CBSE MATH

Made Simple

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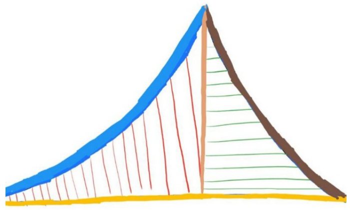

Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Intersection of Conics

1.1. Chords

1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

OR

2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

3. (a) Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

OR

(b) If the area of the regin bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\dfrac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

4. Find the area of the region enclosed by the curves $y^2 = x$, $x = \dfrac{1}{4}$, $y = 0$ and $x = 1$, using integration.

5. If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.

6. (a) Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

OR

(b) Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration.

7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

1.2. Curves

Chapter 2

Tangent And Normal

1. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.

2.1. Construction

Chapter 3

Vectors

3.1. projection vectors

1. If $\overrightarrow{a} = 2\hat{i} + y\hat{j} + \hat{k}$ and $\overrightarrow{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are two vectors for which the vector $(\overrightarrow{a} + \overrightarrow{b})$ is perpendicular to the vector $(\overrightarrow{a} - \overrightarrow{b})$ then find all the possible values of y.

2. Write the projection of the vector $(\overrightarrow{b} + \overrightarrow{c})$ on the vector $\overrightarrow{a}$, where $\overrightarrow{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\overrightarrow{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\overrightarrow{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

3. If $\overrightarrow{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\overrightarrow{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{c} = \hat{i} + 3\hat{j} - \hat{k}$ and the projection of vector $\overrightarrow{c} + \lambda\overrightarrow{b}$ on vector $\overrightarrow{a}$ is $2\sqrt{6}$, find the value of λ.

4. If $\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k}$, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

5. If $\overrightarrow{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{b} = 5\hat{i} - 3\hat{j} - 4\hat{k}$, then find the ratio $\dfrac{projection of vector\, \overrightarrow{a}\, on vector\, \overrightarrow{b}}{projection of vector\, \overrightarrow{b}\, on vector\, \overrightarrow{a}}$

6. Show that the three vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$, and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right-angled triangle. If $\overrightarrow{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\overrightarrow{b} =$

$-\hat{i}+2\hat{j}+\hat{k}$ and $\overrightarrow{c}=3\hat{i}+\hat{j}$ are such that the vector $(\overrightarrow{a}+\lambda\overrightarrow{b})$ is perpendicular to vector $\overrightarrow{c}$, then find the value of λ.

3.2. product vectors

1. $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that $\left|2\overrightarrow{a}+3\overrightarrow{b}\right|=\left|3\overrightarrow{a}-2\overrightarrow{b}\right|$. Find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

2. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that $\overrightarrow{a}=\hat{i}-\hat{j}+\hat{k}$ and $\overrightarrow{b}=2\hat{i}-\hat{j}-3\hat{k}$ then find the vector $\overrightarrow{c}$, given that $\overrightarrow{a}\times\overrightarrow{c}=\overrightarrow{b}$ and $\overrightarrow{a}.\overrightarrow{c}=4$.

3. If $\left|\overrightarrow{a}\times\overrightarrow{b}\right|^2+\left|\overrightarrow{a}.\overrightarrow{b}\right|^2=400$ and $\left|\overrightarrow{b}\right|=5$, find the value of $|\overrightarrow{a}|$.

4. If $\overrightarrow{a}=\hat{i}+\hat{j}+\hat{k}$, $\overrightarrow{a}.\overrightarrow{b}=1$ and $\overrightarrow{a}\times\overrightarrow{b}=\hat{j}-\hat{k}$, then find $\left|\overrightarrow{b}\right|$

5. If $|\overrightarrow{a}|=3$, $\left|\overrightarrow{b}\right|=2\sqrt{3}$ and $\overrightarrow{a}.\overrightarrow{b}=6$, then find the value of $\left|\overrightarrow{a}\times\overrightarrow{b}\right|$.

6. $|\overrightarrow{a}|=8$, $\left|\overrightarrow{b}\right|=3$ and $\overrightarrow{a}.\overrightarrow{b}=12\sqrt{3}$, then the value of $\left|\overrightarrow{a}\times\overrightarrow{b}\right|$ is

   (a) 24

   (b) 144

   (c) 2

   (d) 12

   If $\overrightarrow{a}=2\hat{i}+\hat{j}+3\hat{k}, \hat{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\overrightarrow{c}=3\hat{i}+\hat{j}+2\hat{k}$, then find $\overrightarrow{a}.(\overrightarrow{b}\times\overrightarrow{c})$.

7. $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ and $\overrightarrow{d}$ are four non-zeros vectors such that $\overrightarrow{a}\times\overrightarrow{b}=\overrightarrow{c}\times\overrightarrow{d}$ and $\overrightarrow{a}\times\overrightarrow{c}=4\overrightarrow{b}\times\overrightarrow{d}$, then show that $(\overrightarrow{a}-2\overrightarrow{d}$ is parallel to$(2\overrightarrow{b}-\overrightarrow{c})$ where $\overrightarrow{a}\neq 2\overrightarrow{d}$, $\overrightarrow{c}\neq 2\overrightarrow{b}$

8. If $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a} . \overrightarrow{b} = 1$ and$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k},$ then find $\left|\overrightarrow{b}\right|$

9. (a) If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that $\left|\overrightarrow{a} + \overrightarrow{b}\right| = \left|\overrightarrow{b}\right|$,then prove that $(\overrightarrow{a} + 2\overrightarrow{b})$ is perpendicular to $\overrightarrow{a}$.

10. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are unit vectors and θ is the angle between them , then prove that $\sin\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right|$.

11. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that and θ is the angle between them, then prove that

$$\sin\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right|$$

3.3. position vectors

1. If $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are the position vectors of the points A(2, 3, -4), B(3, -4, -5) and C(3, 2,-3) and respectively, then $\left|\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}\right|$ is equal to
   (a) $\sqrt{113}$
   (b) $\sqrt{185}$
   (c) $\sqrt{203}$
   (d) $\sqrt{209}$

3.4. Section Formula

1. A circle has its center at (4,4). If one end of adiameter is (4,0), then find the coordinates of other end.

3.5. plane vectors

1. Find the values λ, for which the distanceof point (2,1, λ) from plane $3x + 5y + 4z = 11$ is $2\sqrt{2}$ units.

2. Find the coordinates of the point where the line through (3,4,1) crosses the ZX-plane

3.6. geometry vectors

1. Using vectors, find the area of the triangle withvertices A(-1, 0, -2), B(0, 2, 1) and C(-1, 4,1)

2. Using integration, find the area of triangle region whose vertices are (2,0) , (4,5) and (1,4).

3.7. Distance formula

1. The distance between the points (0,0) and (a-b, a+b) is

   (a) $2\sqrt{ab}$

   (b) $\sqrt{2a^2 + ab}$

   (c) $2\sqrt{a^2 + b^2}$

   (d) $\sqrt{2a^2 + 2b^2}$

2. The value of m which makes the point (0,0) , (2m,-4)and (3,6) collinear, is ______

3.8. Direction vectors

1. (a) If a line makes $60°$ and $45°$ angles with the positive directions of X-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line.

2. The Cartesian equation of a line AB is :

$$\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3}.$$

3. Find the directions cosines of a line parallel to line AB.

4. Find the direction cosines of a line whose cartesian equation is given as 3x + 1 = 6y - 2 = 1 - z.

5. A vector of magnitude 9 units in the direction ofthe vector $-2\hat{\imath}-\hat{\jmath}+2\hat{k}$ is ______

3.9. Diagonal vectors

1. The two adajacent sides of a parallelogram are represented by $2\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. Find the unit vectors parallel toits diagonals. Using the diagonal vectors, find the area ofthe parallelogram also.

2. The two adjacent sides of a parallelogram are represented by vectors $2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also,find the area of the parallelogram.

3. If $\overrightarrow{a} = \overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k}$ and $\overrightarrow{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, then find the unit vector parallel to the diagonal of the parallelogram

3.10. Area of triangle

1. Find the area of the quadrilateral ABCD whose vertices are A(-4, -3) , B(3, -1), C(0, 5) and D(-4, 2)

2. If the points A(2,0), B(6,1), and c(p ,q) form a triangle of area 12sq. units (positive only) and 2p + q = 10,then find the values of p and q.