were tested in three groups during a regular school day. Initially, the experimenters introduced the game and explained the process of the game exam to the students. Afterwards, students got their personal passwords for the game that was used to record individual playing behaviour. After that, each student received an iPad and they played the game individually. Students were not allowed to discuss about the game tasks with other students during the playing session. They got 30 min to complete the game.

2.4. Analysis

Prior to analyses, response times were log10-transformed to normalize distributions. To test whether students performed worse in comparisons of fraction magnitudes (Hypothesis 1) than in comparisons of whole number magnitudes a paired *t*-test was conducted. By using a correlation analysis we evaluated the hypothesized positive relation between students' whole number comparison speed and their fraction comparison speed (Hypothesis 2). For the analysis of Hypothesis 3a, 3b, 4a, and 4b, performance in the fraction comparison task was analysed by a 2 × 2 ANOVA with the factors numerical distance (small vs. large) and fraction type (consistent vs. inconsistent). The expected associations between fraction comparison performance and fraction estimation accuracy (Hypothesis 5) as well as the associations of students' performance in fraction comparison (Hypothesis 6a) and fraction estimation accuracy (Hypothesis 6b) with general math achievement were evaluated using correlation analyses. Math achievement was measured by participants' math grade (following the Finnish classification scheme 10 reflects the best and 4 the lowest grade). However, since not all students provided their previous math grades 7 children had to be excluded for the respective analyses. ANOVAs, *t*-tests and correlations were conducted using R (R Core Team, 2016) and the R packages Ez (Anova; (Lawrence, 2015) and corrplot (Correlations; Wei & Simko, 2016)). Data visualization was realized with the package ggplot2 (Wickham, 2009).

Table 1
Fraction comparison pairs separated by fraction type (C vs. IC) and distance (small vs. large).

	Consistent			Inconsistent		
Small distance	1/2	vs.	4/7	2/5	vs.	1/2
	2/3	vs.	4/5	2/3	vs.	4/9
	5/9	vs.	3/8	2/9	vs.	1/3
	3/7	vs.	1/4	3/7	vs.	2/3
	4/7	vs.	2/5	5/8	vs.	3/4
	9/10	vs.	7/9	7/11	vs.	10/19
	3/11	vs.	5/12	6/19	vs.	5/12
Large distance	1/2	vs.	8/9	1/9	vs.	1/2
	2/5	vs.	5/7	3/9	vs.	3/4
	1/3	vs.	3/4	3/4	vs.	4/9
	6/11	vs.	11/13	3/9	vs.	2/3
	13/18	vs.	4/11	7/8	vs.	8/15
	8/17	vs.	1/16	12/22	vs.	11/12
	4/11	vs.	13/18	10/11	vs.	11/23

Table 2
Mean fraction comparison distances separated by fraction type (Consistent vs. Inconsistent) and distance (small vs. large).

	Mean distances		
	Overall	Numerator	Denominator
Consistent			
small distance	0.14	2.14	2.14
large distance	0.36	6.00	3.86
mean	0.25	4.07	3.00
Inconsistent			
small distance	0.14	1.57	5.43
large distance	0.37	0.71	7.43
mean	0.26	1.14	6.43

3. Results

3.1. Descriptive statistics - onboarding phase

Performance during the onboarding phase was used to study the adoption of the user interface (Whole Number Comparison and Whole Number Estimation). During the first three whole number estimation task items a coin on the ground indicated the correct location of the whole number magnitude on the number line. For these three items students achieved an accuracy of 98.87% ($SE = 0.09\%$). When the tutorial elements (a coin indicating the correct solution) were no longer present whole number estimation accuracy decreased (i.e., items 4–7: mean accuracy = 92.74%, $SE = 0.7\%$). Nevertheless, accuracy rates of >90% clearly indicated students ability to handle user interface and tilt control. In line with this claim, students' performance in whole number comparison ($M = 96.60\%$, $SE = 1.02\%$), as indicated by correctly solved comparisons, was very good. However, it seems that some students faced difficulties in the whole number comparison task. The accuracy of the first two comparison items was 94.44% ($SE = 2.16\%$) and at the end, i.e. last two comparisons, of the onboarding phase the accuracy increased to 97.22% ($SE = 1.57\%$). In order to explore reasons for mistakes made in the last two whole number comparison items, players' answers were analysed. The analysis revealed that all the mistakes were wrong because players ordered the stones from largest to smallest. We assume that this is a common careless error that happens also in paper based ordering tasks every now and then. Nevertheless, this finding indicates that the students could handle the stone carrying user interface. Based on these results we argue that the user interface of the game is appropriate for studying students' conceptual fraction knowledge.

3.2. Whole number and fraction comparison

3.2.1. Comparison accuracy

Using a paired t-test we examined potential differences in students' performance between fraction and whole number comparison with regard to the percentage of correctly solved comparisons. As hypothesized (Hypothesis 1), students performed significantly better [$t(53) = 8.39$, $p < 0.001$] in the whole number comparison task (performance: $M = 96.60\%$, $SE = 1.02\%$) as compared to the fraction comparison task (performance: $M = 78.37\%$, $SE = 2.04\%$).

3.2.2. Association of whole number and fraction number comparison speed

A correlation analysis was used to evaluate the expected positive association between whole number comparison speed and fraction comparison speed (Hypothesis 2). The correlation analysis substantiated our expectation revealing a high positive correlation between whole number comparison speed and fraction comparison speed [$r(52) = 0.76$, $p < 0.001$].

3.3. Fraction comparison

3.3.1. Comparison accuracy

The ANOVA revealed a significant main effect of numerical distance [Hypothesis 4a: $F(1,53) = 62.07$, $p < 0.001$] indicating that accuracy was higher for items with a large numerical distance ($M = 85.71\%$, $SE = 2.11\%$) as compared to items with a small numerical distance ($M = 71.03\%$, $SE = 2.42\%$). Neither the main effect of fraction types (Hypothesis 3a, consistent: $M = 79.23\%$, $SE = 1.77\%$ vs. inconsistent: $M = 77.51\%$, $SE = 2.87\%$) nor the interaction was significant (see left chart in Fig. 4).

3.3.2. Comparison speed

The ANOVA revealed significant main effects of numerical distance [Hypothesis 4b: $F(1, 53) = 12.45$, $p < 0.01$] and fraction type [Hypothesis 3b: $F(1, 53) = 18.11$, $p < 0.001$]. In line with our hypothesis, this indicated that responses were faster for items with a large as compared to a small numerical distance ($M = 5887$ ms, $SE = 199$ ms vs. $M = 6394$ ms, $SE = 231$ ms, respectively). However, contrary to our expectations consistent items were responded to slower than inconsistent items ($M = 6456$ ms, $SE = 228$ ms vs. $M = 5824$ ms, $SE = 201$ ms, respectively). The interaction between the fraction type and numerical distance was not significant (see right chart in Fig. 4).

3.4. Fraction comparison performance and fraction estimation accuracy with math achievement

3.4.1. Association of fraction comparison and fraction estimation performance

In line with our expectations (Hypothesis 5) the correlation analysis performed to examine the relation between fraction estimation and comparison indicated a high positive association between these two tasks [$r(52) = 0.69$, $p < 0.001$].

3.4.2. Association of fraction comparison performance and fraction estimation accuracy with math achievement

The correlation analyses revealed that student's fraction comparison performance [$r(45) = 0.57$, $p < 0.001$] as well as their fraction estimation accuracy [$r(45) = 0.71$, $p < 0.001$) were correlated positively and significantly with their general math achievement as indicated by their previous math grade (see Fig. 5).

4. Discussion and future work

The goal of this study was to examine whether a game with tilt control created with the Semideus rational number research engine can be used successfully to assess students' fraction understanding and processing. The current findings corroborate this claim. Our results were in line with most of our hypotheses regarding the processing of fraction magnitudes as derived from basic research on numerical cognition. In particular, the present data indicated that in-game measures provided reliable and valid information about students' conceptual fraction knowledge.

Descriptive analysis of the onboarding phase indicates that students were able to handle the user interface of the Semideus game including the tilt control (with accuracy >90%). While this appears trivial, successful handling of such a new and innovative user interface is crucial to perform valid assessment of students' performance.

To further explore whether Semideus provides reliable and valid

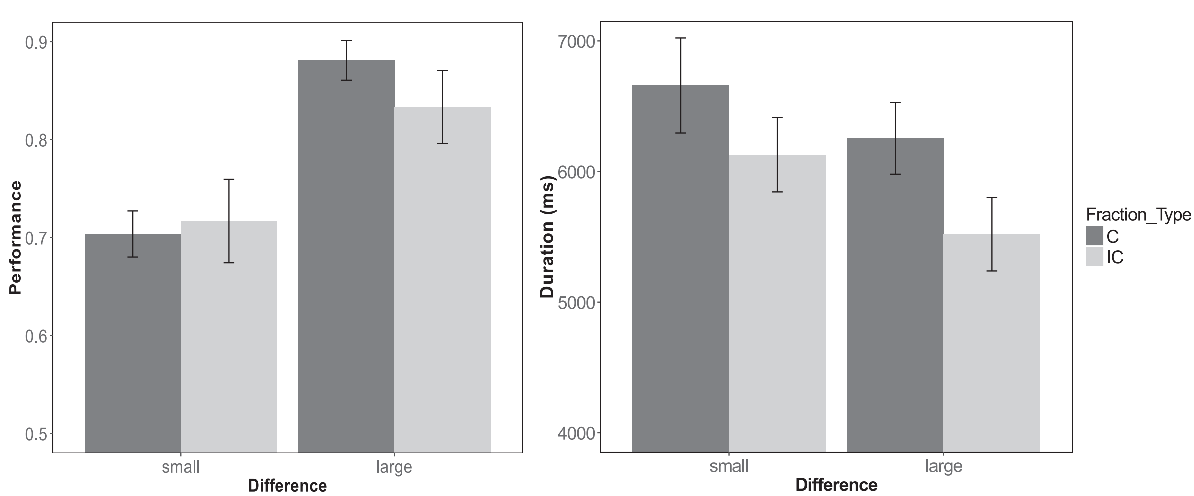

Fig. 4. Left chart: Mean performance for comparison condition (C=Consistent; IC=Inconsistent) and distance (small, large); Right chart: Mean duration in milliseconds (ms) for comparison condition (C=Consistent; IC=Inconsistent) and distance (small, large). Error bars depict the standard error of the mean.

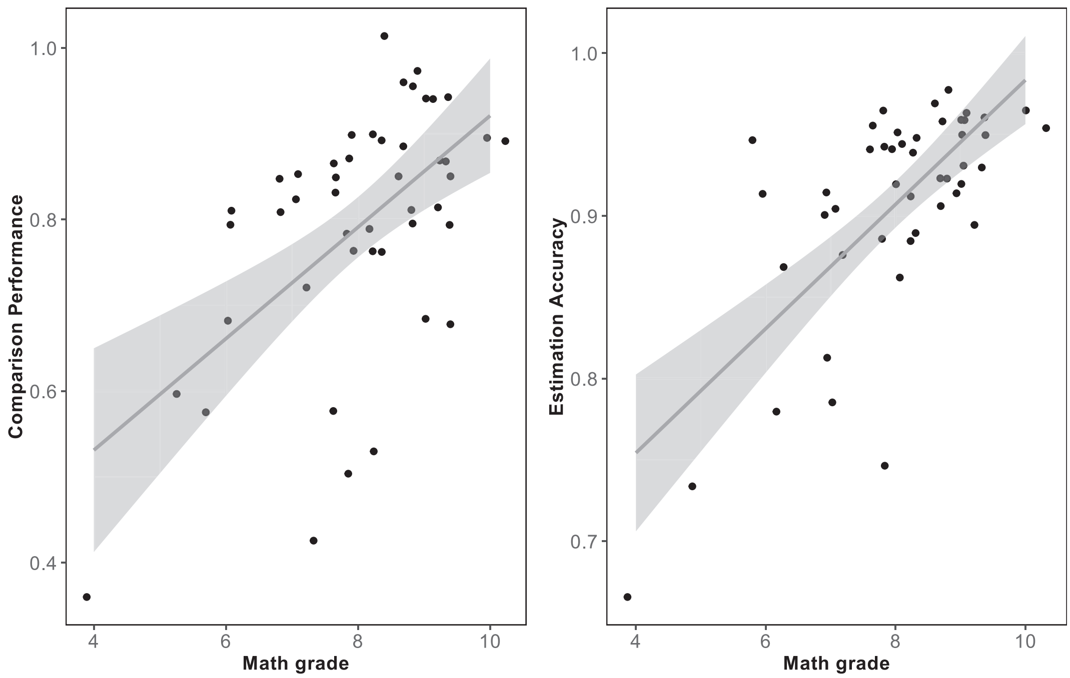

Fig. 5. Left chart: Correlation of mean performance of fraction comparison and math grade; Right chart: Correlation between mean accuracy for fraction estimation and math grade. Grey shaded area indicates 95% confidence region for the correlation.

data for the assessment of players' conceptual fraction knowledge six hypotheses were derived from previous basic research on numerical cognition. Thus, in the following, we will discuss the results in light of each of these hypotheses.

Consistent with *Hypothesis 1*, children performed worse in comparisons of fraction magnitudes than comparisons of whole number magnitudes. Research has shown, that whole number magnitudes are represented rather accurately across age groups and cultures (e.g., (Feigenson, Dehaene, & Spelke, 2004; Geary, 2006). Representations of fractions, however, are considered as a major challenge in mathematics education and even expert mathematicians show a tendency to be biased by properties of whole numbers, and thus show a whole number bias, when comparing the magnitude of fractions (Obersteiner et al., 2013).

Moreover, our correlation analysis evaluating *Hypothesis 2* revealed a strong association between students' speed with whole number comparisons and their speed with fraction comparison. This result is also consistent with other results identified with basic research studies and conventional non-game-based measures, which suggest that whole number skills are related to fraction skills (e.g.,Siegler et al., 2011).

Hypotheses 3 and 4 specifically addressed the comparison of fraction magnitude and the influence of (i) fraction type (i.e., the inconsistency of the fraction's magnitude with its whole number components) as well as (ii) numerical distance. With respect to the former, it is known that fraction processing in adults as well as students seems to be biased by the numerical values of numerators and denominators (e.g., the erroneous assumption that ¼ may be larger than $^1/_3$ because 4 is larger than 3). This phenomenon is referred to as whole number bias (Ni & Zhou, 2005) or natural number bias (Alibali & Sidney, 2015). Accordingly, we expected that students perform better and respond faster when comparing fraction magnitudes for which the magnitudes of numerators and denominators are consistent with whole number ordering as compared to fractions pairs for which the magnitudes of numerators and denominators are inconsistent (Vamvakoussi et al., 2012; Van Hoof et al., 2015). However, no effect of fraction type was found on comparison accuracy (*Hypothesis 3a*). In fact, participants were slower when solving consistent items as compared to inconsistent ones, which is in contrast to what we expected (*Hypothesis 3b*). Recent studies, however, have also revealed inconsistent results regarding the whole number bias on fraction magnitude comparison. For instance, DeWolf and Vosniadou (2015) observed a regular whole number bias for adult participants from the United States (i.e., inconsistent comparisons were responded to less accurately and slower). However, the authors also found that participants from Greece showed the opposite result pattern (i.e., a reversed consistency effect as in the present study on Finnish fifth graders). Importantly, the origin of these inconsistent findings is still not resolved.

One line of research, suggests that participants' educational background (e.g., US psychology students vs. Greek computer science students, DeWolf & Vosniadou, 2015) or the stimulus set used (Huber, Moeller, & Nuerk, 2014) may influence strategies used to compare the magnitude of fractions. In particular, holistic and componential strategies are differentiated. Using a holistic strategies in this context means that participants base their decision of which fraction is the larger one on considering the overall magnitude of the fractions [e.g., 3/8 (0.375) is smaller than 2/5 (0.4)]. In contrast, using a componential strategy would mean to rely on the componential magnitudes of numerators and denominators. For the above inconsistent example, this means that for numerators 3 is larger than 2 and for denominators 8 is larger than 5 which implies that 2/5 is smaller than 3/8, which is incorrect. The use of componential strategies is heavily influenced by task characteristics (e.g., Huber et al., 2014) with the numerical distances for numerators and denominators being critical. The latter might account for the reversed whole number bias effect in the current study. While the overall numerical distance was balanced across consistent and inconsistent fraction pairs (see Table 2) this was not the case for the numerical distances of numerators and denominators. In fact, numerical distances of numerators were larger for consistent as compared to inconsistent pairs, while this was reversed for inconsistent pairs. These unbalanced componential distances might have obscured the expected whole number bias.

With respect to the hypothesized distance effect our results were consistent with our *Hypotheses 4a and 4b*. We identified a significant distance effect in the fraction comparison tasks. Students' performance was less accurate and slower when distances between two fractions were small as compared to when they were large. This result suggests that students calculated or estimated and then compared the two fractions' overall numerical magnitudes utilizing a holistic processing strategy (e.g., Ischebeck, Schocke, & Delazer, 2009). Together with the results concerning Hypothesis 3a and 3b, present findings indicate that the students in this sample compared the magnitude of the fractions based on a combination of holistic and componential strategies (see also e.g. DeWolf & Vosniadou, 2015; M. Schneider & Siegler, 2010).

In line with our expectation that magnitude comparison as well as number line estimation tasks require an understanding of fraction magnitudes a high correlation ($r = 0.69$) between magnitude comparison performance and number line estimation accuracy was observed (*Hypothesis 5*).

As indicated earlier, fraction understanding has been shown to be relevant for math education in general (e.g., Siegler et al., 2013). Previous research indicated that high school students' fraction knowledge correlated very high with their actual mathematics achievement ($r > 0.8$) and also predicts future algebra and overall mathematics achievement in high school (e.g., Bailey et al., 2012; Booth & Newton, 2012). Consistent with *Hypotheses 6a and 6b* and this previous research, we found a strong positive correlation between performance in fraction comparison ($r = 0.57$) as well as fraction estimation accuracy ($r = 0.71$) and students' general math achievement as indicated by their previous math grades.

4.1. Implications

Taken together, present results suggest that a game-based assessment of students' fraction understanding realized with the Semideus game engine is capable of replicating relevant findings from basic research on numerical cognition. In turn, this suggests that students' performance while playing Semideus provides meaningful information on students' fraction understanding. Moreover, this also provides evidence about the usefulness and applicability of the game engine as an assessment and research tool. Additionally, these results extend our understanding of the nature of fraction knowledge suggesting that within multiple contexts, including paper-and-pencil (e.g. McMullen et al., 2014), computer-based basic tasks (e.g. Vamvakoussi et al., 2012), and now game-based number line tasks, students face similar difficulties with understanding fraction magnitudes. Importantly, this even holds when students use tilt-control to maneuver a game character on the number line.

The confirmation of Semideus as a reliable and useful instrument for examining students' fraction knowledge also provides the possibility that it may be used as a diagnostic tool for uncovering deficits in students' conceptual fraction knowledge. Such game-based assessment do not only provide less anxiety-inducing assessment methods, but can also provide teachers and educators with specific diagnostic information about students' performance in order to apply individual support. The opportunity to align gameplay with individual needs suggests that Semideus may be a potentially powerful tool for assessing and training fraction understanding.

Game-based digital solutions to study performance and user behaviour are deployed broadly in the educational sector. The results of the current study further justify the use of games in education. Importantly, in-game measures of conceptual fraction knowledge, that is fraction comparison performance and fraction estimation accuracy, were moderately to highly correlated with an external criterion of math achievement. This demonstration of the applicability and validity of these in-game measures supports the idea of leveraging serious games or game-based learning as a suitable means not only for fostering numerical knowledge but also using them as possible assessment tools (e.g. Baalsrud Hauge et al., 2015; Freire et al., 2016).

4.2. Limitations and future work

The present study yielded promising results regarding the use of a game-based application to assess conceptual knowledge of fractions. However, there are some limitations that should be considered in future studies. For instance, the present study did not examine students' fraction knowledge using a standardized curricular test, especially one that measures fraction magnitude understanding. Such a measure would be desirable for further validation of Semideus and would be helpful in assessing the limitations and affordances of such a game. In particular, this would provide more specific information on students' fraction understanding as compared to the more general index of math grades which we used in the current study.

Moreover, future studies may consider the use of real-time in-game measures or learning analytics, respectively, in more detail. Online analysis of performance and user behaviour provide valuable input for adaptive learning (e.g. Serrano-Laguna, Torrente, Moreno-Ger, & Fernández-Manjón, 2014; for a review see, Serrano-Laguna, Torrente, Moreno-Ger, & Fernández-Manjón, 2012; Bellotti, Kapralos, Lee, Moreno-Ger, & Berta, 2013). Automatic transition and selection of appropriate tasks for individual users would be desirable advanced solutions for learning environments. Moreover, the acquisition of (neuro-)physiological data during learning with games may provide even more information about users' internal states during learning and may also allow for in-game adaptations to the individual user (e.g., Ji, Zhu, & Lan, 2004; Ninaus, Kober, Friedrich, Neuper, & Wood, 2014; Pomplun & Sunkara, 2003; Witte, Ninaus, Kober, Neuper, & Wood, 2015; for reviews see; Ninaus, Kober, Friedrich, Dunwell, et al., 2014; Schneider, Börner, Van Rosmalen, & Specht, 2015).

Furthermore, the current study was not designed as an intervention study to evaluate how gameplay improved students' actual knowledge about fraction magnitudes. In other words, whether there are positive learning outcomes from playing Semideus needs to be evaluated in future studies. The on-going research on this seems promising, but a better controlled experimental design would be desirable to substantiate these results and to specifically answer this question. Within the context of such studies, online processes of motivation, enjoyment, engagement, and math anxiety should also be taken into consideration in order to further investigate the nature of the gaming effects on students' performance and learning.

5. Conclusions

In summary, the present study aimed at evaluating whether students' conceptual fraction knowledge can be assessed by a game-based application. In particular, our research engine Semideus employed number line estimation and magnitude comparison tasks on fractions to be solved using a tilting user interface. Implementation of these tasks in a game-based environment raised the question of whether these tasks yield valid measures of fraction magnitude understanding. The results of the present study suggested that this seems to be the case. We were successful in replicating relevant findings and hallmark effects from basic research on fraction understanding from the domain of numerical cognition using the Semideus game engine. Moreover, in-game performance measures were observed to be highly correlated with an external criterion of general mathematics achievement. These results provide first evidence suggesting that game-based assessment is capable of providing valid information on students' conceptual fraction knowledge.

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