The Delphic forward guidance puzzle in New Keynesian models *

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Abstract

When the central bank has information that can help the private sector better predict the future, should it communicate such information to the public? In purely forward-looking New Keynesian models, such Delphic forward guidance unambiguously reduces ex ante welfare by increasing the variability of inflation and the output gap. We call this phenomenon the Delphic forward guidance puzzle. In more elaborate models with endogenous state variables, a combination of Delphic forward guidance and preemptive policy actions may improve welfare. However, full information revelation is generally not optimal and what information needs to be revealed is highly model-dependent.

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1 Introduction

Central banks have been thought to possess private information about future economic conditions. Romer and Romer (2000) provide empirical evidence of asymmetric information between the central bank and private agents: "the Federal Reserve has considerable information about inflation beyond what is known to commercial forecasters."[1] The possession of such superior information by the central bank raises several questions. How should monetary policy be designed when the central bank has private information about future economic conditions? Does the central bank benefit from managing the private sector's expectations by utilizing such information?

This paper investigates whether central banks should reveal their superior information about future economic conditions, either by communicating it or by undertaking observable policy actions, in representative-agent dynamic stochastic general equilibrium (DSGE) models with nominal rigidities. DSGE models with nominal rigidities are best suited for our analysis because the central bank in the models can manage the expectations of forward-looking agents by conveying its superior information. In addition, these models are widely used in central banks to guide policy.

Central banks' communication of such information is of practical relevance. Campbell, Evans, Fisher, and Justiniano (2012) distinguish between *Delphic* forward guidance, which involves public statements about "a forecast of macroeconomic performance and likely or intended monetary policy actions based on the policymaker's potentially superior information about future macroeconomic fundamentals and its own policy goals," and *Odyssean* forward guidance that involves the policy-maker's commitment to a future, possibly state-contingent, action plan. They found empirical evidence suggesting that the forward guidance employed by the Federal Open Market Committee has "a substantial Delphic component". Although the importance of

[1] Fujiwara (2005) shows that central bank forecasts significantly affect those by professional forecasters.

Odyssean forward guidance has been well established in the New Keynesian monetary policy literature, it is not yet known whether Delphic forward guidance is useful in New Keynesian models. This paper sheds light on this issue.

The present paper argues that improving social welfare through Delphic forward guidance is indeed difficult, if not impossible, in these DSGE models. Our argument is based on a theoretical result in simple New Keynesian models that Delphic forward guidance unambiguously reduces welfare and on numerical experiments in more elaborate models that the welfare implications of Delphic forward guidance are highly model-dependent.

First, we identify a key mechanism in New Keynesian models through which Delphic forward guidance generates a welfare loss. This is done by extending a textbook, purely forward-looking New Keynesian model (Woodford, 2003) to incorporate a direct communication channel from the central bank. In the model, Delphic forward guidance unambiguously decreases ex ante welfare. The benevolent central bank finds it optimal to commit to not revealing its superior information about future shocks, either directly through communication or indirectly through observable policy actions. The result holds for any shocks — a natural rate shock, a cost-push shock, or a shock to the welfare loss function — and even in the presence of an effective lower bound on the nominal interest rate.

The underlying mechanism is simple and operates through the forward-looking New Keynesian Phillips curve. Consider a simple New Keynesian model in which the central bank has preferences for stabilizing inflation and the output gap. When the private sector becomes better informed about future shocks, its inflation expectations vary too and, from an ex ante point of view, become more volatile. Because the increased volatility of inflation expectations acts as an additional source of disturbance in the New Keynesian Phillips curve, it translates into higher variability of inflation and the output gap, and is therefore harmful to the central bank that aims to stabilize these variables.

Second, we consider two variants of New Keynesian models with endogenous state variables. This is because lack of endogenous state variables in the purely forward-looking model rules out the potential benefits of preemptive monetary policy actions to combat future shocks. In the first model, the economy can be in a severe recession in the future because of a negative natural rate (*i.e.*, demand) shock and of a binding effective lower bound on the nominal interest rate. Preemptive monetary policy easing can mitigate the future recession because a rise in current inflation raises future inflation at an effective lower bound through backward price indexation. In the second model, both price and nominal wage are sticky, making the real wage a slow moving variable. The central bank can, potentially, influence the current real wage so as to reduce the effect of future price and wage mark-up shocks upon their realization.

In both models with endogenous state variables, we examine whether information revelation, either through direct communication or through information-dependent policy actions, can improve welfare. In the model with price indexation and an effective lower bound, information revelation can improve welfare, but only when the Phillips curve is sufficiently steep and the elasticity of intertemporal substitution is extremely high. With realistic parameter values, it is optimal for the central bank to reveal no information and to set the current nominal rate independently of its knowledge about the future natural rate shock. In the sticky price and wage model, information revelation improves welfare when either the price or the wage is relatively — but not perfectly — flexible, but reduces welfare when both are sufficiently sticky. Hence, although endogenous state variables open up the possibility that the gains from information revelation and preemptive policy actions exceed the costs, secrecy is still optimal for a range of realistic parameterizations of these models. We also show that, even when secrecy is suboptimal, indirect information revelation through policy actions is often sufficient in these two models. In other words, Delphic forward guidance should not reveal more information than the central bank's actions.

Moreover, the welfare implications of Delphic forward guidance are determined by some complicated interactions between shocks and frictions in the model, and are thus dependent on the details of the model at hands. The shock type by itself does not determine the sign of the welfare effect of Delphic forward guidance: forward guidance about the mark-up shocks may improve welfare in the model with sticky price and wage, whereas it unambiguously reduces welfare in the purely forward-looking New Keynesian models; and forward guidance about the future demand shock may be harmful to social welfare in the presence of the effective lower bound on the nominal interest rate. Even within the same model, the welfare effect becomes either positive or negative, depending on parameter values. Overall, the welfare implications of Delphic forward guidance is model-dependent.

Finally, we complement the previous analysis using a more elaborate, nonlinear DSGE model that is based on Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003, 2007). The model features multiple distortions, frictions and shocks. Both price and wage are sticky and are subject to backward indexation. The household accumulates capital subject to investment adjustment costs, and external habit formation is assumed. Monetary policy follows a Taylor rule with inertia. There are four kinds of shocks: price and wage mark-up shocks, a technology shock, and a monetary policy shock. The model is solved using a second-order approximation, assuming that Delphic forward guidance perfectly reveals future realizations of any of these four shocks. Different versions of the model are also examined, by turning on and off a subset of frictions.

The results reiterate the model-dependency of the welfare effects of Delphic forward guidance: the sign of the welfare effects differ across the shock types and can change easily when some distortions and frictions are included or excluded from the model. In the full-blown model, forward guidance that reveals the price mark-up shocks and the monetary policy shocks improves welfare, but welfare decreases when the wage mark-up shocks or the technology shocks are revealed. However, with weaker wage rigidity, forward guidance about the price mark-up shock reduces welfare; without price and wage rigidities, forward guidance about the technology shock improves welfare; and without policy inertia, forward guidance about the monetary policy shock reduces welfare. These implications are the opposite to what we find in the full-blown model. Therefore, the welfare implications are highly model-dependent and choosing which shocks and how much information to reveal *a priori* is not at all straightforward.

The importance of *managing expectations* has been emphasized in the New Keynesian literature (Woodford, 2003).[2] However, our results overall suggest that, in the

[2]Its importance has been also emphasized in real-world policy-making after many central banks in advanced economies reduced short-term nominal interest rates to the lowest possible level in response to the recent financial crisis. Forward guidance is not necessarily a policy prescription under liquidity trap. Svensson (2014) states that "for many years, some central banks have used forward guidance as a natural part of their normal monetary policy." Its usefulness has been reported even in normal time.

same class of models in which expectations are important, it is difficult to improve welfare using Delphic forward guidance based on the central bank's superior information about the future economic conditions. Therefore we call this property *the Delphic forward guidance puzzle*. The central bank should instead aim to conduct Odyssean forward guidance by communicating its state-contingent policy, *i.e.*, what it will do in response to these shocks after they materialize.

This paper is structured as follows. Section 2 extends a textbook New Keynesian model to incorporate a direct communication channel by the central bank and shows that it is optimal for the benevolent central bank to commit to secrecy. Section 3 analyzes two stylized New Keynesian models with endogenous state variables as well as a more elaborate DSGE model. Section 4 concludes.

1.1 Related literature

Our paper is most closely related to the literature on Bayesian persuasion (Rayo and Segal, 2010; Kamenica and Gentzkow, 2011; Jehiel, 2015). As in the literature, we assume that an informed party, the central bank, can commit to a signal-generating structure before observing private information, and characterize the optimal disclosure policy from the viewpoint of the informed party. Therefore, this paper can be framed as a macroeconomic application of Bayesian persuasion, and we will relate our theoretical results to this literature. A distinct feature of our model is that the informed party also takes action that directly affects the private agents' incentives and that, if the action utilizes private information, may indirectly reveal some private information. Hence, we also examine whether the central bank's ability to use forward guidance is essential or indirect information revelation is sufficient. In an accompanying paper (Fujiwara and Waki, 2020), we investigate whether Delphic forward guidance can be useful for the conduct of fiscal policy using a model without nominal rigidities. We show that it can be harmful to ex ante welfare to convey more accurate information about future policy shocks.

To focus on the central bank's communication on future economic conditions, we assume that contemporaneous shocks are perfectly observed by private agents. We also assume symmetric information among private agents in order to focus on information asymmetry between the central bank and the private sector. There has been a vast number of studies that focus on the role of the central bank's disclosure policy in coordinating the actions of private agents who are heterogeneously informed about contemporaneous economic conditions: for example, Morris and Shin (2002) and Angeletos and Pavan (2007). The works related to our paper in this literature are Hellwig (2005) and Lorenzoni (2010). In both papers, monopolistically competitive sellers are heterogeneously informed about the contemporaneous shock and set their nominal prices under information constraints. In Hellwig (2005), the heterogeneous information is about money supply shocks and public information unambiguously improves welfare by lowering belief dispersion and price dispersion among sellers. In Lorenzoni (2010), the heterogeneous information is about aggregate productivity, and a more precise public signal is shown to improve social welfare when the monetary policy rule is chosen optimally. If we were to introduce heterogeneously informed sellers to our model, then the optimal communication policy would strike a balance between the gain from coordination and the loss from volatility.[3] In this literature, it is also found that increased precision of a public signal can reduce welfare, but the reason is the agents' coordination motives. Our paper shows that information revelation can be detrimental to welfare even without heterogeneous information or coordination motives.[4]

In our paper the central bank is assumed to be able to commit to a signal-generating structure as in the literature of Bayesian persuasion, and there is no strategic interaction between the central bank and the private agents. By contrast, Stein (1989) and Moscarini (2007) analyze strategic information transmission by setting up a cheap-talk game (Crawford and Sobel, 1982). They show that, although full information revelation is desirable, only imperfect communication is possible in equilibrium, thereby

[3]There are other papers that investigate the role of central bank's information revelation when private agents are heterogeneously informed; *e.g.*, Gaballo (2016) and Walsh (2007). Melosi (2017) develops a New Keynesian model with dispersed information to examine the role of monetary policy to convey the central bank's private information about the current shocks. In contrast to our paper, his analysis is a positive one and demonstrates that the dispersed information model outperforms the homogeneous information model.

[4]Svensson (2006) argues that the welfare-reducing property of increased precision of the public signal is rather limited to a small region of the parameter space in the model of Morris and Shin (2002). In the simple New Keynesian model, we show that the undesirability of information revelation is a global property.

providing a theory of imprecise announcement from policy-makers. Moscarini (2007) further shows that the more precise the signal the central bank observes, the more information is revealed and the higher the level of welfare. The reason that information disclosure is desirable is that the central bank in their model has private information about shocks to its current policy objective but not about news shocks. A recent paper by Bassetto (2019) analyzes the role of forward guidance in a dynamic cheap-talk game. His model is not a New Keynesian model and the central bank's private information is not about future shocks.

This paper is also related to the literature on news shocks, including Beaudry and Portier (2006, 2014), Jaimovich and Rebelo (2009), Fujiwara, Hirose, and Shintani (2011) and Schmitt-Grohé and Uribe (2012). These papers give positive analysis and largely focus on the role of news about future technology shocks in accounting for business cycle fluctuations and assume symmetric information between the central bank and the private sector. We depart from the assumption of symmetric information to examine how the central bank should communicate superior information and examine the normative aspect of news shocks.

Representative-agent DSGE models are known to exhibit quantitatively too strong responses to a commitment to a policy rate cut at a future date. Del Negro, Giannoni, and Patterson (2012) have called this the "forward guidance puzzle." The puzzle is, however, about the size of the response and not the sign. Recent studies find ways to reduce this responsiveness by using a heterogeneous-agent incomplete market framework (McKay, Nakamura, and Steinsson, 2016) or by introducing bounded rationality (Gabaix, 2020) or both (Farhi and Werning, 2019). McKay, Nakamura, and Steinsson (2017) demonstrate that a representative-agent model can mimic the aggregate consumption response in their incomplete market model by introducing further "discounting" in the Euler equation. Even with the discounted Euler equation, if the central bank's loss function is a convex function of inflation and the output gap, then the central bank would still find it optimal to commit to secrecy. The welfare loss from transparency, however, would be smaller.[5]

[5] However, once we depart from the representative-agent New Keynesian model, we cannot in general justify such a loss function as a measure of social welfare loss.

2 Delphic forward guidance puzzle in a purely forward-looking New Keynesian model

Our baseline model is an extension of the simple New Keynesian model in which the monetary policy trade-off is given by distortionary cost-push shocks to the New Keynesian Phillips curve, and the central bank is assumed to be better-informed about future shocks. The central bank may send costless messages to the private sector, or the private sector may infer the central bank's private information from central bank actions that depend on its private information (as in, *e.g.*, Cukierman and Meltzer, 1986). The question we ask is, does the central bank find it beneficial to commit to making the private sector better-informed about future shocks? We find that the answer to this question is no: the optimal commitment policy never reveals or exploits superior information possessed by the central bank. This result holds even when the central bank possesses private information about the policy objective or when there is a zero lower bound constraint on nominal interest rates (Online Appendix B.2).

Proofs are simple and based on Jensen's inequality, exploiting the linearity of the New Keynesian Phillips curve and the strict convexity of the loss function.[6] Therefore, the result of the desirability of secrecy about future fundamental shocks holds true in more general, linearized DSGE models without endogenous state variables.

2.1 Environment

We employ the standard analytical framework for optimal monetary policy as in Woodford (2003), Galí (2008) or Walsh (2010). The framework is extended here to incorporate the central bank's superior information about future shocks and communication about it. A representative household consumes the final good, supplies labor to intermediate firms, and trades state-contingent claims in complete markets. There is a unit measure of intermediate firms, and firm $i \in [0, 1]$ supplies the intermediate good i in a monopolistically competitive market. The final good firm produces the final good from intermediate goods, using the Dixit-Stiglitz aggregator. The central

[6] Linearity is stronger than we need. A sufficient condition for our result is that the constraint set of the Ramsey problem is convex.

bank chooses the nominal interest rate as well as how much information to reveal to the private sector.

All exogenous shocks we describe below are modeled as random variables on the probability space $(\Omega, \mathcal{F}, P)$.

At the beginning of each period t, everyone in the economy observes contemporaneous fundamental shocks to the natural rate of interest, $r_t^n \in \mathbb{R}$, and the cost-push shock, $u_t \in \mathbb{R}$, and a common signal about future shocks, $s_t^{COMMON} \in \mathbb{R}^{N_{COM}}$.[7] The fundamental shocks, $\{r_t^n, u_t\}_{t=0}^{\infty}$, are allowed to be correlated with each other and over time. Following this, a private signal about future shocks, $s_t^{CB} \in \mathbb{R}^{N_{CB}}$, and a non-fundamental shock, $e_t \in \mathbb{R}^{N_e}$, realize and are privately observed only by the central bank. The signals, s^{COMMON} and s^{CB}, and the non-fundamental shock, e, can be multidimensional, *i.e.*, both N_{COM} and N_{CB} can be strictly bigger than one. For example, both the central bank and the private sector observe next period's mark-up shock, $s_t^{COMMON} = u_{t+1}$, but the central bank also observes the whole sequence of mark-up shocks, $s_t^{CB} = (u_{t+2}, u_{t+3}, ...)$. The non-fundamental shock is assumed to be independent of the fundamental shocks and can be also multidimensional, $N_e \geq 1$. The central bank may use the non-fundamental shock to add random noise to the message it sends to the private sector.

We denote the vector of exogenous random variables that the central bank observes at the beginning of period t by h_t^{CB}, *i.e.*, $h_t^{CB} = (r_t^n, u_t, s_t^{COMMON}, s_t^{CB}, e_t)$. The central bank's information is represented by a filtration $\mathcal{G}^{CB} = \{\mathcal{G}_t^{CB}\}_{t=0}^{\infty}$, where for each t, $\mathcal{G}_t^{CB} \subset \mathcal{F}$ is the smallest σ-field for which a sequence $h^{CB,t} := (h_0^{CB}, h_1^{CB}, ..., h_t^{CB})$ is measurable.

After observing h_t^{CB}, the central bank chooses the nominal interest rate, i_t, and a public message, m_t. Both i_t and $m_t \in \mathbb{R}^{N_m}$ depend only on the information possessed by the central bank when they are chosen, *i.e.*, they are $\mathcal{G}^{CB}$-adapted. We call a $\mathcal{G}^{CB}$-adapted process $\{(i_t, m_t)\}_{t=0}^{\infty}$ the central bank's policy. We assume that $N_m \geq N_{CB}$ so

[7]Strictly speaking, the natural rate is usually a linear combination of the productivity and the government spending shocks in the current period and the conditional expectations of these shocks in the next period. The central bank's communication policy can influence the contemporaneous natural rate by providing more information about the next period shocks. Because our theoretical results remain true even if we take it into account, we assume that r_t^n is unaffected by communication for notational simplicity.

that the central bank can always communicate the private signal itself if it wants to.

Before making a decision in period t, all private agents observe the central bank's action, (i_t, m_t), without error. Let $h_t^P = (r_t^n, u_t, s_t^{COMMON}, i_t, m_t)$ be the private sector's observation in period t. The private sector's information is represented by a filtration $\mathcal{G}^P = \{\mathcal{G}_t^P\}_{t=0}^{\infty}$. The private agents' decisions in period t depend only on the information that is summarized by $\mathcal{G}_t^P$.

Now we define a rational expectation equilibrium (REE) in this setting. In addition to the standard equilibrium conditions such as the New Keynesian Phillips curve and the Dynamic IS equation, it is required that (i) the private sector's information set is determined by the central bank's policy; and (ii) the output gap and inflation in period t must depend only on the private sector's information. Formally, a REE given the central bank's policy $\{(i_t, m_t)\}_{t=0}^{\infty}$ is a pair of a filtration $\mathcal{G}^P$ and a stochastic process of inflation and the output gap, $\{(\pi_t, x_t)\}_{t=0}^{\infty}$, such that: (i) for each t, $\mathcal{G}_t^P$ is the smallest σ-field for which a sequence of $h^{P,t} := (h_0^P, h_1^P, ..., h_t^P)$ is measurable; (ii) $\{(\pi_t, x_t)\}_{t=0}^{\infty}$ is $\mathcal{G}^P$-adapted; and (iii) the New Keynesian Phillips curve,

$$\pi_t = \kappa x_t + \beta \mathbb{E}[\pi_{t+1} | \mathcal{G}_t^P] + u_t, \tag{1}$$

and the Dynamic IS equation,

$$x_t = \mathbb{E}[x_{t+1} | \mathcal{G}_t^P] - \sigma^{-1}\{i_t - \mathbb{E}[\pi_{t+1} | \mathcal{G}_t^P] - r_t^n\}, \tag{2}$$

are satisfied. Here, κ is the standard slope parameter for the Phillips curve, β is the representative household's preference discount factor, and σ is the inverse of the intertemporal elasticity of substitution.

The central bank's ex ante loss function is given by

$$\mathbb{E}[\sum_{t=0}^{\infty} \delta^t L(\pi_t, x_t)], \tag{3}$$

where L is a strictly convex, momentary loss function and $\delta \in (0, 1)$ is the discount factor. This loss function represents the idea that the central bank pursues some kind of "dual mandate" — the central bank benefits from stabilizing inflation and the output gap. This specification nests the standard linear-quadratic model with a benevolent central bank minimizing the loss function which is obtained from the second-order approximation of the representative household's utility. The approximation in a Calvo-type sticky-price model is given by:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\frac{1}{2}\left(\pi_t^2+\frac{\kappa}{\varepsilon}x_t^2\right)+\text{ t.i.p.}\right], \tag{4}$$

where ε denotes the CES parameter for intermediate goods and t.i.p. refers to the terms independent of policy.[8] Online Appendix B.1 shows that the above standard quadratic loss function is valid in the present setting with communication.

Note that the expectation in (3) is the unconditional one and thus the loss is evaluated before any shocks realize and before either the private sector or the central bank receives any information at time 0. In standard symmetric information cases, it is more common to use the expectation conditional on time-0 information, *i.e.*, after the time-0 shocks have realized. All our theoretical results hold even if we replace the unconditional expectation with the conditional one given commonly observed period-0 shocks, $(r_0^n, u_0, s_0^{COMMON})$. Such a specification is justified under the assumption that the central bank evaluates its loss after the private sector and the central bank receive the common time-0 information but before the central bank receives any superior information.

2.2 Desirability of committing to secrecy: an illustrative example

We first use a simple example to illustrate a key mechanism through which information revelation reduces welfare.

Consider a quadratic loss function $\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\pi_t^2+bx_t^2\right)/2\right]$, and then let $b\to\infty$ holding other parameters fixed. This makes it extremely costly to have a non-zero

[8]This approximation is obtained when the steady-state distortion associated with monopolistic competition is offset by a tax or subsidy, with x denoting the welfare relevant output gap. Approximation is realization-by-realization, *i.e.*, it only uses backward-looking equations and never uses equations that involve conditional expectations. See, *e.g.*, Woodford (2003) and Online Appendix B.1. Therefore, t.i.p. does not include terms affected by information, *e.g.*, forecast errors by the private sector. It is not only independent of the central bank's interest rate strategy but also of its messaging strategy.

output gap, and, in the limit, the loss-minimizing central bank must conduct policy so that the output gap is always zero. Given a complete stabilization of the output gap at zero,

$$\pi_t = u_t + \beta\mathbb{E}[\pi_{t+1}|\mathcal{G}_t^P] = u_t + \mathbb{E}[\sum_{j=1}^{\infty}\beta^j u_{t+j}|\mathcal{G}_t^P]$$

must hold in any REE. Therefore, inflation is solely driven by exogenous shocks and the private sector's expectations about future shocks.

Suppose that the cost-push shock $\{u_t\}_{t=0}^{\infty}$ is a sequence of i.i.d. shocks with zero mean and a finite variance of σ_u^2 and that a common signal is uninformative, $s_t^{COMMON} = \emptyset$. Imagine that the central bank observes N-period ahead shocks, $s_t^{CB} = (u_{t+1}, u_{t+2}, ..., u_{t+N})$ in addition to contemporaneous shocks (r_t^n, u_t).

Now we ask whether and how fully revealing n-period ahead shocks, i.e. setting $m_t = (u_{t+1}, u_{t+2}, ..., u_{t+n})$, affects the inflation outcome and ex ante welfare for $n = 0, 1, \cdots, N$. When $n = 0$, we have

$$\pi_t = u_t \text{ and } \mathbb{E}[\pi_t^2] = \sigma_u^2.$$

For general n, we have

$$\pi_t = u_t + \beta u_{t+1} + \cdots + \beta^n u_{t+n}$$

and

$$\mathbb{E}[\pi_t^2] = \sigma_u^2 + \beta^2\sigma_u^2 + \cdots + \beta^{2n}\sigma_u^2 = \frac{1-\beta^{2(n+1)}}{1-\beta^2}\sigma_u^2 > \sigma_u^2.$$

Hence, the welfare loss is monotonically increasing in n.

Therefore, the ex ante welfare loss strictly increases if the private sector becomes able to observe future cost-push shocks perfectly. Hence, the benevolent central bank also wants to commit to not sending perfect signals about future shocks.

The mechanism at work is quite simple. Notice that the additional variability of inflation above comes from the increased volatility of expected, one-period ahead inflation. Future inflation varies with a future shock, and so when the private sector becomes better-informed about a future shock, its inflation expectations move due to previously unavailable information. Inflation expectations thus become more volatile.

The increased volatility of inflation expectations translates into higher variability of inflation through the New Keynesian Phillips curve. As we show in the following, this mechanism is also at work in our general setting, in which the output gap is a meaningful choice variable of the central bank and the central bank can send imperfect, noisy signals about its private information.

2.3 Optimal commitment policy is secretive

The Ramsey problem for the central bank is to choose a policy and a REE given the policy so that the welfare loss is minimized. We say that a policy, $\{(i_t, m_t)\}$, and a process of inflation and the output gap, $\{(\pi_t, x_t)\}$, solve the Ramsey problem if (i) $\big(\mathcal{G}^P, \{(\pi_t, x_t)\}\big)$ is the best REE given the policy; and if (ii) no other pair of a policy and a REE under that policy that achieves lower welfare loss than $\{(\pi_t, x_t)\}$.

As a benchmark, let us consider a situation where the central bank commits to secrecy — it neither reveals private information through message nor responds to it by adjusting the nominal interest rate. The private sector's information is then defined by the filtration generated by commonly observed shocks $\{(r_t^n, u_t, s_t^{COMMON})\}$, which we denote by $\mathcal{G}^{SEC}$. A policy $\{(i_t, m_t)\}$ is said to be *secretive* if and only if it is $\mathcal{G}^{SEC}$-adapted, *i.e.*, they depend only on the history of commonly observed shocks.

Define the *optimal secretive commitment policy*, denoted by $\{(i_t^{SEC}, m_t^{SEC}, \pi_t^{SEC}, x_t^{SEC})\}$, as the optimal commitment policy in this situation. Formally, the process of inflation and the output gap, $\{(\pi_t^{SEC}, x_t^{SEC})\}$, is a solution to the following problem:

$$\min_{\{(\pi_t, x_t)\}} \mathbb{E}[\sum_{t=0}^{\infty} \delta^t L(\pi_t, x_t)],$$

subject to the New Keynesian Phillips curve: for all t,

$$\pi_t = \kappa x_t + \beta \mathbb{E}[\pi_{t+1} | \mathcal{G}_t^{SEC}] + u_t,$$

and the information constraint: $\{(\pi_t, x_t)\}$ is $\mathcal{G}^{SEC}$-adapted. The process of the nominal interest rate, $\{i_t^{SEC}\}$, is obtained by solving

$$x_t^{SEC} = \mathbb{E}[x_{t+1}^{SEC}|\mathcal{G}_t^{SEC}] - \sigma^{-1}\{i_t^{SEC} - \mathbb{E}[\pi_{t+1}^{SEC}|\mathcal{G}_t^{SEC}] - r_t^n\}.$$

The message process, $\{m_t^{SEC}\}$, is uninformative, *i.e.*, $m_t^{SEC} = \emptyset$ always. Because the constraint set is convex, the optimal secretive commitment policy is unique almost surely.

The following proposition claims that the optimal secretive commitment policy is *the* solution to the Ramsey problem.

Proposition 1 *The optimal secretive commitment policy is a solution to the Ramsey problem. Any solution to the Ramsey problem equals the optimal secretive commitment policy with probability one.*

The proof can be found in Appendix A.1. Because this proposition implies that making the private sector better informed is welfare-reducing, it is also undesirable when the common signal becomes more informative:

Corollary 1 *The ex ante welfare loss increases when the common signal, $\{s_t^{COMMON}\}$, becomes more informative about future shocks.*

2.4 Intuition

To obtain some intuition, let us rewrite equation (1) as

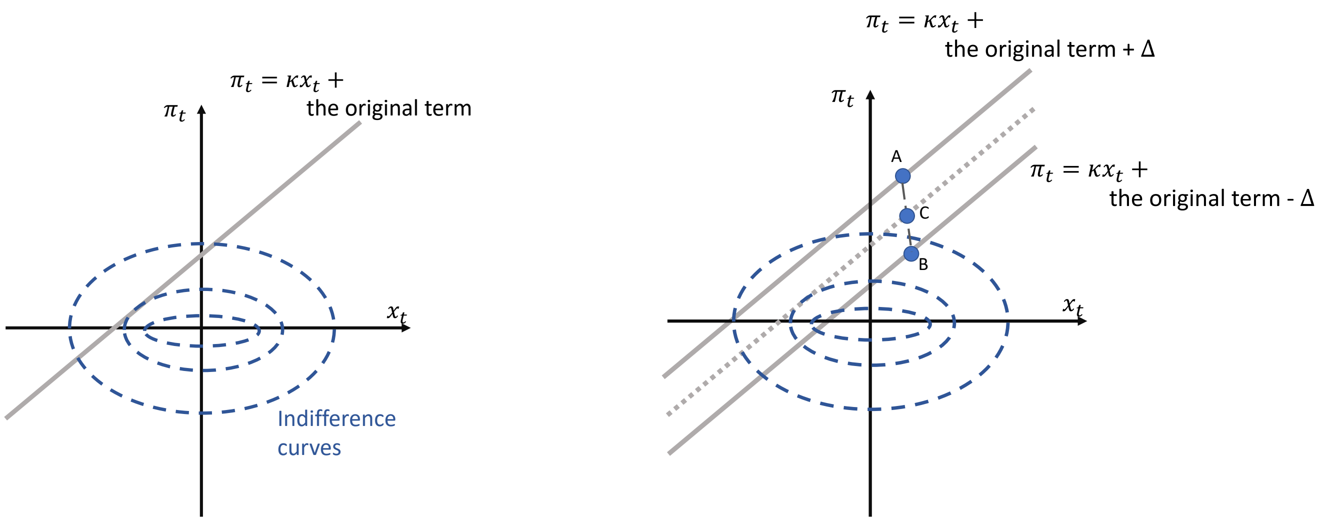
$$\pi_t - \kappa x_t = \underbrace{\{\beta\mathbb{E}[\pi_{t+1}|\mathcal{G}_t^{SEC}] + u_t\}}_{\text{“original” term}} + \underbrace{\beta\{\mathbb{E}[\pi_{t+1}|\mathcal{G}_t^{P}] - \mathbb{E}[\pi_{t+1}|\mathcal{G}_t^{SEC}]\}}_{\text{“updating” term}}. \tag{5}$$

Observe that the central bank that minimizes the expected loss in equation (3) benefits from stabilizing the right-hand side of equation (5), because it can then stabilize current inflation, π_t, and the current output gap, x_t. The right-hand side consists of two terms, the “original” term and the “updating” term. The former collects the terms that are present even when the private agents are left uninformed (when their information is given by $\mathcal{G}_t^{SEC}$), and the latter captures how inflation expectations are updated when the central bank's messages reveal some information (and the information is updated from $\mathcal{G}_t^{SEC}$ to any $\mathcal{G}_t^{P}$). Therefore, taking the probability distribution of the next period's inflation, π_{t+1}, as given, the updating term represents the effects of information revelation.

The decomposition in equation (5) implies that the presence of the updating term increases the variability of the right-hand side, and hence that the social loss increases with information revelation. To see this, note that the original term depends only on the information available to the private agents originally, *i.e.*, without additional information provided by the central bank, whereas the updating term is orthogonal to the original information set, $\mathcal{G}_t^{SEC}$. This orthogonality implies that the variance of the right-hand side of equation (5) is the sum of the variances of the original and the updating terms, which is minimized when the latter is zero, *i.e.*, when no additional information is provided. Roughly speaking, the updating term effectively acts as an additional orthogonal disturbance term in the New Keynesian Phillips curve, which exacerbates the inflation-output trade-off the central bank faces. Therefore, *any* information that helps predict future inflation is harmful for ex ante welfare.[9]

Figure 1 provides another way to understand intuitively Proposition 1. Holding the probability distribution of π_{t+1} fixed, Panel (a) in Figure 1 depicts some indifference curves based on the quadratic loss function and the New Keynesian Phillips curve when no information is provided by the central bank. Therefore, the updating term in equation (5) is zero. The original term in equation (5) is assumed to be positive. The origin is the bliss point at which the loss is minimized. In Panel (b), the updating term is no longer zero, and is assumed to be either positive (Δ) or negative ($-\Delta$) with equal probability. When the updating term is positive, the New Keynesian Phillips curve shifts upward from the no-information case, and when it is negative,

[9] Wohltmann and Winkler (2008) obtain a similar result in a perfect foresight economy. They derive ex post welfare under an optimal policy when a cost-push shock hits in a known period T. They find that welfare loss is minimized at $T = 0$ unless prices are implausibly flexible, and offer intuition along the lines of ours. However, under the assumption of perfect foresight, agents always perfectly anticipate an infinite sequence of cost-push shocks, regardless of the value of T. The effect they identify is not that of anticipation of future shocks, but rather that of the delayed materialization of a shock as in Del Negro, Giannoni, and Patterson (2012). Our framework is more suitable to analyze the role of the anticipation of future shocks, because the shock process is held fixed and only the private agents' information set is affected by the signal structure, and is more general because it permits, *e.g.*, imperfect signals.



(a) Without additional information

(b) With additional information

Figure 1: Indifference curves and the NKPCs

Panel (a) depicts some indifference curves and the New Keynesian Phillips curve when no information is provided by the central bank, with the original term being positive. In Panel (b), the updating term is either Δ or $-\Delta$ with equal probability. Take any points A and B on each of these two NKPCs. Then the probability-weighted average, point C, is on the original NKPC, and achieves lower loss than the mean loss achieved by A and B.

the Phillips curve shifts downward. How can we see that the central bank can achieve lower welfare in Panel (a)? In Panel (b), the central bank in period t can implement any point on the above New Keynesian Phillips curve (*e.g.*, point A in the figure) when the updating term is positive, and any point on the below New Keynesian Phillips curve (*e.g.*, point B) when the updating term is negative. No matter what points the central bank achieves in Panel (b), their probability-weighted convex combination (point C) lies exactly on the original Phillips curve in Panel (a). Because the loss function is convex, expected loss is lower when point C is achieved with certainty than when points A and B are achieved with equal probability. In other words, no matter what the central bank's choices are when some information is provided (Panel (b)), their mean is always attainable when no information is provided (Panel (a)) and achieves lower expected welfare loss. Hence, facing the Phillips curve in Panel (a) is strictly more desirable for the central bank than in Panel (b).

2.5 Where do the gains from better information go?

There are two reasons why ex ante welfare unambiguously deteriorates with additional information in this model. The first reason is a misalignment of incentives between the central bank and the price setters, and the second reason is the lack of endogenous state variables in the model.

2.5.1 Misalignment of incentives

If the private sector obtains more information, it may appear that private agents — both the household and goods producers — must not lose anything because they can still choose not to use the additional information. This assertion is incorrect because the price setters' incentives are not perfectly aligned with the household's (*i.e.*, social welfare) or with the central bank's. Price setters in a Calvo model do not internalize the inefficiency associated with price dispersion and their profit-maximizing responses to news shocks increase expected inefficiency.

To see this, consider a benevolent central bank that minimizes the loss in equation (4). Ideally, it wants to conduct policy so that both inflation and the output gap are always zero. For any given process of inflation and any information the household has, the central bank can indeed conduct policy so that the output gap is always zero. However, there is an incentive for the price setters to deviate from price stability even if the output gap is fully stabilized at zero, when a mark-up shock and inflation expectations deviate from zero. In this sense, price setters' incentives are not aligned with the social objective.[10] When price setters have more information about future shocks, they tailor their current prices based on additional information to increase profits.[11] However, as a result, prices then tend to move with future shocks and social welfare decreases.

[10] This is the reason why the optimal commitment policy problem has to take the New Keynesian Phillips curve, which summarizes price setters' incentives, as a constraint.

[11] The price setters take certain prices as given, *e.g.*, the aggregate nominal price, the real wage, etc. Taking these prices as given, the profits of the price setters weakly increase with information they possess. Because these objects change in an equilibrium when all firms change their prices using additional information, the price setters' equilibrium profits may not increase.

2.5.2 Lack of endogenous state variables

Can the central bank, by reacting preemptively, mitigate the negative effects of anticipated shocks? Not in this model. This model is purely forward-looking, and has no endogenous state variables. As a result, changes in the current nominal interest rate do not have any effects on future economic outcomes. Future economic losses cannot be reduced by taking preemptive policy actions in response to anticipated future disturbances. For example, a monetary policy tightening today won't reduce future inflation, and inflation at a future date can be reduced only by tightening monetary policy from that date onwards. In Online Appendix B.2 we show that the main result in this section holds true in some other purely forward-looking models, including a model with the zero lower bound on the nominal interest rate.

In contrast, in models with endogenous state variables, preemptive policy actions that react to anticipated future shocks may be able to reduce welfare loss in the future. At the same time, such preemptive actions reveal some of the central bank's private information and thus increase the economy's variability through the mechanism that is identified in the above model. Policymakers thus face a trade-off between these benefits and costs when choosing whether to react to its superior information. Section 3 examines this trade-off using two models with endogenous state variables.

2.6 Relation to the Bayesian persuasion literature

Before moving on to the models with endogenous state variables, let us emphasize the relationship between our theoretical results and the literature of Bayesian persuasion.

In the Bayesian persuasion literature, the informed party (Sender) commits to a signal-generating device before observing private information, and the uninformed party (Receiver) takes an action after observing a signal. A signal (or a message) affects Receiver's action through his posterior belief. Whether disclosure benefits Sender depends on how her payoff changes with Receiver's posterior belief. Kamenica and Gentzkow (2011) has shown that Sender benefits from disclosure if her interim payoff written as a function of Receiver's posterior belief is non-concave, because disclosure amounts to "concavifying" such a payoff function.

In our model, Sender is the central bank and Receiver is the private agents. As shown in Figure 1, an informative signal shifts the Phillips curve by affecting Receiver's posterior. It results in larger volatility of inflation and the output gap. Sender's interim payoff (loss) as a function of Receiver's posterior belief is, therefore, concave (convex, respectively).

Therefore our result can be interpreted as an application of Bayesian persuasion to a macroeconomic question of forward guidance. However, our model differs from standard models in the Bayesian persuasion literature: the central bank not only sends messages but also sets interest rates that affect Receiver's incentives, and Receiver is not a single agent but atomless private agents that interact with each other and with the central bank through markets. Despite these differences, the basic insight of Bayesian persuasion holds true.

Fujiwara and Waki (2020) use a neoclassical growth model without nominal rigidities to analyze the welfare effect of forward guidance about future fiscal shocks. They find that the optimal disclosure policy features selective transparency — forward guidance should be transparent about future non-distortionary spending shocks while secretive about future distortionary tax shocks. In contrast to the present paper in which the New Keynesian Phillips curve is crucial for the optimality of non-transparency, the key mechanism in their paper is the Euler equation that can be distorted by future taxes.

3 Models with endogenous state variables

Now we consider three models in which preemptive policy actions can reduce future welfare loss by reacting to anticipated future disturbances, through adjustments of endogenous state variables. Such preemptive actions may raise current loss for two reasons. First, preemptive actions may reveal some information about future shocks and thus destabilize the current economy through more variable expectations in forward-looking equations. Second, preemptive actions themselves may be costly. Hence, there is a trade-off, and we examine it in different models.

The first model is a model with an effective lower bound (ELB) on the nominal interest rate and with backward price indexation. It is the demand shock (the natural rate shock) that is anticipated. The second model features both price stickiness and wage stickiness. Two mark-up shocks hit each of the Phillips curves. The third model is a more elaborate DSGE model with many real and nominal frictions and multiple shocks.

3.1 A model with backward indexation and an effective lower bound

In a model with backward price indexation, the Dynamic IS equation is still given by equation (2) but the New Keynesian Phillips curve (without a cost-push shock) is given by

$$\pi_t - \gamma\pi_{t-1} = \kappa x_t + \beta \mathbb{E}_t \left(\pi_{t+1} - \gamma\pi_t\right),$$

where $\gamma \in [0, 1)$ denotes the degree of price indexation. The difference from the model without price indexation is that inflation, π_t, is replaced with its quasi-difference, $\pi_t - \gamma\pi_{t-1}$. As shown in Woodford (2003), inflation in the loss function is also replaced by the quasi-difference, and the quadratic loss function takes the following form:

$$L(y_t, \pi_t - \gamma\pi_{t-1}) = \left(\pi_t - \gamma\pi_{t-1}\right)^2 + b x_t^2,$$

for some $b > 0$.

When the optimal monetary policy is considered, the IS equation is again redundant and thus is dropped from the set of constraints in the planner's problem. Then, denoting the quasi-difference of inflation, $\pi_t - \gamma\pi_{t-1}$, by $\hat{\pi}_t$, the optimal commitment policy problem is isomorphic to that in the purely forward-looking New Keynesian model studied in the previous section. It follows that the optimal commitment policy features secrecy. Hence, price indexation per se cannot overturn our result in the previous section. When the effective lower bound (ELB) on the nominal interest rate is imposed, then the Dynamic IS equation is no longer redundant whenever the ELB binds. Because inflation expectations in the Dynamic IS equation contains a geometric sum of past quasi-differences of inflation, there is a backward-looking element in the Dynamic IS equation.[12]

Thus, the central bank may be able to mitigate the severity of a future recession due to a binding ELB, by cutting the nominal rate preemptively. Imagine that the central

[12] This is because $\pi_{t+1} = \hat{\pi}_{t+1} + \gamma\pi_t = \hat{\pi}_{t+1} + \gamma\hat{\pi}_t + \gamma^2\pi_{t-1} = \cdots = \sum_{j=0}^{t} \gamma^j \hat{\pi}_{t+1-j} + \gamma^t \pi_0$.

bank anticipates that the natural rate will become negative and that the nominal rate will be constrained at the ELB in the near future. If the central bank cuts the nominal rate today and generates inflation, then holding other things equal, expected inflation in a future date at the ELB also rises. This lowers the real interest rate at the ELB, the recession is ameliorated, and welfare loss at the ELB is reduced.

However, cutting the nominal rate today may send a negative signal about the future state of the economy, and may worsen the economic outcome today through worsened expectations about future economic conditions. For example, suppose that the central bank cuts the nominal rate today only when it receives a news shock that the ELB is going to bind in near future. Given this strategy of the central bank, after observing a nominal interest rate cut, private agents become more certain that the future economy will be in a severe recession, thereby putting a downward pressure on the current economy. To prevent private agents' expectations from deteriorating, the central bank may need to use a strategy so that the private agents can infer its superior information only imperfectly or not at all.

We illustrate this trade-off using a simple model. The only shock in the economy is the period-1 natural rate shock. The natural rate in period 1, r_1^n, becomes negative with probability p. We denote the negative value of the natural rate by $r_{elb}^n < 0$. The period-1 natural rate equals its steady-state value of r^n with probability $1 - p$. Again the central bank possesses superior information at the beginning of period 0. The period-1 natural rate r_1^n becomes public information at the beginning of period 1, while the central bank observes it in advance at the beginning of period 0.

Further simplifying assumptions are in order. First, the initial condition for previous period's inflation is set to zero, $\pi_{-1} = 0$. Its initial value is never crucial in our numerical experiments, and thus it is set to zero to simplify mathematical expressions. The quasi-difference of inflation in period 0 is the same as the period-0 inflation itself, $\pi_0 = \hat{\pi}_0$. Second, the ELB is set to zero and is imposed only in period 1.

Our third simplifying assumption is that, from period 1 onwards, the central bank uses the optimal discretionary policy. It follows that, whenever $r_t^n = r^n$ from period 1 onwards, both the output gap, x_t and the quasi-difference of inflation, $\hat{\pi}_t$ become zero. As a result, the associated loss is also zero. This assumption also implies that the nominal rate hits the ELB when the natural rate becomes negative in period 1. The quasi-difference of inflation and the output gap in period 1 if $r_t^n = r_{elb}^n$ are denoted by $\hat{\pi}_{1,elb}(\pi_0)$ and $x_{1,elb}(\pi_0)$, respectively, and are given by

$$(x_{1,elb}(\pi_0), \hat{\pi}_{1,elb}(\pi_0)) = \left(\frac{\gamma^2/\sigma}{1-\gamma\kappa/\sigma}\pi_0 + \frac{1/\sigma}{1-\gamma\kappa/\sigma} r_{elb}^n, \frac{\kappa\gamma^2/\sigma}{1-\gamma\kappa/\sigma}\pi_0 + \frac{\kappa/\sigma}{1-\gamma\kappa/\sigma} r_{elb}^n \right).$$

Derivation can be found in Online Appendix B.3. We assume $1 - \gamma\kappa/\sigma > 0$ so that the negative natural rate shock is contractionary and that positive inflation in period 0 is expansionary.[13] The continuation welfare loss from period 1 onwards if $r_t^n = r_{elb}^n$ is, therefore, a quadratic function of π_0, which we denote by $\mathcal{L}_{1,elb}(\pi_0)$. The continuation loss is zero if $r_t^n = r^n$ instead.

Having characterized what happens from period 1 on, now we move on to the analysis of period 0. Before observing its private information, the central bank commits to a strategy to minimize ex ante welfare loss. We compare three options that the central bank may take. First, the central bank never utilizes its superior information; second, the central bank uses its action (the period-0 interest rate) but not messages; third, the central bank use both messages and actions. To simplify the analysis, the zero lower bound is not imposed in period 0.

3.1.1 Optimal interest rate setting under secrecy

First consider the optimal secretive policy. In order to keep private information secret, the central bank needs to use the same (potentially mixed) strategy, regardless of its private information. With such strategies, messages must be uninformative, and, therefore, we assume, without loss of generality, that the central bank always sends the same message and that the private agents' belief that the ELB is binding in period 1 is fixed at the prior probability p.

The central bank's problem given the private agents' prior probability p is to solve

$$C(p) := \min_{(\pi_0, x_0)} L(\pi_0, x_0) + \beta p \mathcal{L}_{1,elb}(\pi_0)$$

[13] Graphically, this assumption can be stated equivalently as follows. Taking the inflation quasi-difference on the vertical axis and the output gap on the horizontal axis, the Aggregate Demand curve (the IS equation with a fixed interest rate) is steeper than the Aggregate Supply curve (the New Keynesian Phillips curve).

subject to the Phillips curve,

$$\pi_0 = \kappa x_0 + \beta p \hat{\pi}_{1,elb}(\pi_0).$$

The optimal interest rate is obtained using the solution to the problem and the Dynamic IS equation,

$$x_0 = p x_{1,elb}(\pi_0) - \sigma^{-1} \{ i_0 - (\gamma \pi_0 + p \hat{\pi}_{1,elb}(\pi_0)) - r^n \}.$$

Note that $p\hat{\pi}_{1,elb}(\pi_0)$ and $p x_{1,elb}(\pi_0)$ are the unconditional expectations of the future quasi-difference in inflation, $\mathbb{E}[\hat{\pi}_1]$, and of the output gap, $\mathbb{E}[x_1]$, and that $\gamma\pi_0 + p\hat{\pi}_{1,elb}(\pi_0)$ in the Dynamic IS equation is the expected inflation, $\mathbb{E}[\pi_1]$. Because all the constraints are linear and the loss function is strictly convex, there is no additional gain from allowing for mixed strategies.

3.1.2 Optimal messaging and interest rate setting strategy

If the central bank can use both messages and actions, what is the optimal strategy? In this case, any information that is conveyed through the setting of the nominal interest rate can be also communicated using messages. Therefore, without loss of generality, we assume that the central bank first sends messages and that when setting the nominal interest rate the central bank utilizes no more information than the sent messages.

Once the private agents receive messages, they update their beliefs using Bayes' rule, taking the messaging strategy of the central bank as given. Because the nominal interest rate is not allowed to convey more information than messages, for each posterior belief, we can consider a belief-dependent optimal interest rate.

Let $\rho \in [0,1]$ be the posterior probability of the period-1 natural rate shock being negative. The optimal belief-dependent policy is a solution to the following problem:

$$C(\rho) := \min_{(\pi_0, x_0)} L(\pi_0, x_0) + \beta \rho \mathcal{L}_{1,elb}(\pi_0)$$

subject to the Phillips curve,

$$\pi_0 = \kappa x_0 + \beta \rho \hat{\pi}_{1,elb}(\pi_0).$$

Again, the optimal interest rate is obtained using the solution to the problem and the Dynamic IS equation,

$$x_0 = \rho x_{1,elb}(\pi_0) - \sigma^{-1}\left\{i_0 - (\gamma\pi_0 + \rho\hat{\pi}_{1,elb}(\pi_0)) - r^n\right\}.$$

The structure of the problem is identical to that in the previous section, with the prior probability p being replaced with the posterior probability ρ.

However, unlike in the problem under secrecy, the posterior ρ is not exogenous and the central bank can influence it through a messaging strategy. The problem faced by the central bank is then indeed a problem of Bayesian persuasion (Kamenica and Gentzkow, 2011), and is formulated as follows:

$$C^*(p) := \min_{\Psi\text{:a probability distribution over }[0,1]} \int C(\rho)d\Psi \qquad \text{subject to} \quad p = \int \rho d\Psi.$$

In words, the central bank can induce a probability distribution of posteriors, ρ, over an interval $[0,1]$ using a messaging strategy, subject to the constraint that the mean of the distribution must be equal to the prior, p.

A set of standard results in Bayesian persuasion applies. For example, if C is a convex function, then by Jensen's inequality, the solution to the problem puts a unit mass at $\rho = p$, meaning that secrecy is optimal and that $C^*(p) = C(p)$. If C is a concave function, then again by Jensen's inequality, it is optimal to put a probability weight of p at $\rho = 1$ and a weight $1-p$ at $\rho = 0$. If C is neither concave nor convex, then secrecy may be optimal for some prior values but may not be for other prior values.

What is then crucial for desirability of communication is whether C is convex or non-convex. Bayesian persuasion improves welfare if and only if C is non-convex.

3.1.3 Optimal interest rate strategy without messages

If the central bank cannot use messages, then its private information may only be communicated through its actions, *i.e.*, the nominal interest rate. What is the optimal strategy for the central bank in this setting? It turns out that actions are nearly sufficient for the central bank. In Online Appendix B.3 we show that the central bank can *virtually* achieve C^* even without messages, *i.e.*, for any prior probability p and any

$\epsilon > 0$, the central bank can achieve an ex ante loss that is lower than $C^*(p) + \epsilon$ without sending messages.

It follows that in the optimal policy we have characterized, the central bank conveys no more information through messages than its action reveals.

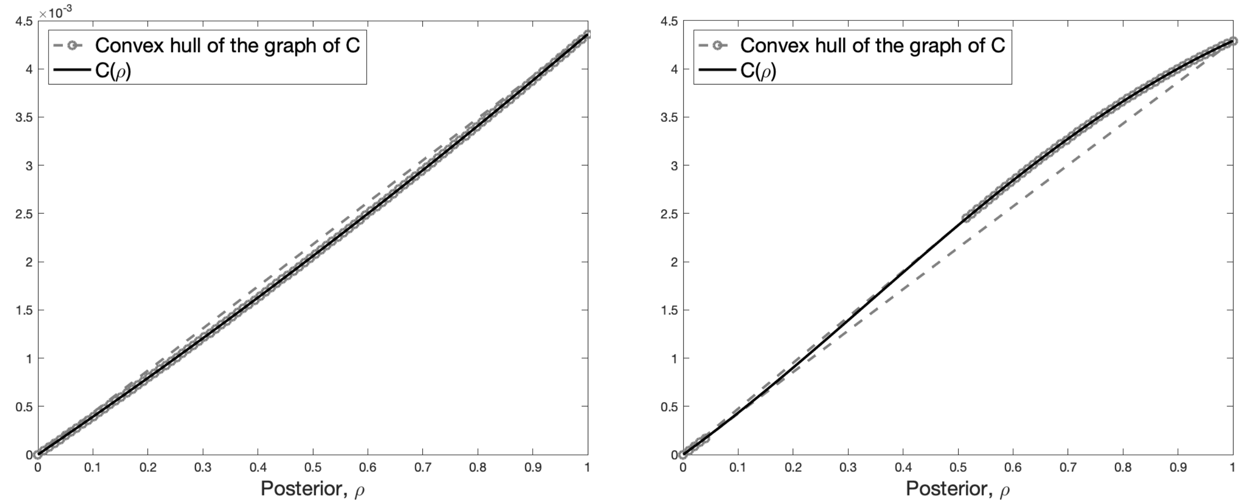
3.1.4 Numerical examples

Here we present two numerical examples. The first example features a convex C, *i.e.*, secrecy is optimal. In the second example, C is neither convex nor concave, and forward guidance improves welfare.

It turns out that it is difficult to find an example in which the Delphic forward guidance is beneficial. Examples are find only when the Phillips curve is sufficiently steep and when the elasticity of intertemporal substitution is extremely high. With realistic parameter values, secrecy is optimal and the central bank finds it optimal to set the current nominal rate independently of its superior information about the future natural rate shock.

Example 1: convex C. Consider $\beta = 0.99$, $\kappa = 0.04$, $r^n = 100 \times (1/\beta - 1)$ (%), $\gamma = 0.6$, $\varepsilon = 3$, $\sigma = 1$, $L(\pi, x) = \pi^2 + \kappa/\varepsilon x^2$, and $r^n_{elb} = -0.5$ (%). Figure 2a depicts the function C with the red line and the convex full of its graph with a blue dashed line. The lower envelope of the convex hull, which is C^*, coincide with the function C. In this example, secrecy is optimal. Indeed, with standard parameter values, we could only find convex C.

Example 2: non-convex C. We could find examples in which C^* is non-convex but they assume extremely unrealistic parameter values. Using high values of the slope of the Phillips curve, κ, and low values for the inverse of elasticity of intertemporal substitution, σ, is key for obtaining non-convex C. Here we set $\kappa = 0.2$ and $\sigma = 0.2$ and use the same values for other parameters as in Example 1. Hence, the elasticity of intertemporal substitution is as high as 5. In Figure 2b, again the function C and the convex hull of its graph are drawn. The lower envelope of the convex hull, which is C^*, is lower than the function C for a wide range of posterior, ρ. Hence, a messaging strategy can strictly improve welfare.



(a) Example 1: convex C.

(b) Example 2: non-convex C.

Figure 2: Welfare implication of forward guidance
These panels draw the function $C(\rho)$ and the convex hull of its graph. The lower envelope of the latter corresponds to $C^*(\rho)$.

3.2 A model with sticky price and wage

The second model with endogenous state variables is a New Keynesian model with price and wage stickiness in the spirit of Erceg, Henderson, and Levin (2000). The model is simplified by assuming that the household supplies differentiated labor to a labor union where they are aggregated into composite labor. We incorporate mark-up shocks for price and for wage.

A REE given policy $\{(i_t, m_t)\}$ is a pair of a filtration $\mathcal{G}^P$ that represents the private agents' information and a process of price inflation, wage inflation, the output gap, and the real wage, $\{(\pi_t, \pi_{W,t}, x_t, w_t)\}$, which satisfies the following four equations: the price Phillips curve,

$$\pi_t = \kappa_P (w_t + \mu_t) + \beta \mathbb{E}[\pi_{t+1} | \mathcal{G}_t^P],$$

where μ_t is the price mark-up shock, the wage Phillips curve,

$$\pi_{W,t} = \kappa_W [(\eta + \sigma) x_t - w_t + \mu_{W,t}] + \beta \mathbb{E}[\pi_{W,t+1} | \mathcal{G}_t^P],$$

where $\mu_{W,t}$ is the wage mark-up shock, the real wage dynamics,

$$w_t = w_{t-1} + \pi_{W,t} - \pi_t,$$

and the Dynamic IS equation (2), as well as the information requirement, which is the same as in the purely forward-looking model in Section 2: (i) the private sector's information set is determined by the central bank's policy; and (ii) the endogenous variables in period t must depend only on the private sector's information.

The loss function is the standard quadratic one:

$$\mathbb{E}\left[\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left\{\frac{\varepsilon}{\kappa_P}\pi_t^2 + \frac{\theta}{\kappa_W}\pi_{W,t}^2 + (\sigma+\eta)\,x_t^2\right\}\right].$$

Parameters ε and θ denote the elasticities of substitution across intermediate goods and across intermediate labor, respectively. The coefficient on the output gap squared is the sum of the inverse of the elasticity of intertemporal substitution, σ, and the inverse of Frisch elasticity, η.

Several simplifying assumptions are now in order. First, the mark-up shocks hit the economy only in period 1 and take the value of zero in all other periods. Hence, the source of uncertainty in the model is $(\mu_1, \mu_{W,1})$ only, and we assume that their distributions are independent, standardized normal distributions. The central bank observes $(\mu_1, \mu_{W,1})$ at the beginning of period 0, whereas the private agents do not observe them until they realize, *i.e.*, at the beginning of period 1. The private agents' prior is the same as the true distributions of shocks, *i.e.*, it is given by independent, standardized normal distributions.

The second simplifying assumption is that, from period 1 onwards, the central bank conducts the optimal discretionary policy. Once the period-1 shocks realize, there is no longer information asymmetry. Hence, the definition of the optimal discretionary policy here is a standard one. Thanks to the linear-quadratic structure, there is a time-stationary linear mapping from the pair of the previous period's real wage and the current shock to all endogenous variables — price inflation, wage inflation, the output gap, and the current real wage.[14] Specifically, we write the mappings for

[14]Period 1 is different from all the following periods — there are shocks in period 1 but not in other

price and wage inflation as $\pi_t = g_{P,w}w_{t-1} + g_{P,\mu}\mu_t + g_{P,\mu_W}\mu_{W,t}$ and $\pi_{W,t} = -g_{W,w}w_{t-1} + g_{W,\mu}\mu_t + g_{W,\mu_W}\mu_{W,t}$ for all $t \geq 1$. The continuation loss from period 1 onwards under the optimal discretionary policy is a quadratic function of the period-0 real wage and the period 1 shocks, and is then succinctly written as $\mathcal{L}(w_0, \mu_1, \mu_{W,1}) + F(\mu_1, \mu_{W,1})$. Here, the first function $\mathcal{L}$ collects terms that vary with the period-0 real wage and is given by

$$\mathcal{L}(w_0, \mu_1, \mu_{W,1}) = L_w w_0^2 + 2L_{w,\mu}w_0\mu_1 + 2L_{w,\mu_W}w_0\mu_{W,1} \tag{6}$$

for some constants $L_w > 0$, $L_{w,\mu}$, L_{w,μ_W}. In contrast, the second function F is a collection of terms that are independent of the endogenous variable (the real wage).

3.2.1 Phillips curves and the expected continuation loss in period 0

Before analyzing the optimal policy problem, let us describe what the above simplifying assumptions imply for the period-0 Phillips curves and the period-0 expected continuation loss.

Let $\mathcal{G}_0^P$ denote the period-0 information set of the private agents. Taking as given that the optimal discretionary policy determines what happens from period 1 onwards, the Phillips curves can be rewritten as

$$\pi_0 = (\kappa_P + \beta g_{P,w})w_0 + \beta g_{P,\mu}\mathbb{E}[\mu_1|\mathcal{G}_0^P] + \beta g_{P,\mu_W}\mathbb{E}[\mu_{W,1}|\mathcal{G}_0^P] \tag{7}$$

and

$$\pi_{W,0} = \kappa_W(\sigma+\eta)x_0 - (\kappa_W + \beta g_{W,w})w_0 + \beta g_{W,\mu}\mathbb{E}[\mu_1|\mathcal{G}_0^P] + \beta g_{W,\mu_W}\mathbb{E}[\mu_{W,1}|\mathcal{G}_0^P]. \tag{8}$$

Noting that the continuation loss function $\mathcal{L}$ is linear in shocks, $(\mu_1, \mu_{W,1})$, as shown in equation (6) and that the period-0 real wage is known in period 0, we have

$$\mathbb{E}[\mathcal{L}(w_0, \mu_1, \mu_{W,1})|\mathcal{G}_0^P] = \mathcal{L}(w_0, \mathbb{E}[\mu_1|\mathcal{G}_0^P], \mathbb{E}[\mu_{W,1}|\mathcal{G}_0^P]),$$

periods. However, this does not affect the linear mapping, because the certainty-equivalence holds thanks to the linear-quadratic structure of this model. We can apply the same linear mapping for all periods following period 1 and substitute the value of zero to the current shocks from period 2 onwards.

and thus the expected continuation loss is given by:

$$\mathcal{L}(w_0, \mathbb{E}[\mu_1|\mathcal{G}_0^P], \mathbb{E}[\mu_{W,1}|\mathcal{G}_0^P]) + \mathbb{E}[F(\mu_1, \mu_{W,1})|\mathcal{G}_0^P]. \tag{9}$$

These equations are used in the optimal policy problem below.

3.2.2 Optimal interest rate setting under secrecy

Suppose that the central bank, before observing its private information, commits to secrecy, *i.e.*, to sending an empty message no matter what. Then, what is the optimal interest rate under secrecy?

Because the private agents' prior distribution of shocks is identical to the unconditional distribution, which is independent standard normal, their expectations of future shocks under a secretive policy are zero. Hence, the conditional expectations of μ_1 and $\mu_{W,1}$ in equations (7), (8), and (9) are all identical to zero. The conditional expectation of $F(\mu_1, \mu_{W,1})$ equals its unconditional expectation, $\mathbb{E}[F(\mu_1, \mu_{W,1})]$, which is an exogenous constant.

The optimal interest rate under secrecy can be found by solving

$$\min_{\pi_0, \pi_{W,0}, x_0, w_0} \frac{1}{2}\left(\frac{\varepsilon}{\kappa_P}\pi_0^2 + \frac{\theta}{\kappa_W}\pi_{W,0}^2 + (\sigma+\eta)\, x_0^2\right) + \beta\mathcal{L}(w_0, 0, 0)$$

subject to the price and the wage Phillips curves,

$$\pi_0 = (\kappa_P + \beta g_{P,w}) w_0,$$

$$\pi_{W,0} = \kappa_W(\sigma+\eta)x_0 - (\kappa_W + \beta g_{W,w}) w_0,$$

and the real wage dynamics,

$$w_0 = \pi_{W,0} - \pi_0. \tag{10}$$

The initial condition, w_{-1}, is set to zero. The optimal interest rate is backed out from the solution to this problem and the dynamic IS equation.

The solution is trivial, $\pi_0 = \pi_{W,0} = x_0 = w_0 = i_0 - r^n = 0$, because the expected continuation loss is minimized at $w_0 = 0$: $\mathcal{L}(w_0, 0, 0) = L_w w_0^2$ with $L_w > 0$. Hence, under secrecy, there is no action in period 0, and the shocks have effects only after they materialize in period 1.

3.2.3 Optimal messaging and interest rate setting strategy

When the central bank can use a messaging strategy flexibly, we can assume, without loss of generality, that its actions (*i.e.*, setting of the nominal interest rate) do not reveal more information than messages. The private agents update their beliefs upon observing a message, but their beliefs are fixed afterwards throughout period 0. Hence, we first consider the optimal interest rate setting strategy given the private agents have arbitrary beliefs, and then take a step back to characterize the optimal messaging strategy that induces a desirable distribution of posteriors.

Let us consider the period-0 optimal policy problem given private agents' beliefs. Given the private agents' beliefs, and given that the central bank does not utilize information that is not known by them when choosing the interest rate, the second term in the expression (9) is independent of the central bank's actions and can be omitted in the minimization problem. Hence, the private agents' beliefs in the problem are summarized by a pair of two conditional expectations, $\mathbb{E}[\mu_1|\mathcal{G}_0^P]$ and $\mathbb{E}[\mu_{W,1}|\mathcal{G}_0^P]$.

Let (μ^e, μ_W^e) denote the private agents' posterior expectations of shocks, and we identify the pair as the private agents' beliefs. The period-0 problem given beliefs (μ^e, μ_W^e) is then given by

$$C(\mu^e, \mu_W^e) := \min_{\pi_0, \pi_{W,0}, x_0, w_0} \frac{1}{2}\left(\frac{\varepsilon}{\kappa_P}\pi_0^2 + \frac{\theta}{\kappa_W}\pi_{W,0}^2 + (\sigma+\eta)\, x_0^2\right) + \beta\mathcal{L}(w_0, \mu^e, \mu_W^e)$$

subject to the price and the wage Phillips curve,

$$\pi_0 = (\kappa_P + \beta g_{P,w})w_0 + \beta g_{P,\mu}\mu^e + \beta g_{P,\mu_W}\mu_W^e,$$

$$\pi_{W,0} = \kappa_W(\sigma+\eta)x_0 - (\kappa_W + \beta g_{W,w})w_0 + \beta g_{W,\mu}\mu^e + \beta g_{W,\mu_W}\mu_W^e,$$

and the real wage dynamics in equation (10). Again, the optimal interest rate can be found by the solution to the above problem and the Dynamic IS equation. Clearly, the problem under secrecy is a special case of this problem, with $(\mu^e, \mu_W^e) = (0, 0)$.

Because the problem is a linear-quadratic one, its solution is characterized by a set of first-order conditions. In the first-order conditions, the conditional expectations of shocks, (μ^e, μ_W^e), appear only in an additively-separable manner. This is because they do not appear in the coefficients of the quadratic terms of the endogenous variables. Hence, the minimized loss, $C(\mu^e, \mu_W^e)$, is a quadratic function in (μ^e, μ_W^e), and the solution is linear in (μ^e, μ_W^e). The implied nominal rate's deviation from the natural rate, $i_0 - r^n$, is also linear in them.

Now we consider the optimal messaging strategy that minimizes ex ante loss, *i.e.*, unconditional expectation of loss. It is obtained by integrating $C(\mu^e, \mu_W^e)$ with respect to the distribution of beliefs, (μ^e, μ_W^e), that is induced by a messaging strategy, and by adding the unconditional expectation $\mathbb{E}[F(\mu_1, \mu_{W,1})]$. Because the unconditional expectation of F is independent from the messaging strategy, we can characterize the optimal messaging strategy only by focusing on the expected value of $C(\mu^e, \mu_W^e)$.

The following proposition relates the shape of C and the optimal message strategy:

Proposition 2 (Optimal messaging strategy) *(i) If C is a convex function, then secrecy, i.e. $m_0 = \emptyset$, is optimal. (ii) If C is a concave function, then full disclosure, i.e. $m_0 = (\mu_1, \mu_{W,1})$, is optimal. (iii) If C is neither convex nor concave, then there exists a pair of coefficients, $g_{m,\mu}$ and g_{m,μ_W}, such that sending a message $m_0 = g_{m,\mu}\mu_1 + g_{m,\mu_W}\mu_{W,1}$ is optimal.*

The proof is in Appendix A.2. Both (i) and (ii) in the above proposition follow immediately from Jensen's inequality. Regarding (iii), this messaging strategy implies that the conditional expectation of shocks is linear in the message, *i.e.*, $\mathbb{E}[\mu_1|m_0] = g_{m,\mu}m_0/(g_{m,\mu}^2 + g_{m,\mu_W}^2)$ and $\mathbb{E}[\mu_{W,1}|m_0] = g_{m,\mu_W}m_0/(g_{m,\mu}^2 + g_{m,\mu_W}^2)$. Hence, the conditional expectation of shocks is distributed on a straight line in two-dimensional space. The coefficients $g_{m,\mu}$ and g_{m,μ_W} are chosen in such a way that the quadratic function C restricted onto this line is concave. (See Appendix A.2.)

3.2.4 Optimal interest rate strategy without messages

In the previous model with an ELB, it is shown that the central bank's ability to use messages is (virtually) irrelevant. In the current model, this ability is relevant if and only if C is concave, as the following proposition shows.

Proposition 3 ((Ir)relevance of messages) *The central bank's ability to send a message is irrelevant if C is convex or if C is neither convex nor concave: the central bank can achieve the same ex ante welfare loss only through an interest rate strategy. If C is concave, then the ability to send a message is essential: the attainable ex ante loss is higher without the messaging strategy.*

The proof is in Appendix A.3. This result is obvious when secrecy is optimal, *i.e.*, when C is convex. When C is concave, then the optimal messaging strategy is to convey both μ_1 and $\mu_{W,1}$. To minimize ex ante loss under the optimal messaging strategy, the interest rate strategy must be linear in μ_1 and $\mu_{W,1}$. It follows that the private agents cannot recover the pair $(\mu_1, \mu_{W,1})$ only by observing the realized interest rate. Therefore, the minimal ex ante loss that is achievable without a messaging strategy is strictly higher than that attainable with the optimal messaging strategy. When C is neither convex nor concave, it turns out that, under the optimal message strategy, the interest rate strategy is linear in the message m_0. Hence, the private agents can back out the optimal message by observing only the realized interest rate.

According to this proposition, unless the function C is concave, the central bank should convey no more information using messages than revealed through the nominal interest rate it chooses.

3.2.5 A model with one shock

What would happen if, instead of two mark-up shocks, the economy is hit by only one shock? In such a model, the function C is still a quadratic function of the expected value of the shock and is symmetric around zero, *i.e.*, it is a constant times the squared expected value of the shock. Hence, it is either concave or convex. It follows that the optimal messaging strategy is either full disclosure or no disclosure. In a model with only one shock, the central bank's ability to send messages is irrelevant, because the private agents can perfectly infer the shock if the nominal interest rate is a linear, non-constant function of the shock.

Parameter	Description	Value
β	Subjective discount factor	0.99
θ	Elasticity of substitution among labor	6
ε	Elasticity of substitution among goods	6
σ	Inverse of intertemporal elasticity of substitution	1
η	Inverse of Frisch elasticity	1
ω	Price stickiness	0.75
ϕ	Wage stickiness	0.75

Table 1: Benchmark calibration

Slopes of the two Phillips curves, κ_P and κ_W, are determined by the Calvo parameters for price, ω, and for wage, ϕ, as $\kappa_P := (1-\omega)(1-\omega\beta)/\omega$ and $\kappa_W := (1-\phi)(1-\phi\beta)/\phi$.

3.2.6 Numerical examples

To understand how different shapes of C are obtained in this model, we conduct some numerical experiments. The benchmark calibration of parameters is as shown in Table 1. Both price and wage stickiness are set to somewhat high values in the baseline, and we vary these parameters to see how they affect the shape of C and, hence, the optimal messaging strategy.

Our benchmark calibration implies that the function C is convex, and, therefore, that secrecy is optimal. However, using different parameterizations we also find cases in which C is neither convex nor concave. In Figure 3, we vary the wage and the price stickiness parameters between 0.001 and 0.999 to see how the shape of the function C changes. Other parameters are fixed as in Table 1. Because the function C is a quadratic form, there is a 2-by-2 symmetric real matrix C_{matrix} such that

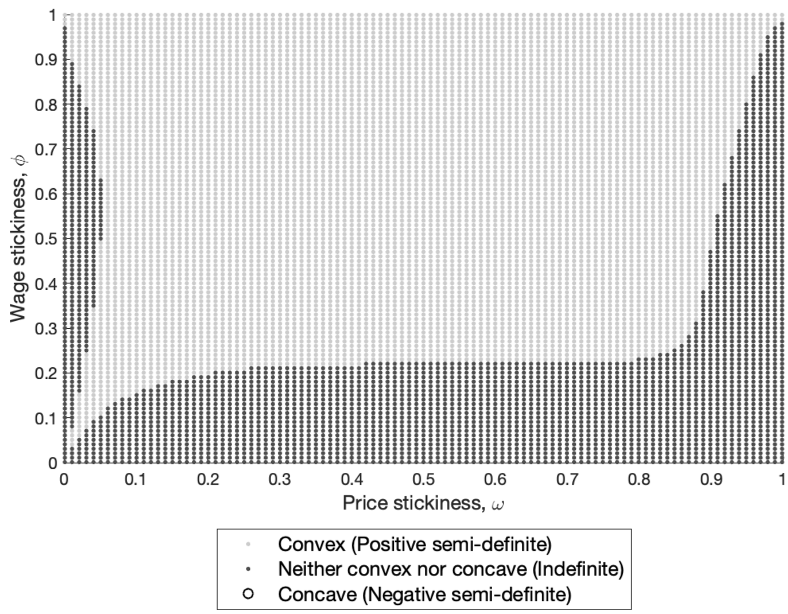
$$C(\mu^e, \mu^e_W) = [\mu^e, \mu^e_W] \times C_{matrix} \times \begin{bmatrix} \mu^e \\ \mu^e_W \end{bmatrix}. \tag{11}$$

We can determine whether C is convex or concave or neither by examining the eigenvalues of C_{matrix} (see Table 2).

An important observation is that, conditional on the baseline parameter values, the function C is never a concave function even if we vary two stickiness parameters. Hence, it is never optimal to reveal all information no matter how we vary the price and the wage stickiness parameters.

C	C_{matrix}	eigenvalues of C_{matrix}
(weakly) convex	positive semi-definite (PSD)	both are non-negative
(weakly) concave	negative semi-definite (NSD)	both are non-positive
neither convex nor concave	neither PSD nor NSD	one positive one negative

Table 2: Properties of C_{matrix} and the shape of C



We vary the wage and the price stickiness parameters between 0.001 and 0.999, to see how the shape of the function C changes. Other parameters are fixed.

Figure 3: Price and Wage Stickiness and the Shape of C

Another observation is that secrecy is not optimal when one of the stickiness parameters is sufficiently low. In Figure 3, the function C is neither convex nor concave in areas near the horizontal or the vertical axis. When both stickiness parameters are low, however, it is possible that secrecy is optimal. Revealing partial information also tends to be optimal when price stickiness is extremely high. However, as long as the Calvo parameter for price stickiness is not too high, say, less than 0.9, then secrecy tends to be optimal for a modest-to-high degree of price and wage stickiness.

It may be puzzling that secrecy is not optimal near the horizontal axis or the vertical axis, where either the price or the wage is almost flexible, in Figure 3. If either the price or the wage is perfectly flexible, then the model becomes a purely forward-looking model, thereby implying that secrecy is optimal. Does it contradict our results of optimality of secrecy in purely forward-looking models? Not at all. Near the two axises, a negative eigenvalue of the C_{matrix} is almost zero, suggesting that C_{matrix} is converging to a positive semi-definite matrix as one of the stickiness parameters is let to approach zero. In the limit, therefore, secrecy is indeed optimal. This is consistent with our previous results.

3.3 Numerical experiments with a canonical DSGE model

The models considered so far are much simpler than those used for forecasting and policy simulation in many policy institutions. In what follows, we use a more elaborate DSGE model that is based on Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003, 2007). The model features distortions such as price and wage rigidities and the external habit in consumption. Frictions such as investment adjustment costs and price and wage indexation to past inflation are also introduced. A complete description of the model is provided in Online Appendix B.4.

Because the model is much more complicated than the previous models, analytical characterization of optimal messaging policy is difficult. Hence, we solve the model numerically, using a second-order approximation and the parameter values reported in Table 3. For tractability, we also restrict our attention to simple policy strategies. First, the nominal interest rate is assumed to follow a Taylor rule with inertia. Hence, the central bank's policy actions (the nominal interest rate) do not directly respond to the central bank's superior information. Second, the central bank is assumed to convey, without noise, n-period ahead shocks to the private sector and to choose which shocks to convey and how many periods in advance (*i.e.*, n). In addition to the price and the wage mark-up shocks, the model has a technology shock as well as a monetary policy shock. For each shock, we examine how ex ante welfare changes as n is increased and interpret the changes as the welfare effect of Delphic forward guidance.

In Figure 4, four different models are examined, and each panel depicts the welfare effect of increasing the news horizon, n, for a particular shock. For each model and for each value of n, the welfare number is calculated as the consumption equivalent

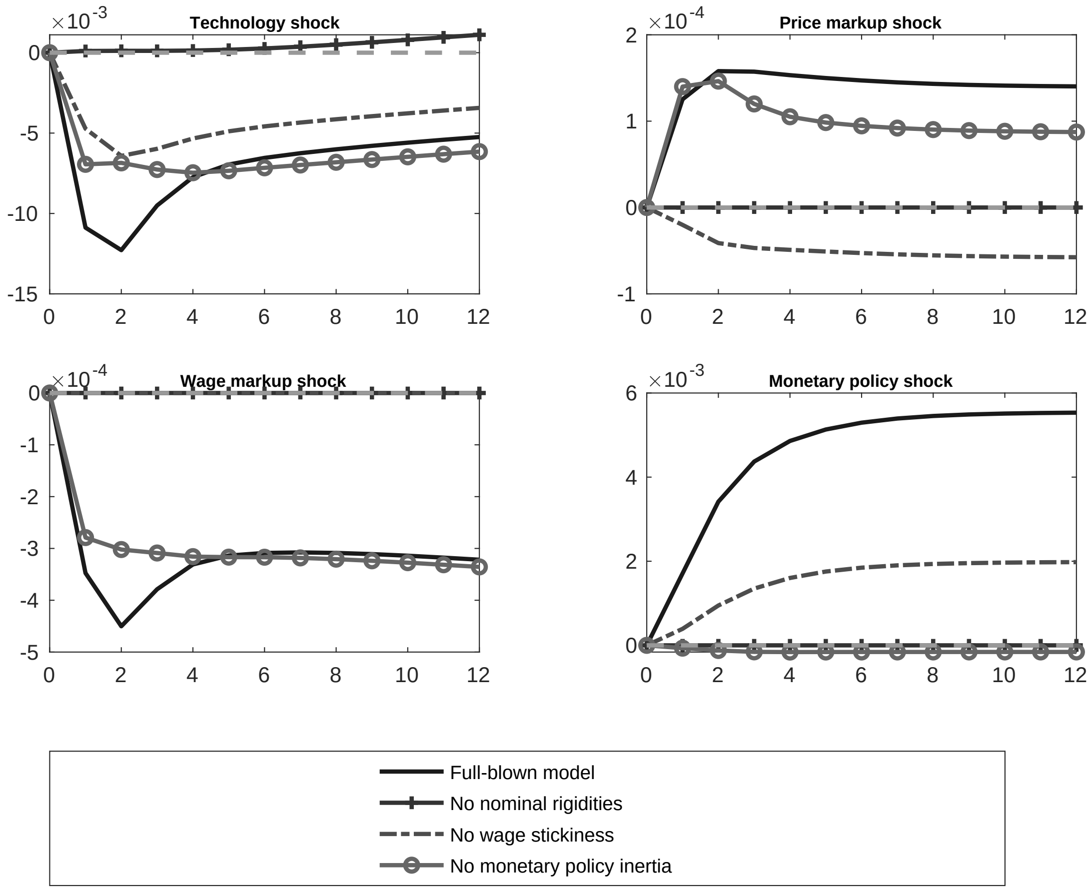


Figure 4: Welfare Changes in CEV (%) by News Horizon across Different Models
On the horizontal axis is the news horizon, n. Each panel shows how welfare changes with n for four different models.

variation (CEV) relative to when $n = 0$, *i.e.*, when no information about future shocks are provided. Hence, positive numbers indicate welfare gains relative to secrecy and negative numbers indicate relative losses.

Let us first look at the full-blown model, which is drawn with solid lines. In this model, revealing future realizations of the price mark-up shock and the monetary policy shock improves welfare, but the welfare effect of information revelation is negative for the technology shock and the wage mark-up shock. Therefore, to maximize social welfare, the central bank needs to be selective about which shocks to reveal. In other words, selective transparency is desirable as shown in Fujiwara and Waki (2020) for forward guidance about fiscal policy in neoclassical growth models.

However, choosing which shocks to reveal is not a simple task, because the sign of welfare effects can be reversed when the model specification is altered. For example, revealing future technology shocks becomes welfare-improving when both the price and the wage stickiness are removed, as shown in the top-left panel in Figure 4. the welfare effect of revealing future price mark-up shocks turns negative when the wage stickiness is assumed away (the top-right panel in Figure 4). The Delphic forward guidance about the future monetary policy also reduces welfare when the monetary policy inertia is removed (the bottom-right panel in Figure 4). Although not shown here, revealing the technology shocks or the wage mark-up shocks can improve welfare when their shock persistence are reduced sufficiently, and revealing the price mark-up shocks or the monetary policy shocks reduces welfare when the price stickiness is sufficiently increased.

3.4 Taking stock

Let us now summarize what we have learned from these three models with endogenous state variables.

First, endogenous state variables open up the possibility that the benefits of preemptive policy reactions dominate the costs associated with destabilization due to fluctuations in expectations. When the benefits dominate the costs, Delphic forward guidance about and early policy reactions to news shocks can improve welfare. Even when the monetary policy follows a suboptimal exogenous rule, Delphic forward guidance may improve welfare.

However, a mere existence of endogenous state variables does not reverse the welfare implication of Delphic forward guidance. Even in models with endogenous state variables, the optimal policy may be secretive. Secrecy is found to be optimal for a wide range of realistic parameterizations of the models. This is particularly so in the model with an ELB, which requires some unrealistic parameterizations to generate welfare gains from Delphic forward guidance. In the sticky price and wage model, secrecy is found to be optimal when both price and wage are sufficiently sticky. In a more elaborate DSGE model, it is found that being secretive about some shocks is indeed desirable.

Second, the welfare implication of Delphic forward guidance depends on some complicated interaction between shocks and frictions in the model, and is thus model-dependent. In the model with an ELB, the only shock is the demand shock (the natural rate shock), but Delphic forward guidance about this shock is harmful to ex ante welfare. In the model without an ELB, such Delphic forward guidance has no effect as far as the policy action neutralizes fluctuations in the natural rate. Delphic forward guidance about mark-up shocks can be welfare-improving in the model with sticky price and wage, whereas it unambiguously reduces welfare in the textbook sticky price model. Using the DSGE model, we demonstrate that the welfare implication of Delphic forward guidance can change when some frictions are turned off.

There are studies that offer some simple criterion regarding the social value of information. In a frictionless model, Hirshleifer (1971) finds that access to better information before trading takes place will reduce risk-averse households' expected utility through higher variability of the prices of state-contingent claims, unless information itself is of direct social value. Angeletos, Iovino, and La'O (2016) use a model with nominal and real rigidities that are based on informational friction to examine the social value of information and offer a simple criterion as to which shocks ought to be revealed. They find "[w]hen the business cycle is driven by non-distortionary forces such as technology shocks, welfare unambiguously increases with either private or public information. When instead the business cycle is driven by distortionary forces such as shocks to monopoly markups, welfare unambiguously decreases with either type of information."[15] In contrast, simple criteria such as these seem unobtainable in the New Keynesian models with endogenous variables. For example, it is possible that forward guidance about future price mark-up shocks improves welfare, whereas that about technology shocks reduces welfare.

Given the difficulty in identifying which shocks are good to reveal, it is a nontrivial task to offer central banks general and simple policy prescriptions for a messag-

[15] This point is also made in Angeletos and Pavan (2007): "if business cycles are driven primarily by shocks in markups or other distortions that induce a countercyclical efficiency gap, it is possible that providing markets with information that helps predict these shocks may reduce welfare."

ing strategy about their superior information about future, particularly when realistic frictions and rigidities are possibly present. We therefore conclude that there is no pressing need for Delphic forward guidance according to standard DSGE models. [16]

4 Conclusion

When the central bank possesses private information about future economic conditions, should it reveal and/or react to it? In the simple New Keynesian model, a central bank that has a dual-mandate-type objective function finds it optimal to commit to secrecy, by neither revealing nor reacting to superior information about future. In a more elaborate DSGE model with a large number of distortions, frictions, and shocks, Delphic forward guidance may improve welfare, but the sign of the welfare effect depends crucially on the shock type, distortions, and frictions: the sign of the welfare effect of a news shock can flip when a particular distortion or friction is removed from or added to the model. Therefore, according to standard DSGE models, improving welfare through Delphic forward guidance is difficult.

There are mechanisms that are absent in the models in this paper but that are likely to counteract the negative effects of information revelation. For example, when the representative household is not an expected utility maximizer but instead has a preference for early resolution of uncertainty, then there can be a direct, positive effect on social welfare from revealing information regarding future shocks to the household. If price setters receive idiosyncratic, noisy private signals regarding future shocks, then the resulting price distribution can be more dispersed than it would be when they have homogeneous information. Providing a public signal may improve welfare by reducing the dispersion of inflation expectations and, hence, that of the price dispersion as in Hellwig (2005), which is the source of inefficiency in the New Keynesian model. It would be interesting to examine whether these mechanisms can more than offset the negative welfare effect found in the present paper, for a set of reasonable parameter values. It would also be interesting to examine the optimal

[16] This result is contrary to the common view which tends to appraise transparency about future shocks. Hirose and Kurozumi (2017) find that anticipated monetary policy disturbances play a larger role in monetary policy transmission mechanism after 1999 and conclude that this is "consistent with the rise in the academic views on central banking as management of expectations."

time-consistent communication policy in the present setting. These questions are left for our future research.

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A Appendix

A.1 Proof of Proposition 1

Let $\{(i_t, m_t, \pi_t, x_t)\}_{t=0}^{\infty}$ denote a solution to the Ramsey problem. Let $\mathcal{G}^P$ be a filtration that represents the private agents' information under the policy $\{(i_t, m_t)\}$. We show that the optimal secretive commitment policy achieves no higher ex ante loss than does $\{(i_t, m_t, \pi_t, x_t)\}_{t=0}^{\infty}$.

To this end, first construct an alternative, secretive policy, $\{(i_t^{ALT}, m_t^{ALT})\}$, from $\{(i_t, m_t)\}$ as $i_t^{ALT} := \mathbb{E}[i_t|\mathcal{G}_t^{SEC}]$ and $m_t^{ALT} := \emptyset$. Because the alternative process of nominal interest rates, $\{i_t^{ALT}\}$, is constructed by taking conditional expectation given the private agents' information set under secrecy, it contains no information that is observed only by the central bank. Hence, together with an empty message process, this policy is secretive. An alternative process of inflation and the output gap, $\{(\pi_t^{ALT}, x_t^{ALT})\}$, is constructed in the same way, as $\pi_t^{ALT} := \mathbb{E}[\pi_t|\mathcal{G}_t^{SEC}]$ and $x_t^{ALT} := \mathbb{E}[x_t|\mathcal{G}_t^{SEC}]$.

Next, we show that $\left(\mathcal{G}^{SEC}, \{(\pi_t^{ALT}, x_t^{ALT})\}\right)$ is a REE given the alternative policy, $\{(i_t^{ALT}, m_t^{ALT})\}$. Because the alternative policy is secretive, the private agents' information is represented by a filtration $\mathcal{G}^{SEC}$. Inflation and the output gap processes, $\{(\pi_t^{ALT}, x_t^{ALT})\}$, are $\mathcal{G}^{SEC}$-adapted because (π_t^{ALT}, x_t^{ALT}) are constructed as conditional expectations given $\mathcal{G}_t^{SEC}$ for each period t. What remains to be shown is that the New Keynesian Phillips curve and the Dynamic IS equation are satisfied. Observe that, because $\mathcal{G}_t^{SEC} \subseteq \mathcal{G}_t^P$,

$$\begin{aligned}
\pi_t^{ALT} = \mathbb{E}[\pi_t|\mathcal{G}_t^{SEC}] &= \mathbb{E}\left[\kappa x_t + \beta\mathbb{E}[\pi_{t+1}|\mathcal{G}_t^P] + u_t|\mathcal{G}_t^{SEC}\right], \\
&= \kappa x_t^{ALT} + \beta\mathbb{E}[\pi_{t+1}|\mathcal{G}_t^{SEC}] + u_t, \\
&= \kappa x_t^{ALT} + \beta\mathbb{E}[\pi_{t+1}^{ALT}|\mathcal{G}_t^{SEC}] + u_t.
\end{aligned}$$

The last equality holds because of the law of iterated expectations:

$$\mathbb{E}[\pi_{t+1}|\mathcal{G}_t^{SEC}] = \mathbb{E}\left[\mathbb{E}[\pi_{t+1}|\mathcal{G}_{t+1}^{SEC}]|\mathcal{G}_t^{SEC}\right] = \mathbb{E}\left[\pi_{t+1}^{ALT}|\mathcal{G}_t^{SEC}\right].$$

By the same token, the Dynamic IS equation is satisfied: $x_t^{ALT} = \mathbb{E}[x_{t+1}^{ALT}|\mathcal{G}_t^{SEC}] - \sigma^{-1}\{i_t^{ALT} - \mathbb{E}[\pi_{t+1}^{ALT}|\mathcal{G}_t^{SEC}] - r_t^n\}$. Hence, $\left(\mathcal{G}^{SEC}, \{(\pi_t^{ALT}, x_t^{ALT})\}\right)$ is a REE given policy $\{(i_t^{ALT}, m_t^{ALT})\}$.

Because $\{(i_t^{ALT}, m_t^{ALT})\}$ is a secretive policy, the ex ante welfare loss achieved by $\{(\pi_t^{ALT}, x_t^{ALT})\}$ cannot be lower than that achieved by the optimal secretive commitment policy:

$$\mathbb{E}[\sum_{t=0}^{\infty} \delta^t L(\pi_t^{ALT}, x_t^{ALT})] \geq \mathbb{E}[\sum_{t=0}^{\infty} \delta^t L(\pi_t^{SEC}, x_t^{SEC})]. \tag{12}$$

Equality holds if and only if $\{(\pi_t^{ALT}, x_t^{ALT})\}$ equals the optimal secretive commitment policy with probability one, because the latter is unique almost surely. Now we show that the loss achieved by $\{(\pi_t, x_t)\}$ cannot be lower than that achieved by $\{(\pi_t^{ALT}, x_t^{ALT})\}$. Because the loss function L is strictly convex, Jensen's inequality implies

$$\mathbb{E}[L(\pi_t, x_t)] = \mathbb{E}\left[\mathbb{E}[L(\pi_t, x_t)|\mathcal{G}_t^{SEC}]\right] \geq \mathbb{E}\left[L(\mathbb{E}[\pi_t|\mathcal{G}_t^{SEC}], \mathbb{E}[x_t|\mathcal{G}_t^{SEC}])\right] = \mathbb{E}[L(\pi_t^{ALT}, x_t^{ALT})], \tag{13}$$

where equality holds if and only if $(\pi_t, x_t) = (\pi_t^{ALT}, x_t^{ALT})$ with probability one.

By combining inequalities in equations (12) and (13) we have established

$$\mathbb{E}[\sum_{t=0}^{\infty} \delta^t L(\pi_t, x_t)] \geq \mathbb{E}[\sum_{t=0}^{\infty} \delta^t L(\pi_t^{ALT}, x_t^{ALT})] \geq \mathbb{E}[\sum_{t=0}^{\infty} \delta^t L(\pi_t^{SEC}, x_t^{SEC})],$$

and the leftmost and the rightmost expressions are equal if and only if $\{(\pi_t, x_t)\} = \{(\pi_t^{SEC}, x_t^{SEC})\}$ with probability one. Therefore, for $\{(i_t, m_t, \pi_t, x_t)\}$ to be a solution to the Ramsey problem, $\{(\pi_t, x_t)\} = \{(\pi_t^{SEC}, x_t^{SEC})\}$ must hold with probability one.

A.2 Proof of Proposition 2

Both (i) and (ii) follow immediately from Jensen's inequality and the proof is omitted. Suppose that C is neither convex nor concave. Then the real symmetric matrix C_{matrix} in the quadratic form in equation (11) is neither a positive semi-definite nor a negative semi-definite. It follows that C_{matrix} has one strictly positive eigenvalue and one strictly negative eigenvalue. Consider an eigen-decomposition

$$C_{matrix} = Q\Lambda Q^T = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \begin{bmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{bmatrix} \begin{bmatrix} q_{11} & q_{21} \\ q_{12} & q_{22} \end{bmatrix},$$

where the eigenvalues λ_+ and λ_- satisfy $\lambda_- < 0 < \lambda_+$ and where QQ^T equals the 2-by-2 identity matrix, I_2. Note that

$$Q^T \begin{bmatrix} \mu_1 \\ \mu_{W,1} \end{bmatrix} = \begin{bmatrix} q_{11}\mu_1 + q_{21}\mu_{W,1} \\ q_{12}\mu_1 + q_{22}\mu_{W,1} \end{bmatrix} \sim N(0, QQ^T) = N(0, I_2). \tag{14}$$

It follows that $q_{11}\mu_1 + q_{21}\mu_{W,1}$ and $q_{12}\mu_1 + q_{22}\mu_{W,1}$ are independent.

The interim welfare given belief (μ^e, μ^e_W), $C(\mu^e, \mu^e_W)$, can be rewritten as

$$[\mu^e, \mu^e_W] \times C_{matrix} \times \begin{bmatrix} \mu^e \\ \mu^e_W \end{bmatrix} = \begin{bmatrix} q_{11}\mu^e + q_{21}\mu^e_W & q_{12}\mu^e + q_{22}\mu^e_W \end{bmatrix} \Lambda \begin{bmatrix} q_{11}\mu^e + q_{21}\mu^e_W \\ q_{12}\mu^e + q_{22}\mu^e_W \end{bmatrix}.$$

The right-hand side can be further simplified as

$$\lambda_+(q_{11}\mu^e + q_{21}\mu^e_W)^2 + \lambda_-(q_{12}\mu^e + q_{22}\mu^e_W)^2. \tag{15}$$

Let us first establish the lower bound for ex ante welfare loss. Let $\mathcal{G}_0^P$ denote a σ-field that represents the private agents' information in period 0 after receiving a message from the central bank. Then the ex ante loss is

$$\begin{aligned} & \mathbb{E}\left[\lambda_+(\mathbb{E}[q_{11}\mu_1 + q_{21}\mu_{W,1}|\mathcal{G}_0^P])^2 + \lambda_-(\mathbb{E}[q_{12}\mu_1 + q_{22}\mu_{W,1}|\mathcal{G}_0^P])^2\right]. \\ = \; & \underbrace{\mathbb{E}\left[\lambda_+(\mathbb{E}[q_{11}\mu_1 + q_{21}\mu_{W,1}|\mathcal{G}_0^P])^2\right]}_{\geq 0} + \underbrace{\mathbb{E}\left[\lambda_-(\mathbb{E}[q_{12}\mu_1 + q_{22}\mu_{W,1}|\mathcal{G}_0^P])^2\right]}_{\geq \mathbb{E}\left[\lambda_-(q_{12}\mu_1 + q_{22}\mu_{W,1})^2\right]} \\ \geq \; & \mathbb{E}\left[\lambda_-(q_{12}\mu_1 + q_{22}\mu_{W,1})^2\right] = \lambda_-. \end{aligned}$$

The first inequality in the second line follows from $\lambda_+ > 0$. The second inequality in the second line follows from $\lambda_- < 0$ and Jensen's inequality. The rightmost equality in the bottom line is obtained because $q_{12}\mu_1 + q_{22}\mu_{W,1}$ is distributed according to $N(0, 1)$ as shown in equation (14).

Now we construct a messaging strategy that achieves the lower bound for ex ante loss which we have just established. Consider the following messaging strategy:

$$m_0 = q_{12}\mu_1 + q_{22}\mu_{W,1}.$$

This messaging strategy induces the following belief:

$$(\mathbb{E}[\mu_1|m_0], \mathbb{E}[\mu_{W,1}|m_0]) = \left(\frac{q_{12}}{q_{12}^2 + q_{22}^2} m_0, \frac{q_{22}}{q_{12}^2 + q_{22}^2} m_0\right).$$

Therefore, the ex ante welfare loss is given by $\mathbb{E}\left[\lambda_+(q_{11}\mathbb{E}[\mu_1|m_0] + q_{21}\mathbb{E}[\mu_{W,1}|m_0])^2 + \lambda_-(m_0)^2\right]$ under this messaging strategy and it is equal to

$$\mathbb{E}\left[\lambda_+(\mathbb{E}[q_{11}\mu_1 + q_{21}\mu_{W,1}|m_0])^2 + \lambda_-(m_0)^2\right] = \lambda_-\mathbb{E}[m_0^2] = \lambda_-.$$

The leftmost equality holds because $\mathbb{E}[q_{11}\mu_1 + q_{21}\mu_{W,1}|m_0] = 0$, which follows from the fact that $q_{11}\mu_1 + q_{21}\mu_{W,1}$ and $m_0 = q_{12}\mu_1 + q_{22}\mu_{W,1}$ are independent normal random variables. The rightmost equality holds because the message $m_0 = q_{12}\mu_1 + q_{22}\mu_{W,1}$ is distributed according to $N(0,1)$. Hence, the proposed messaging strategy is shown to minimize the ex ante loss. By setting $g_{m,\mu} = q_{12}$ and $g_{m,\mu_W} = q_{22}$, Proposition 2-(iii) is proved. Intuitively, this messaging strategy convexifies the concave part of the loss function, the second term in equation (15), while leaving untouched the convex part, the second term in equation (15).

A.3 Proof of Proposition 3

When the function C is either convex or concave, the proposition is obvious (see the discussion that follows the statement of Proposition 3). Therefore, suppose that the function C is neither convex nor concave.

First, consider the period-0 problem of the central bank given the private agents' belief (μ^e, μ_W^e). Due to the linear-quadratic structure of the problem, its solution as well as the optimal belief-dependent interest rate (less the natural rate) are linear in the belief (μ^e, μ_W^e).

Second, under the optimal messaging strategy, the belief (μ^e, μ_W^e) is linear in the sent message, m_0. To see this, recall that the optimal messaging strategy takes the following form,

$$m_0 = g_{m,\mu}\mu_1 + g_{m,\mu_W}\mu_{W,1},$$

which is a linear combination of shocks. Given this message strategy, the conditional expectation of shocks is linear in the message, *i.e.*, $\mathbb{E}[\mu_1|m_0] = g_{m,\mu}m_0/(g_{m,\mu}^2 + g_{m,\mu_W}^2)$ and $\mathbb{E}[\mu_{W,1}|m_0] = g_{m,\mu_W}m_0/(g_{m,\mu}^2 + g_{m,\mu_W}^2)$.

Taken together, the optimal belief-dependent interest rate (less the natural rate) is linear in the message, m_0. Hence, the interest rate set by the central bank can convey the same information even without the central bank's message.

A.4 Parameter values used in the DSGE model

Parameters are set following Fujiwara, Hirose, and Shintani (2011). Table 3 reports the parameter values used in the DSGE model.

Table 3: Parameter Values

Parameters	Values	Explanation
β	.9983	Subjective discount factor
σ	1.72	Inverse of intertemporal elasticity of substitution
η	2.23	Inverse of Frisch elasticity
ε	10	Elasticity of substitution among differentiated products
θ	.4	Calvo parameter for price
δ	.025	Depreciation rate
s''	4.82	Investment growth adjustment costs
b	.38	Consumption habit
α	.21	Capital share
θ_h	.26	Calvo parameter for wage
γ	.18	Price indexation
γ_h	.51	Wage indexation
ε_h	10	Elasticity of substituion among differentiated labor
ρ	.75	Policy inertia
ϕ^π	2.1	Policy reaction to inflation rates
ϕ^y	.17	Policy reaction to output growth
ρ_z	.98	AR(1) parameter for technology shock
ρ_u	.86	AR(1) parameter for price mark-up shock
ρ_μ	.96	AR(1) parameter for wage mark-up shock
ρ_η	.36	AR(1) parameter for monetary policy shock
σ_z	.0043	Standard deviation of technology shock
σ_u	.0014	Standard deviation of price mark-up shock
σ_μ	.0022	Standard deviation of wage mark-up shock
σ_η	.0016	Standard deviation of monetary policy shock

Online Appendix to Ippei Fujiwara and Yuichiro Waki,
"The Delphic forward guidance puzzle in New Keynesian models"
(Not to be published with the paper)

B Appendix

B.1 Derivation of the quadratic social welfare in equation (4)

Is the standard quadratic approximation in equation (4) valid in the present setting with communication? A short answer is yes. This is because the standard approximation is realization-by-realization and never uses equations that involve conditional expectations. More specifically, the standard procedure approximates $\mathbb{E}_0[\sum_{t=0}^{\infty}\beta^t u(c_t, h_t)]$, where c is consumption and h is labor, by first obtaining a quadratic approximation of a momentary utility function in period t, $u(c_t, h_t)$, using deterministic equations only, sums them up from time 0 to infinity with discounting, and then take expectation based on the initial information set. When computing ex ante utility, the last step is replaced to that taking unconditional expectation, and there is no step in which conditional expectations based on the private sector's information set are taken. We have an additively separable term that captures unconditional expectation of *terms independent of policy* (t.i.p.) from period 0 on, but the term is unaffected by communication as it is unconditional expectation and, therefore, we can drop it in our analysis. This is a benefit of using ex ante utility. In contrast, if one were to evaluate the representative household's expected utility from period t on based on the period-t information available to it, then there is an additively separable term that captures *conditional* expectation of t.i.p. based on the household's information set in period t, and it is affected by the communication policy. In this case, one should not drop t.i.p. when examine the effect of communication policy.

To illustrate the above point, we follow Woodford (2010) and derive the second-order approximation step by step. When necessary, we refer to equation numbers in his handbook chapter. Throughout, we will assume that the steady state is efficient. Therefore, Section 3.4.1 in Woodford (2010) is the relevant section. He uses a model in which the household supplies a set of differentiated labor to intermediate goods producers, and shows that the representative household's utility in equilibrium, evaluated in period 0, can be expressed as a function of output, Y, and the measure of price dispersion, Δ, as

$$\mathbb{E}_0^P \left[\sum_{t=0}^{\infty} \beta^t \{u(Y_t; \xi_t) - v(Y_t; \xi_t)\Delta_t\} \right],$$

where ξ_t is a preference shock. The price dispersion measure Δ_t is defined as

$$\Delta_t := \int_0^1 \left(\frac{p_t(i)}{P_t} \right)^{-\theta(1+\nu)} di,$$

where $\theta > 1$ is the Dixit-Stiglitz elasticity of substitution parameter and $\nu > 0$ is the inverse of Frisch elasticity of labor supply (equation 61).[1]

Let $U(Y, \Delta; \xi) := u(Y; \xi) - v(Y; \xi)\Delta$. Its second order Taylor expansion around the steady state yields (equation 92):

$$\begin{aligned} U(Y_t, \Delta_t; \xi_t) &= \overline{Y} U_Y \hat{Y}_t + U_\Delta \hat{D}_t + \frac{1}{2}(\overline{Y} U_Y + \overline{Y}^2 U_{YY})\hat{Y}_t^2 \\ &\quad + \overline{Y} U_{Y\Delta} \hat{Y}_t \hat{\Delta}_t + \overline{Y} U'_{Y\xi} \hat{\xi}_t \hat{Y}_t + \text{t.i.p.} + \mathcal{O}(||\xi||^3). \end{aligned}$$

Here t.i.p. refers to terms "that do not involve endogenous variables" and, therefore, consists of linear and quadratic terms in $\hat{\xi}_t$ and a constant.

When the steady state is efficient, we have $U_Y = 0$ and therefore

$$\begin{aligned} U(Y_t, \Delta_t; \xi_t) &= U_\Delta \hat{D}_t + \frac{1}{2}\overline{Y}^2 U_{YY}(\hat{Y}_t - \hat{Y}_t^e)^2 \\ &\quad + \overline{Y} U_{Y\Delta} \hat{Y}_t \hat{\Delta}_t + \text{t.i.p.} + \mathcal{O}(||\xi||^3) \end{aligned}$$

(equation 94). Assuming that the initial $\hat{\Delta}$ is sufficiently small, i.e. $\hat{\Delta}_{-1} = \mathcal{O}(||\xi||^2)$, we

[1] Here we assume, for simplicity, that the intermediate goods production function is linear in labor. Woodford (2010) allows for diminishing marginal product of labor for the intermediate goods production, and ν is a composite of the Frisch elasticity and the production function curvature parameter.

obtain $\hat{\Delta}_t = \mathcal{O}(||\xi||^2)$ for all t, hence

$$\begin{aligned} U(Y_t, \Delta_t; \xi_t) &= \frac{1}{2}\overline{Y}^2 U_{YY}(\hat{Y}_t - \hat{Y}_t^e)^2 - \overline{v}\hat{\Delta}_t + \\ &\quad + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \end{aligned}$$

where $\overline{v} := v(\overline{Y}; \overline{\xi}) > 0$ (equation 95).

Finally, using the second order approximation of the dynamic equation for Δ, i.e. $\Delta_t = h(\Delta_{t-1}, 1 + \pi_t)$, Woodford (2010) shows

$$\sum_{t=0}^{\infty} \beta^t \hat{\Delta}_t = \frac{1}{2} \frac{h_{\pi\pi}}{1 - \alpha\beta} \sum_{t=0}^{\infty} \beta^t \pi_t^2 + \text{t.i.p.} + \mathcal{O}(||\xi||^3),$$

(equation 97). Because the dynamic equation for Δ is deterministic, the term t.i.p. refers only to terms proportional to $\hat{\Delta}_{-1}$ and does not involve any exogenous variables.

As a result, we have a realization-by-realization approximation:

$$\sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t; \xi_t) = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \{\overline{Y}^2 U_{YY} x_t^2 - (1 - \alpha\beta)^{-1} \overline{v} h_{\pi\pi} \pi_t^2\} + \text{t.i.p.} + \mathcal{O}(||\xi||^3),$$

where $x_t = \hat{Y}_t - \hat{Y}_t^e$ is the welfare-relevant measure of the output gap.

Observe that so far we have not used any equations that involve conditional expectations based on either the private sector's information or the central bank's information. All equations that are used are either static or backward-looking. Hence, the term t.i.p. does not include conditional expectations of variables.

By taking the unconditional expectation (given the initial condition $\hat{\Delta}_{-1}$), we obtain

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t; \xi_t)\right] = \mathbb{E}\left[\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \{\overline{Y}^2 U_{YY} x_t^2 - (1 - \alpha\beta)^{-1} \overline{v} h_{\pi\pi} \pi_t^2\}\right] + \mathbb{E}[\text{t.i.p.}] + \mathbb{E}[\mathcal{O}(||\xi||^3)].$$

Recall that the term t.i.p. is the sum of two terms: one that comes from the second order approximation of the utility function and the other that comes from the approximation of the dynamic equation of Δ. The former includes the sum of constants and first and second order terms in $\{\hat{\xi}_t\}_{t=0}^{\infty}$. The latter includes linear terms in $\hat{\Delta}_{-1}$, which is exogenously given. Neither includes endogenous variables. Hence, the unconditional expectation of the sum of these terms, $\mathbb{E}[\text{t.i.p.}]$, is independent of how much information the private sector obtains along the way. Therefore, we can treat it as constant in our setting with communication, as far as we are concerned with ex ante welfare.

If one instead attempts to evaluate the household's expected utility in equilibrium from period s on, based on the information available to the household, it is expressed as:

$$\mathbb{E}_s^P\left[\frac{1}{2}\sum_{t=s}^{\infty}\beta^t\{\overline{Y}^2 U_{YY}x_t^2-(1-\alpha\beta)^{-1}\overline{v}h_{\pi\pi}\pi_t^2\}\right]+\mathbb{E}_s^P[\text{t.i.p.}]+\mathbb{E}_s^P[\mathcal{O}(||\xi||^3)].$$

Then the term $\mathbb{E}_s^P[\text{t.i.p.}]$ depends on the information available to the household in period s and, therefore, depends on communication policy as well as what the household has observed up to period s.

B.2 Optimality of secrecy in other purely forward-looking models

Commitment to secrecy remains optimal even if we augment the model with an effective lower bound on the nominal interest rate and with a shock to the social loss function.

B.2.1 A New Keynesian model with the zero lower bound

We use a model along the lines of Eggertsson and Woodford (2003) and Adam and Billi (2006), in which the zero lower bound on nominal interest rates can bind when a large, negative shock to the natural rate of interest hits the economy. Our model is more general than the conventional, simple one. The zero lower bound may bind multiple times and may not be binding at time 0, and the central bank can act differently when it foresees that the zero bound will bind or that it will cease to bind in the near future.

The Ramsey problem is the same as before except that it must respect the nonnegativity constraint on nominal interest rates:

$$i_t \geq 0. \tag{B.1}$$

An optimal secretive commitment policy is $\{(\pi_t^{SEC}, x_t^{SEC}, i_t^{SEC})\}_{t=0}^{\infty}$ that minimizes the loss function in equation (3) subject to the New Keynesian Phillips curve in equation (1), the dynamic IS equation in equation (2), and the zero lower bound (ZLB) constraint in equation (B.1). Then we have the following result:

Corollary B.1 *Proposition 1 and Corollary 1 hold in the presence of the zero lower bound.*

Our proof of Proposition 1 is valid in the presence of the ZLB because the process of the nominal interest rate that is constructed in the proof, $\{i_t^{ALT}\}$, satisfies the ZLB if $\{i_t\}$ does.

This proposition implies that, from the ex ante point of view, the central bank should be secretive even if the zero lower bound is already binding at time 0 and if it may, for example, receive private news that a negative natural rate shock disappears in near future or that a future cost-push shock is positive. This might appear to contradict with the literature, which has shown that raising inflation expectations can be welfare-improving at the zero lower bound, but it is not. From the ex post point of view, once the central bank observes, *e.g.*, the short duration of a negative natural rate shock or a positive future cost-push shock, ex post welfare improves if the private sector is also informed about the information. However, from the ex ante point of view, the duration of a negative natural rate shock may be much longer and a future cost-push shock may be negative; transparency lowers ex post welfare in that scenario. On average, it is better to leave the private sector uninformed.

B.2.2 A three-equation model with the Taylor rule

It is straightforward to extend our theoretical results in Section 2 to the case where the nominal interest rate follows a simple Taylor rule:

$$i_t = \phi_t^{\pi} \pi_t + \phi_t^{x} x_t + \eta_t, \quad \forall t, \tag{B.2}$$

where its coefficients, (ϕ_t^π, ϕ_t^x), and the monetary policy shock, η_t, are stochastic and their realized values in period t are assumed to be observed by the private sector at the beginning of period t.

In this setting, the only choice of the central bank is the message it sends to the private sector, and we need to alter the definition of a rational expectation equilibrium by including (B.2) as an equilibrium condition. Using an argument similar to the proof of Proposition 1, we can show that the equilibrium ex ante welfare loss is minimized with central bank secrecy.

B.2.3 Forward guidance about the central bank's future policy goals

Delphic forward guidance can be used to communicate information not only about future cost-push shocks but also about the central bank's objective in the future. Let $\{\theta_t\}_{t=0}^{\infty}$ be an exogenous stochastic process. Ex ante welfare loss is now given by

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \delta^t L(\pi_t, x_t, \theta_t)\right].$$

Proposition B.1 *Suppose that θ_t is observed by the private sector at the beginning of period t. Then Proposition 1 and Corollary 1 hold in the presence of shocks to the social welfare function.*

If θ_t is publicly observed at the beginning of period t, we can replace equation (13) in the proof of Proposition 1 to

$$\begin{aligned}
\mathbb{E}[L(\pi_t, x_t, \theta_t)] &= \mathbb{E}[\mathbb{E}[L(\pi_t, x_t, \theta_t)|\mathcal{G}_t^{SEC}]] \\
&\geq \mathbb{E}[L(\mathbb{E}[\pi_t|\mathcal{G}_t^{SEC}], \mathbb{E}[x_t|\mathcal{G}_t^{SEC}], \theta_t)] = \mathbb{E}[L(\tilde{\pi}_t, \tilde{x}_t, \theta_t)],
\end{aligned}$$

and Proposition B.1 follows.

Our focus on the future shock is crucial for this result. When a contemporaneous shock to θ is observed by the central bank but not by the private sector, then the central bank generally faces a trade-off: there are gains from making period-t actions contingent on θ_t, but that can reveal to the private sector some information about θ_t

and possibly about future θ's, which is detrimental to welfare. Therefore, for contemporaneous shocks, secrecy is not in general optimal.

B.3 A model with backward indexation and an ELB

B.3.1 Derivation of the quasi-difference inflation and the output gap in period 1

Our assumption is that the endogenous variables are determined by the optimal discretionary policy from period 1 onwards. From period 1 onwards, once the natural rate is normalized (*i.e.*, $r_t^n = r^n$) there is no state variable or shocks in the economy. Hence, both the output gap and the quasi-difference of inflation become zero.

Now consider the period-1 endogenous variables when $r_1^n = r_{elb}^n < 0$. The New Keynesian Phillips curve is given by

$$\hat{\pi}_1 = \kappa x_1$$

because the one-period-ahead quasi-difference in inflation is zero. The Dynamic IS equation is given by

$$x_1 = \sigma^{-1}\{i_1 - (\gamma\hat{\pi}_1 + \gamma^2\pi_0) - r_{elb}^n\}$$

because the output gap in period 2 is zero and because the period-2 inflation equals $\gamma\hat{\pi}_1 + \gamma^2\pi_0$.

As long as the zero lower bound is binding when $r_1^n = r_{elb}^n$, these two equations can be solved, using $i_1 = 0$, and the solution, $(x_{1,elb}(\pi_0), \hat{\pi}_{1,elb}(\pi_0))$, is given by

$$(x_{1,elb}(\pi_0), \hat{\pi}_{1,elb}(\pi_0)) \quad = \quad \left(\frac{\gamma^2/\sigma}{1-\gamma\kappa/\sigma}\pi_0 + \frac{1/\sigma}{1-\gamma\kappa/\sigma}r_{elb}^n, \frac{\kappa\gamma^2/\sigma}{1-\gamma\kappa/\sigma}\pi_0 + \frac{\kappa/\sigma}{1-\gamma\kappa/\sigma}r_{elb}^n\right).$$

B.3.2 Messages are virtually irrelevant

This section provides the proof that the central bank can *virtually* achieve C^* even without messages, *i.e.*, for any prior probability p and any $\epsilon > 0$, the central bank can achieve an ex ante loss that is lower than $C^*(p) + \epsilon$ without sending messages.

To understand the reason, it is instructive to consider two cases: (i) $C^*(p) = C(p)$ at p and (ii) $C^*(p) < C(p)$ at p. In Case (i), secrecy is optimal at p and, therefore, prohibiting the central bank from sending messages is irrelevant for welfare. What about Case (ii)? Because C is a continuous function on a compact interval $[0,1]$ in $\mathbb{R}$ and because C^* is a convexification of C, for each such $p \in (0,1)$, we can find p_1, p_2, and α between 0 and 1 such that $p = \alpha p_1 + (1-\alpha)p_2$ and $C^*(p) = \alpha C(p_1) + (1-\alpha)C(p_2)$.

Let $i^*(\rho)$ denote the optimal belief-dependent nominal interest rate given belief ρ. If $i^*(p_1)$ and $i^*(p_2)$ are different, the central bank can exactly achieve $C^*(p)$ by setting the nominal interest rate as follows: if $r_1^n = r_{elb}^n$, then

$$i_0 = \begin{cases} i^*(p_1) & \text{with probability } \frac{p_1(p_2-p)}{p(p_2-p_1)} \\ i^*(p_2) & \text{with the remaining probability} \end{cases}$$

and if $r_1^n = r^n$, then

$$i_0 = \begin{cases} i^*(p_1) & \text{with probability } \frac{(1-p_1)(p_2-p)}{(1-p)(p_2-p_1)} \\ i^*(p_2) & \text{with the remaining probability.} \end{cases}$$

Then, the probability of $i_0 = i^*(p_1)$ equals

$$p \times \frac{p_1(p_2-p)}{p(p_2-p_1)} + (1-p) \times \frac{(1-p_1)(p_2-p)}{(1-p)(p_2-p_1)} = \alpha,$$

and the probability of $i_0 = i^*(p_2)$ equals $1-\alpha$. Note also that, after observing $i_0 = i^*(p_1)$, the private agents' posterior probability of the period-1 natural rate shock being negative becomes p_1, and that after observing $i_0 = i^*(p_2)$ the posterior becomes p_2.

Even if $i^*(p_1)$ and $i^*(p_2)$ happen to be identical, the central bank can achieve the ex ante loss that is arbitrarily close to $C^*(p)$. Fix an arbitrary $\delta \neq 0$ such that $i^*(p_2+\delta) \neq i^*(p_2) = i^*(p_1)$. Imagine that the central bank sets the nominal rate as follows: if $r_1^n = r_{elb}^n$, then

$$i_0 = \begin{cases} i^*(p_1) & \text{with probability } \frac{p_1(p_2-p)}{p(p_2-p_1)} \\ i^*(p_2+\delta) & \text{with the remaining probability} \end{cases}$$

and if $r_1^n = r^n$, then

$$i_0 = \begin{cases} i^*(p_1) & \text{with probability } \frac{(1-p_1)(p_2-p)}{(1-p)(p_2-p_1)} \\ i^*(p_2+\delta) & \text{with the remaining probability.} \end{cases}$$

Then, as before, after observing $i_0 = i^*(p_1)$, the private agents' posterior probability of the period-1 natural rate shock being negative becomes p_1, and that after observing $i_0 = i^*(p_2+\delta)$ the posterior becomes p_2.

As shown in lemmas below, i^* is continuous and there is no interval on which i^* is constant. It then follows that we can let δ arbitrarily close to zero while maintaining $i^*(p_2+\delta) \neq i^*(p_2)$. Continuity of i^* implies that, by letting $\delta \to 0$, the achieved ex ante loss converges to $C^*(p)$. This completes the proof.

Lemma B.1 *i^* is differentiable.*

Proof. Recall that the optimal belief-dependent policy solves the following problem:

$$C(\rho) := \min_{(\pi_0,x_0)} L(\pi_0,x_0) + \beta\rho\mathcal{L}_{1,elb}(\pi_0)$$

subject to the Phillips curve,

$$\pi_0 = \kappa x_0 + \beta\rho\hat{\pi}_{1,elb}(\pi_0).$$

Because this is a linear-quadratic problem, its solution, which we denote by $\pi^*(\rho)$ and $x^*(\rho)$, can be obtained analytically as follows. Let

$$\begin{aligned} A_\pi &= \frac{\gamma^2/\sigma}{1-\gamma\kappa/\sigma} \\ A_r &= +\frac{1/\sigma}{1-\gamma\kappa/\sigma}, \end{aligned}$$

and the solution is given by

$$\pi^*(\rho) = -\frac{\beta\rho \times F_1 - 2b\left(\frac{1}{\kappa} - \beta A_\pi\rho\right)\rho\beta A_r r_{elb}^n}{2\left[1 + b\left(\frac{1}{\kappa} - \beta A_\pi\rho\right)^2 + \beta\rho F_2\right]} \tag{B.3}$$

and

$$x^*(\rho) = \left(\frac{1}{\kappa} - \beta A_\pi \rho\right) \pi^*(\rho) - \rho\beta A_r r^n_{elb}.$$

From the Dynamic IS equation, the optimal belief-dependent interest rate is given by

$$i^*(\rho) = \gamma\pi^*(\rho) + \rho\hat{\pi}_{1,elb}(\pi^*(\rho)) + r^n - \sigma\left(x^*(\rho) - \rho x_{1,elb}(\pi^*(\rho))\right).$$

Clearly, π^*, x^*, and i^* are all differentiable. ■

Lemma B.2 *There is no interval on which i^* is constant.*

Proof. Suppose to the contrary that there is such a subinterval. Because i^* is differentiable, its derivative must be zero on that subinterval. Differentiating i^*, we obtain

$$(i^*)'(\rho) = (\pi^*)'(\rho) \times \left\{\gamma - \frac{\sigma}{\kappa} + (\kappa + \sigma(1+\beta))A_\pi \rho\right\}.$$

Hence, the aforementioned subinterval must be contained in the following union of two sets:

$$\{\rho \in (0,1) | (\pi^*)'(\rho) = 0\} \cup \left\{-\frac{\gamma - \frac{\sigma}{\kappa}}{(\kappa + \sigma(1+\beta))A_\pi}\right\}.$$

Because the latter set is a singleton, the former must contain an interval. Consider such an interval. Then π^* is constant on it. Let π^{const} denote the value of π^* on the interval. Then, from equation (B.3), the following equality must hold for any ρ on the interval:

$$2\left[1 + b\left(\frac{1}{\kappa} - \beta A_\pi \rho\right)^2 + \beta\rho F_2\right]\pi^{const} \quad = \quad -\beta\rho \times F_1 - 2b\left(\frac{1}{\kappa} - \beta A_\pi \rho\right)\rho\beta A_r r^n_{zlb}.$$

This implies that the quadratic equations for ρ on both sides must have the same set of coefficients. However, this cannot be true. If $\pi^{const} \neq 0$, the left-hand side has a constant while the right-hand side does not. If $\pi^{const} = 0$, then the coefficients on ρ and ρ^2 on the right-hand side must be zero, but they are not. This is a contradiction and, therefore, there is no subinterval of $(0,1)$ on which i^* is constant. ■

B.4 A canonical DSGE model

B.4.1 The representative household

The representative household's expected utility at the beginning of period 0 is:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t - X_t\right) - v\left(L_t\right) \right].$$

where C_t is consumption, L_t is labor, and X_t is external habit that equals bC_{t-1} $(b \geq 0)$ in equilibrium.

The household accumulates capital according to the capital accumulation equation:

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(I_t, I_{t-1}\right)\right] I_t,$$

where I_t is investment and K_t is capital. The function S represents investment adjustment costs. The flow budget constraint is given by:

$$A_{t+1} + P_t\left(C_t + I_t\right) \quad \leq \quad \tilde{W}_t L_t + R_{t-1} A_t + P_t r_t^K K_t + \text{profits and transfers},$$

where A_t is the nominal risk-free bond, P_t the nominal price index, $\tilde{W}_t$ the nominal wage, R_t the gross nominal interest rate, and r_t^K the rental rate of capital, respectively. Although it is not included in the equation, the household also trades a complete set of state-contingent claims.

We assume the following functional forms:

$$\begin{aligned}
u\left(C_t - X_t\right) &= \frac{\left(C_t - X_t\right)^{1-\sigma}}{1-\sigma}, \\
v\left(L_t\right) &= \frac{L_t^{1+\eta}}{1+\eta}, \\
S\left(I_t, I_{t-1}\right) &= \frac{s}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2.
\end{aligned}$$

B.4.2 Labor union

We model sticky nominal wages following Smets and Wouters (2007). There is a unit measure of labor unions, each of which is indexed by $l \in [0, 1]$. A union l collects the homogeneous labor supplied from households and transforms it into a differentiated labor good indexed by l using a linear technology. It faces a downward-sloping demand curve for its labor good and chooses the nominal wage subject to the Calvo-style probability. Firms combine differentiated labor goods into a single, composite labor using a CES aggregator $h_t = [\int_0^1 h_t(l)^{\frac{\epsilon_h - 1}{\epsilon_h}} dl]^{\frac{\epsilon_h}{\epsilon_h - 1}}$. The nominal wage of this composite labor good is denoted by W_t.

When the union l is given an opportunity to change its wage in period t, it solves

$$\max_{W_t^*} \mathbb{E}_t \sum_{s=t}^{\infty} \theta_h^{s-t} m_{t,s} \left[W_s(l) - \frac{\tilde{W}_s}{1+\tau_s^h} \right] h_s(l),$$

subject to

$$\begin{aligned} h_s(l) &= \left[\frac{W_s(l)}{W_s} \right]^{-\varepsilon_h} h_s, \forall s \geq t, \\ W_{t+i}(l) &= W_t(l) \prod_{n=1}^{i} \Pi_{t+n-1}^{\gamma_h}, \forall i \geq 1, \\ W_t(l) &= W_t^*. \end{aligned}$$

Here, $m_{t,s}$ denotes the stochastic discount factor for nominal payoffs, τ_t^h the subsidy to the union, and $\Pi_t = P_t/P_{t-1}$ the gross consumer price inflation rates, respectively. θ_h, ε_h andγ_h are the Calvo parameter for staggered nominal wages, elasticity of substitution among differentiated labor, and the degree of nominal wage indexation on past inflation rates.

B.4.3 Intermediate goods producer

The sticky price friction is modeled in a standard fashion. When the intermediate-goods producer f is given an opportunity to change its price in period t, it solves:

$$\max_{P_t^*} \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} m_{t,s} \left[(1+\tau_s) P_s(f) - P_s MC_s \right] Y_t(f),$$

subject to

$$\begin{aligned} Y_s(f) &= \left[\frac{P_s(f)}{P_s} \right]^{-\varepsilon} Y_s, \\ P_{t+i}(f) &= P_t(f) \prod_{n=1}^{i} \Pi_{t+n-1}^{\gamma}, \forall i \geq 1, \\ P_t(f) &= P_t^*. \end{aligned}$$

The sales subsidy is denoted by τ_t. γ is the degree of price indexation on past inflation rates. Real marginal cost MC_t is the Lagrange multiplier in the cost minimization problem:

$$\min_{h_t, K_t} \frac{W_t}{P_t} h_t + r_t^K K_t,$$

subject to the production technology:

$$Y_t = K_t^{\alpha} \left[\exp(z_t) h_t \right]^{1-\alpha},$$

where z_t denotes the aggregate technology shock.

B.4.4 Final good producer

A final good producer minimizes the total cost $\int_0^1 P_t(f) Y_t(f) \mathrm{d}f$ subject to the aggregating technology:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}f \right]^{\frac{\epsilon}{\epsilon-1}}.$$

B.4.5 Monetary policy

The central bank set nominal interest rates following the Taylor type rule:

$$R_t - 1 = \rho(R_{t-1} - 1) + (1-\rho)\left[\phi^{\pi}\left(\Pi_t - 1\right) + \phi^{y}\left(\frac{Y_t}{Y_{t-1}} - 1\right)\right] + \eta_t,$$

where ρ denotes the degree of policy inertia.

B.4.6 System of equations

We have 19 equations for 19 endogenous variables: Y_t, λ_t, Π_t, w_t, q_t, r_t^K, I_t, MC_t, K_t, $\bar{F}_t$, $\bar{K}_t$, C_t, Δ_t, $\Pi_{W,t}$, $\bar{F}_t^h$, $\bar{K}_t^h$, Δ_t^h, and R_t.

$$\begin{aligned}
K_{t+1} &= (1-\delta)K_t + \left\{1 - \frac{s}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right\} I_t, \\
Y_t &= C_t + I_t, \\
\lambda_t &= \left(C_t - bC_{t-1}\right)^{-\sigma}, \\
\lambda_t &= \beta \mathbb{E}_t \frac{R_t}{\Pi_{t+1}} \lambda_{t+1}, \\
q_t &= \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[r_{t+1}^K + q_{t+1}(1-\delta)\right], \\
1 &= q_t \left\{1 - \frac{s}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - s\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right\} + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} s \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2, \\
w_t &= (1-\alpha)\exp\left(z_t\right)^{1-\alpha} MC_t K_t^{\alpha} h_t^{-\alpha}, \\
r_t^K &= \alpha \exp\left(z_t\right)^{1-\alpha} MC_t K_t^{\alpha-1} h_t^{1-\alpha}, \\
\bar{F}_t &= 1 + \theta\beta \mathbb{E}_t \frac{\lambda_{t+1} Y_{t+1}}{\lambda_t Y_t} \pi_{t+1}^{\varepsilon} \pi_t^{\gamma} \bar{F}_{t+1}, \\
\bar{K}_t &= \exp\left(u_t\right) MC_t + \theta\beta \mathbb{E}_t \frac{\lambda_{t+1} Y_{t+1}}{\lambda_t Y_t} \pi_{t+1}^{1+\varepsilon} \bar{K}_{t+1}, \\
\bar{K}_t &= \left[\frac{1 - \theta\left(\frac{\pi_{t-1}^{\gamma}}{\pi_t}\right)^{1-\varepsilon}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \bar{F}_t,
\end{aligned}$$

$$
\begin{aligned}
\Delta_t &= (1-\theta)\left[\frac{1-\theta\left(\frac{\pi_{t-1}^{\gamma}}{\pi_t}\right)^{1-\varepsilon}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} + \theta\left(\frac{\pi_t}{\pi_{t-1}^{\gamma}}\right)^{\varepsilon}\Delta_{t-1}, \\
\Delta_t Y_t &= K_t^{\alpha}\left[\exp\left(z_t\right)h_t\right]^{1-\alpha}, \\
\Pi_t^{w} &= \Pi_t w_t / w_{t-1}, \\
\bar{F}_t^{h} &= \lambda_t w_t h_t + \beta\theta_h \mathbb{E}_t\left[\bar{F}_{t+1}^{h}(\Pi_{t+1}^{W})^{\epsilon_h-1}\right]\Pi_t^{(1-\epsilon_h)\gamma_h}, \\
\bar{K}_t^{h} &= \exp\left(\mu_t\right)h_t^{1+\eta}(\Delta_t^{h})^{\eta} + \beta\theta_h \mathbb{E}_t\left[\bar{K}_{t+1}^{h}(\Pi_{t+1}^{W})^{\epsilon_h}\right]\Pi_t^{-\epsilon_h\gamma_h}, \\
\bar{K}_t^{h} &= \left[\frac{1-\theta_h\left(\frac{\pi_{t-1}^{\gamma_h}}{\pi_{W,t}}\right)^{1-\varepsilon_h}}{1-\theta_h}\right]^{\frac{1}{1-\varepsilon_h}}\bar{F}_t^{h},
\end{aligned}
$$

$$
\begin{aligned}
\Delta_t^{h} &= (1-\theta_h)\left[\frac{1-\theta_h\left(\frac{\pi_{t-1}^{\gamma_h}}{\pi_{W,t}}\right)^{1-\varepsilon_h}}{1-\theta_h}\right]^{\frac{\varepsilon_h}{\varepsilon_h-1}} + \theta_h\left(\frac{\pi_{W,t}}{\pi_{t-1}^{\gamma_h}}\right)^{\varepsilon_h}\Delta_{t-1}^{h}, \\
R_t - 1 &= \rho(R_{t-1}-1) + (1-\rho)\left[\phi^{\pi}\left(\pi_t-1\right) + \phi^{y}\left(\frac{Y_t}{Y_{t-1}}-1\right)\right] + \eta_t.
\end{aligned}
$$

Here Δ_t and Δ_t^h denote relative price dispersion terms for prices and wages, defined as

$$
\begin{aligned}
\Delta_t &:= \int_0^1 \left[\frac{P_t\left(f\right)}{P_t}\right]^{-\varepsilon}\mathrm{d}f, \\
\Delta_t^{h} &:= \int_0^1 \left[\frac{W_t\left(l\right)}{W_t}\right]^{-\varepsilon_h}\mathrm{d}l.
\end{aligned}
$$

$\bar{F}_t$, $\bar{K}_t$, $\bar{F}_t^h$, and $\bar{K}_t^h$ are auxiliary variables. Price and wage markup shocks are defined as

$$
\begin{aligned}
\exp\left(u_t\right) &:= \frac{\varepsilon}{\left(1+\tau_t\right)\left(\varepsilon-1\right)}, \\
\exp\left(\mu_t\right) &:= \frac{\varepsilon_h}{\left(1+\tau_t^{h}\right)\left(\varepsilon_h-1\right)}.
\end{aligned}
$$

All shocks are assumed to follow AR(1) processes:

$$
\begin{aligned}
z_t &= \rho_z z_{t-1} + \omega_{z,t}, \\
u_t &= \rho_u u_{t-1} + \omega_{u,t}, \\
\mu_t &= \rho_\mu \mu_{t-1} + \omega_{\mu,t}, \\
\eta_t &= \rho_\eta \eta_{t-1} + \omega_{\eta,t},
\end{aligned}
$$

where

$$
\begin{aligned}
\omega_{z,t} &\sim N\left(0, \sigma_z^2\right), \\
\omega_{u,t} &\sim N\left(0, \sigma_u^2\right), \\
\omega_{\mu,t} &\sim N\left(0, \sigma_\mu^2\right), \\
\omega_{\eta,t} &\sim N\left(0, \sigma_\eta^2\right).
\end{aligned}
$$

B.4.7 Welfare cost

Let $\{(C_t^n, L_t^n)\}$ be the equilibrium consumption and labor in an economy where the agents observe n-period ahead shocks. Ex ante welfare of the household is

$$V^n = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \{u(C_t^n - bC_{t-1}^n) - v(L_t^n)\}\right].$$

For each n, we measure welfare gain/loss in consumption unit defined as

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \{u((1+\lambda_n)C_t^0 - b(1+\lambda_n)C_{t-1}^0) - v(L_t^0)\}\right] = V^n.$$

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