CBSE MATH

Made Simple

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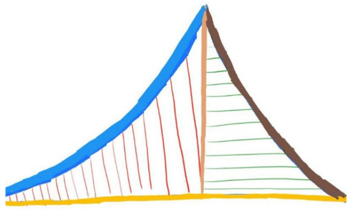

Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Intersection of Conics

1.1. Chords

1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

OR

2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

3. (a) Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

OR

(b) If the area of the regin bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\dfrac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

4. Find the area of the region enclosed by the curves $y^2 = x$, $x = \dfrac{1}{4}$, $y = 0$ and $x = 1$, using integration.

5. If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.

6. (a) Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

OR

(b) Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration.

7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

1.2. Curves

Chapter 2

Tangent And Normal

1. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.

2.1. Construction

Chapter 3

Vectors

3.1. Product vectors

1. $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that

$$\left|2\overrightarrow{a}+3\overrightarrow{b}\right| = \left|3\overrightarrow{a}-2\overrightarrow{b}\right|. \tag{3.1}$$

Find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

2. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that

$$\overrightarrow{a} = \hat{i} - \hat{j} + \hat{k} \tag{3.2}$$

and

$$\overrightarrow{b} = 2\hat{i} - \hat{j} - 3\hat{k} \tag{3.3}$$

then find the vector $\overrightarrow{c}$, given that

$$\overrightarrow{a} \times \overrightarrow{c} = \overrightarrow{b} \tag{3.4}$$

and

$$\overrightarrow{a}.\overrightarrow{c} = 4. \tag{3.5}$$

3.

$$If \left|\overrightarrow{a} \times \overrightarrow{b}\right|^2 + \left|\overrightarrow{a}.\overrightarrow{b}\right|^2 = 400 \tag{3.6}$$

and

$$\left|\overrightarrow{b}\right| = 5 \tag{3.7}$$

find the value of $|\overrightarrow{a}|$.

4. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a}.\overrightarrow{b} = 1 \tag{3.8}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k} \tag{3.9}$$

, then find $\left|\overrightarrow{b}\right|$

5. If

$$|\overrightarrow{a}| = 3, \left|\overrightarrow{b}\right| = 2\sqrt{3} \tag{3.10}$$

and

$$\overrightarrow{a}.\overrightarrow{b} = 6, \tag{3.11}$$

then find the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$.

6. $|\overrightarrow{a}| = 8, \left|\overrightarrow{b}\right| = 3$ and $\overrightarrow{a}.\overrightarrow{b} = 12\sqrt{3}$, then the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$ is
   (a) 24
   (b) 144
   (c) 2
   (d) 12

7. If

$$\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{3.12}$$

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{3.13}$$

, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

8. $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ and $\overrightarrow{d}$ are four non-zeros vectors such that $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c} \times \overrightarrow{d}$ and

$$\overrightarrow{a} \times \overrightarrow{c} = 4\overrightarrow{b} \times \overrightarrow{d} \tag{3.14}$$

, then show that $(\overrightarrow{a} - 2\overrightarrow{d}$ is parallel to$(2\overrightarrow{b} - \overrightarrow{c})$ where

$$\overrightarrow{a} \neq 2\overrightarrow{d}, \overrightarrow{c} \neq 2\overrightarrow{b} \tag{3.15}$$

9. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a}.\overrightarrow{b} = 1 \tag{3.16}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k}, \tag{3.17}$$

then find $\left|\overrightarrow{b}\right|$

10. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that

$$\left|\overrightarrow{a} + \overrightarrow{b}\right| = \left|\overrightarrow{b}\right|, \tag{3.18}$$

then prove that $(\overrightarrow{a} + 2\overrightarrow{b})$ is perpendicular to $\overrightarrow{a}$.

11. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are unit vectors and θ is the angle between them , then prove that sin

$$\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{3.19}$$

12. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that and θ is the ang le between

them, then prove that

$$sin\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{3.20}$$

3.2. Projection vectors

13. If

$$\overrightarrow{a} = 2\hat{i} + y\hat{j} + \hat{k} \tag{3.21}$$

and

$$\overrightarrow{b} = \hat{i} + 2\hat{j} + 3\hat{k} \tag{3.22}$$

are two vectors for which the vector $(\overrightarrow{a} + \overrightarrow{b})$ is perpendicular to the vector $(\overrightarrow{a} - \overrightarrow{b})$ then find all the possible values of y.

14. Write the projection of the vector $(\overrightarrow{b} + \overrightarrow{c})$ on the vector $\overrightarrow{a}$, where

$$\overrightarrow{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + 2\hat{j} - 2\hat{k} \tag{3.23}$$

and

$$\overrightarrow{c} = 2\hat{i} - \hat{j} + 4\hat{k}. \tag{3.24}$$

15. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + \hat{j} - 2\hat{k} \tag{3.25}$$

and

$$\overrightarrow{c} = \hat{i} + 3\hat{j} - \hat{k} \tag{3.26}$$

and the projection of vector $\overrightarrow{c} + \lambda\overrightarrow{b}$ on vector $\overrightarrow{a}$ is $2\sqrt{6}$, find the value of λ.

16. If $\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{3.27}$$

, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

17. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + 2\hat{k} \tag{3.28}$$

and

$$\overrightarrow{b} = 5\hat{i} - 3\hat{j} - 4\hat{k} \tag{3.29}$$

, then find the ratio $\dfrac{projection of vector \overrightarrow{a} on vector \overrightarrow{b}}{projection of vector \overrightarrow{b} on vector \overrightarrow{a}}$

18. Show that the three vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$, and $3\hat{i} - 4\hat{j} - 4\hat{k}$

form the vertices of a right-angled triangle. If $\overrightarrow{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\overrightarrow{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} \tag{3.30}$$

are such that the vector $(\overrightarrow{a} + \lambda\overrightarrow{b})$ is perpendicular to vector $\overrightarrow{c}$, then find the value of λ.

3.3. Position vectors

19. If $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are the position vectors of the points $\mathbf{A}(2, 3, -4)$, $\mathbf{B}(3, -4, -5)$ and $\mathbf{C}(3, 2, -3)$ and respectively, then $\left|\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}\right|$ is equal to

    (a) $\sqrt{113}$

    (b) $\sqrt{185}$

    (c) $\sqrt{203}$

    (d) $\sqrt{209}$

3.4. Section formula

20. **A** circle has its center at $(4, 4)$. If one end ofa diameter is $(4, 0)$, then find the coordinates ofother end.

3.5. Plane vectors

21. Find the values λ, for which the distance of point $(2, 1, \lambda)$ from plane

$$3x + 5y + 4z = 11 \tag{3.31}$$

is $2\sqrt{2}$ units.

22. Find the coordinates of the point where the line through $(3, 4, 1)$ crosses the ZX-plane

3.6. Geometry vectors

23. Using vectors, find the area of the triangle withvertices $\mathbf{A}(-1, 0, -2)$, $\mathbf{B}(0, 2, 1)$ and $\mathbf{C}(-1, 4, 1)$

24. Using integration, find the area of triangle region whose vertices are $(2, 0)$, $(4, 5)$ and $(1, 4)$.

3.7. Distance formula

25. The distance between the points $(0, 0)$ and $(a - b, a + b)$ is

    (a) $2\sqrt{ab}$

    (b) $\sqrt{2a^2 + ab}$

    (c) $2\sqrt{a^2 + b^2}$

(d) $\sqrt{2a^2 + 2b^2}$

26. The value of m which makes the point $(0, 0)$, $(2m, -4)$and $(3, 6)$ collinear, is ____

3.8. Direction vectors

27. If a line makes $60°$ and $45°$ angles with the positive directions of X-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direct6on cosines of the line.

28. The Cartesian equation of a line AB is :

$$\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3} \tag{3.32}$$

.

29. Find the directions cosines of a line parallel to line AB.

30. Find the direction cosines of a line whose cartesian equation is given as

$$3x + 1 = 6y - 2 = 1 - z. \tag{3.33}$$

31. A vector of magnitude 9 units in the direction of the vector $-2\hat{i}-\hat{j}+2\hat{k}$ is ____

3.9. Diagonal vectors

32. The two adajacent sides of a parallelogram are represented by $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.

33. The two adjacent sides of a parallelogram are represented by vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also,find the area of the parallelogram.

34. If

$$\overrightarrow{a} = \overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k} \tag{3.34}$$

and

$$\overrightarrow{b} = 2\hat{i} + 4\hat{j} - 5\hat{k} \tag{3.35}$$

represent two adjacent sides of a parallelogram, then find the unit vector parallel to the diagonal of the parallelogram

3.10. Area of triangle

35. Find the area of the quadrilateral $ABCD$ whose vertices are $\mathbf{A}(-4,-3)$, $\mathbf{B}(3,-1)$, $\mathbf{C}(0,5)$ and $\mathbf{D}(-4,2)$

36. If the points $\mathbf{A}(2,0)$, $\mathbf{B}(6,1)$, and $\mathbf{C}(p,q)$ form a triangle of area 12sq.

units (positive only) and

$$2p + q = 10, \tag{3.36}$$

then find the values of p and q.