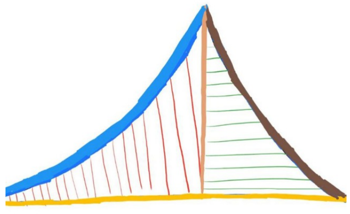
CBSE MATH

Made Simple

G. V. V. Sharma



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Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

1.1. 2022

1.1.1. $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that

$$\left|2\overrightarrow{a}+3\overrightarrow{b}\right|=\left|3\overrightarrow{a}-2\overrightarrow{b}\right|. \tag{1.1.1.1}$$

Find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

1.1.2. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that

$$\overrightarrow{a}=\hat{i}-\hat{j}+\hat{k} \tag{1.1.2.1}$$

and

$$\overrightarrow{b}=2\hat{i}-\hat{j}-3\hat{k} \tag{1.1.2.2}$$

then find the vector $\overrightarrow{c}$, given that

$$\overrightarrow{a}\times\overrightarrow{c}=\overrightarrow{b} \tag{1.1.2.3}$$

and

$$\overrightarrow{a}.\overrightarrow{c} = 4. \tag{1.1.2.4}$$

1.1.3.

$$If\left|\overrightarrow{a}\times\overrightarrow{b}\right|^2+\left|\overrightarrow{a}.\overrightarrow{b}\right|^2 = 400 \tag{1.1.3.1}$$

and

$$\left|\overrightarrow{b}\right| = 5 \tag{1.1.3.2}$$

find the value of $|\overrightarrow{a}|$.

1.1.4. If

$$\overrightarrow{a} = \hat{i}+\hat{j}+\hat{k}, \overrightarrow{a}.\overrightarrow{b} = 1 \tag{1.1.4.1}$$

and

$$\overrightarrow{a}\times\overrightarrow{b} = \hat{j}-\hat{k} \tag{1.1.4.2}$$

, then find $\left|\overrightarrow{b}\right|$

1.1.5. If

$$|\overrightarrow{a}| = 3, \left|\overrightarrow{b}\right| = 2\sqrt{3} \tag{1.1.5.1}$$

and

$$\overrightarrow{a}.\overrightarrow{b} = 6, \tag{1.1.5.2}$$

then find the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$.

1.1.6. $|\overrightarrow{a}| = 8, \left|\overrightarrow{b}\right| = 3$ and $\overrightarrow{a}.\overrightarrow{b} = 12\sqrt{3}$, then the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$ is

(a) 24

(b) 144

(c) 2

(d) 12

1.1.7. If

$$\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{1.1.7.1}$$

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.1.7.2}$$

, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

1.1.8. $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ and $\overrightarrow{d}$ are four non-zeros vectors such that $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c} \times \overrightarrow{d}$ and

$$\overrightarrow{a} \times \overrightarrow{c} = 4\overrightarrow{b} \times \overrightarrow{d} \tag{1.1.8.1}$$

, then show that $(\overrightarrow{a} - 2\overrightarrow{d}$ is parallel to$(2\overrightarrow{b} - \overrightarrow{c})$ where

$$\overrightarrow{a} \neq 2\overrightarrow{d}, \overrightarrow{c} \neq 2\overrightarrow{b} \tag{1.1.8.2}$$

1.1.9. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a}.\overrightarrow{b} = 1 \tag{1.1.9.1}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k}, \tag{1.1.9.2}$$

then find $\left|\overrightarrow{b}\right|$

1.1.10. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that

$$\left|\overrightarrow{a} + \overrightarrow{b}\right| = \left|\overrightarrow{b}\right|, \tag{1.1.10.1}$$

then prove that $(\overrightarrow{a} + 2\overrightarrow{b})$ is perpendicular to $\overrightarrow{a}$.

1.1.11. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are unit vectors and θ is the angle between them , then prove that sin

$$\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{1.1.11.1}$$

1.1.12. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that and θ is the ang le between them, then prove that

$$sin\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{1.1.12.1}$$

1.1.13. If

$$\overrightarrow{a} = 2\hat{i} + y\hat{j} + \hat{k} \tag{1.1.13.1}$$

and

$$\overrightarrow{b} = \hat{i} + 2\hat{j} + 3\hat{k} \tag{1.1.13.2}$$

are two vectors for which the vector $(\overrightarrow{a} + \overrightarrow{b})$ is perpendicular to the vector $(\overrightarrow{a} - \overrightarrow{b})$ then find all the possible values of y.

1.1.14. Write the projection of the vector $(\overrightarrow{b} + \overrightarrow{c})$ on the vector $\overrightarrow{a}$, where

$$\overrightarrow{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + 2\hat{j} - 2\hat{k} \tag{1.1.14.1}$$

and

$$\overrightarrow{c} = 2\hat{i} - \hat{j} + 4\hat{k}. \tag{1.1.14.2}$$

1.1.15. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + \hat{j} - 2\hat{k} \tag{1.1.15.1}$$

and

$$\overrightarrow{c} = \hat{i} + 3\hat{j} - \hat{k} \tag{1.1.15.2}$$

and the projection of vector $\overrightarrow{c} + \lambda\overrightarrow{b}$ on vector $\overrightarrow{a}$ is $2\sqrt{6}$, find the value of λ.

1.1.16. If $\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.1.16.1}$$

, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

1.1.17. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + 2\hat{k} \tag{1.1.17.1}$$

and

$$\overrightarrow{b} = 5\hat{i} - 3\hat{j} - 4\hat{k} \tag{1.1.17.2}$$

, then find the ratio $\frac{projection of vector \overrightarrow{a} on vector \overrightarrow{b}}{projection of vector \overrightarrow{b} on vector \overrightarrow{a}}$

1.1.18. Show that the three vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$, and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right-angled triangle. If $\overrightarrow{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\overrightarrow{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} \tag{1.1.18.1}$$

are such that the vector $(\overrightarrow{a} + \lambda \overrightarrow{b})$ is perpendicular to vector $\overrightarrow{c}$, then find the value of λ.

1.1.19. If $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are the position vectors of the points $\mathbf{A}(2, 3, -4)$, $\mathbf{B}(3, -4, -5)$ and $\mathbf{C}(3, 2, -3)$ and respectively, then $\left|\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}\right|$ is equal to

(a) $\sqrt{113}$

(b) $\sqrt{185}$

(c) $\sqrt{203}$

(d) $\sqrt{209}$

1.1.20. **A** circle has its center at $(4, 4)$. If one end ofa diameter is $(4, 0)$, then find the coordinates ofother end.

1.1.21. Find the values λ, for which the distance of point $(2, 1, \lambda)$ from plane

$$3x + 5y + 4z = 11 \tag{1.1.21.1}$$

is $2\sqrt{2}$ units.

1.1.22. Find the coordinates of the point where the line through $(3, 4, 1)$ crosses the ZX-plane

1.1.23. Using vectors, find the area of the triangle withvertices $\mathbf{A}(-1, 0, -2)$, $\mathbf{B}(0, 2, 1)$ and $\mathbf{C}(-1, 4, 1)$

1.1.24. Using integration, find the area of triangle region whose vertices are $(2, 0)$, $(4, 5)$ and $(1, 4)$.

1.1.25. The distance between the points $(0,0)$ and $(a-b, a+b)$ is

(a) $2\sqrt{ab}$

(b) $\sqrt{2a^2+ab}$

(c) $2\sqrt{a^2+b^2}$

(d) $\sqrt{2a^2+2b^2}$

1.1.26. The value of m which makes the point $(0,0)$, $(2m,-4)$and $(3,6)$ collinear, is _____

1.1.27. If a line makes $60°$ and $45°$ angles with the positive directions of X-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direct6on cosines of the line.

1.1.28. The Cartesian equation of a line AB is :

$$\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3} \tag{1.1.28.1}$$

.

1.1.29. Find the directions cosines of a line parallel to line AB.

1.1.30. Find the direction cosines of a line whose cartesian equation is given as

$$3x+1 = 6y-2 = 1-z. \tag{1.1.30.1}$$

1.1.31. A vector of magnitude 9 units in the direction of the vector $-2\hat{\imath}-\hat{\jmath}+2\hat{k}$ is ______

1.1.32. The two adajacent sides of a parallelogram are represented by $2\hat{\imath}-4\hat{\jmath}-5\hat{k}$ and $\hat{\imath}+2\hat{\jmath}+3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.

1.1.33. The two adjacent sides of a parallelogram are represented by vectors $2\hat{\imath}-4\hat{\jmath}+5\hat{k}$ and $\hat{\imath}-2\hat{\jmath}-3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also,find the area of the parallelogram.

1.1.34. If

$$\overrightarrow{a} = \overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k} \tag{1.1.34.1}$$

and

$$\overrightarrow{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k} \tag{1.1.34.2}$$

represent two adjacent sides of a parallelogram, then find the unit vector parallel to the diagonal of the parallelogram

1.1.35. Find the area of the quadrilateral $ABCD$ whose vertices are $\mathbf{A}(-4,-3)$, $\mathbf{B}(3,-1)$, $\mathbf{C}(0,5)$ and $\mathbf{D}(-4,2)$

1.1.36. If the points $\mathbf{A}(2,0)$, $\mathbf{B}(6,1)$, and $\mathbf{C}(p,q)$ form a triangle of area 12sq. units (positive only) and

$$2p + q = 10, \tag{1.1.36.1}$$

then find the values of p and q.

Chapter 2

Linear Forms

2.1. 2022

2.1.1. Solve the equations $x + 2y = 6$ and $2x - 5y = 12$ graphically.

2.1.2. Solve the following equations for x and y using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \tag{2.1.2.1}$$

$$(bx + ay) + (b - 4a) = 0 \tag{2.1.2.2}$$

2.1.3. Find the co-ordinates of the point where the line $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ crosses the plane passing through the points $\left(\frac{7}{2}, 0, 0\right)$, $(0, 7, 0)$, $(0, 0, 7)$.

2.1.4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

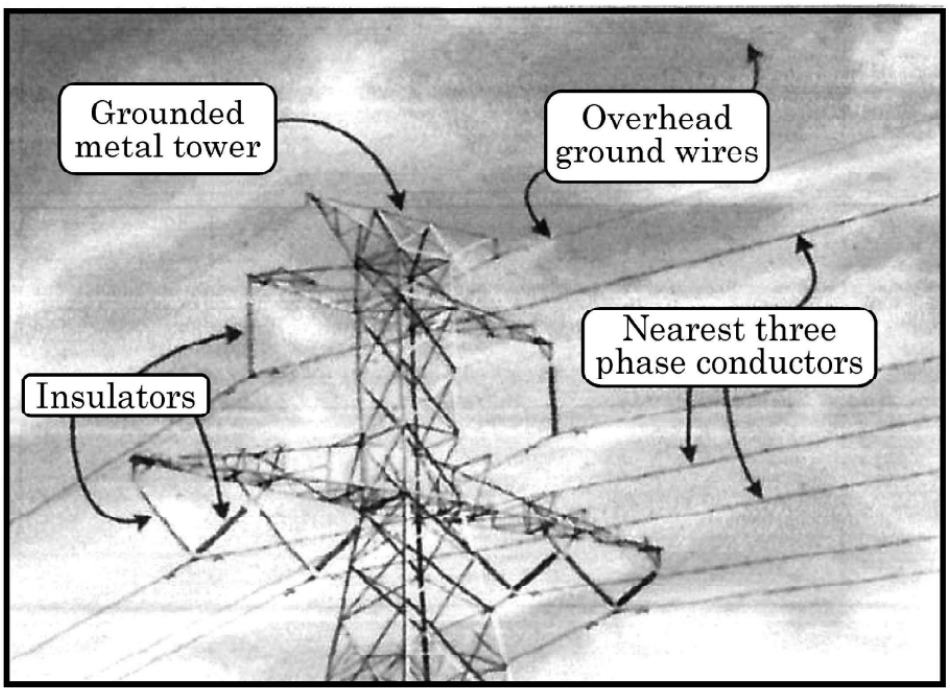

Figure 2.1.4.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 2.1.4.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \tag{2.1.4.1}$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \tag{2.1.4.2}$$

Based on the given information, answer the following questions:

(a) Are the l_1 and l_2 coplanar? Justify your answer.

(b) Find the point of intersection of lines l_1 and l_2.

2.1.5. Write the cartesian equation of the line PQ passing through points

P$(2,2,1)$ and Q$(5,1,-2)$. Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is -2.

2.1.6. Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$.

2.1.7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + \hat{k}) \tag{2.1.7.1}$$

$$\mathbf{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(7\hat{\imath} - 6\hat{\jmath} + \hat{k}) \tag{2.1.7.2}$$

2.1.8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 2.1.8.1) represented by the following lines

$$\mathbf{r} = \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k}) \tag{2.1.8.1}$$

$$\mathbf{r} = (3\hat{\imath} + 3\hat{\jmath}) + \mu(2\hat{\imath} + \hat{\jmath} + \hat{k}) \tag{2.1.8.2}$$

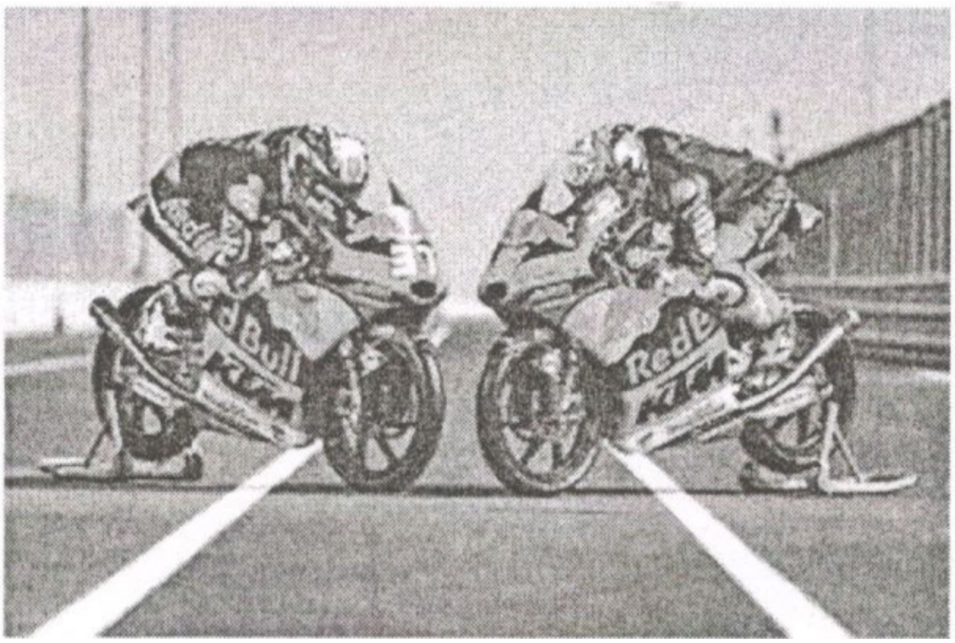

Figure 2.1.8.1: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

(a) Find the shortest distance between the given lines.

(b) Find a point at which the motorcycles may collide.

2.1.9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{i} + (\lambda + 4)\hat{j} - (\lambda - 3)\hat{k} \tag{2.1.9.1}$$

$$\mathbf{r} = (3 - \mu)\hat{i} + (2\mu + 2)\hat{j} + (\mu + 6)\hat{k} \tag{2.1.9.2}$$

2.1.10. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \tag{2.1.10.1}$$

2.1.11. Find the equation of the plane passing through the points $(2, 1, 0), (3, -2, -2)$ and $(1, 1, 7)$. Also, obtain its distance from the origin.

2.1.12. The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane.

2.1.13. Find the cartesian and the vector equation of a plane which passes through the point $(3, 2, 0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

2.1.14. The distance between the planes $4x-4y+2z+5 = 0$ and $2x-2y+z+6 = 0$ is

(a) $\frac{1}{6}$
(b) $\frac{7}{6}$
(c) $\frac{11}{6}$
(d) $\frac{16}{6}$

2.1.15. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r} \cdot (\hat{\imath} + 3\hat{\jmath}) + 6 = 0 \tag{2.1.15.1}$$

$$\mathbf{r} \cdot (3\hat{\imath} - \hat{\jmath} - 4\hat{k}) = 0 \tag{2.1.15.2}$$

which is at a unit distance from the origin.

2.1.16. If the distance of the point $(1, 1, 1)$ from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ.

2.1.17. Find the distance of the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$.

2.1.18. Find the distance of the point $P(4, 3, 2)$ from the plane determined by the points $A(-1, 6, -5), B(-5, -2, 3)$ and $C(2, 4, -5)$.

2.1.19. The distance of the line

$$\mathbf{r} = (\hat{\imath} - \hat{\jmath}) + \lambda(\hat{\imath} + 5\hat{\jmath} + \hat{k}) \tag{2.1.19.1}$$

from the plane

$$\mathbf{r} \cdot (\hat{\imath} - \hat{\jmath} + 4\hat{k}) = 5 \tag{2.1.19.2}$$

is

(a) $\sqrt{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{3\sqrt{2}}$

(d) $\frac{-2}{3\sqrt{2}}$

2.1.20. Find a unit vector perpendicular to each of the vectors $(\mathbf{a} + \mathbf{b})$ and

$(\mathbf{a} - \mathbf{b})$ where

$$\mathbf{a} = \hat{\imath} + \hat{\jmath} + \hat{k} \tag{2.1.20.1}$$

$$\mathbf{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} \tag{2.1.20.2}$$

2.1.21. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{\imath} + 2\hat{\jmath} + 7\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}) \tag{2.1.21.1}$$

and the plane

$$\mathbf{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 10. \tag{2.1.21.2}$$

2.1.22. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.

2.1.23. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.

2.1.24. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 2.1.24.1:

Table 2.1.24.1: Table showing yearly trend of production of goods in lakh tonnes

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40

Chapter 3

Intersection of Conics

3.1. 2022

3.1.1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

3.1.2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

3.1.3. Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

3.1.4. If the area of the regin bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

3.1.5. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration.

3.1.6. If the area of the region bounded bythe line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.

3.1.7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

3.1.8. Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

3.1.9. Find the area of the region $\{(x, y) : x^2 \le y \le x\}$, using integration

Chapter 4

Tangent And Normal

4.1. 2022

4.1.1. Draw a circle of radius 2.5 cm. Take a point **P** outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point **P**.

4.1.2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point **P**, at a distance of 7 cm from its center **O**.

4.1.3. In Figure 4.1.3.1, there are two concentric circles with centre **O**. If ARC and AQB are tangents to the smaller circle from the point **A** lying on the larger circle, find the length of AC, if $AQ = 5$ cm.

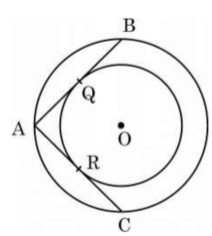

Figure 4.1.3.1: Two concentric circles with **O** as centre

4.1.4. In Figure 4.1.4.1, if a circle touches the side QR of ΔPQR at **S** and extended sides PQ and PR at **M** and **N**, respectively,

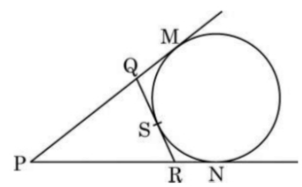

Figure 4.1.4.1: Two tangents are drawn from point **P** to the circle

prove that $PM = \frac{1}{2}(PQ + QR + PR)$

4.1.5. In Figure 4.1.5.1, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact **D** are of lengths 6 cm and 8 cm respectively. If the area of ΔABC is 84 cm^2, find the lengths of sides AB and AC.

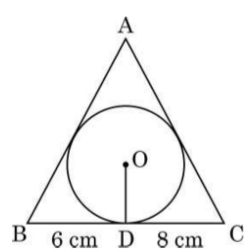

Figure 4.1.5.1: Circle with **O** as center circumscribed in triangle ABC

4.1.6. In Figure 4.1.6.1, PQ and PR are tangents to the circle centered at **O**. If $\angle OPR = 45°$, then prove that $ORPQ$ is a square.

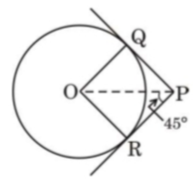

Figure 4.1.6.1: Two tangents drawn from point **P** to a circle whose centre is **O**

4.1.7. In Figure 4.1.7.1, **O** is the centre of a circle of radius 5 cm. PA and BC are tangents to the circle at **A** and **B** respectively. If OP is 13 cm, then find the length of tangents PA and BC.

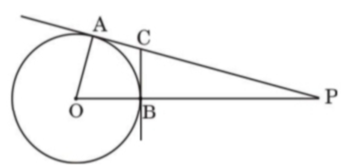

Figure 4.1.7.1: Two tangents drawn from point **C** to a circle whose centre is **O**

4.1.8. In Figure 4.1.8.1, AB is diameter of a circle centered at **O**. BC is tangent to the circle at **B**. If OP bisects the chord AD and $\angle AOP = 60°$, then find $m\angle C$.

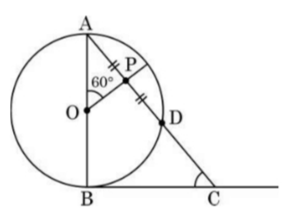

Figure 4.1.8.1: Tangent BC is drawn from point **C** to a circle whose centre is **O**

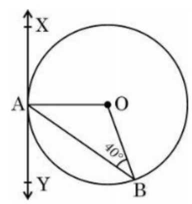

Figure 4.1.9.1: The line XAY is tangent to the circle centered at **O**

4.1.9. In Figure 4.1.9.1, XAY is a tangent to the circle centered at **O**. If $\angle ABO = 60°$,then find $m\angle BAY$ and $m\angle AOB$.

4.1.10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

4.1.11. In Figure 4.1.11.1, a triangle ABC with $\angle B = 90°$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point **P**. Prove that the tangent drawn at point **P** bisects BC.

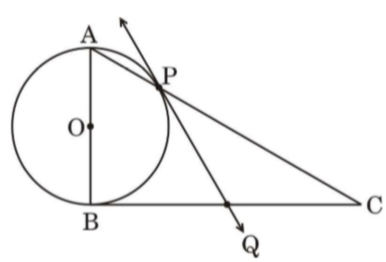

Figure 4.1.11.1: PQ is tangent to the circle centered at **O**. AB is the diameter and $\angle B = 90°$

4.1.12. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.

Chapter 5

Probability

5.1. 2022

1. Let A and B be two events such that $P(A) = \frac{5}{8}, PB = p(A/B) = \frac{3}{4}$. Find the value of P(B/A).

2. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variables X denotes the number of red balls. Find the probabillity distribution of X.

3. A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace.

4. probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved.

5. A pair of dice is thrown. It is given that the sum of numbbers appearing on both dice is an even number. Find the probability that the number

apprearing on at least one die is 3.

6. At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. such a coin is unbaised with equal probabilities of getting head and tail. Based on

Figure 6.1: Toss before the match

the above information, answer the following question:

   (a) If such a coin is tossed 2 times, then find the probability distibution of numbers of tails.

   (b) Find the probability of getting at least one head in three tosses of such a coin.

7. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards.

8. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die.

9. The probability that **A** hits the target is $\frac{a}{3}$ and the probability that

B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probabillity that the target is hit.

10. A shopkeeper sells three of flower seeds **A1**,**A2**,**A3**. They are sold in the form of a mixture, where the proportions of these seeds are **4** : 4 : 2, respectively. The germinaton rates of the three types of seeds are 45%, 60% and 35% respectively. Based on the above information :

Figure 10.1: Three types of flowers

   (a) Calculate the probability that a randomly chosen seed will germinate;

   (b) Calculate the probability that the seed is of type A2, given that a randomly choosen seed germinates.

   (c) Three friends **A**, **B**, and **C** got their photograph clicked. Find the probability that **B** is standing at the central position, given that **A** is standing at the left corner.

11. In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are alloted

points **9** to **1** in sequential order moving outwards. Archer **A** is likely to earn 10 points with a probability of 0.8 and Archer **B** is likely the earn 10 points with a probability of 0.9. Based on the above innfor-

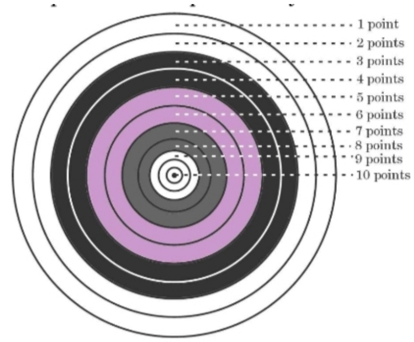

Figure 11.1: centermost ring

mation, answer the following questions :

(a) exactly one of them earns 10 points .

(b) both of them earn 10 point.

12. Event **A** and **B** are such that

$$p(A) = \frac{1}{2}, p(B) = \frac{7}{12} and p(\bar{A} \cup \bar{B}) = \frac{1}{4} \tag{12.1}$$

Find whether the events **A** and **B** are independent or not.

13. A box B_1 contain 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2, then find the probability that the two balls drawn areof the same colour.

14. Let X be random variable which assumes values x_1 , x_2 , x_3 , x_4 such that

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4). \quad (14.1)$$

Find the probability distribution of X.

15. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes , is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II.

16. In a toss of three different coins, find the probability of comming up of three heads, if it is known that at least one head comes up.

17. Two rotten apples are mixed with 8 fresh apples. Find the probability distribution of number of rotten apples, if two apples are drawn at random, one-by-one without replacement.

18. A laboratory blood text is 98% effective in detecting a certain disease when it is fact, present. However, the text also yeilds a false positive result for 0.4% of the healthy person tested. From a large population, it is given that 0.2% of the population actually has the diseases. Based on the above, answer the following questtion :

    (a) one person, from the population, is taken at random and given the test. Find the probabiliy of his getting a positive test result.

(b) what is the probability that the person actually has the disease, given that his test result is positive ?

19. Two cards are drawn from a well-shuffled pack of playing cards one-by-one with replacement. The probability that the first card is a king and the second card is a queen is

(a) $\frac{1}{13} + \frac{1}{13}$

(b) $\frac{1}{13} \times \frac{4}{51}$

(c) $\frac{4}{52} \times \frac{3}{51}$

(d) $\frac{1}{13} \times \frac{1}{13}$

20. For two events A and B if p(A) $= \frac{4}{10}, PB = \frac{8}{10}$ and P(B—A) = then find $P(A \cup B)$.

21. Bag I contain 4 red and 3 black balls Bag II contains 3 red and 5 black balls. One of two bags is selected at random and a ball is drawn from the bag, which if found to be red. Find the probability thta the bill is drawn from bag II.

22. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces and hences find its mean.

23. The probability of solving a specific question independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively . If both try to solve the question independently, the probability that the question is solved is

(a) $\dfrac{7}{15}$

(b) $\dfrac{8}{15}$

(c) $\dfrac{2}{15}$

(d) $\dfrac{14}{15}$

24. A card is packed at random from a pack of 52 playing cards. Given that the picked up card is a queen, the probability equation $x\dfrac{dy}{dx}$ - y = log x is _____.

25. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the numberof even numbers on the ticket.

26. Find the probability distribution of the numbers of successes in two tosses of a die, when a success is defined as "number greater than 5".

27. Ten cartoons are taken at random from an automatic packing machine. The mean net weight of the ten carton is 11.8 kg and standard deviation is 0.15 kg. Does the sample mean differ significantly from the intended mean of 12 kg ? [Given that for $d.f. = 9, t_0.05 = 2.26$]

28. coin is tossed twice. The following table shows the probability distribution of numbers of tails:

29. If X is a random variable with probability distribution as given below:

Table 28.1: Table shows the probability distribution of numbers of tails

X	0	1	2
P(X)	K	6K	9K

Table 29.1: table shows the proability distribution

X	0	1	2
P(X)	K	4K	K

30. The random variable X has a probability function P(x) as defined below, where K is some number : P(X)=$\begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$

Chapter 6

Construction

6.1. 2022

1. In figure-4, BN and CM are medians of a $\triangle$ ABC right-angled at A. Prove that $4(BN^2 + CM^2) = 5BC^2$

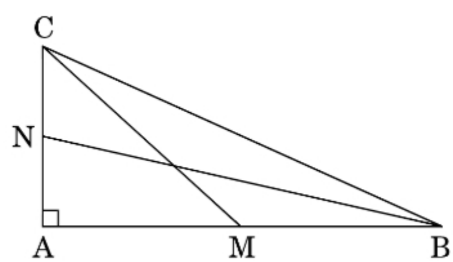

Figure 1.1: Right-angled triangle

2. $\mathbf{CaseStudy - 1}$:

KiteFestival

Kite festival is celebrated in many countries at different times of the year. in India, every year 14th January is celebrated as international kite Day. on his day many people visit India and participate in the festival by flying various kinds of kites. The picture given below , three kites flying together.

In Fig. 5, the angles of elevation of two kites (point C) are found to

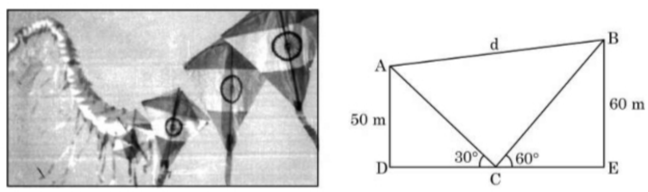

Figure 2.1: kites flying to gether

be °30 and °60 respectively. Taking AD = 50 m and BE = 60 m, find

(a) the length of string used (take them straight) for kites A and B as shown in the figure.

(b) the distance 'd' between these two kites