
CBSE MATH

Made Simple

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Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

1.1. Product vectors

1. \vec{a} and \vec{b} are two unit vectors such that

$$\left| 2\vec{a} + 3\vec{b} \right| = \left| 3\vec{a} - 2\vec{b} \right|. \quad (1.1)$$

Find the angle between \vec{a} and \vec{b} .

2. If \vec{a} and \vec{b} are two vectors such that

$$\vec{a} = \hat{i} - \hat{j} + \hat{k} \quad (1.2)$$

and

$$\vec{b} = 2\hat{i} - \hat{j} - 3\hat{k} \quad (1.3)$$

then find the vector \vec{c} , given that

$$\vec{a} \times \vec{c} = \vec{b} \quad (1.4)$$

and

$$\vec{a} \cdot \vec{c} = 4. \quad (1.5)$$

3.

$$\text{If } \left| \vec{a} \times \vec{b} \right|^2 + \left| \vec{a} \cdot \vec{b} \right|^2 = 400 \quad (1.6)$$

and

$$\left| \vec{b} \right| = 5 \quad (1.7)$$

find the value of $|\vec{a}|$.

4. If

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{a} \cdot \vec{b} = 1 \quad (1.8)$$

and

$$\vec{a} \times \vec{b} = \hat{j} - \hat{k} \quad (1.9)$$

, then find $\left| \vec{b} \right|$

5. If

$$|\vec{a}| = 3, \left| \vec{b} \right| = 2\sqrt{3} \quad (1.10)$$

and

$$\vec{a} \cdot \vec{b} = 6, \quad (1.11)$$

then find the value of $\left| \vec{a} \times \vec{b} \right|$.

6. $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 12\sqrt{3}$, then the value of $\left| \vec{a} \times \vec{b} \right|$ is

(a) 24

(b) 144

(c) 2

(d) 12

7. If

$$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k} \quad (1.12)$$

and

$$\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k} \quad (1.13)$$

, then find $\vec{a} \cdot (\vec{b} \times \vec{c})$.

8. \vec{a} , \vec{b} , \vec{c} and \vec{d} are four non-zeros vectors such that $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$

and

$$\vec{a} \times \vec{c} = 4\vec{b} \times \vec{d} \quad (1.14)$$

, then show that $(\vec{a} - 2\vec{d})$ is parallel to $(2\vec{b} - \vec{c})$ where

$$\vec{a} \neq 2\vec{d}, \vec{c} \neq 2\vec{b} \quad (1.15)$$

9. If

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{a} \cdot \vec{b} = 1 \quad (1.16)$$

and

$$\vec{a} \times \vec{b} = \hat{j} - \hat{k}, \quad (1.17)$$

then find $|\vec{b}|$

10. If \vec{a} and \vec{b} are two vectors such that

$$|\vec{a} + \vec{b}| = |\vec{b}|, \quad (1.18)$$

then prove that $(\vec{a} + 2\vec{b})$ is perpendicular to \vec{a} .

11. If \vec{a} and \vec{b} are unit vectors and θ is the angle between them, then prove that \sin

$$\frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}| \quad (1.19)$$

12. If \vec{a} and \vec{b} are two unit vectors such that and θ is the angle between

them, then prove that

$$\sin \frac{\theta}{2} = \frac{1}{2} \left| \vec{a} - \vec{b} \right| \quad (1.20)$$

1.2. Projection vectors

13. If

$$\vec{a} = 2\hat{i} + y\hat{j} + \hat{k} \quad (1.21)$$

and

$$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k} \quad (1.22)$$

are two vectors for which the vector $(\vec{a} + \vec{b})$ is perpendicular to the vector $(\vec{a} - \vec{b})$ then find all the possible values of y .

14. Write the projection of the vector $(\vec{b} + \vec{c})$ on the vector \vec{a} , where

$$\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k} \quad (1.23)$$

and

$$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}. \quad (1.24)$$

15. If

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + \hat{j} - 2\hat{k} \quad (1.25)$$

and

$$\vec{c} = \hat{i} + 3\hat{j} - \hat{k} \quad (1.26)$$

and the projection of vector $\vec{c} + \lambda \vec{b}$ on vector \vec{a} is $2\sqrt{6}$, find the value of λ .

16. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\hat{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and

$$\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k} \quad (1.27)$$

, then find $\vec{a} \cdot (\vec{b} \times \vec{c})$.

17. If

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k} \quad (1.28)$$

and

$$\vec{b} = 5\hat{i} - 3\hat{j} - 4\hat{k} \quad (1.29)$$

, then find the ratio $\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}}$

18. Show that the three vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$, and $3\hat{i} - 4\hat{j} - 4\hat{k}$

form the vertices of a right-angled triangle. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and

$$\vec{c} = 3\hat{i} + \hat{j} \quad (1.30)$$

are such that the vector $(\vec{a} + \lambda \vec{b})$ is perpendicular to vector \vec{c} , then find the value of λ .

1.3. Position vectors

19. If \vec{a} , \vec{b} and \vec{c} are the position vectors of the points $\mathbf{A}(2, 3, -4)$, $\mathbf{B}(3, -4, -5)$ and $\mathbf{C}(3, 2, -3)$ and respectively, then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to

(a) $\sqrt{113}$

(b) $\sqrt{185}$

(c) $\sqrt{203}$

(d) $\sqrt{209}$

1.4. Section formula

20. A circle has its center at $(4, 4)$. If one end of a diameter is $(4, 0)$, then find the coordinates of the other end.

1.5. Plane vectors

21. Find the values λ , for which the distance of point $(2, 1, \lambda)$ from plane

$$3x + 5y + 4z = 11 \quad (1.31)$$

is $2\sqrt{2}$ units.

22. Find the coordinates of the point where the line through $(3, 4, 1)$ crosses the ZX-plane

1.6. Geometry vectors

23. Using vectors, find the area of the triangle with vertices $\mathbf{A}(-1, 0, -2)$, $\mathbf{B}(0, 2, 1)$ and $\mathbf{C}(-1, 4, 1)$
24. Using integration, find the area of triangle region whose vertices are $(2, 0)$, $(4, 5)$ and $(1, 4)$.

1.7. Distance formula

25. The distance between the points $(0, 0)$ and $(a - b, a + b)$ is

(a) $2\sqrt{ab}$

(b) $\sqrt{2a^2 + ab}$

(c) $2\sqrt{a^2 + b^2}$

(d) $\sqrt{2a^2 + 2b^2}$

26. The value of m which makes the point $(0, 0)$, $(2m, -4)$ and $(3, 6)$ collinear, is _____

1.8. Direction vectors

27. If a line makes 60° and 45° angles with the positive directions of X-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line.

28. The Cartesian equation of a line AB is :

$$\frac{2x - 1}{12} = \frac{y + 2}{2} = \frac{z - 3}{3} \quad (1.32)$$

.

29. Find the direction cosines of a line parallel to line AB .
30. Find the direction cosines of a line whose cartesian equation is given as

$$3x + 1 = 6y - 2 = 1 - z. \quad (1.33)$$

31. A vector of magnitude 9 units in the direction of the vector $-2\hat{i} - \hat{j} + 2\hat{k}$ is _____

1.9. Diagonal vectors

32. The two adjacent sides of a parallelogram are represented by $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.
33. The two adjacent sides of a parallelogram are represented by vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find the area of the parallelogram.
34. If

$$\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k} \quad (1.34)$$

and

$$\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k} \quad (1.35)$$

represent two adjacent sides of a parallelogram, then find the unit vector parallel to the diagonal of the parallelogram

1.10. Area of triangle

35. Find the area of the quadrilateral $ABCD$ whose vertices are $\mathbf{A}(-4, -3)$, $\mathbf{B}(3, -1)$, $\mathbf{C}(0, 5)$ and $\mathbf{D}(-4, 2)$
36. If the points $\mathbf{A}(2, 0)$, $\mathbf{B}(6, 1)$, and $\mathbf{C}(p, q)$ form a triangle of area 12sq.

units (positive only) and

$$2p + q = 10, \tag{1.36}$$

then find the values of p and q .

Chapter 2

Linear Forms

1. Solve the equations $x + 2y = 6$ and $2x - 5y = 12$ graphically.
2. Solve the following equations for x and y using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \quad (2.1)$$

$$(bx + ay) + (b - 4a) = 0 \quad (2.2)$$

3. Find the co-ordinates of the point where the line $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ crosses the plane passing through the points $\left(\frac{7}{2}, 0, 0\right), (0, 7, 0), (0, 0, 7)$.

4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

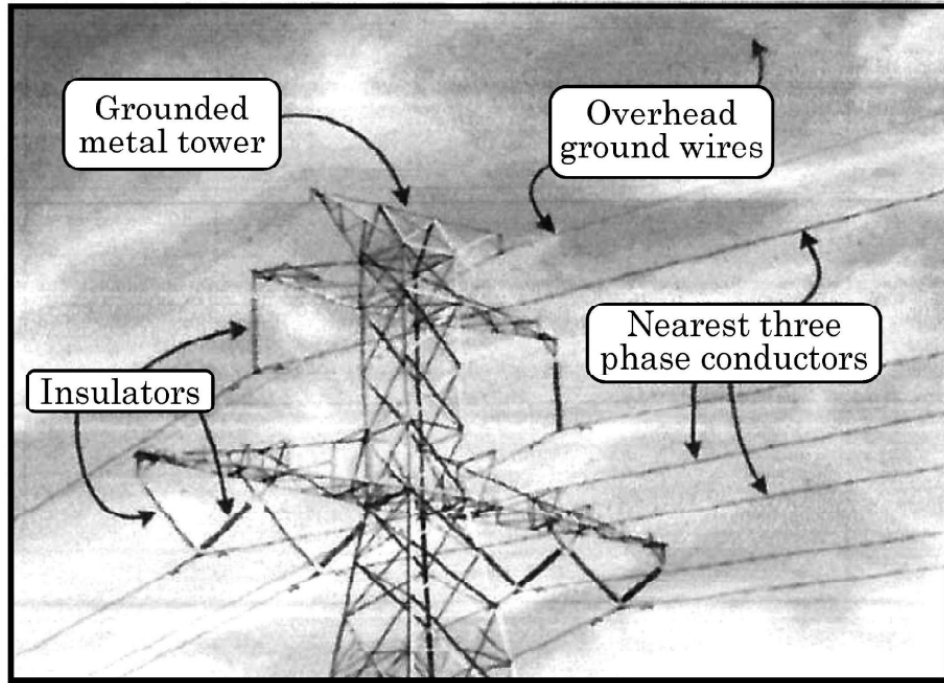


Figure 2.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 2.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \quad (2.3)$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \quad (2.4)$$

Based on the given information, answer the following questions:

- (a) Are the l_1 and l_2 coplanar? Justify your answer.
 - (b) Find the point of intersection of lines l_1 and l_2 .
5. Write the cartesian equation of the line PQ passing through points P(2, 2, 1) and Q(5, 1, -2). Hence, find the y-coordinate of the point on

the line PQ whose z-coordinate is -2.

6. Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$.

7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k}) \quad (2.5)$$

$$\mathbf{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k}) \quad (2.6)$$

8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 2.2) represented by the following lines

$$\mathbf{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k}) \quad (2.7)$$

$$\mathbf{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k}) \quad (2.8)$$

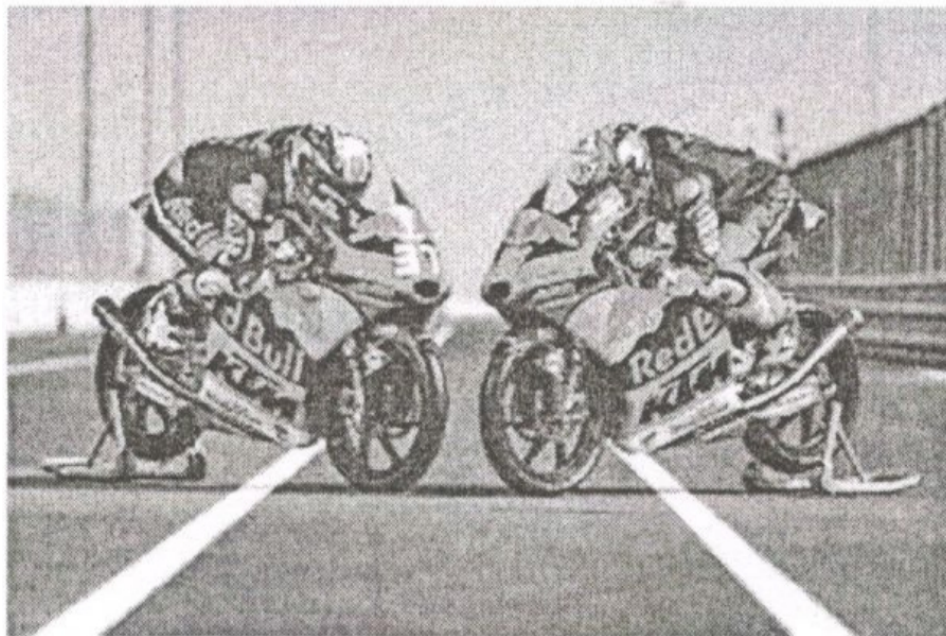


Figure 2.2: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

- (a) Find the shortest distance between the given lines.
- (b) Find a point at which the motorcycles may collide.

9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{i} + (\lambda + 4)\hat{j} - (\lambda - 3)\hat{k} \quad (2.9)$$

$$\mathbf{r} = (3 - \mu)\hat{i} + (2\mu + 2)\hat{j} + (\mu + 6)\hat{k} \quad (2.10)$$

10. Find the shortest distance between the following lines and hence write

whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \quad (2.11)$$

1. If the distance of the point $(1, 1, 1)$ from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ .
2. Find the distance of the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$.
3. Find the distance of the point $P(4, 3, 2)$ from the plane determined by the points $A(-1, 6, -5)$, $B(-5, -2, 3)$ and $C(2, 4, -5)$.
4. The distance of the line

$$\mathbf{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + 5\hat{j} + \hat{k}) \quad (2.12)$$

from the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + 4\hat{k}) = 5 \quad (2.13)$$

is

- (a) $\sqrt{2}$
- (b) $\frac{1}{\sqrt{2}}$
- (c) $\frac{1}{3\sqrt{2}}$
- (d) $\frac{-2}{3\sqrt{2}}$

5. Find a unit vector perpendicular to each of the vectors $(\mathbf{a} + \mathbf{b})$ and $(\mathbf{a} - \mathbf{b})$ where

$$\mathbf{a} = \hat{i} + \hat{j} + \hat{k} \quad (2.14)$$

$$\mathbf{b} = \hat{i} + 2\hat{j} + 3\hat{k} \quad (2.15)$$

1. Find the equation of the plane passing through the points $(2, 1, 0)$, $(3, -2, -2)$ and $(1, 1, 7)$. Also, obtain its distance from the origin.
2. The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane.
3. Find the cartesian and the vector equation of a plane which passes through the point $(3, 2, 0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.
4. The distance between the planes $4x-4y+2z+5=0$ and $2x-2y+z+6=0$ is
 - (a) $\frac{1}{6}$
 - (b) $\frac{7}{6}$
 - (c) $\frac{11}{6}$
 - (d) $\frac{16}{6}$

5. Find the equation of the plane through the line of intersection of the

planes

$$\mathbf{r} \cdot (\hat{i} + 3\hat{j}) + 6 = 0 \quad (2.16)$$

$$\mathbf{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0 \quad (2.17)$$

which is at a unit distance from the origin.

1. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \quad (2.18)$$

and the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10. \quad (2.19)$$

2. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.
3. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.
4. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 2.1:

Table 2.1: Table showing yearly trend of production of goods in lakh tonnes

| Year | Production (in lakh tonnes) |
|------|-----------------------------|
| 2001 | 30 |
| 2002 | 35 |
| 2003 | 36 |
| 2004 | 32 |
| 2005 | 37 |
| 2006 | 40 |

1. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.

Chapter 3

Intersection of Conics

1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.
2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.
3. Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.
4. If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a , where $a > 0$.
5. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration.
6. If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m , using integration.
7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a , using integration.

8. Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.
9. Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration

Chapter 4

Tangent And Normal

1. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.
1. Draw a circle of radius 2.5 cm. Take a point **P** outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point **P**.
2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point **P**, at a distance of 7 cm from its center **O**.
3. In Figure 4.1, there are two concentric circles with centre **O**. If ARC and AQB are tangents to the smaller circle from the point **A** lying on the larger circle, find the length of AC , if $AQ = 5$ cm.

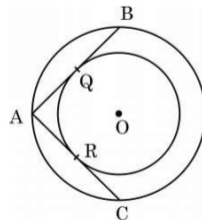


Figure 4.1: Two concentric circles with **O** as centre

4. In Figure 4.2, if a circle touches the side QR of $\triangle PQR$ at S and extended sides PQ and PR at M and N , respectively,

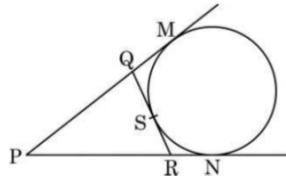


Figure 4.2: Two tangents are drawn from point P to the circle

prove that $PM = \frac{1}{2}(PQ + QR + PR)$

5. In Figure 4.3, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 8 cm respectively. If the area of $\triangle ABC$ is 84 cm^2 , find the lengths of sides AB and AC .

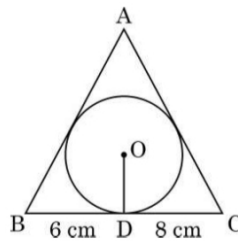


Figure 4.3: Circle with O as center circumscribed in triangle ABC

6. In Figure 4.4, PQ and PR are tangents to the circle centered at O . If $\angle OPR = 45^\circ$, then prove that $ORPQ$ is a square.

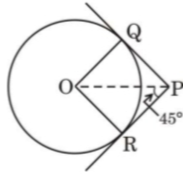


Figure 4.4: Two tangents drawn from point **P** to a circle whose centre is **O**

7. In Figure 4.5, **O** is the centre of a circle of radius 5 cm. PA and BC are tangents to the circle at **A** and **B** respectively. If OP is 13 cm, then find the length of tangents PA and BC .

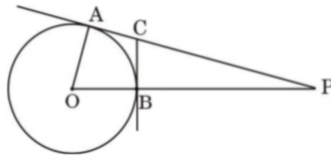


Figure 4.5: Two tangents drawn from point **C** to a circle whose centre is **O**

8. In Figure 4.6, AB is diameter of a circle centered at **O**. BC is tangent to the circle at **B. If OP bisects the chord AD and $\angle AOP = 60^\circ$, then find $m\angle C$.**

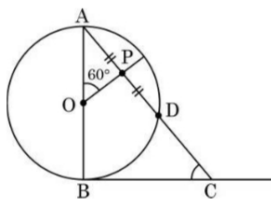


Figure 4.6: Tangent BC is drawn from point **C** to a circle whose centre is **O**

9. In Figure 4.7, XAY is a tangent to the circle centered at **O**. If $\angle ABO =$

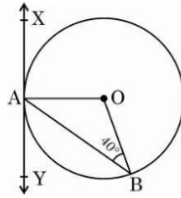


Figure 4.7: The line XAY is tangent to the circle centered at O

60° , then find $m\angle BAY$ and $m\angle AOB$.

10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
11. In Figure 4.8, a triangle ABC with $\angle B = 90^\circ$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point P . Prove that the tangent drawn at point P bisects BC .

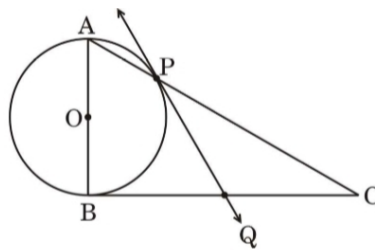


Figure 4.8: PQ is tangent to the circle centered at O . AB is the diameter and $\angle B = 90^\circ$

4.1. Construction