First step

In[1]:=
```
Clear[all];
```
очистить

```
uu = List[-u*, +u*];
```
список

In[3]:=
```
sol1 =
 Flatten[Simplify[DSolve[{Θ'[x] == ω[x], ω'[x] == m[x] + Θ[x], m'[x] == uu[[1]], Θ[0] == 1,
```
уплостить | упростить | решить дифференциальные уравнения

```
    ω[0] == Ω0, m[0] == 0}, {Θ[x], ω[x], m[x]}, x], Element[x, Reals]]]
```
принадлежи… | множество действительных чи

Out[3]=
$$\left\{m[x] \to -x\, u_*,\ \Theta[x] \to \frac{1}{2} e^{-x}\left(1 + e^{2x} + \left(-1 + e^{2x}\right)\Omega_0 + \left(1 - e^{2x} + 2\, e^{x}\, x\right) u_*\right),\right.$$
$$\left.\omega[x] \to \frac{1}{2} e^{-x}\left(\left(1 + e^{2x}\right)\Omega_0 - \left(-1 + e^{x}\right)\left(-1 - e^{x} + \left(-1 + e^{x}\right) u_*\right)\right)\right\}$$

In[4]:=
```
linkingStage1 =
 {m[x] /. sol1[[1]] /. x → τ1, Θ[x] /. sol1[[2]] /. x → τ1, ω[x] /. sol1[[3]] /. x → τ1}
```

Out[4]=
$$\left\{-\tau_1\, u_*,\ \frac{1}{2} e^{-\tau_1}\left(1 + e^{2\tau_1} + \left(-1 + e^{2\tau_1}\right)\Omega_0 + \left(1 - e^{2\tau_1} + 2\, e^{\tau_1}\, \tau_1\right) u_*\right),\right.$$
$$\left.\frac{1}{2} e^{-\tau_1}\left(\left(1 + e^{2\tau_1}\right)\Omega_0 - \left(-1 + e^{\tau_1}\right)\left(-1 - e^{\tau_1} + \left(-1 + e^{\tau_1}\right) u_*\right)\right)\right\}$$

In[5]:=
```
sol2 = Simplify[DSolve[
```
упростить | решить дифференциальные уравнения

```
    {Θ'[x] == ω[x], ω'[x] == m[x] + Θ[x], m'[x] == uu[[2]], Θ[τf] == 0, ω[τf] == 0, m[τf] == 0},
    {Θ[x], ω[x], m[x]}, x], Element[{τ1, τ2, τf, x}, Reals]] // Flatten
```
принадлежит множеству | множество … | уплостить

Out[5]=
$$\left\{m[x] \to (x - \tau_f)\, u_*,\ \Theta[x] \to \frac{1}{2}\left(e^{x-\tau_f} - e^{-x+\tau_f} - 2x + 2\tau_f\right) u_*,\ \omega[x] \to \frac{1}{2} e^{-x-\tau_f}\left(e^{x} - e^{\tau_f}\right)^2 u_*\right\}$$

In[6]:=
```
linkingStage2 = {m[x] /. sol2[[1]] /. x → τ1,
  Θ[x] /. sol2[[2]] /. x → τ1 // Simplify, ω[x] /. sol2[[3]] /. x → τ1 // Simplify}
```
упростить | упростить

Out[6]=
$$\left\{(\tau_1 - \tau_f)\, u_*,\ \frac{1}{2}\left(e^{\tau_1-\tau_f} - e^{-\tau_1+\tau_f} - 2\tau_1 + 2\tau_f\right) u_*,\ \frac{1}{2} e^{-\tau_1-\tau_f}\left(e^{\tau_1} - e^{\tau_f}\right)^2 u_*\right\}$$

Full system

In[7]:=
```
first = (linkingStage1[[1]] - linkingStage2[[1]] == 0 // FullSimplify)
```
упростить в полном

```
second = (linkingStage1[[2]] - linkingStage2[[2]] == 0 // Simplify)
```
упростить

```
third = (linkingStage1[[3]] - linkingStage2[[3]] == 0 // Simplify)
```
упростить

Out[7]=
$$2\,\tau_1\, u_* == \tau_f\, u_*$$

Out[8]=
$$e^{-\tau_1} + e^{\tau_1} + \left(-e^{-\tau_1} + e^{\tau_1}\right)\Omega_0 + \left(e^{-\tau_1-\tau_f}\left(1 + e^{\tau_f}\right)\left(-e^{2\tau_1} + e^{\tau_f}\right) + 4\,\tau_1 - 2\,\tau_f\right) u_* == 0$$

Out[9]=
$$e^{-\tau_1} + \left(-4 + e^{-\tau_1} + e^{\tau_1} + e^{\tau_1-\tau_f} + e^{-\tau_1+\tau_f}\right) u_* == e^{\tau_1} + \left(e^{-\tau_1} + e^{\tau_1}\right)\Omega_0$$

Make variable replace $e^{\tau_1}=x$; $e^{\tau_f} = z$

In[10]:=
```
first1 = (x^2 == z)

Collect[second /. {e^τ1 → x, e^τf → z, e^-τ1 → 1/x, e^-3 τ1 → 1/x^3, τf → 2 τ1} // Simplify,
 {e^τf, e^τ1}, FullSimplify]

Collect[third /. {e^τ1 → x, e^τf → z, e^-τ1 → 1/x, e^-3 τ1 → 1/x^3, τf → 2 τ1} // Simplify,
 {e^τf, e^τ1}, FullSimplify]
```

Out[10]= $x^2 == z$

Out[11]= $\frac{1 + x^2 + \left(-1 + x^2\right) \Omega_0}{x} - e^{-\tau_1} (1 + z) u_* + e^{-3\tau_1} z (1 + z) u_* == 0$

Out[12]= $e^{-\tau_1} u_* + e^{\tau_1} u_* + \frac{1 + u_* + (-4 + x)\, x\, u_*}{x} == x + \left(\frac{1}{x} + x\right) \Omega_0$

In[13]:=
```
second1 = ((1 + x^2 + (-1 + x^2) Ω0)/x - 1/x (1 + z) u* + 1/x^3 z (1 + z) u* == 0) // Simplify

third1 = (1/x u* + x * u* + (1 + u* + (-4 + x) x u*)/x == x + (1/x + x) Ω0) // Simplify
```

Out[13]= $x + x^3 + x \left(-1 + x^2\right) \Omega_0 == \frac{\left(x^2 - z\right) (1 + z) u_*}{x}$

Out[14]= $\frac{1 + 2 (-1 + x)^2 u_*}{x} == x + \left(\frac{1}{x} + x\right) \Omega_0$

In[15]:=
```
Together[Collect[second1, {x}]]

Together[Collect[third1, {z}]]
```

Out[15]= $x + x^3 - x\, \Omega_0 + x^3\, \Omega_0 == \frac{x^2 u_* - z u_* + x^2 z u_* - z^2 u_*}{x}$

Out[16]= $\frac{1 + 2 u_* - 4 x u_* + 2 x^2 u_*}{x} == \frac{x^2 + \Omega_0 + x^2 \Omega_0}{x}$

In[17]:=
```
t* = 0.174; ω0 = 0.149; ϕ0 = 0.021; u* = +0.63; Ω0 = t*/ϕ0 ω0;

Print["Ω0=", Ω0];
```

Ω0=1.23457

In[19]:=
```
Solve[third1 && second1, {x, z}, Reals]
```
решить уравнения | множество

```
NSolve[third1 && second1, {x, z}, Reals]
```
численное решение уравнений | множести

Solve : Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[19]= $\{\{x \to 0.357491, z \to -0.99075\}, \{x \to 0.357491, z \to 0.11855\}\}$

Out[20]= $\{\{x \to 0.357491, z \to -0.99075\}, \{x \to 0.357491, z \to 0.11855\}\}$