Vector Algebra

1. **Problem statement :** The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one find the value of λ
   Solution:

   Let

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} \tag{1}$$

   The sum of vectors

$$(\mathbf{b}+\mathbf{c}) = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} + \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix} \tag{2}$$

$$\tag{3}$$

   Let $\mathbf{r}$ to be the unit vector along with $(\mathbf{b}+\mathbf{c})$ and $\mathbf{a}$

$$\hat{\mathbf{r}} = \frac{(\mathbf{b}+\mathbf{c})\cdot\mathbf{a}}{\|(\mathbf{b}+\mathbf{c})\cdot\mathbf{a}\|} \tag{4}$$

   $\hat{\mathbf{r}} = 1$

$$1 = \frac{\begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix} . \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}{\left\| \begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\|} \tag{5}$$

$$1 = \frac{2+\lambda+6-2}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}} \tag{6}$$

$$\tag{7}$$

Squaring on both sides

$$\left(\sqrt{(2+\lambda)^2+6^2+(-2)^2}\right)^2 = (\lambda+6)^2 \tag{8}$$

$$4+\lambda^2+4\lambda+36+4 = \lambda^2+36+12\lambda \tag{9}$$

$$44+4\lambda = 36+12\lambda \tag{10}$$

$$8 = 8\,(\lambda) \tag{11}$$

$$\lambda = 1 \tag{12}$$

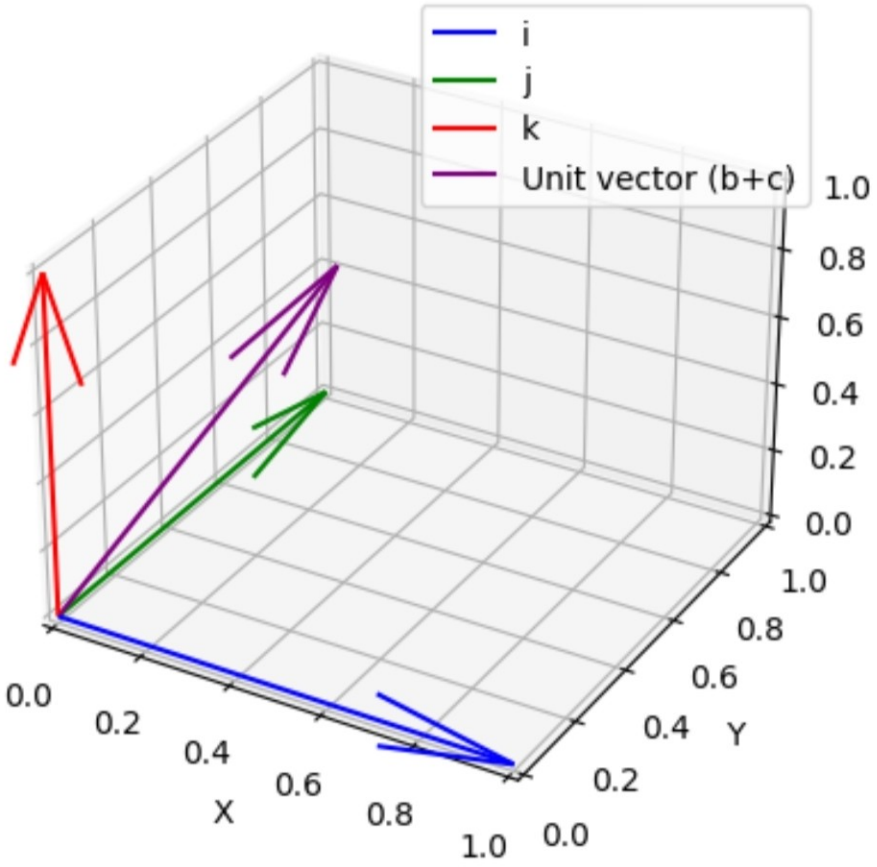

Figure 1