

Vector Algebra

1. **Problem statement :** The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one, Find the value of λ .

Solution:

Let

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} \quad (1)$$

Then

$$(\mathbf{b} + \mathbf{c}) = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} + \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} \quad (2)$$

$$\|\mathbf{b} + \mathbf{c}\| = \sqrt{\lambda^2 + 4\lambda + 44} \quad (3)$$

$$(4)$$

Let \mathbf{r} be the unit vector along with $(\mathbf{b} + \mathbf{c})$

$$\hat{\mathbf{r}} = \frac{(\mathbf{b} + \mathbf{c})}{\|\mathbf{b} + \mathbf{c}\|} \quad (5)$$

Given $\mathbf{a}^\top (\hat{\mathbf{r}}) = 1$

$$\mathbf{a}^\top \frac{(\mathbf{b} + \mathbf{c})}{\|\mathbf{b} + \mathbf{c}\|} = 1 \quad (6)$$

$$\mathbf{a}^\top (\mathbf{b} + \mathbf{c}) = \|\mathbf{b} + \mathbf{c}\| \quad (7)$$

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} = \sqrt{\lambda^2 + 4\lambda + 44} \quad (8)$$

$$\lambda + 6 = \sqrt{\lambda^2 + 4\lambda + 44} \quad (9)$$

$$(10)$$

Squaring on both sides

$$(\lambda + 6)^2 = \left(\sqrt{\lambda^2 + 4\lambda + 44} \right)^2 \quad (11)$$

$$\lambda^2 + 12(\lambda) + 36 = \lambda^2 + 4\lambda + 44 \quad (12)$$

$$8(\lambda) = 8 \quad (13)$$

$$\lambda = 1 \quad (14)$$