

Global Constraint Catalog

[PDF](#)[PL](#)[PNG](#)[XML](#)

5.318. peak

[DESCRIPTION](#) [LINKS](#) [AUTOMATON](#)

Origin

Derived from [inflexion](#).

Constraint

`peak(N, VARIABLES)`

Arguments

`N` `dvar`
`VARIABLES` `collection(var – dvar)`

Restrictions

$N \geq 0$
 $2 * N \leq \max(|VARIABLES| - 1, 0)$
[required](#)(`VARIABLES`, `var`)

Purpose

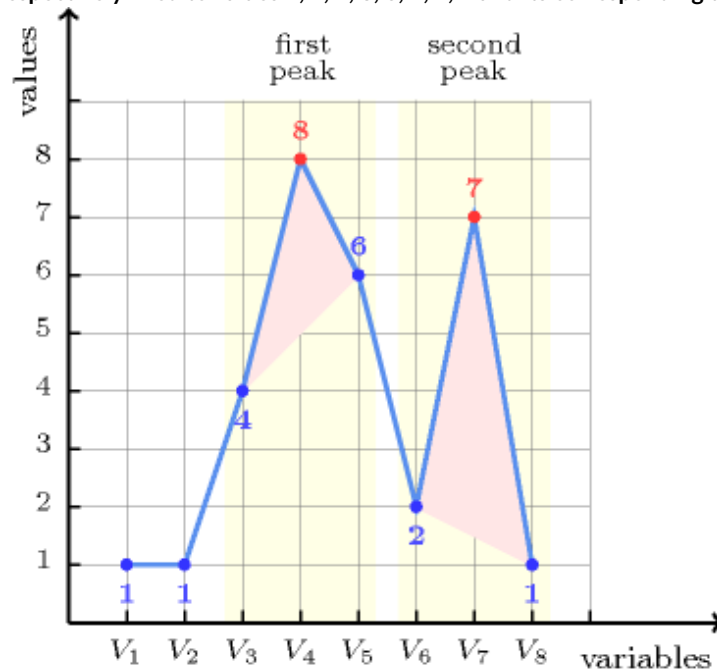
A variable V_k ($1 < k < m$) of the sequence of variables $VARIABLES = V_1, \dots, V_m$ is a *peak* if and only if there exists an i (with $1 < i \leq k$) such that $V_{i-1} < V_i$ and $V_i = V_{i+1} = \dots = V_k$ and $V_k > V_{k+1}$. N is the total number of peaks of the sequence of variables $VARIABLES$.

Example

(2, $\langle 1, 1, 4, 8, 6, 2, 7, 1 \rangle$)
 (0, $\langle 1, 1, 4, 4, 4, 6, 7, 7 \rangle$)
 (4, $\langle 1, 5, 4, 9, 4, 6, 2, 7, 6 \rangle$)

The first peak constraint holds since the sequence 11486271 contains two peaks that respectively correspond to the variables that are assigned to values 8 and 7.

Figure 5.318.1. Illustration of the first example of the Example slot: a sequence of eight variables $V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8$ respectively fixed to values 1, 1, 4, 8, 6, 2, 7, 1 and its corresponding two peaks ($N = 2$)



All solutions

Figure 5.318.2 gives all solutions to the following non ground instance of the peak constraint: $N \in [1, 2]$, $V_1 \in [1, 2]$, $V_2 = 2$, $V_3 \in [1, 2]$, $V_4 \in [1, 2]$, $V_5 \in [2, 3]$, $\text{peak}(N, \langle V_1, V_2, V_3, V_4, V_5 \rangle)$.

Figure 5.318.2. All solutions corresponding to the non ground example of the peak constraint of the All solutions slot where each peak is coloured in orange



Typical

$|\text{VARIABLES}| > 2$
 $\text{range}(\text{VARIABLES.var}) > 1$

Symmetries

- Items of **VARIABLES** can be **reversed**.
- One and the same constant can be **added** to the **var** attribute of all items of **VARIABLES**.

Arg. properties

- **Functional dependency:** N determined by $VARIABLES$.
- **Contractible** wrt. $VARIABLES$ when $N = 0$.

Usage

Useful for constraining the number of *peaks* of a sequence of domain variables.

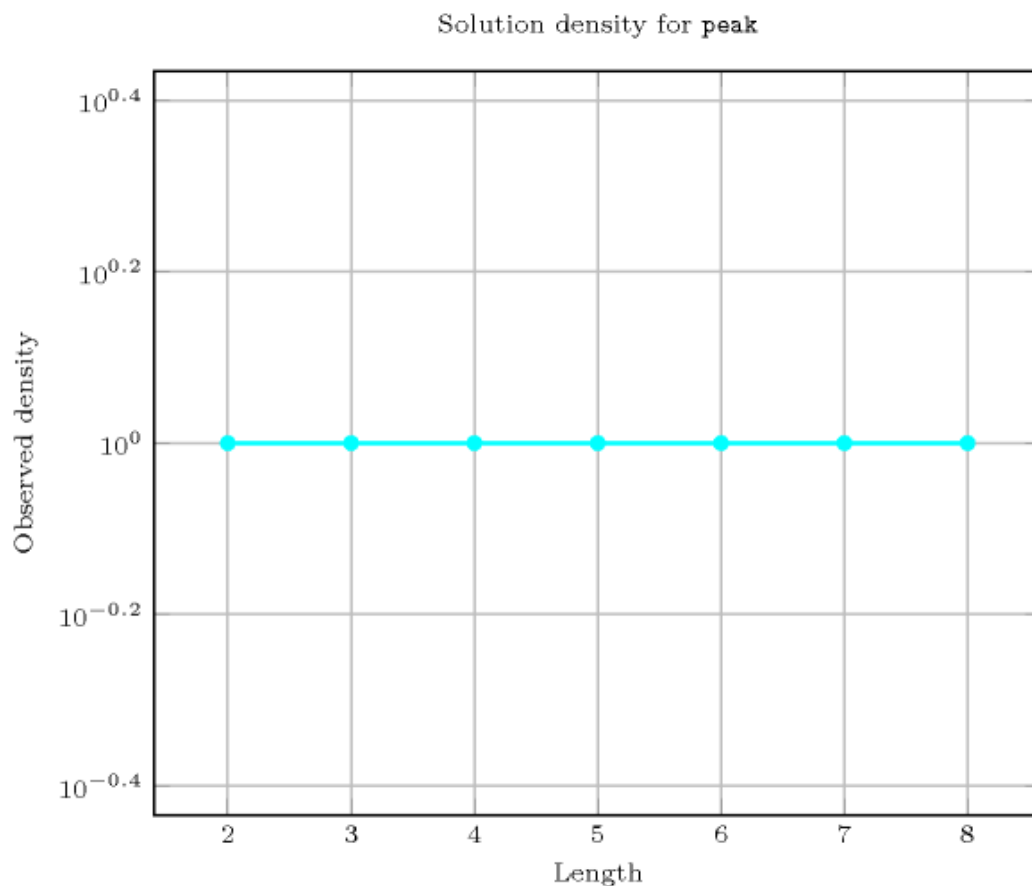
Remark

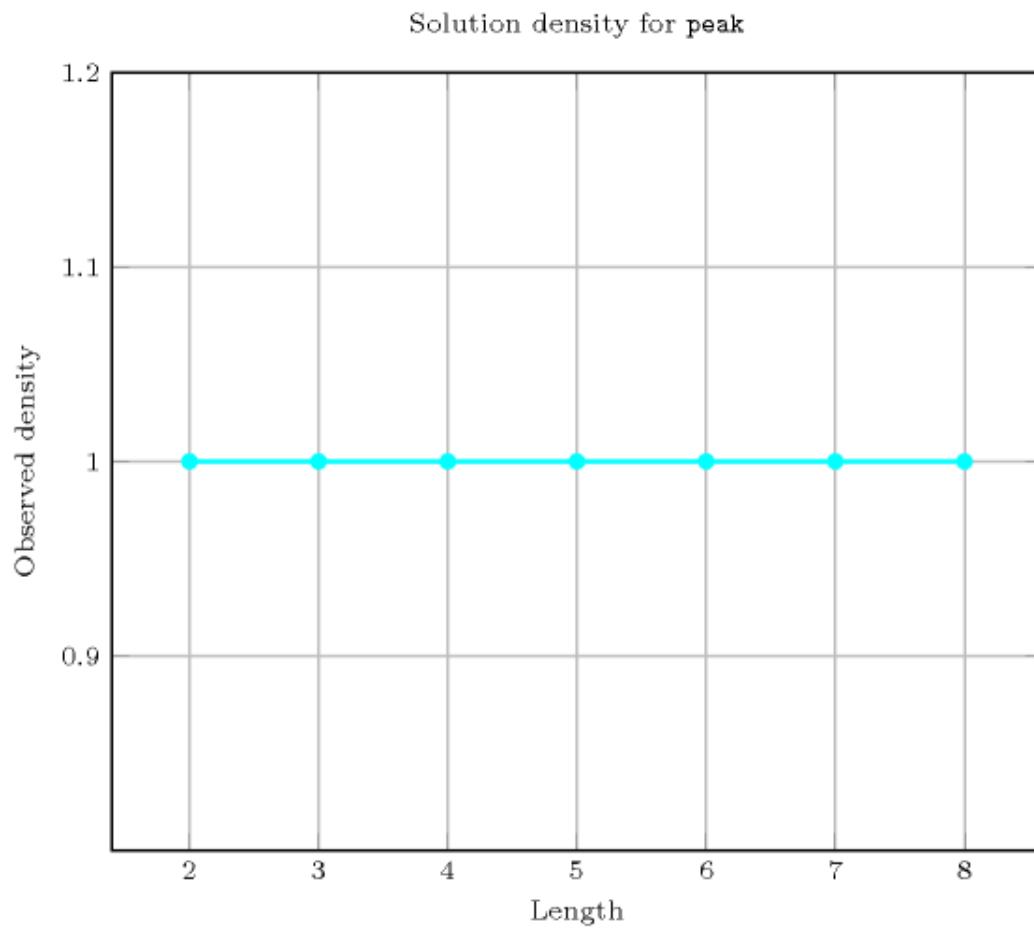
Since the arity of the arc constraint is not fixed, the *peak* constraint cannot be currently described with the graph-based representation. However, this would not hold anymore if we were introducing a slot that specifies how to merge adjacent vertices of the final graph.

Counting

Length (n)	2	3	4	5	6	7	8
Solutions	9	64	625	7776	117649	2097152	43046721

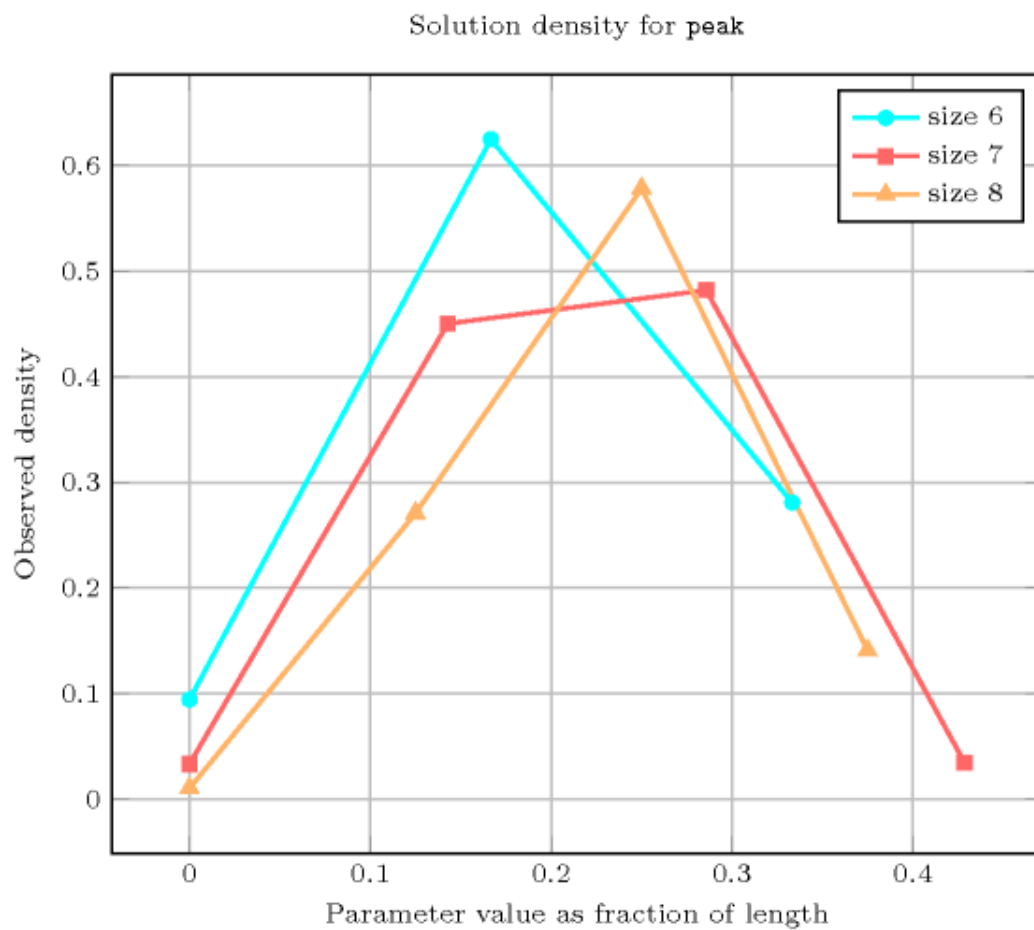
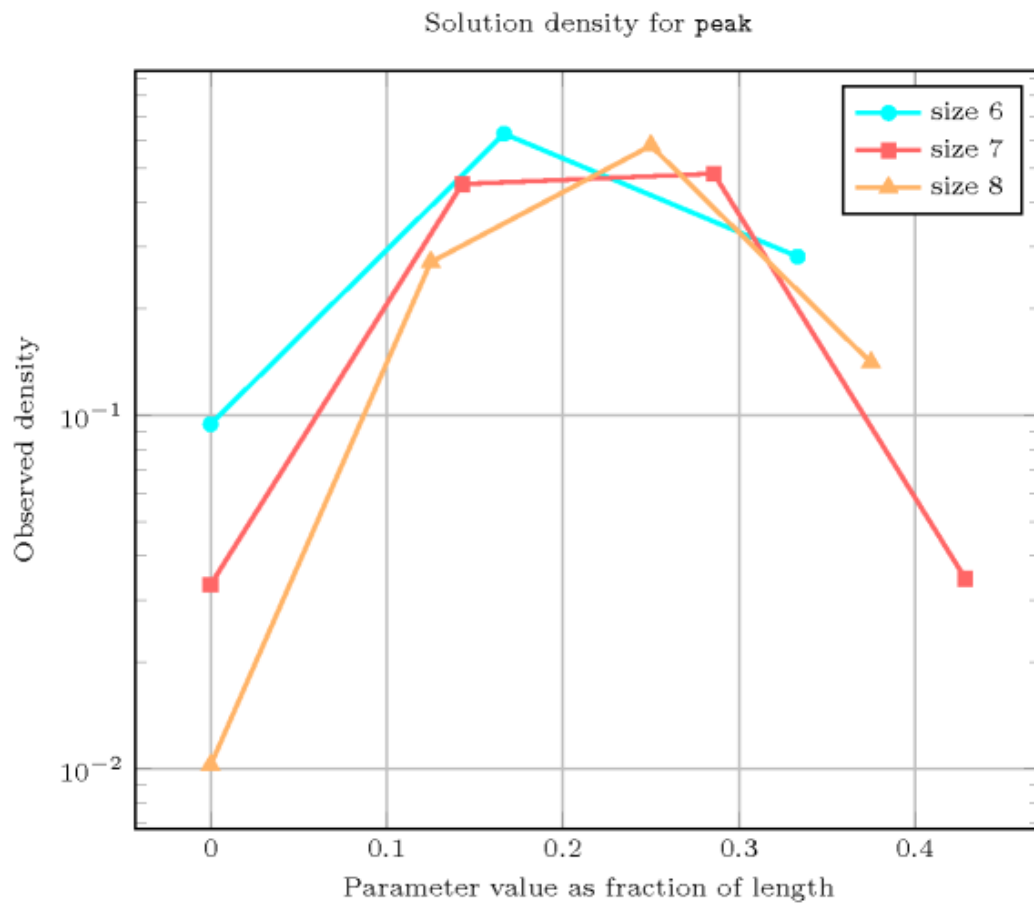
Number of solutions for *peak*: domains $0..n$





Length (n)	2	3	4	5	6	7	8	
Total	9	64	625	7776	117649	2097152	43046721	
Parameter value	0	9	50	295	1792	11088	69498	439791
	1	-	14	330	5313	73528	944430	11654622
	2	-	-	-	671	33033	1010922	24895038
	3	-	-	-	-	-	72302	6057270

Solution count for peak: domains 0. . n



See also **common keyword:** `highest _ peak`, `inflexion`, `min _ dist _ between _ inflexion`, `min _ width _ peak` (*sequence*).

comparison swapped: `valley`.

generalisation: `big _ peak` (*a tolerance parameter is added for counting only big peaks*).

related: `all _ equal _ peak`, `all _ equal _ peak _ max`, `decreasing _ peak`, `increasing _ peak`, `no _ valley`.

specialisation: `no _ peak` (*the variable counting the number of peaks is set to 0 and removed*).

Keywords

characteristic of a constraint: `automaton`, `automaton with counters`, `automaton with same input symbol`.

combinatorial object: `sequence`.

constraint arguments: `reverse of a constraint`, `pure functional dependency`.

constraint network structure: `sliding cyclic(1) constraint network(2)`.

filtering: `glue matrix`.

modelling: `functional dependency`.

Cond. implications

- `peak(N, VARIABLES)`
with $N > 0$
implies `atleast _ nvalue` (`NVAL`, `VARIABLES`)
when $NVAL = 2$.
- `peak(N, VARIABLES)`
implies `inflexion` (`N`, `VARIABLES`)
when $N = \text{peak}(\text{VARIABLES.var}) + \text{valley}(\text{VARIABLES.var})$.

Automaton

Figure 5.318.3 depicts the automaton associated with the `peak` constraint. To each pair of consecutive variables (VAR_i, VAR_{i+1}) of the collection `VARIABLES` corresponds a signature variable S_i . The following signature constraint links VAR_i , VAR_{i+1} and S_i :
 $(VAR_i < VAR_{i+1} \Leftrightarrow S_i = 0) \wedge (VAR_i = VAR_{i+1} \Leftrightarrow S_i = 1) \wedge (VAR_i > VAR_{i+1} \Leftrightarrow S_i = 2)$.

Figure 5.318.3. Automaton of the `peak` constraint

STATES SEMANTICS

s : stationary/decreasing mode ($\{> | =\}^*$)
 u : increasing mode ($\{< | =\}^*$)

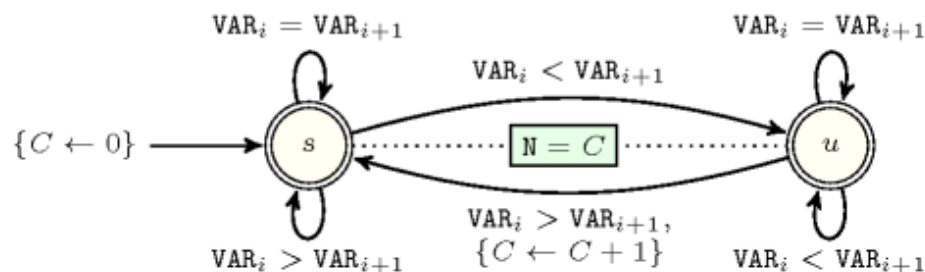


Figure 5.318.4. Hypergraph of the reformulation corresponding to the automaton of the peak constraint (since all states of the automaton are accepting there is no restriction on the last variable Q_{n-1})

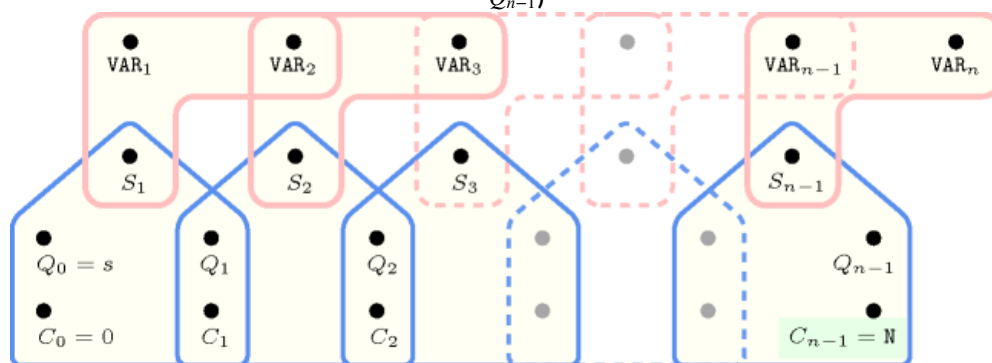
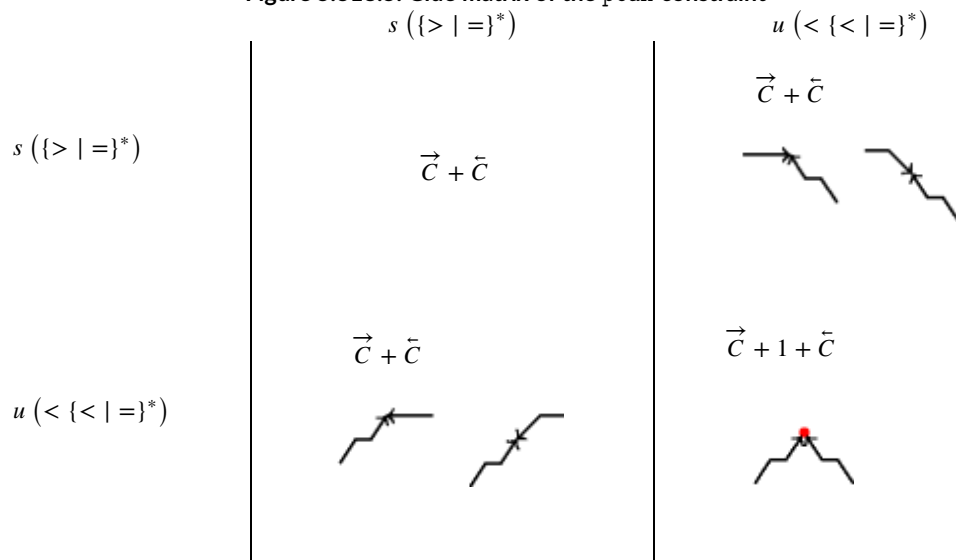
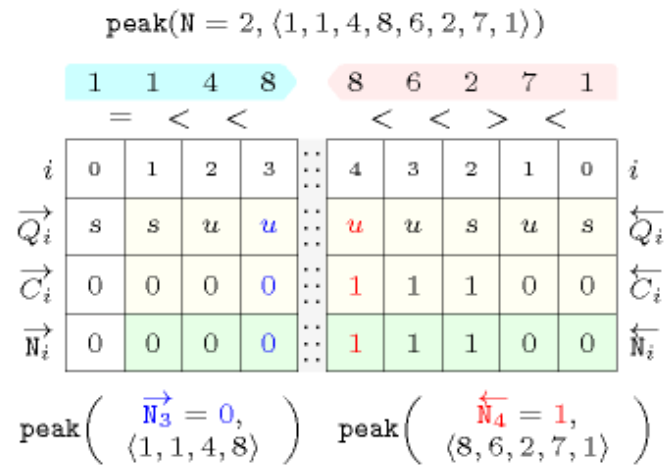


Figure 5.318.5. Glue matrix of the peak constraint



Glue matrix where \vec{C} and \bar{C} resp. represent the counter value C at the end of a prefix and at the end of the corresponding reverse suffix that partitions the sequence VARIABLES.

Figure 5.318.6. Illustrating the use of the state pair (u, u) of the glue matrix for linking N with the counters variables obtained after reading the prefix 1, 1, 4, 8 and corresponding suffix 8, 6, 2, 7, 1 of the sequence 1, 1, 4, 8, 6, 2, 7, 1; note that the suffix 8, 6, 2, 7, 1 (in pink) is proceed in reverse order; the left (resp. right) table shows the initialisation (for $i = 0$) and the evolution (for $i > 0$) of the state of the automaton and of its counter C upon reading the prefix 1, 1, 4, 8 (resp. the suffix 1, 7, 2, 6, 8).



glue matrix entry associated with the state pair (u, u) :

$$N = \vec{C}_3 + 1 + \overleftarrow{C}_4 = 0 + 1 + 1 = 2$$

W3C: [XHTML](#) - last update: 2014-6-10. [SD](#).