Gravity Simulation

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Problem Overview

The Problem

Integrate Newtonian equations of motion

$$F = \frac{m_1 m_2}{r^2}$$

$$F = ma$$

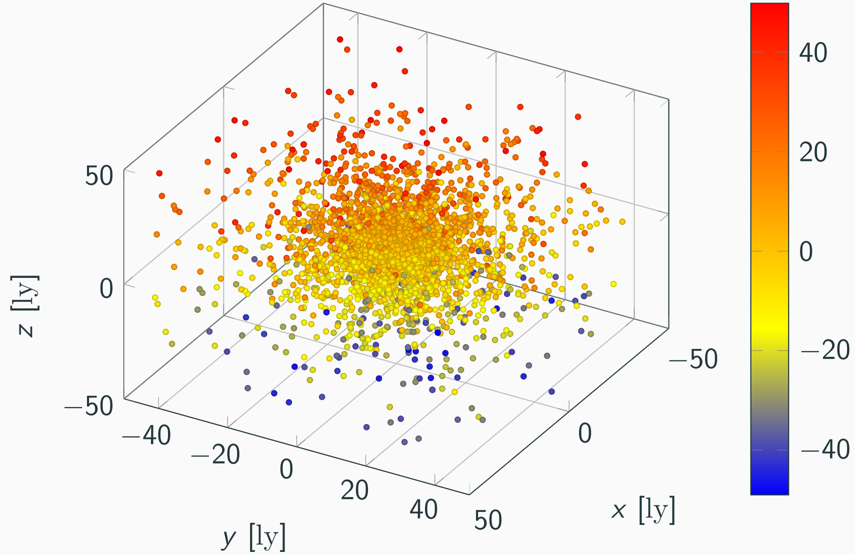
System of units

Constant	Dimensions	Value Conversion
Mass	M	$1[M] = 1M_{\odot} = 1.99 \cdot 10^{30}\text{kg}$
Length	L	$1[L] = 100\text{ly} = 9.46 \cdot 10^{17}\text{m}$
Time	T	$1[T] = 2.53\text{Gyr} = 7.99 \cdot 10^{16}\text{s}$
G	$M^{-1}L^{3}T^{-2}$	$1\text{G} = 1\frac{[L]^3}{[M]\cdot[T]^2} = 6.67 \cdot 10^{-11}\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$
Velocity	LT^{-1}	$1\frac{[L]}{[T]} = 11.85\frac{\text{m}}{\text{s}}$
Acceleration	LT^{-2}	$1\frac{[L]}{[T]^2} = 1.48 \cdot 10^{-16}\frac{\text{m}}{\text{s}^2}$
Force	MLT^{-2}	$1\frac{[M]\cdot[L]}{[T]^2} = 2.95 \cdot 10^{14}\text{N}$

The Dataset

- $N = 50000$
- $M = 5 \cdot 10^6 M_\odot$
- $R(< 0.99M) = 1400\text{ly}$
- $R(< 0.5M) = 19\text{ly}$

The Dataset - Visualization



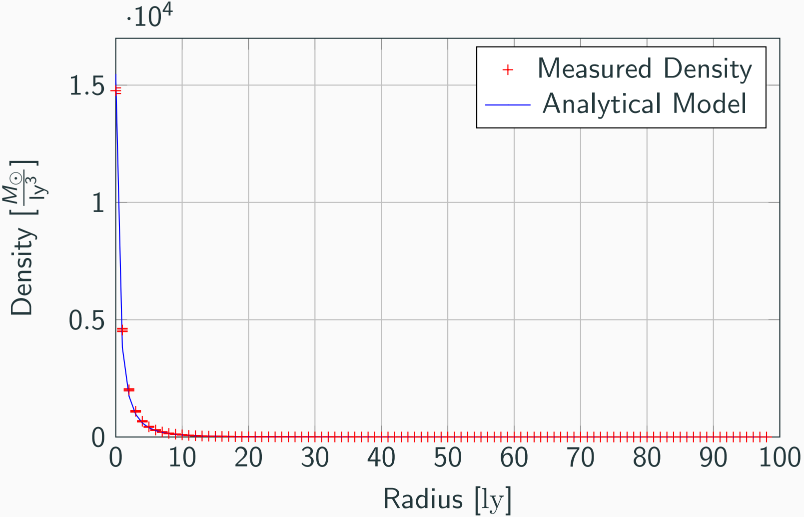
The Dataset - Density Function

A model for spherical galaxies and bulges, HERNQUIST (1990):

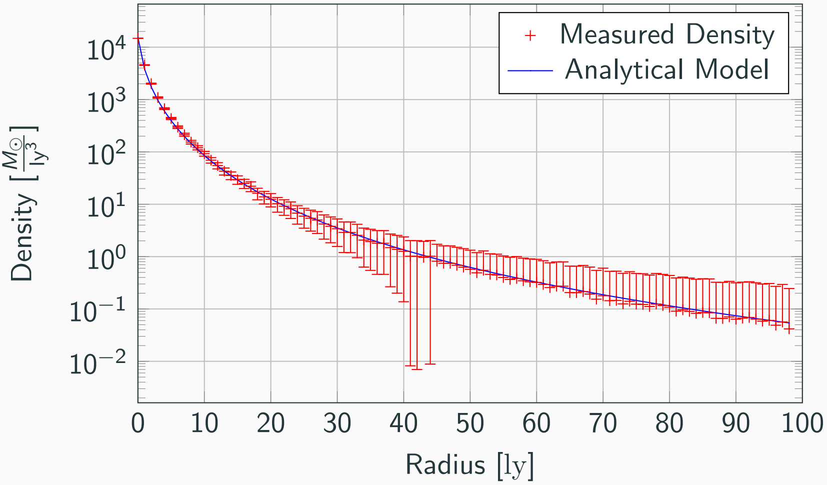
$$\rho(r) = \frac{M}{2\pi} \frac{1}{(r+a)^3}$$

with $a = 0.09$

The Dataset - Density Function



The Dataset - Density Function



The Dataset - Kepler Orbits

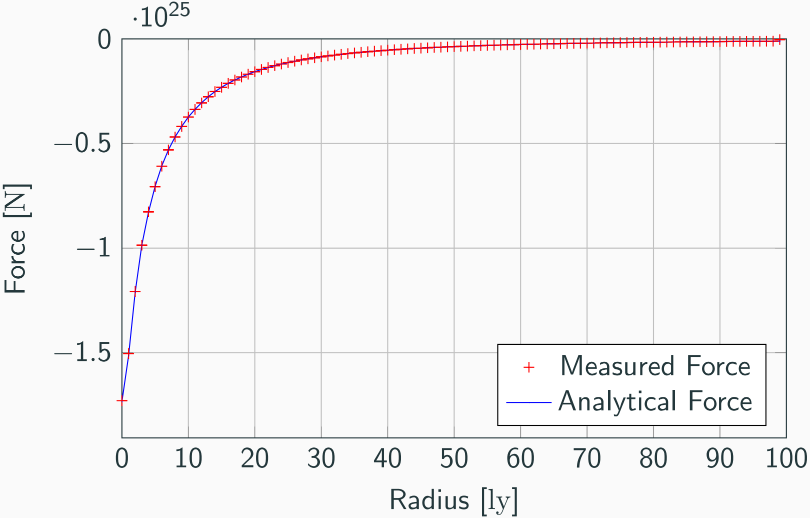
Force of a spherically symmetric distribution:

$$F_p = M_{r<r_p} \frac{m_p}{r_p^2}$$

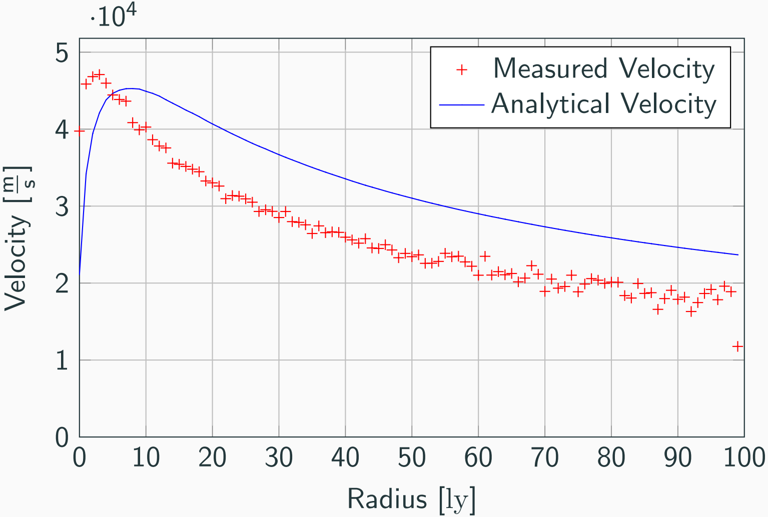
Orbital velocity for a circular orbit:

$$v(p) = \sqrt{\frac{M_{r<r_p}}{r_p}}$$

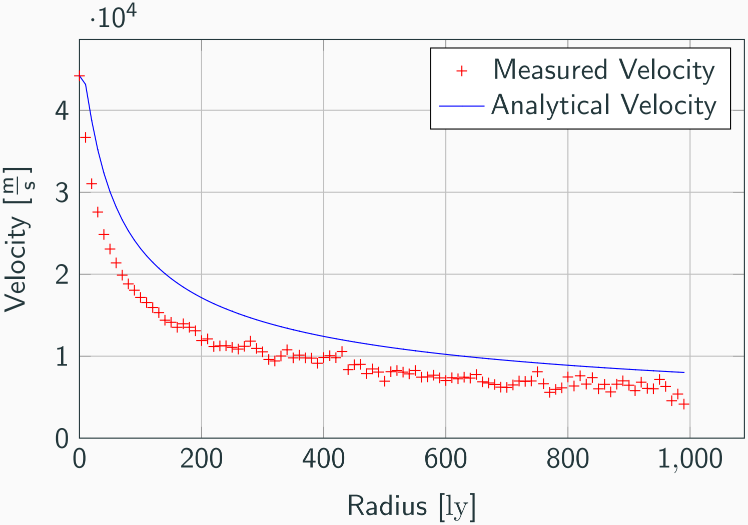
The Dataset - Kepler Orbits



The Dataset - Kepler Orbits



The Dataset - Kepler Orbits



The Dataset - Relaxation Time

Relaxation time is defined as

$$t_{\text{relax}} = n_{\text{relax}} t_{\text{cross}} \approx \frac{0.1N}{\ln N} \cdot 2\pi \sqrt{\frac{2r^3_{M_r<0.5M_{\text{tot}}}}{M_{tot}}}$$

With $N = 50000$:

$$t_{\text{relax}} = 397\text{Myr}$$

The Dataset - Relaxation Time

Not every particle is a single star!

With $N = 100 \cdot 50000$:

$$t_{\mathrm{relax}} = 27.9\mathrm{Gyr}$$

With $N = 1000 \cdot 50000$:

$$t_{\mathrm{relax}} = 242\mathrm{Gyr}$$

Non-Collisional System

Add softening of interparticle distance $\epsilon = 0.2\text{ly}$.

$$\Phi(p_a) = \sum_{b \neq a} m_b S(|\mathbf{x}_b - \mathbf{x}_a|)$$

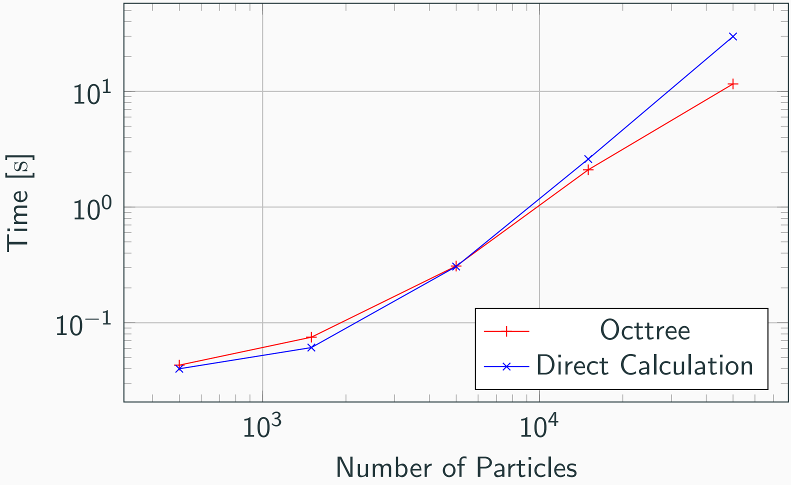
$$S(r) = -\frac{1}{\sqrt{r^2 + \epsilon^2}}$$

$$F(p_a) = m_a \sum_{b \neq a} m_b S_F(|\mathbf{x}_b - \mathbf{x}_a|) \frac{\mathbf{x}_b - \mathbf{x}_a}{|\mathbf{x}_b - \mathbf{x}_a|}$$

$$S_F(r) = \frac{\mathrm{d}}{\mathrm{d}r} S(r) = \frac{r}{(r^2 + \epsilon^2)^{\frac{3}{2}}}$$

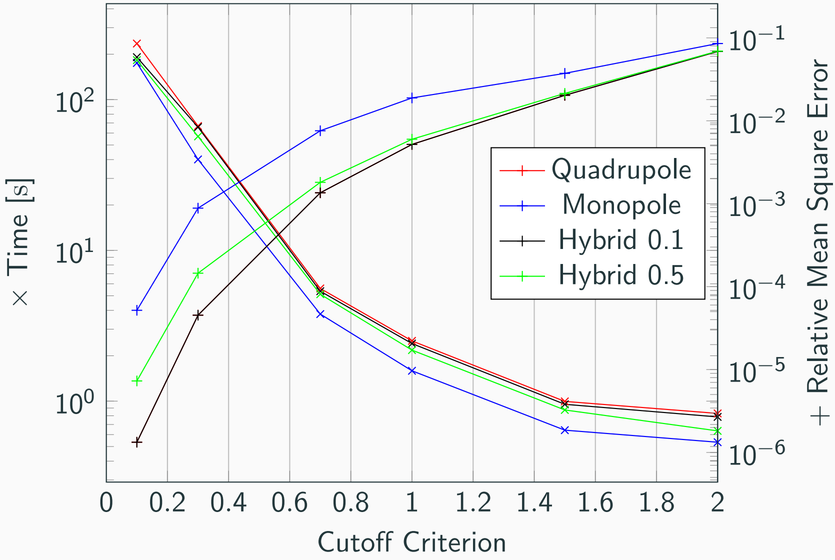
Performance Optimization

Octtree



Time [s]
10^{1}
10^{0}
10^{-1}
10^{3}
10^{4}
Number of Particles
Octtree
Direct Calculation

Octtree tuning



General Optimizations

- Single Precision Floating Point -**26%**
- Unsafe Operations -**75%**
- Vectorization -**7%**
- **Total -83%**

Time integration

Integration Scheme

Leapfrog Kick-Drift-Kick

1. $v_{i+\frac{1}{2}} = v_i + a_i \frac{\Delta t}{2}$
2. $x_{i+1} = x_i + v_{i+\frac{1}{2}} \Delta t$
3. $v_{i+1} = v_{i+\frac{1}{2}} + a_{i+1} \frac{\Delta t}{2}$

Integration sequence for time T

1. Calculate target $\Delta t = C \sqrt{\frac{1}{a_i}}$
2. If target $\Delta t < T$, integrate twice recursively over $\frac{T}{2}$
3. Otherwise calculate x_{i+1}, a_{i+1}, v_{i+1}

Stability