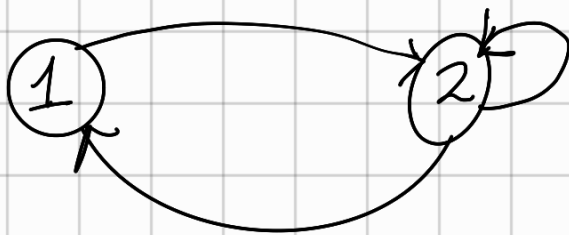


Tiempo promedio de recurrencia

μ_{ij} : Número de etapas promedio en pasar desde Estado i hasta j .

μ_{ii} : Número de etapas promedio en retornar al estado i .



$$\mu_{12} = \frac{2+3}{2} = \frac{5}{2}$$

1, 1, 2, 1, 1, 2

Los valores de μ_{ii} y μ_{ij} se obtienen de las siguientes ecuaciones:

$$\mu_{ij} = \frac{1}{\pi_i}$$

$$\pi = \pi P$$

$$\mu_{ij} = 1 + \sum_{k, k \neq j} p_{ik} \cdot \mu_{kj}$$

Ejemplo: Calcular μ_{11} , μ_{22} , μ_{21} , μ_{12}

para $P = \begin{pmatrix} 2/3 & 1/3 \\ 1/2 & 1/2 \end{pmatrix}$

i) μ_{11}, μ_{22} Resolvamos el sistema:

$$(\pi_1 \pi_2) = (\pi_1 \pi_2) \begin{pmatrix} 2/3 & 1/3 \\ 1/2 & 1/2 \end{pmatrix}$$

$$\pi_1 = \frac{3}{5}$$

$$\therefore \mu_{11} = \frac{1}{\pi_1} = \frac{5}{3} = 1,6$$

$$\pi_2 = \frac{2}{5}$$

$$\mu_{22} = \frac{1}{\pi_2} = \frac{5}{2} = 2,6$$

$$\mu_{12} = 1 + \sum_{\substack{k=1 \\ j=2}} P_{1k} \cdot \mu_{k2} = 1 + P_{11} \cdot \mu_{12} = \mu_{12} = 1 + \frac{2}{3} \cdot \mu_{12} \Rightarrow \mu_{12} = 3$$

$$\mu_{21} = 1 + \sum_{\substack{k=2 \\ j=1}} P_{2k} \cdot \mu_{k1} = 1 + P_{22} \cdot \mu_{21} \Rightarrow \mu_{21} = 1 + \frac{1}{2} \mu_{21} \Rightarrow \mu_{21} = 2$$

Ejercicio:

Sea la Cadena de Markov con matriz $P = \begin{pmatrix} 0.2 & 0.2 & 0.6 \\ 0.1 & 0.4 & 0.5 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$

$$\begin{aligned} \mu_{12} &= 1 + \sum_{\substack{k=1 \\ k \neq 2}} P_{1k} \cdot \mu_{k2} = 1 + P_{11} \cdot \mu_{12} + P_{13} \cdot \mu_{32} = 1 + 0,2 \mu_{12} + 0,6 \mu_{32} \Rightarrow \mu_{12} = 1 + 0,2 \mu_{12} + 0,6 \mu_{32} \\ \mu_{31} &= 1 + \sum_{\substack{k=1 \\ k \neq 2}} P_{3k} \cdot \mu_{k2} = 1 + P_{31} \cdot \mu_{12} + P_{33} \cdot \mu_{32} = 1 + 0,3 \mu_{12} + 0,3 \mu_{32} \Rightarrow \mu_{32} = 1 + 0,3 \mu_{12} + 0,3 \mu_{32} \end{aligned}$$

$$\begin{aligned} 0,8 \mu_{12} - 0,6 \mu_{32} &= 1 \quad \checkmark \\ -0,3 \mu_{12} + 0,7 \mu_{32} &= 1 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \mu_{12} &= 3,42 \\ \mu_{32} &= 2,90 \end{aligned} //$$

$$\mu_{15} = ?$$

$$\mu_{13} = 1,73$$

$$\mu_{23} = 1,95$$

