Online Appendix to Ippei Fujiwara and Yuichiro Waki,
"Private news and monetary policy"

A Appendix

A.1 Derivation of the quadratic social welfare in equation (4)

Is the standard quadratic approximation in equation (4) valid in the present setting with communication? A short answer is yes. This is because the standard approximation is realization-by-realization and never uses equations that involve conditional expectations. More specifically, the standard procedure approximates $\mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t u(c_t, h_t)]$, where c is consumption and h is labor, by first obtaining a quadratic approximation of a momentary utility function in period t, $u(c_t, h_t)$, using deterministic equations only, sums them up from time 0 to infinity with discounting, and then take expectation based on the initial information set. When computing ex ante utility, the last step is replaced to that taking unconditional expectation, and there is no step in which conditional expectations based on the private sector's information set are taken. We have an additively separable term that captures unconditional expectation of *terms independent of policy* (t.i.p.) from period 0 on, but the term is unaffected by communication as it is unconditional expectation and, therefore, we can drop it in our analysis. This is a benefit of using ex ante utility. In contrast, if one were to evaluate the representative household's expected utility from period t on based on the period-t information available to it, then there is an additively separable term that captures *conditional* expectation of t.i.p. based on the household's information set in period t, and it is affected by the communication policy. In this case, one should not drop t.i.p. when examine the effect of communication policy.

Let us illustrate the above point by following Woodford (2010). When necessary, we refer to equation numbers in his chapter. Throughout, we will assume that the steady state is efficient. Therefore, Section 3.4.1 in Woodford (2010) is the relevant section. He uses a model in which the household supplies a set of differentiated labor to intermediate goods producers, and shows that the representative household's utility in equilibrium, evaluated in period 0, can be expressed as a function of output, Y, and the measure of price dispersion, Δ, as

$$\mathbb{E}_0^P\left[\sum_{t=0}^{\infty}\beta^t\{u(Y_t;\xi_t)-v(Y_t;\xi_t)\Delta_t\}\right],$$

where ξ_t is a preference shock. The price dispersion measure Δ_t is defined as

$$\Delta_t := \int_0^1\left(\frac{p_t(i)}{P_t}\right)^{-\theta(1+\nu)}di,$$

where $\theta > 1$ is the Dixit-Stiglitz elasticity of substitution parameter and $\nu > 0$ is the inverse of Frisch elasticity of labor supply (equation 61).[1]

Let $U(Y,\Delta;\xi) := u(Y;\xi) - v(Y;\xi)\Delta$. Its second order Taylor expansion around the steady state yields (equation 92):

$$\begin{aligned} U(Y_t,\Delta_t;\xi_t) &= \overline{Y}U_Y\hat{Y}_t + U_\Delta\hat{D}_t + \frac{1}{2}(\overline{Y}U_Y + \overline{Y}^2U_{YY})\hat{Y}_t^2 \\ &\quad +\overline{Y}U_{Y\Delta}\hat{Y}_t\hat{\Delta}_t + \overline{Y}U'_{Y\xi}\hat{\xi}_t\hat{Y}_t + \text{t.i.p.} + \mathcal{O}(||\xi||^3). \end{aligned}$$

Here t.i.p. refers to terms "that do not involve endogenous variables" and, therefore, consists of linear and quadratic terms in $\hat{\xi}_t$ and a constant.

When the steady state is efficient, we have $U_Y = 0$ and therefore

$$\begin{aligned} U(Y_t,\Delta_t;\xi_t) &= U_\Delta\hat{D}_t + \frac{1}{2}\overline{Y}^2U_{YY}(\hat{Y}_t - \hat{Y}_t^e)^2 \\ &\quad +\overline{Y}U_{Y\Delta}\hat{Y}_t\hat{\Delta}_t + \text{t.i.p.} + \mathcal{O}(||\xi||^3) \end{aligned}$$

(equation 94). Assuming that the initial $\hat{\Delta}$ is sufficiently small, i.e. $\hat{\Delta}_{-1} = \mathcal{O}(||\xi||^2)$, we obtain $\hat{\Delta}_t = \mathcal{O}(||\xi||^2)$ for all t, hence

$$\begin{aligned} U(Y_t,\Delta_t;\xi_t) &= \frac{1}{2}\overline{Y}^2U_{YY}(\hat{Y}_t - \hat{Y}_t^e)^2 - \overline{v}\hat{D}_t + \\ &\quad +\text{t.i.p.} + \mathcal{O}(||\xi||^3), \end{aligned}$$

[1]Here we assume, for simplicity, that the intermediate goods production function is linear in labor. Woodford (2010) allows for diminishing marginal product of labor for the intermediate goods production, and ν is a composite of the Frisch elasticity and the production function curvature parameter.

where $\overline{v} := v(\overline{Y}; \overline{\xi}) > 0$ (equation 95).

Finally, using the second order approximation of the dynamic equation for Δ, i.e. $\Delta_t = h(\Delta_{t-1}, 1 + \pi_t)$, Woodford (2010) shows

$$\sum_{t=0}^{\infty} \beta^t \hat{\Delta}_t = \frac{1}{2} \frac{h_{\pi\pi}}{1 - \alpha\beta} \sum_{t=0}^{\infty} \beta^t \pi_t^2 + \text{t.i.p.} + \mathcal{O}(||\xi||^3),$$

(equation 97). Because the dynamic equation for Δ is deterministic, the term t.i.p. refers only to terms proportional to $\hat{\Delta}_{-1}$ and does not involve any exogenous variables.

As a result, we have a realization-by-realization approximation:

$$\sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t; \xi_t) = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \{\overline{Y}^2 U_{YY} x_t^2 - (1 - \alpha\beta)^{-1} \overline{v} h_{\pi\pi} \pi_t^2\} + \text{t.i.p.} + \mathcal{O}(||\xi||^3),$$

where $x_t = \hat{Y}_t - \hat{Y}_t^e$ is the welfare-relevant measure of the output gap.

Observe that so far we have not used any equations that involve conditional expectations based on either the private sector's information or the central bank's information. All equations that are used are either static or backward-looking. Hence, the term t.i.p. does not include conditional expectations of exogenous variables.

By taking the unconditional expectation (given the initial condition $\hat{\Delta}_{-1}$), we obtain

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t U(Y_t, \Delta_t; \xi_t)\right] = \mathbb{E}\left[\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \{\overline{Y}^2 U_{YY} x_t^2 - (1 - \alpha\beta)^{-1} \overline{v} h_{\pi\pi} \pi_t^2\}\right] + \mathbb{E}[\text{t.i.p.}] + \mathbb{E}[\mathcal{O}(||\xi||^3)].$$

Recall that the term t.i.p. is the sum of two terms: one that comes from the second order approximation of the utility function and the other that comes from the approximation of the dynamic equation of Δ. The former includes the sum of constants and first and second order terms in $\{\hat{\xi}_t\}_{t=0}^{\infty}$. The latter includes linear terms in $\hat{\Delta}_{-1}$, which is exogenously given. Neither includes endogenous variables. Hence, the unconditional expectation of the sum of these terms, $\mathbb{E}$[t.i.p.], is independent of how much information the private sector obtains along the way. Therefore, we can treat it as constant in our setting with communication, as far as we are concerned with ex ante welfare.

If we attempt to evaluate the household's expected utility in equilibrium from period s on, based on the information available to the household, it is expressed as:

$$\mathbb{E}_s^P\left[\frac{1}{2}\sum_{t=s}^{\infty}\beta^t\{\overline{Y}^2U_{YY}x_t^2-(1-\alpha\beta)^{-1}\overline{v}h_{\pi\pi}\pi_t^2\}\right]+\mathbb{E}_s^P[\text{t.i.p.}]+\mathbb{E}_s^P[\mathcal{O}(||\xi||^3)].$$

Quite obviously, the term $\mathbb{E}_s^P[\text{t.i.p.}]$ depends on the information available to the household in period s and, therefore, depends on communication policy as well as what the household has observed up to period s.

A.2 A three-equation model with a lagged interest rate

A.2.1 When the Taylor rule is not inertial

It is straightforward to extend our theoretical results in Section 2 to the case where the nominal interest rate follows a simple Taylor rule:

$$i_t=\phi_t^{\pi}\pi_t+\phi_t^{x}x_t+\eta_t,\quad \forall t, \tag{A.1}$$

where its coefficients, $(\phi_t^{\pi},\phi_t^{x})$, and the monetary policy shock, η_t, are stochastic and their realized values in period t are assumed to be observed by the private sector at the beginning of period t.

In this setting, the only choice of the central bank is the message it sends to the private sector, and we need to alter the definition of a rational expectation equilibrium by including (A.1) as an equilibrium condition. Using an argument similar to the proof of Proposition 1, we can show that the equilibrium ex ante welfare loss is minimized with central bank secrecy.

A.2.2 When the Taylor rule is inertial

Bianchi and Melosi (2018) use this model with a Taylor rule that has a lagged nominal interest rate. In their model the Taylor rule coefficients change with a policy regime that switches between three regimes (one active, and two passive with different per-

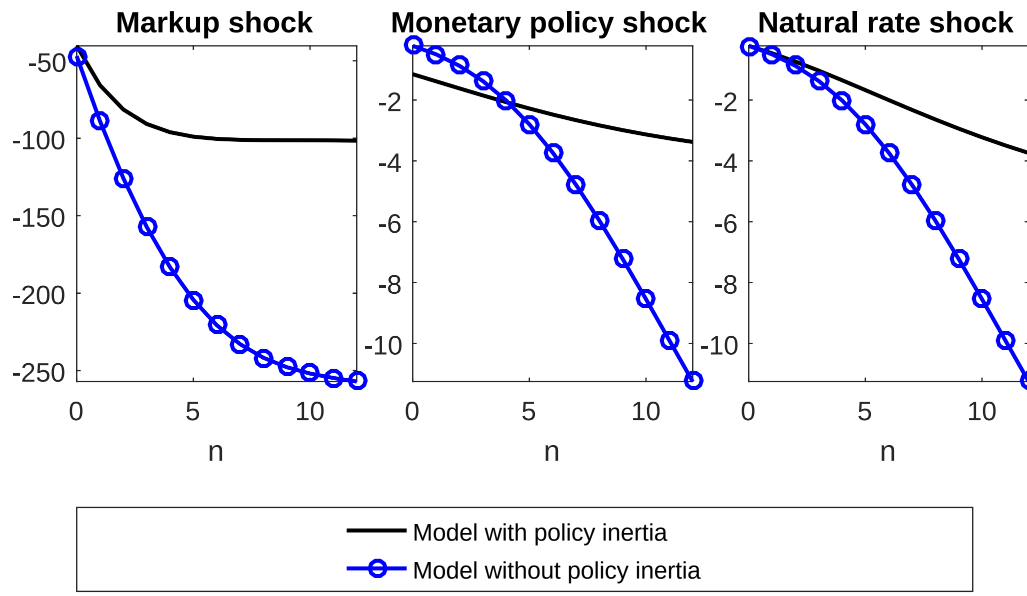

Figure 1: Ex Ante Welfare in the Three Equation Model

sistence). Under the no-transparency policy, the private sector is unable to distinguish between the two passive regimes and is unsure about the persistence of the current passive regime. Under the transparency policy, whenever the policy switches to a passive regime, the private sector is informed about the exact end date of the passive regime. This is a particular kind of communication about private news the central bank receives, and they argue that steady state welfare improves under transparency.

With policy inertia, the model has a backward-looking equation and our theoretical results cannot be applied directly. Instead, here we demonstrate numerically that policy inertia itself does not overturn the desirability of secrecy. The Taylor rule is

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_x x_t\right) + \eta_t, \quad 0 < \rho < 1.$$

Unlike Bianchi and Melosi (2018), we do not model time-varying coefficients. Structural parameters are the same as in Table 1 and we use $(\rho, \phi_\pi, \phi_x) = (0.6, 1.6, 0.3)$.

Figure 1 reports how the ex ante loss varies with n, for the mark-up shock, for the natural rate shock, and for the monetary policy shock, both with and without policy inertia. In all cases, welfare loss is increasing in n, and therefore revealing information about future shocks is detrimental to welfare. This pattern does not change with ρ, and higher values for ρ (e.g. $\rho = 0.9$) produce the same pattern. Although the model considered here is not identical to Bianchi and Melosi (2018), the above result suggests that the lagged nominal interest rate in the Taylor rule by itself does not have significant implications for the welfare consequences of information revelation. The welfare improving property of transparency in Bianchi and Melosi (2018) may depend crucially on their use of the steady-state welfare. Because we are interested in whether it is beneficial for the central bank to commit to secrecy, it is our view that ex ante expected loss is a more natural criterion.

A.3 Undesirability of information revelation without commitment

We first define an equilibrium under discretion *for a given information structure*. Although it is conventional to focus on a Markov perfect equilibrium when considering discretionary policy, we do not require a Markov property here.

Definition 1 *Let $\mathcal{G} = \{\mathcal{G}_t\}_{t=0}^{\infty}$ be a filtration with $\mathcal{G}^{MIN} \subset \mathcal{G}$. A $\mathcal{G}$-adapted stochastic process $\{(\pi_t, x_t)\}_{t=0}^{\infty}$ is a $\mathcal{G}$-discretionary policy equilibrium if and only if, for all t, (π_t, x_t) solves $\min_{(\pi,x)} L(\pi, x)$ subject to $\pi = \kappa x + \beta \mathbb{E}[\pi_{t+1}|\mathcal{G}_t] + u_t$.*[2]

The next proposition shows that, for any filtrations $\mathcal{F}$ and $\mathcal{G}$ that satisfy $\mathcal{G}^{MIN} \subset \mathcal{F} \subset \mathcal{G}$ and for any $\mathcal{G}$-discretionary policy equilibrium, we can find an $\mathcal{F}$-discretionary policy equilibrium that achieves (weakly) lower ex ante loss. In this sense, information revelation is undesirable even without commitment.

Proposition 1 *Suppose that L is quadratic: $L(\pi, x) = (\pi^2 + bx^2)/2$ with $b > 0$. Let $\mathcal{F}$ and $\mathcal{G}$ and be filtrations such that $\mathcal{G}_t^{MIN} \subset \mathcal{F}_t \subset \mathcal{G}_t$ for all t. Then the following holds.*

1. *For any $\mathcal{G}$-discretionary policy equilibrium $\{(\pi_t, x_t)\}_{t=0}^{\infty}$,there exists an $\mathcal{F}$-discretionary policy equilibrium $\{(\tilde{\pi}_t, \tilde{x}_t)\}_{t=0}^{\infty}$ such that $\mathbb{E}[L(\pi_t, x_t)|\mathcal{F}_t] \geq \mathbb{E}[L(\tilde{\pi}_t, \tilde{x}_t)|\mathcal{F}_t]$ (hence*

[2] Because L is continuous and strictly convex, a $\mathcal{G}$-discretionary policy equilibrium is $\mathcal{G}$-adapted.

$\mathbb{E}[L(\pi_t, x_t)] \geq \mathbb{E}[L(\tilde{\pi}_t, \tilde{x}_t)]$) *for all* t. *Equality holds if and only if* $\{(\pi_t, x_t)\}_{t=0}^{\infty} = \{(\tilde{\pi}_t, \tilde{x}_t)\}_{t=0}^{\infty}$ *almost everywhere.*

2. *Let* $\{(\pi_t^*, x_t^*)\}_{t=0}^{\infty}$ *be the best* $\mathcal{F}$*-discretionary policy equilibrium, i.e. it minimizes the loss among all* $\mathcal{F}$*-discretionary policy equilibria. If*

$$\textit{Probability of } \left\{\mathbb{E}[\pi_{t+1}^*|\mathcal{G}_t] \neq \mathbb{E}[\pi_{t+1}^*|\mathcal{F}_t] \textit{ for some } t\right\} > 0, \tag{A.2}$$

then the best $\mathcal{G}$*-discretionary policy equilibrium yields strictly larger minimized loss than* $\{(\pi_t^*, x_t^*)\}_{t=0}^{\infty}$.

Proof. [Proof of Proposition 1] Let $\{(\pi_t, x_t)\}_{t=0}^{\infty}$ be a $\mathcal{G}$-discretionary policy equilibrium. Then, for all t, it satisfies the first-order necessary and sufficient condition for the problem $\min_{\pi,x} L(\pi, x)$ subject to $\pi = \kappa x + \beta\mathbb{E}[\pi_{t+1}|\mathcal{G}_t] + u_t$, which is summarized by

$$(\pi_t, x_t) = \left(\frac{b/\kappa}{\kappa + b/\kappa}\{\beta\mathbb{E}[\pi_{t+1}|\mathcal{G}_t] + u_t\}, -\frac{1}{\kappa + b/\kappa}\{\beta\mathbb{E}[\pi_{t+1}|\mathcal{G}_t] + u_t\}\right).$$

Define $\{(\tilde{\pi}_t, \tilde{x}_t)\}_{t=0}^{\infty}$ as in Proposition 1. Then it satisfies

$$(\tilde{\pi}_t, \tilde{x}_t) = \left(\frac{b/\kappa}{\kappa + b/\kappa}\{\beta\mathbb{E}[\tilde{\pi}_{t+1}|\mathcal{F}_t] + u_t\}, -\frac{1}{\kappa + b/\kappa}\{\beta\mathbb{E}[\tilde{\pi}_{t+1}|\mathcal{F}_t] + u_t\}\right),$$

implying that $\{(\tilde{\pi}_t, \tilde{x}_t)\}_{t=0}^{\infty}$ is a $\mathcal{F}$-discretionary policy equilibrium. It follows from Jensen's inequality that $\mathbb{E}[L(\pi_t, x_t)] \geq \mathbb{E}[L(\tilde{\pi}_t, \tilde{x}_t)]$ for all t. Because L is quadratic, equality holds if and only if $\{(\pi_t, x_t)\}_{t=0}^{\infty} = \{(\tilde{\pi}_t, \tilde{x}_t)\}_{t=0}^{\infty}$ almost everywhere. This proves the part 1. The proof of the part 2 is the same as that of Proposition 1 and thus is omitted. ∎

A.4 A canonical DSGE model

A.4.1 The representative household

The representative household's expected utility at the beginning of period 0 is:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t - X_t\right) - v\left(L_t\right)\right].$$

where C_t is consumption, L_t is labor, and X_t is external habit that equals bC_{t-1} $(b \geq 0)$ in equilibrium.

The household accumulates capital according to the capital accumulation equation:

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(I_t, I_{t-1}\right)\right] I_t,$$

where I_t is investment and K_t is capital. The function S represents investment adjustment costs. The flow budget constraint is given by:

$$A_{t+1} + P_t\left(C_t + I_t\right) \leq \tilde{W}_t L_t + R_{t-1} A_t + P_t r_t^K K_t + \text{profits and transfers},$$

where A_t is the nominal risk-free bond, P_t the nominal price index, $\tilde{W}_t$ the nominal wage, R_t the gross nominal interest rate, and r_t^K the rental rate of capital, respectively. Although it is not included in the equation, the household also trades a complete set of state-contingent claims.

We assume the following functional forms:

$$\begin{aligned} u\left(C_t - X_t\right) &= \frac{\left(C_t - X_t\right)^{1-\sigma}}{1-\sigma}, \\ v\left(L_t\right) &= \frac{L_t^{1+\eta}}{1+\eta}, \\ S\left(I_t, I_{t-1}\right) &= \frac{s}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2. \end{aligned}$$

A.4.2 Labor union

We model sticky nominal wages following Smets and Wouters (2007). There is a unit measure of labor unions, each of which is indexed by $l \in [0,1]$. A union l collects the homogeneous labor supplied from households and transforms it into a differentiated labor good indexed by l using a linear technology. It faces a downward-sloping demand curve for its labor good and chooses the nominal wage subject to the Calvo-style probability. Firms combine differentiated labor goods into a single, composite labor using a CES aggregator $h_t = [\int_0^1 h_t(l)^{\frac{\epsilon_h - 1}{\epsilon_h}} dl]^{\frac{\epsilon_h}{\epsilon_h - 1}}$. The nominal wage of this composite labor good is denoted by W_t.

When the union l is given an opportunity to change its wage in period t, it solves

$$\max_{W_t^*} \mathbb{E}_t \sum_{s=t}^{\infty} \theta_h^{s-t} m_{t,s} \left[W_s(l) - \frac{\tilde{W}_s}{1+\tau_s^h} \right] h_s(l),$$

subject to

$$\begin{aligned} h_s(l) &= \left[\frac{W_s(l)}{W_s} \right]^{-\varepsilon_h} h_s, \forall s \geq t, \\ W_{t+i}(l) &= W_t(l) \prod_{n=1}^{i} \Pi_{t+n-1}^{\gamma_h}, \forall i \geq 1, \\ W_t(l) &= W_t^*. \end{aligned}$$

Here, $m_{t,s}$ denotes the stochastic discount factor for nominal payoffs, τ_t^h the subsidy to the union, and $\Pi_t = P_t/P_{t-1}$ the gross consumer price inflation rates, respectively. θ_h, ε_h andγ_h are the Calvo parameter for staggered nominal wages, elasticity of substitution among differentiated labor, and the degree of nominal wage indexation on past inflation rates.

A.4.3 Intermediate goods producer

The sticky price friction is modeled in a standard fashion. When the intermediate-goods producer f is given an opportunity to change its price in period t, it solves:

$$\max_{P_t^*} \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} m_{t,s} \left[(1+\tau_s) P_s(f) - P_s MC_s \right] Y_t(f),$$

subject to

$$\begin{aligned} Y_s(f) &= \left[\frac{P_s(f)}{P_s} \right]^{-\varepsilon} Y_s, \\ P_{t+i}(f) &= P_t(f) \prod_{n=1}^{i} \Pi_{t+n-1}^{\gamma}, \forall i \geq 1, \\ P_t(f) &= P_t^*. \end{aligned}$$

The sales subsidy is denoted by τ_t. γ is the degree of price indexation on past inflation rates. Real marginal cost MC_t is the Lagrange multiplier in the cost minimization problem:

$$\min_{h_t, K_t} \frac{W_t}{P_t} h_t + r_t^K K_t,$$

subject to the production technology:

$$Y_t = K_t^{\alpha} \left[\exp(z_t) h_t \right]^{1-\alpha},$$

where z_t denotes the aggregate technology shock.

A.4.4 Final good producer

A final good producer minimizes the total cost $\int_0^1 P_t(f) Y_t(f) \mathrm{d}f$ subject to the aggregating technology:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}f \right]^{\frac{\epsilon}{\epsilon-1}}.$$

A.4.5 Monetary policy

The central bank set nominal interest rates following the Taylor type rule:

$$R_t - 1 = \rho(R_{t-1} - 1) + (1 - \rho)\left[\phi^{\pi}\left(\Pi_t - 1\right) + \phi^{y}\left(\frac{Y_t}{Y_{t-1}} - 1\right)\right] + \eta_t,$$

where ρ denotes the degree of policy inertia.

A.4.6 System of equations

We have 19 equations for 19 endogenous variables: Y_t, λ_t, Π_t, w_t, q_t, r_t^K, I_t, MC_t, K_t, $\bar{F}_t$, $\bar{K}_t$, C_t, Δ_t, $\Pi_{W,t}$, $\bar{F}_t^h$, $\bar{K}_t^h$, Δ_t^h, and R_t.

$$
\begin{aligned}
K_{t+1} &= (1-\delta)K_t + \left\{1 - \frac{s}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right\} I_t, \\
Y_t &= C_t + I_t, \\
\lambda_t &= \left(C_t - bC_{t-1}\right)^{-\sigma}, \\
\lambda_t &= \beta \mathbb{E}_t \frac{R_t}{\Pi_{t+1}} \lambda_{t+1}, \\
q_t &= \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[r_{t+1}^K + q_{t+1}(1-\delta)\right], \\
1 &= q_t \left\{1 - \frac{s}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - s\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}}\right\} + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} s \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2, \\
w_t &= (1-\alpha)\exp\left(z_t\right)^{1-\alpha} MC_t K_t^{\alpha} h_t^{-\alpha}, \\
r_t^K &= \alpha \exp\left(z_t\right)^{1-\alpha} MC_t K_t^{\alpha-1} h_t^{1-\alpha}, \\
\bar{F}_t &= 1 + \theta\beta\mathbb{E}_t \frac{\lambda_{t+1}Y_{t+1}}{\lambda_t Y_t}\pi_{t+1}^{\varepsilon}\pi_t^{\gamma}\bar{F}_{t+1}, \\
\bar{K}_t &= \exp\left(u_t\right) MC_t + \theta\beta\mathbb{E}_t \frac{\lambda_{t+1}Y_{t+1}}{\lambda_t Y_t}\pi_{t+1}^{1+\varepsilon}\bar{K}_{t+1}, \\
\bar{K}_t &= \left[\frac{1 - \theta\left(\frac{\pi_{t-1}^{\gamma}}{\pi_t}\right)^{1-\varepsilon}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \bar{F}_t, \\
\Delta_t &= (1-\theta)\left[\frac{1 - \theta\left(\frac{\pi_{t-1}^{\gamma}}{\pi_t}\right)^{1-\varepsilon}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} + \theta\left(\frac{\pi_t}{\pi_{t-1}^{\gamma}}\right)^{\varepsilon}\Delta_{t-1}, \\
\Delta_t Y_t &= K_t^{\alpha}\left[\exp\left(z_t\right) h_t\right]^{1-\alpha}, \\
\Pi_t^w &= \Pi_t w_t / w_{t-1}, \\
\bar{F}_t^h &= \lambda_t w_t h_t + \beta\theta_h \mathbb{E}_t\left[\bar{F}_{t+1}^h (\Pi_{t+1}^W)^{\epsilon_h - 1}\right]\Pi_t^{(1-\epsilon_h)\gamma_h}, \\
\bar{K}_t^h &= \exp\left(\mu_t\right) h_t^{1+\eta}(\Delta_t^h)^{\eta} + \beta\theta_h \mathbb{E}_t\left[\bar{K}_{t+1}^h (\Pi_{t+1}^W)^{\epsilon_h}\right]\Pi_t^{-\epsilon_h\gamma_h}, \\
\bar{K}_t^h &= \left[\frac{1 - \theta_h\left(\frac{\pi_{t-1}^{\gamma_h}}{\pi_{W,t}}\right)^{1-\varepsilon_h}}{1-\theta_h}\right]^{\frac{1}{1-\varepsilon_h}} \bar{F}_t^h,
\end{aligned}
$$

$$
\begin{aligned}
\Delta_t^h &= (1-\theta_h)\left[\frac{1-\theta_h\left(\frac{\pi_{t-1}^{\gamma_h}}{\pi_{W,t}}\right)^{1-\varepsilon_h}}{1-\theta_h}\right]^{\frac{\varepsilon_h}{\varepsilon_h-1}} + \theta_h\left(\frac{\pi_{W,t}}{\pi_{t-1}^{\gamma_h}}\right)^{\varepsilon_h}\Delta_{t-1}^h, \\
R_t - 1 &= \rho(R_{t-1}-1) + (1-\rho)\left[\phi^\pi(\pi_t - 1) + \phi^y\left(\frac{Y_t}{Y_{t-1}} - 1\right)\right] + \eta_t.
\end{aligned}
$$

Here Δ_t and Δ_t^h denote relative price dispersion terms for prices and wages, defined as

$$
\begin{aligned}
\Delta_t &:= \int_0^1 \left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} \mathrm{d}f, \\
\Delta_t^h &:= \int_0^1 \left[\frac{W_t(l)}{W_t}\right]^{-\varepsilon_h} \mathrm{d}l.
\end{aligned}
$$

$\bar{F}_t$, $\bar{K}_t$, $\bar{F}_t^h$, and $\bar{K}_t^h$ are auxiliary variables. Price and wage markup shocks are defined as

$$
\begin{aligned}
\exp(u_t) &:= \frac{\varepsilon}{(1+\tau_t)(\varepsilon-1)}, \\
\exp(\mu_t) &:= \frac{\varepsilon_h}{\left(1+\tau_t^h\right)(\varepsilon_h-1)}.
\end{aligned}
$$

All shocks are assumed to follow AR(1) processes:

$$
\begin{aligned}
z_t &= \rho_z z_{t-1} + \omega_{z,t}, \\
u_t &= \rho_u u_{t-1} + \omega_{u,t}, \\
\mu_t &= \rho_\mu \mu_{t-1} + \omega_{\mu,t}, \\
\eta_t &= \rho_\eta \eta_{t-1} + \omega_{\eta,t},
\end{aligned}
$$

where

$$\begin{aligned}
\omega_{z,t} &\sim N\left(0, \sigma_z^2\right), \\
\omega_{u,t} &\sim N\left(0, \sigma_u^2\right), \\
\omega_{\mu,t} &\sim N\left(0, \sigma_\mu^2\right), \\
\omega_{\eta,t} &\sim N\left(0, \sigma_\eta^2\right).
\end{aligned}$$

A.4.7 Welfare cost

Let $\{(C_t^n, L_t^n)\}$ be the equilibrium consumption and labor in an economy where the agents observe n-period ahead shocks. Ex ante welfare of the household is

$$V^n = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \{u(C_t^n - bC_{t-1}^n) - v(L_t^n)\}\right].$$

For each n, we measure welfare gain/loss in consumption unit defined as

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \{u((1+\lambda_n)C_t^0 - b(1+\lambda_n)C_{t-1}^0) - v(L_t^0)\}\right] = V^n.$$

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