Online Appendix for "Fiscal Forward Guidance: A Case for Selective Transparency"

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A Proof of Proposition 1

Let

$$\phi(s) := \frac{Rv'(Rs - G_2) - u'(Y_1 - s)}{\frac{\partial}{\partial S} MRS(s)}.$$

where

$$MRS(S) := \frac{u'(Y_1 - S)}{v'(RS - G_2)R}.$$

The following proposition shows that there always exists an interval on which ϕ is strictly decreasing.

Lemma 1 *Suppose that u and v are C^3 functions. Let S_{FB} be the first-best saving, i.e. $S_{FB} := S(0)$. There exists S_L and S_U with $S_L < S_{FB} < S_U$ and ϕ is decreasing on $[S_L, S_U]$.*

Proof. Note that

$$\begin{aligned}
\phi(s) &= \frac{Rv'(Rs - G_2) - u'(Y_1 - s)}{\frac{\partial}{\partial S} MRS(s)} \\
&= \frac{Rv'(Rs - G_2) - u'(Y_1 - s)}{\frac{-u''(Y_1-s)v'(Rs-G_2)R - u'(Y_1-s)v''(Rs-G_2)R^2}{(v'(Rs-G_2)R)^2}} \\
&= \frac{v'(Rs - G_2)R \times \left(\frac{Rv'(Rs-G_2)}{u'(Y_1-s)} - 1\right)}{-\frac{u''(Y_1-s)}{u'(Y_1-s)} - \frac{v''(Rs-G_2)}{v'(Rs-G_2)}R}.
\end{aligned}$$

Because u and v are C^3, it follows that ϕ is a C^1 function.

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Let $a(s) := \frac{Rv'(Rs-G_2)}{u'(Y_1-s)} - 1$ and $b(s) = -\frac{u''(Y_1-s)}{u'(Y_1-s)} - \frac{v''(Rs-G_2)}{v'(Rs-G_2)}R$. Then

$$\phi'(s) = \frac{b(s)\left\{a(s)v''(Rs-G_2)R^2 + a'(s)v'(Rs-G_2)R\right\} - Rv'(Rs-G_2)a(s)b'(s)}{b(s)^2}.$$

Observe that the first-best saving satisfies $a(S_{FB}) = 0$. Hence,

$$\phi'(S_{FB}) = \frac{a'(S_{FB})v'(RS_{FB}-G_2)R}{b(S_{FB})}.$$

Because $a'(s) < 0$, $b(s) > 0$, and $v'(Rs-G_2) > 0$ for any s, we have $\phi'(S_{FB}) < 0$. Because ϕ' is continuous, there is a closed interval around S_{FB} on which $\phi' < 0$. ■

The next lemma uses the Jensen's inequality argument to complete the proof.

Lemma 2 *Suppose that ϕ is strictly decreasing on an interval $[S_L, S_U]$ and that τ_2^K is distributed between $x_L := MRS(S_L) - 1$ and $x_U := MRS(S_U) - 1$. Then no matter what the distribution of τ_2^K is, more information reduces ex ante welfare: for any σ-field $\mathcal{G}$ such that $\mathcal{F}_0 \subset \mathcal{G} \subset \mathcal{F}_1^P$, we have*

$$\mathbb{E}_0\left[W(-\mathbb{E}[\tau_2^K|\mathcal{F}_1^P])\right] \leq \mathbb{E}_0\left[W(-\mathbb{E}[\tau_2^K|\mathcal{G}])\right],$$

with strict inequality if and only if $\mathbb{E}[\tau_2^K|\mathcal{G}] \neq \mathbb{E}[\tau_2^K|\mathcal{F}_1^P]$ with non-zero probability.

Proof. Because $MRS(S(X)) = 1 + X$, the implicit function theorem implies

$$S'(X) = \frac{1}{\frac{\partial}{\partial S}MRS(S(X))} > 0.$$

By differentiating the function $W(x)$, we obtain

$$W'(x) = \left\{-u'(Y_1 - S(x)) + v'(RS(x) - G_2)\right\} S'(x) = \phi(S(x)).$$

Note that S is strictly increasing and that, by definition, $S(x_L) = S_L$ and $S(x_U) = S_U$. Because ϕ is strictly decreasing on $[S(x_L), S(x_U)] = [S_L, S_U]$, W' is strictly decreasing on $[x_L, x_U]$. Therefore, W is concave on $[x_L, x_U]$.

Hence, as far as $-\tau_2^K$ is distributed on $[x_L, x_U]$, for any σ-field $\mathcal{G}$ such that $\mathcal{F}_0 \subset \mathcal{G} \subset \mathcal{F}_1^P$, Jensen's inequality implies:

$$\begin{aligned}
\mathbb{E}_0[W(-\mathbb{E}[\tau_2^K|\mathcal{F}_1^P])] &= \mathbb{E}_0\left[\mathbb{E}\left[W(-\mathbb{E}[\tau_2^K|\mathcal{F}_1^P])|\mathcal{G}\right]\right] \\
&\leq \mathbb{E}_0\left[W(\mathbb{E}\left[-\mathbb{E}[\tau_2^K|\mathcal{F}_1^P]|\mathcal{G}\right])\right] = \mathbb{E}_0\left[W(-\mathbb{E}[\tau_2^K|\mathcal{G}])\right].
\end{aligned}$$

The equality holds if and only if $\mathbb{E}[\tau_2^K|\mathcal{F}_1^P] = \mathbb{E}[\tau_2^K|\mathcal{G}]$ with probability one. ■

Finally, for S_L and S_U satisfying $S_L < S_{FB} < S_U$, and for $x_L := MRS(S_L) - 1$ and $x_U := MRS(S_U) - 1$, we have

$$x_L < 0 < x_U,$$

because $MRS(S_{FB}) - 1 = 0$. Therefore Proposition 1 follows.

B Additional results

B.1 Analytical results in the Brock-Mirman model

In this section we argue that our analytical results for the three period model hold true in an infinite horizon stochastic growth model in Brock and Mirman (1972). Here we only consider shocks to capital income tax. Government spending is assumed to be zero.

The only restriction we impose on the stochastic process of capital income tax rates $\{\tau_t^K\}_{t=0}^{\infty}$ is that an equilibrium exists. Therefore, our specification encapsulates various scenarios that cannot be analyzed with three-period models. For example, we can consider a situation in which private agents know that the tax rate will be increased to a certain level at some future date but are unsure about the exact timing. Another example is that the autocovariance structure of the tax changes stochastically over time.

The representative household maximizes

$$\mathbb{E}_0^P[\sum_{t=0}^{\infty}\beta^t \ln c_t]$$

subject to the budget constraint:

$$c_t + k_{t+1} = (1-\tau_t^K)r_t k_t + w_t + T_t, \quad \forall t,$$

taking the prices $\{r_t, w_t\}_{t=0}^{\infty}$ and the fiscal policy $\{\tau_t^K, T_t\}_{t=0}^{\infty}$ as given. The household supplies one unit of labor inelastically, and capital fully depreciates after production. The representative firm owns a Cobb-Douglas production function, $Y = K^{\alpha}L^{1-\alpha}$, and its first-order condition is given by

$$r_t = \alpha K_t^{\alpha-1} \text{ and } w_t = (1-\alpha)K_t^{\alpha}. \tag{1}$$

The government budget constraint is

$$\tau_t^K r_t k_t = T_t. \tag{2}$$

The equilibrium condition is:

$$\begin{aligned} \frac{1}{C_t} &= \beta \mathbb{E}_t^P[\frac{1}{C_{t+1}}(1-\tau_{t+1}^K)\alpha K_{t+1}^{\alpha-1}], \quad \forall t, \\ C_t + K_{t+1} &= K_t^{\alpha}, \quad \forall t, \end{aligned}$$

and the transversality condition $\lim_{T\to\infty}\beta^T\mathbb{E}_t^P[K_{T+1}/C_T] = 0$.[1]

[1] When we have consumption taxes that converges almost surely, the equilibrium condition is identical except that the term $1-\tau_{t+1}^K$ in the Euler equation is replaced with $(1-\tau_{t+1}^K)(1+\tau_t^C)/(1+\tau_{t+1}^C)$. Therefore, all the following results hold in the presence of consumption tax if we replace $1-\tau_{t+1}^K$ with

Let $Y_t = K_t^\alpha$ and $s_t := K_{t+1}/Y_t$. Then we have

$$\frac{s_t}{1-s_t} = \alpha\beta\mathbb{E}_t^P[\frac{1}{1-s_{t+1}}(1-\tau_{t+1}^K)] \Rightarrow \frac{1}{1-s_t} = 1 + \alpha\beta\mathbb{E}_t^P[\frac{1}{1-s_{t+1}}(1-\tau_{t+1}^K)].$$

Iterating forward, we obtain

$$\frac{1}{1-s_t} = 1 + \mathbb{E}_t^P\Big[\sum_{j=1}^{\infty}(\alpha\beta)^j\prod_{i=1}^{j}(1-\tau_{t+i}^K)\Big] + \lim_{J\to\infty}(\alpha\beta)^J\mathbb{E}_t^P[\frac{1}{1-s_{t+J}}\prod_{j=1}^{J}(1-\tau_{t+j}^K)].$$

Assume that the last term is zero.[2] Then we have

$$\frac{s_t}{1-s_t} = \mathbb{E}_t^P\Big[\sum_{j=1}^{\infty}(\alpha\beta)^j\prod_{i=1}^{j}(1-\tau_{t+i}^K)\Big].$$

This is clearly a generalization of the two-period model with log utility.

Let $X_t = \mathbb{E}_t^P[\sum_{j=1}^{\infty}(\alpha\beta)^j\prod_{i=1}^{j}(1-\tau_{t+i}^K)]$. Then ex ante utility of the representative household can be written in terms of $\{X_t\}_{t=0}^\infty$ and parameters only.

Lemma 3 *Ex ante welfare equals*

$$\frac{\alpha\beta}{1-\alpha\beta}\mathbb{E}\Big[\sum_{t=0}^{\infty}\beta^t f(X_t)\Big] + \frac{\alpha\ln K_0}{1-\alpha\beta},$$

where the function $f(x) := \ln x - (1/\alpha\beta)\ln(1+x)$ is strictly concave on $\big(0, \sqrt{\alpha\beta}/(1-\sqrt{\alpha\beta})\big)$ and is strictly convex on $(\sqrt{\alpha\beta}/(1-\sqrt{\alpha\beta}), \infty)$.

As in the three period model with log utility, when the after-tax rate $1-\tau_t^K$ resides in the interval on which f is strictly concave, more information is harmful for ex ante welfare.

Proposition 1 *Let $\mathcal{F}$ and $\mathcal{G}$ be filtrations such that $\mathcal{F}$ is coarser than $\mathcal{G}$ (i.e. $\mathcal{F}_t \subset \mathcal{G}_t$ for all t) and that $\{\tau_t^K\}$ is $\mathcal{F}$-adapted. Suppose for all t,*

$$0 < \sum_{j=1}^{\infty}(\alpha\beta)^j\prod_{i=1}^{j}(1-\tau_{t+i}^K) < \frac{\sqrt{\alpha\beta}}{1-\sqrt{\alpha\beta}}$$

$(1-\tau_{t+1}^K)(1+\tau_t^C)/(1+\tau_{t+1}^C)$.

[2] A sufficient condition for this is that $1-\tau_t^K$ is bounded above by $1/\alpha$. Then the transversality condition implies the convergence to zero.

holds almost everywhere.[3] *Then,*

$$\mathbb{E}[f(X_t^{\mathcal{F}})] \geq \mathbb{E}[f(X_t^{\mathcal{G}})]$$

for all t, where $X_t^{\mathcal{F}} := \mathbb{E}[\sum_{j=1}^{\infty}(\alpha\beta)^j \prod_{i=1}^{j}(1-\tau_{t+i}^K)|\mathcal{F}_t]$ *and* $X_t^{\mathcal{G}} := \mathbb{E}[\sum_{j=1}^{\infty}(\alpha\beta)^j \prod_{i=1}^{j}(1-\tau_{t+i}^K)|\mathcal{G}_t]$. *Inequality is strict when* $X_t^{\mathcal{F}} \neq X_t^{\mathcal{G}}$ *with positive probability.*

If the representative household is provided with information that improves its expectations of future taxes with positive probability, then the stochastic process of $\{X_t\}_{t=0}^{\infty}$ changes with positive probability. In such situations, Proposition 1 together with Lemma 3 implies that ex ante utility of the household strictly decreases.

B.1.1 Uncertainty shock

The above model encapsulates specifications with uncertainty shocks. Consider the following stochastic process $\{z_t\}_{t=0}^{\infty}$ with time-varying risk:

$$z_t = z_{t-1} \times (1 + \sigma_t \epsilon_t),$$

where ϵ_t is an I.I.D. random variable that is distributed over $[-1, 1]$ symmetrically around 0. Here $\{\sigma_t\}_{t=0}^{\infty}$ is a potentially persistent process in the interval $[\underline{\sigma}, \overline{\sigma}] \subset \mathbb{R}_+$. Let $\underline{z}$ and $\overline{z}$ be such that $0 < \underline{z} < \overline{z} < 1/\sqrt{\alpha\beta}$. Suppose that $z_0 \in (\underline{z}, \overline{z})$. Let S be the stopping time such that

$$S := \inf\{t \geq 0 : z_t < \underline{z} \text{ or } z_t > \overline{z}\}.$$

We assume

$$1 - \tau_t^K = \begin{cases} z_t & \text{if } S > t, \\ \underline{z} & \text{if } S \leq t \text{ and } z_S < \underline{z}, \\ \overline{z} & \text{if } S \leq t \text{ and } z_S > \overline{z}. \end{cases}$$

In other words, $1 - \tau_t^K$ follows the same process as z_t until the latter variable hits the boundary of $[\underline{z}, \overline{z}]$ and stays at the boundary after that. Therefore, when $\{\sigma_t, \epsilon_t\}$ is adapted to the filtration of the household, all the assumptions in Proposition 1 are satisfied. The term $\mathbb{E}_t^P[\sum_{j=1}^{\infty}(\alpha\beta)^j \prod_{i=1}^{j}(1-\tau_{t+i}^K)]$ involves higher-order moments of $\{\sigma_{t+j}\epsilon_{t+j}\}_{j=1}^{\infty}$ and, therefore, it changes when the household is provided more information about $\{\sigma_{t+j}\epsilon_{t+j}\}_{j=1}^{\infty}$. Therefore, it follows from Proposition 1 that ex ante welfare decreases when the government provides information about future uncertainty shocks.

[3] A sufficient condition is that $1 - \tau_t^K$ is bounded above by $1/\sqrt{\alpha\beta}$, because

$$0 < \sum_{j=1}^{\infty}(\alpha\beta)^j \prod_{i=1}^{j}(1-\tau_{t+i}^K) \leq \sum_{j=1}^{\infty} \frac{(\alpha\beta)^j}{(\sqrt{\alpha\beta})^j} = \sum_{j=1}^{\infty}(\sqrt{\alpha\beta})^j = \frac{\sqrt{\alpha\beta}}{1-\sqrt{\alpha\beta}}$$

holds. Hence, as far as $1 - \tau_t^K$ is distributed between 0 and $1/\sqrt{\alpha\beta} > 0$ for all t, this assumption is satisfied.

B.1.2 Proofs

Proof of Lemma 3. Because $K_{t+1} = s_t K_t^\alpha$ by definition, we have

$$\ln K_t = \alpha^t \ln K_0 + \sum_{j=0}^{t-1} \alpha^{t-1-j} \ln s_j.$$

Therefore,

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t \ln C_t &= \sum_{t=0}^{\infty} \beta^t \{\ln(1-s_t) + \alpha \ln K_t\} \\ &= \sum_{t=0}^{\infty} \beta^t \{\ln(1-s_t) + \alpha^{t+1} \ln K_0 + \sum_{j=0}^{t-1} \alpha^{t-j} \ln s_j\}\end{aligned}$$

Collecting the terms that involve s_t,

$$\begin{aligned}\beta^t \ln(1-s_t) + \sum_{l=t+1}^{\infty} \beta^l \alpha^{l-t} \ln s_t &= \frac{\beta^t}{1-\alpha\beta}\{(1-\alpha\beta)\ln(1-s_t) + \alpha\beta \ln s_t\} \\ &= \frac{\beta^t}{1-\alpha\beta}\Big\{\alpha\beta \ln X_t - \ln(1+X_t)\Big\}.\end{aligned}$$

Ex ante welfare of the household is thus

$$\mathbb{E}\Big[\sum_{t=0}^{\infty} \beta^t \frac{\alpha\beta}{1-\alpha\beta}\Big\{\ln X_t - \frac{1}{\alpha\beta}\ln(1+X_t)\Big\}\Big] + \frac{\alpha \ln K_0}{1-\alpha\beta}.$$

The function $f(x) := \ln x - (1/\alpha\beta)\ln(1+x)$ is strictly concave on $\big(0, \sqrt{\alpha\beta}/(1-\sqrt{\alpha\beta})\big)$ and is strictly convex on $(\sqrt{\alpha\beta}/(1-\sqrt{\alpha\beta}), \infty)$. ■

Proof of Proposition 1. Under the stated condition, $X_t^{\mathcal{F}}$ and $X_t^{\mathcal{G}}$ are in the interval $[0, \sqrt{\alpha\beta}/(1-\sqrt{\alpha\beta})]$. Because f is strictly concave on the same interval, Jensen's inequality implies that

$$\mathbb{E}[f(X_t^{\mathcal{G}})] = \mathbb{E}\left[\mathbb{E}[f(X_t^{\mathcal{G}})|\mathcal{F}_t]\right] \le \mathbb{E}\left[f\left(\mathbb{E}[X_t^{\mathcal{G}}|\mathcal{F}_t]\right)\right] = \mathbb{E}[f(X_t^{\mathcal{F}})].$$

($\mathbb{E}[X_t^{\mathcal{G}}|\mathcal{F}_t] = X_t^{\mathcal{F}}$ follows from the law of iterated expectations and $\mathcal{F}_t \subset \mathcal{G}_t$.) Because f is strictly concave, inequality is strict if and only if $X_t^{\mathcal{F}} \neq X_t^{\mathcal{G}}$ with positive probability. ■

B.2 Numerical results in the model without lump-sum taxes

In the paper we have assumed that the lump-sum tax adjusts to balance the budget when a shock hits either the tax rate or the spending level. The purpose of this section is to provide an example in which the forward guidance about future distortionary tax reduces welfare even if tax revenue is not rebated back to the household. In the model we study in this section, the lump-sum tax and transfer are not available. Instead, we add one more period to the three period model so that the government adjusts the period-3 tax rate in response to the period-2 tax shock to satisfy the intertemporal budget constraint. The model also features endogenous labor supply, labor income tax, and a Cobb-Douglas production function.

Time is discrete and $t \in \{0, 1, 2, 3\}$. As in the previous model, the household receives a constant endowment in period 1, but labor supply is endogenous in periods 2 and 3. Tax rates in $t = 2$ are random variables, and the household may be informed about them in $t = 1$, as before. The government spends an exogenously given level, G, in period 1 and finances it by issuing debt, $B_1 = G$. The government is not allowed to use lump-sum tax/transfer to balance the budget. Therefore, it must adjust distortionary taxes in $t = 3$ to repay the debt that is outstanding in period 3.

In period 1, the representative household chooses a state-contingent plan to maximize the expected utility given by:

$$\ln c_1 + \mathbb{E}_1^P[\sum_{t=2}^{3} \beta^t \ln c_t - v(l_t)]$$

subject to

$$\begin{aligned} c_1 + k_1 + b_1 &= Y_1, \\ c_2 + k_2 + b_2 &= (1-\delta)k_1 + (1-\tau_2^K)R_2^K k_1 + (1-\tau_2^L)W_2 l_2 + R_2 b_1, \\ c_3 &= (1-\delta)k_2 + (1-\tau_3^K)R_3^K k_2 + (1-\tau_3^L)W_3 l_3 + R_3 b_2. \end{aligned}$$

Here, R_t is the gross (real) interest rates on government bond in periods t, and R_t^K is the rental rate of capital in period t. Period-t capital income is taxed at the rate τ_t^K. The wage rate in period t is denoted by W_t and is taxed at the rate τ_t^L.

The representative firm maximizes

$$F(K_t, AL_t) - W_t L_t - R_t^K K_t$$

in period t, which yields the standard FOCs:

$$R_t^K = F_K(K_t, AL_t) \text{ and } W_t = F_L(K_t, AL_t)A,$$

where F is the production function and F_K and F_L denote its partial derivatives.

The resource constraint is:

$$\begin{aligned} C_1 + K_1 + G &= Y_1, \\ C_2 + K_2 &= F(K_1, AL_2) + (1-\delta)K_1, \\ C_3 &= F(K_2, AL_3) + (1-\delta)K_2. \end{aligned}$$

The government budget constraint is given by:

$$
\begin{aligned}
G &= B_1, \\
R_2 B_1 &= \tau_2^K R_2^K K_1 + \tau_2^L W_2 L_2 + B_2, \\
R_3 B_2 &= \tau_3^K R_3^K K_2 + \tau_3^L W_3 L_3.
\end{aligned}
$$

The government spends a fixed amount G in period 1, and finances the spending through the government bond. The period 2 tax rates, (τ_2^K, τ_2^L), are random (exogenous), and (τ_3^K, τ_3^L) adjust endogenously so that the government budget balances in period 3. We will examine several ways in which the period 3 taxes are adjusted, e.g. either the capital or the labor income tax adjusts, or both adjust in a perfectly correlated fashion.

We assume that the government bond is risk-free. Hence R_2 is known in period 1 and R_3 is known in period 2.

Only uncertainty in this economy is the randomness of (τ_2^K, τ_2^L). Therefore, once these taxes realize at the beginning of period 2, no uncertainty is left and the household knows (τ_3^K, τ_3^L) perfectly. Hence, once (τ_2^K, τ_2^L) has realized, the economy becomes a perfect foresight economy.

The equilibrium condition consists of

$$
\begin{aligned}
\frac{1}{C_1} &= \beta \mathbb{E}_1^P [\frac{1}{C_2}\{1 - \delta + (1 - \tau_2^K) F_K(K_1, AL_2)\}], \\
\frac{1}{C_1} &= \beta \mathbb{E}_1^P [\frac{1}{C_2}] R_2, \\
\frac{1}{C_2} &= \beta \frac{1}{C_3}\{1 - \delta + F_K(K_2, AL_3)(1 - \tau_3^K)\}, \\
\frac{1}{C_2} &= \beta \frac{1}{C_3} R_3, \\
C_2 v'(L_2) &= (1 - \tau_2^L) F_L(K_1, AL_2) A, \\
C_3 v'(L_3) &= (1 - \tau_3^L) F_L(K_2, AL_3) A, \\
R_2 G &= \tau_2^K F_K(K_1, AL_2) K_1 + \tau_2^L F_L(K_1, AL_2) AL_2 \\
&\quad + \frac{\tau_3^K F_K(K_2, AL_3) K_2 + \tau_3^L F_L(K_2, AL_3) AL_3}{R_3},
\end{aligned}
$$

and the resource constraints. Except for the first two equations, these equations are defined state by state.

How do we show the welfare effect of forward guidance? We assume that there are only two possibilities in period 2, $\{L, H\}$, and the period 2 tax rates take on either $\{\tau_2^K(L), \tau_2^L(L)\}$ or $\{\tau_2^K(H), \tau_2^L(H)\}$. It is assumed that $\tau_2^K(L) < \tau_2^K(H)$ and $\tau_2^L(L) < \tau_2^L(H)$. I.e. the state L corresponds to a "low tax" state and the state H corresponds to a "high tax" state. Let $\rho_1 \in [0, 1]$ be the probability that the state is L, conditional on the household's information set in period 1. Then all equilibrium objects are function of ρ_1 and, in particular, the equilibrium utility evaluated in period 1 can be written as a function W of ρ_1. Let ρ_0 be the prior probability assigned to the state L. Then the Bayes rule implies

$$\mathbb{E}_0[\rho_1] = \rho_0.$$

If f is a concave function, then the Jensen's inequality implies

$$\mathbb{E}_0[W(\rho_1)] \leq W(\mathbb{E}_0[\rho_1]) = W(\rho_0),$$

and the welfare effect of forward guidance is non-positive. It is the concavity of W we will demonstrate below.

In this model we cannot express equilibrium welfare using the expected tax rate. We solve the model numerically to obtain the function W. The preference discount factor, β, is set to 0.99. The labor disutility function $v(L)$ is an isoelastic function, $v(L) = L^{1+\eta}/(1+\eta)$, with $\eta = 2$. The production function is assumed to be the Cobb-Douglas function, $F(K, AL) = K^{\alpha}(AL)^{1-\alpha}$, with $\alpha = 1/3$. The technology parameter A is normalized so that in the infinite horizon version of this model the steady state output equals one without tax. The initial endowment, Y_1, is set to one. Capital depreciation rate, δ, is 0.1. The government spending is set to $G = 0.1$. Tax rates are given by $\tau_2^K(H) = \tau_2^L(H) = 0.2$ and $\tau_2^K(L) = \tau_2^L(L) = 0.1$.

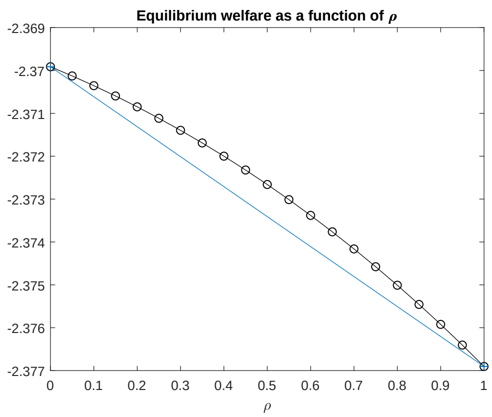
Regarding the way the period-3 taxes adjust, we consider two cases. First, the labor income tax stays at the period-2 level, and the period-3 capital income tax rate adjusts to balance the government budget. Second, the capital income tax remains constant between periods 2 and 3, while the labor income tax adjusts in period 3. Figure 1 displays how welfare and tax change with the posterior belief ρ_1 in each of these cases. In the top two panels, ex post welfare and the period-3 capital income tax are drawn as functions of ρ_1. Ex post welfare is a concave function of ρ_1 (left panel) and the period-3 capital income tax is higher when the period-2 taxes are lower and increases with ρ_1.

B.2.1 Robustness

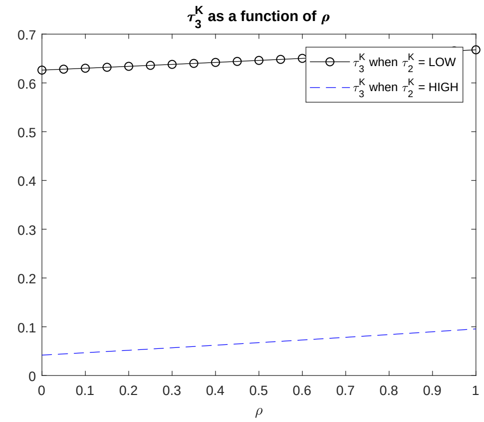
Parameter values: Qualitative implications are unchanged when we use different yet standard parameter values for preference and technology. For example, concavity obtains when a general CRRA utility function $c^{1-\sigma}/(1-\sigma)$ instead of $\ln c$. Changing the Frisch elasticity or the production function parameters does not overturn the result either.

It appears that equilibrium does not exist for some fiscal policy parameter specifications. For example, when G is set to a relatively large number, we could not find an equilibrium. This could be due to the Laffer curve: the government budget constraint cannot be satisfied if the initial spending exceeds the present value of maximal possible tax revenue.

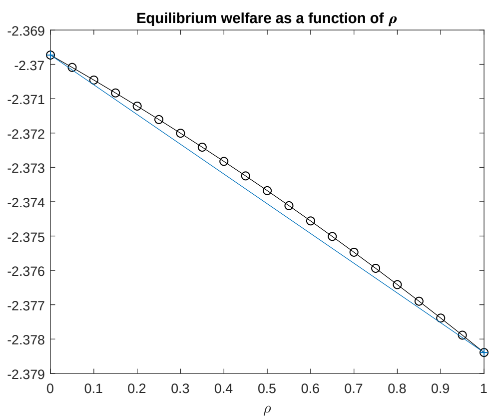
Equilibrium multiplicity: If the Laffer curve has a peak, then there may be multiple equilibria. This is because, given everything else being equal, different tax rates may be able to generate the same tax revenue. Because the model is solved only numerically, we cannot directly show the uniqueness of an equilibrium. However, we



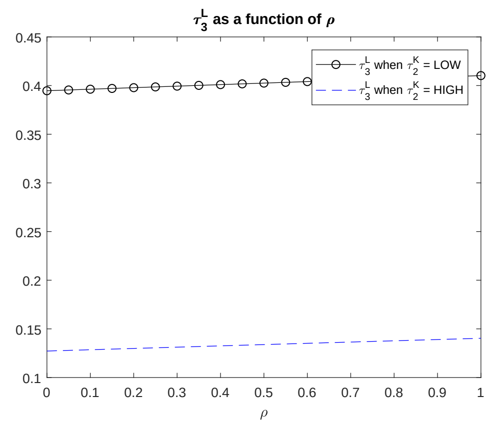
(a) Welfare $W(\rho_1)$ when τ_3^K adjusts



(b) Period-3 capital income tax τ_3^K



(c) Welfare $W(\rho_1)$ when τ_3^L adjusts



(d) Period-3 labor income tax τ_3^L

Figure 1: Welfare and the period-3 tax when no lump-sum tax is used.

think that it is likely that the equilibrium is unique because, even when we used different initial guesses, the numerical algorithm converged to the same solution. In addition, the equilibrium we found appears to change continuously with the belief parameter ρ_1. Hence, even if multiple equilibria exist, it appears that we are able to select a particular kind of equilibrium consistently.

State-contingent government bond: In the above baseline model the government bond is risk-free and the real return is pinned down by equilibrium condition, or more specifically by the corresponding Euler equation. When the government bond offers state-contingent real return, however, additional assumptions are needed because a single Euler equation holds only in expectations.

We also examined a case in which the government bond and capital are perfect substitutes. In other words, they must offer exactly the same ex post return between period 1 and period 2:

$$R_2 = 1 - \delta + (1 - \tau_2^K) F_K(K_1, AL_2).$$

Clearly, when $\rho_1 = 0$ or $\rho_1 = 1$, ex post welfare is unchanged, because effectively there is only one state. Interestingly, when the government bond is perfect substitute to capital, ex post welfare becomes *convex* when the capital income tax adjusts. Hence, more information about the period-2 tax rates improves ex ante welfare in this case. However, observe that, for $\rho_1 \in (0, 1)$, ex ante welfare is strictly lower than in the model with risk-free government bond. Therefore, if the government is benevolent, there is no reason for it to issue state-contingent bond instead of risk-free bond. When the labor income tax adjusts, the function W remains concave even in this case.

C Computation

We use the endogenous grid method (see Carroll (2006)) for computation. We follow Barillas and Fernandez-Villaverde (2007), but the separability of utility in consumption and labor simplifies the algorithm.

We focus on an equilibrium in which aggregate capital K and aggregate labor L follow the laws of motion:

$$\begin{aligned} K' &= X^K(K, z) \\ L &= X^L(K, z). \end{aligned} \tag{3}$$

The representative household's problem is written recursively as follows:

$$V(k, K, z) = \max_{c,k',l} u(c) - v(l) + \beta \mathbb{E}[V(k', K', z')|z]$$

subject to

$$(1+\tau^C(z))c + k' = (1-\tau^K(z))\alpha K^{\alpha-1} L^{1-\alpha} k + (1-\tau^L(z))(1-\alpha) K^\alpha L^{-\alpha} l + (1-\delta)k + T(K, z)$$

and the aggregate law of motion (3). Because L depends only on K and z, we do not need to include L as a state variable for the household problem. The solution to this problem is given by $C(k, K, z)$, $K'(k, K, z)$, and $L(k, K, z)$.

The government budget constraint requires

$$\tau^K(z)\alpha K^\alpha L^{1-\alpha} + \tau^C(z) C + \tau^L(z)(1-\alpha) K^\alpha L^{1-\alpha} = G(K, z) + T(K, z).$$

The government spending is allowed to vary with (K, z). Our specification therefore nests one in which the government spending share in output fluctuates randomly: $G(K, z) = \phi(z) K^\alpha (X^L(K, z))^{1-\alpha}$.

A recursive equilibrium consists of the policy function (C, K', L), the value function v, the aggregate law of motion (X^K, X^L), and the fiscal policy (G, T) such that (a) the pair of (C, K', L) and v solves the household's recursive problem given (X^K, X^L) and T, (b) for all (K, z), (X^K, X^L) satisfies $X^K(K, z) = K'(K, K, z)$ and $X^L(K, z) = L(K, K, z)$, and (c) markets clear.

The equilibrium condition is summarized by the following three equations:

$$\begin{aligned} u'(C(K,K,z)) &= \beta\mathbb{E}[V_1(K',K',z')|z] \\ \frac{v'(L(K,K,z))}{u'(C(K,K,z))} &= \frac{(1-\tau^L(z))(1-\alpha)K^\alpha L(K,K,z)^{-\alpha}}{1+\tau^C(z)} \\ V_1(K,K,z) &= \frac{1}{1+\tau^C(z)}\{1-\delta+(1-\tau^K(z))\alpha K^{\alpha-1}L(K,K,z)^{1-\alpha}\}u'(C(K,K,z)) \end{aligned}$$

Let $W(K,z) := V(K,K,z)$, $D(K,z) := V_1(K,K,z)$, $\tilde{C}(K,z) := C(K,K,z)$, $\tilde{K}'(K,z) := K'(K,K,z)$, and $\tilde{L}(K,z) = L(K,K,z)$.

$$\begin{aligned} u'(\tilde{C}(K,z)) &= \beta\mathbb{E}[D(K',z')|z](1+\tau^C(z)) \\ v'(\tilde{L}(K,z))\tilde{L}(K,z)^\alpha &= \frac{(1-\tau^L(z))(1-\alpha)K^\alpha}{1+\tau^C(z)}u'(\tilde{C}(K,z)) \\ D(K,z) &= \frac{1}{1+\tau^C(z)}\{1-\delta+(1-\tau^K(z))\alpha K^{\alpha-1}\tilde{L}(K,z)^{1-\alpha}\}u'(\tilde{C}(K,z)) \\ W(K,z) &= u(\tilde{C}(K,z)) - v(\tilde{L}(K,z)) + \beta\mathbb{E}[W(\tilde{K}'(K,z),z')|z] \end{aligned}$$

Algorithm 1 (Endogenous grid method with endogenous labor) *First fix the grid $Kend$ for k'.*

- *Initial guess: D_0 and W_0.*
- *For $n \geq 0$, take (D_n, W_n) as given and compute, for each z and $K' \in Kend$,*
  1. *$c_{n+1}(k', z)$ using*

     $$c_{n+1}(K',z) = \left[\beta\mathbb{E}[D_n(K',z')|z](1+\tau^C(z)\right]^{-1/\sigma},$$

  2. *$k_{n+1}(K', z)$ and $l_{n+1}(K', z)$ using*

     $$c_{n+1}(K',z) + K' = k_{n+1}(K',z)^\alpha l_{n+1}(K',z)^{1-\alpha},$$

     and

     $$v'(l_{n+1}(K',z))l_{n+1}(K',z)^\alpha = \frac{(1-\tau^L(z))(1-\alpha)u'(c_{n+1}(K',z))}{1+\tau^C(z)}k_{n+1}(K',z)^\alpha,$$

     [this part requires a nonlinear equation solver]
  3. *The derivative of the value function:*

     $$D_{n+1}(k_{n+1}(K',z),z) = \frac{u'(c_{n+1}(K',z))}{1+\tau^C(z)}\{1-\delta+(1-\tau^K(z))\alpha k_{n+1}(K',z)^{\alpha-1}l_{n+1}(K',z)^{1-\alpha}\},$$

4. *The value function:*

$$W_{n+1}(k_{n+1}(K',z),z) = u(c_{n+1}(K',z)) - v(l_{n+1}(K',z)) + \beta\mathbb{E}[W_n(K',z')|z].$$

5. *For each z, interpolate (D_{n+1}, W_{n+1}) to obtain their values on $Kend$.*
6. *Terminate the iteration if $||W_{n+1} - W_n||$ becomes smaller than the pre-specified tolerance level. Otherwise increase n by 1 and repeat the previous computation.*

When $v(l) = \chi l^{1+\eta}/(1+\eta)$, then in Step 2 we can express

$$\frac{k_{n+1}(K',z)}{l_{n+1}(K',z)} = \left[\frac{(1-\tau^L(z))(1-\alpha)u'(c_{n+1}(K',z))}{\chi(1+\tau^C(z))}\right]^{\frac{-1}{\alpha+\eta}} k_{n+1}(K',z)^{\frac{\eta}{\alpha+\eta}},$$

and substitute this into the resource constraint to obtain

$$c_{n+1}(K',z)+K' = k_{n+1}(K',z)^{\frac{\alpha(1+\eta)}{\alpha+\eta}} \times \left[\frac{(1-\tau^L(z))(1-\alpha)u'(c_{n+1}(K',z))}{\chi(1+\tau^C(z))}\right]^{\frac{1-\alpha}{\alpha+\eta}} + (1-\delta)k_{n+1}(K',z),$$

and solve the second equation for $k_{n+1}(K',z)$ and then the first equation for $l_{n+1}(K',z)$.

D Timing uncertainty: labor income tax and consumption tax

Due to space limitation, we are unable to report in the main paper the results for labor income tax and consumption tax in the model with timing uncertainty.

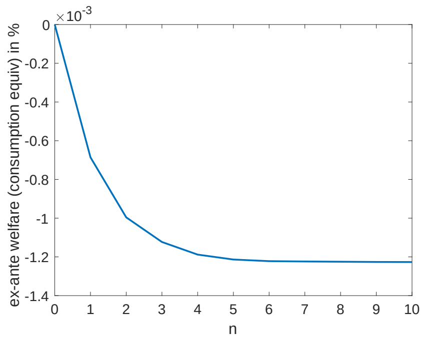
D.1 Labor income tax

Now the capital income tax rate is constant at 20% and the labor income tax is assumed to follow the aforementioned two-state Markov chain with the initial tax rate being 30%.

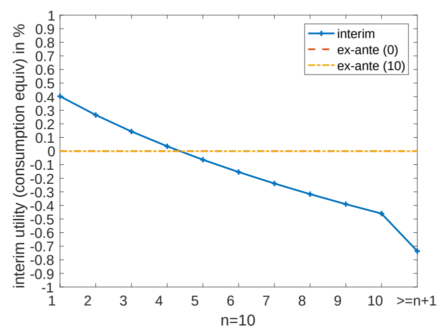
Panels 2a and 2b in Figure 2 displays ex ante and interim welfare. Results are qualitatively the same as before — ex ante welfare decreases with n and interim welfare when $n = 10$ decreases with the anticipated timing of tax cut. Interestingly, although the effect on ex ante welfare is much smaller than in the capital income tax case, the effect on interim welfare, as measured by the slope of interim welfare graph, is larger. Unlike in the capital income tax case, learning that the tax cut will occur very soon has a strong positive effect on interim welfare and weakens the negative effect of information on ex ante welfare.

D.2 Consumption tax

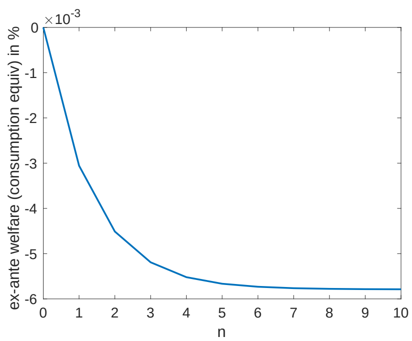
For a consumption tax cut from 20% to 10%, the results are qualitatively similar to the capital income tax cut case as shown in panels 2c and 2d in Figure 2.



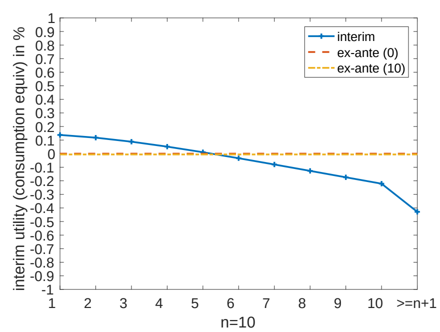
(a) Labor income tax: ex ante welfare



(b) Labor income tax: interim welfare



(c) Consumption tax: ex ante welfare



(d) Consumption tax: interim welfare

Left panels show ex ante welfare when the private sector observes n-period ahead shocks. Welfare are expressed in consumption unit, relative to welfare at $n = 0$. Right panels display interm welfare for $n = 10$ for different values of private information.

Figure 2: Ex ante and interim welfare measured in consumption unit: labor income and consumption taxes

E Partial disclosure

In the numerical exercises in the paper, we assumed that the fiscal forward guidance allows the private agents to observe the future policy shocks perfectly. This section relaxes this assumption, allowing the government to send an imperfect signal about future policy shocks. Even when partial disclosure is allowed, our main finding stands: more information about tax news reduces welfare while more information about spending news increases it.

E.1 Timing uncertainty

First consider the model in Section 3.1. The information structure is now parameterized by two parameters, n and ϵ. The private agents in period t observe:

- Tax rates from t to $t+n-1$ exactly, and
- An imperfect signal of the period $t+n$ tax rate.

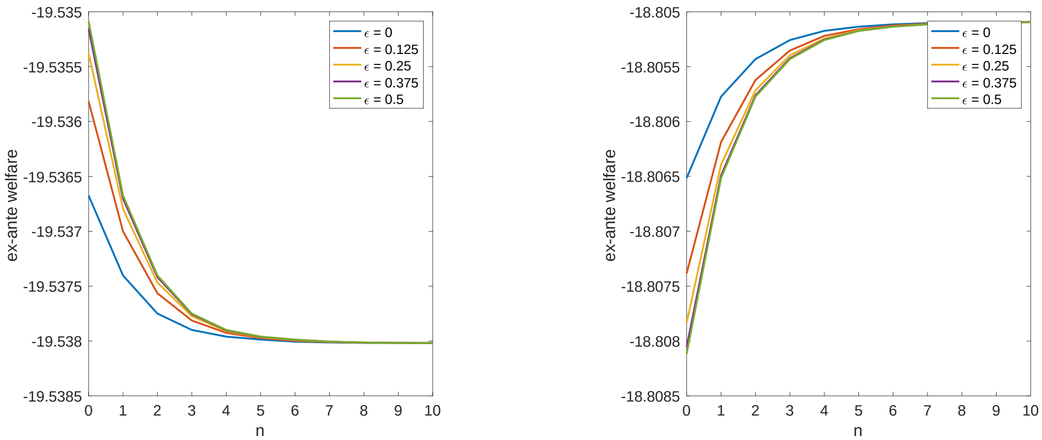
For the tax shock, the conditional distribution of the imperfect signal, $m_t \in \{c, u\}$, is as follows:

$$\begin{aligned}
\pi(c|\tau_{t+n} \neq \tau_{t+n-1} = \tau_0) &= 1-\epsilon, \\
\pi(u|\tau_{t+n} \neq \tau_{t+n-1} = \tau_0) &= \epsilon, \\
\pi(c|\tau_{t+n} = \tau_{t+n-1} = \tau_0) &= \epsilon, \\
\pi(u|\tau_{t+n} = \tau_{t+n-1} = \tau_0) &= 1-\epsilon,
\end{aligned}$$

for some $\epsilon \in [0, 1/2]$. Here c stands for "changed" and u for "unchanged." The signal is assumed to be conditionally independent over time. When $\epsilon = 0$, then private agents can learn perfectly from a signal what the next period's tax rate will be. When $\epsilon = 1/2$, the government's signal is uninformative. (For $\epsilon > 1/2$, we can just switch the role of u and that of c.) Note that the period $t+n$ tax rate is only imperfectly observed in period t but is perfectly observed in period t. This simplifies the belief dynamics and the analysis. When considering the spending shock, the above τ's are replaced with g's.

Panel (a) of Figure 3 shows how ex ante welfare varies with the aforementioned two parameters, n (on the horizontal axis) and ϵ, when the only fiscal policy shock is the capital income tax shock. As in the main paper's analysis, for a given value of ϵ, welfare decreases with n. Also noteworthy is that, for a given value of n, welfare decreases as we increase the precision by lowering ϵ. Qualitative results are unchanged when the consumption and the labor income taxes are concerned.

Panel (a) of Figure 3 does the same for the spending news shock. Here, it is clear that welfare is increasing in both n and the signal precision.



(a) Capital income tax shock

(b) Spending shock

Figure 3: Ex ante welfare vs. n: the case of imperfect signal

E.2 Level uncertainty

Under partial information disclosure, the government can send a signal that is imperfectly correlated with the true news shock. The government determines both the timing: when the information about future shocks are disclosed, and the accuracy: how accurate the information is.

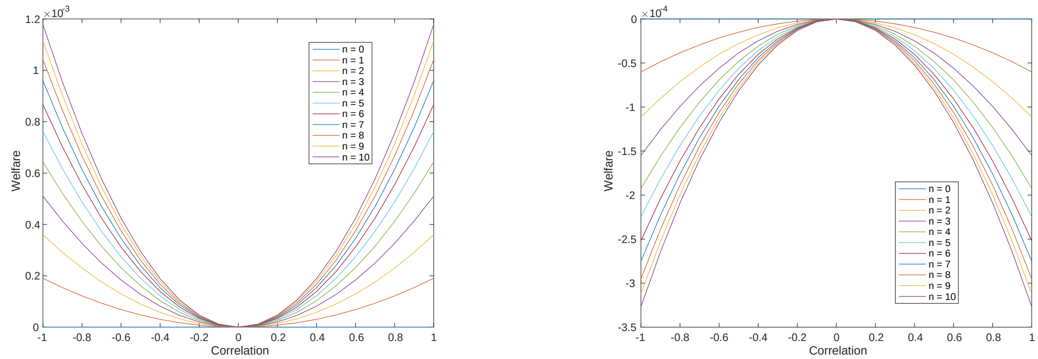
More specifically, the policy shocks are now given by

$$\epsilon_t^i = \sqrt{v}(\sqrt{1-(\gamma^p)^2}u_t^i + \gamma^p u_{t-n}^{i,news}),$$

for $i \in \{g, \tau\}$.

The government chooses two parameters, n and γ^p. The government in period $t-n$ publicly sends a signal that is correlated with the period t shock with the correlation coefficient $\gamma^p \in [-1.1]$. This is equivalent to the situation where either the tax rate or the government spending in period t is hit by two normally distributed shocks: one is a news shock $u_{t-n}^{i,news}$ which is *perfectly* observed in period $t-n$, and the other is a surprise shock u_t^i which is observed for the first time in period t and independent of the news shock. As $|\gamma^p|$ increases, future ϵ_t^i can become more predictable. Note that the distribution of ϵ_t^i is always $N(0, v)$, regardless of the value of γ^p.

Figures 4a and 4b illustrate ex ante welfare with different news horizon n and correlation γ^p. As shown in the experiments examined in the paper, the government expenditure (distortinoary tax) news increases (decreases) ex ante welfare. Also, being consistent with the propositions obtained from simple analytical models, partial information disclosure or intermediate levels of disclosure are never optimal with the distortionary tax news. Similarly, full information revelation of the government expenditure news is optimal at any new horizon.



(a) ex ante welfare: g

(b) ex ante welfare: τ

Figure 4: Ex ante Welfare from news at different levels of n and γ^p

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