Strategic Guessing: Measuring Uncertainty with Entropy in Bulls and Cows

https://github.com/DeivanaiThiyagarajan/Mathematics--1-for-Data-Science

Introduction to Information Theory and Entropy:

- Information Theory is a branch of mathematics that studies the quantification, storage, and communication of information.
- Entropy is the measure of uncertainty or randomness in a set of possible outcomes.

  $H(x) = \sum_{x \epsilon X} \log_2 \frac{1}{p(x)} p(x)$ bits

  $p(x)$ = probability mass function of a discrete random variable
- Bits is the unit of measure for the entropy.
- The term by itself usually refers to the Shannon entropy, which quantifies in the sense of an expected value.
- Shannon Entropy is a measure of average information content one is missing when one does not know the value of the random variable.

Background: Bulls and Cows Game

- What is this Game?
  - ❑ A deductive logic game where the guesser attempts to uncover the secret number chosen by the opponent.
  - ❑ Feedback on each guess helps narrow down the possibilities through logical reasoning
  - ❑ The guesser gives feedback in the form of:

  - Bulls: Correct digits in correct positions

  - Cows: Correct digits in wrong position
- Rules of the game
  - ❑ The secret number has 4 unique digits
  - ❑ Each guess gets a response indicating the number of bulls and cows
  - ❑ The goal is to get to the secret number in the fewest attempts
- Why is it so interesting?
  - ❑ Highlights strategic problem-solving and logic deduction
  - ❑ Demonstrates principles of probability and uncertainty reduction in decision-making

Application of Entropy in Bulls and Cows

- Connecting Entropy to the Game

- Bulls and Cows game is a real-world example of uncertainty reduction

- Each guess provides feedback which reduces the number of possible secret numbers

    - Entropy helps quantify this reduction in uncertainty in each step

- Role of Entropy in Guessing

    - At the start of the game all the possible combinations are equally likely, leading to less probability for each possibility and entropy is high

    - As the feedback is received, we have received some information on the secret number and can reduce the total number of possibilities.

    - Still, the game continues where all possibilities are equally likely to occur, but the possibility count is reduced which in turn increase the probability for each occurrence and decreased the entropy.

$$Probability \propto \frac{1}{Entropy}$$

    - Probability is inversely proportional to Entropy, when probability increases entropy decreases and vice versa.

Computer guessing the numbers

- In this project the computer will try to guess the secret numbers you have in mind.
- Let's see how the computer tries to do it.
- The system generates all possible combinations of 4-digit numbers without repetition.
- It takes randomly a number from the 5040 combinations and asks the player if this is the number.
- The player should respond by the number of bulls and cows the computer got correctly.
- The system then with this feedback eliminates few possibilities and then gets a random choice from the remaining set.
- This cycle continues until the system correctly guesses the secret number.
- Finally, the entropy is determined, and graph is plotted to show in how many guesses the system predicted correctly.

PseudoCode

1. In this start of the game there are 5040(10x9x8x7) unique possibilities each with equal probability of occurence.

2. Initial Entropy is also calculated with the probability and number of possibilities.

3. As the guess increases the number of possibilities decreases--> probability increases and entropy descreases.

4. The number of possibilities decreases with each guess and the rate of decrease depends on the following conditions.

a. If b=0 and c=0, then remove all the possibilities if any one of the number is present in the possibility considered.

b. If b=0 and c=1, then concatenate the index and number of the guess and current possibility and find the common occurences of these should be 0(Indicating 0 bull) and common numbers between guess and current possibility should be exactly 1(Cow).

c. If b=0 and c=2, then do the same as before condition but the common numbers between guess and current possibility should be exactly 2(Cow).

d. If b=0 and c=3, same as above but common occurences should be 3(Cow).

e. If b=0 and c=4, same as above but common occurences should be 4(Cow).

f. If b=1 and c=0, atleast one of the number in the guess and current possibility should be in same index.

g. If b=2 and c=0, atleast 2 of the number in the guess and current possibility should be in the same index.

h. If b=3 and c=0,atleast 3 of the number in the guess and current possibility should be in the same index.

i. If b=1 and c=1, one of the element in the guess and current possibility should be in same index and out of all other elements in other index should have one common occurence in different indices.

j. If b=1 and c=2, one of the element in the guess and current possibility should be in same index and out of all other elements in other indices should have 2 common occurences in different indices.

k. If b=1 and c=3, same as above condition but should have 3 common occurrences all in different indices.

l. If b=2 and c=1, Two of the elements in the guess and current possibility should be in same index and out of all other elements in other indices should have 1 common occurences in different indices.

m. If b=2 and c=2, same as above, but 2 common occurences should be present in different indices

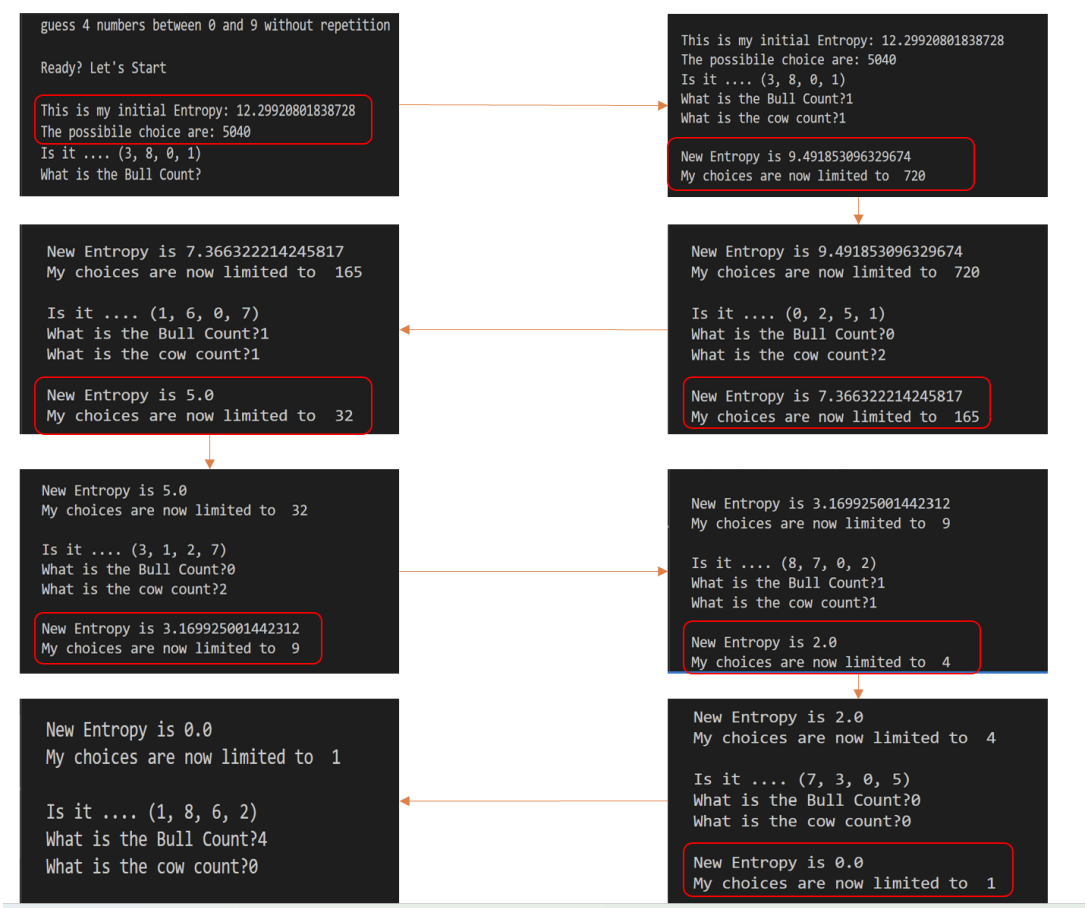
n. If b=4 and c=0, then the guess is correctly predicted and end the game.

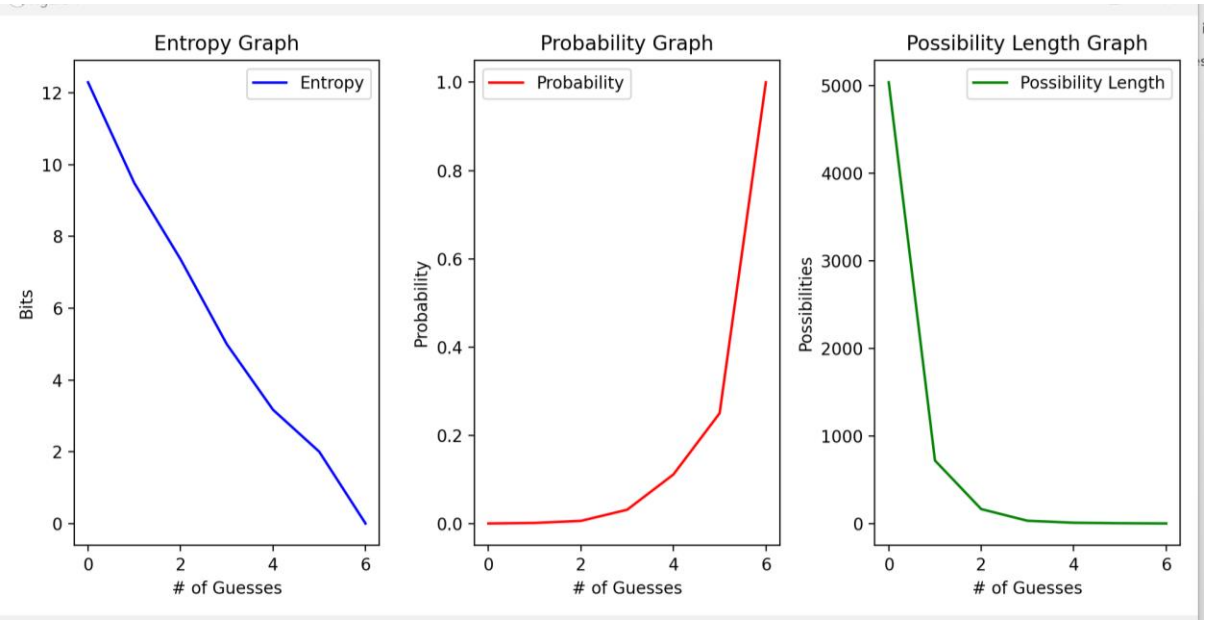
5. After each guess reduce the number of possibilities this inturn increases the probability and decreases entropy

6. After winning, show the graph on how the probability increases, entropy decreases and number of possibility decreases.

Entropy Reduction: Step-by-Step in Bulls and Cows

- Let's see a working example of how the possibility and entropy gets reduced
- Let's assume the secret number is 0123
- Initial set of possibilities: 5040, Initial Entropy: 12.29
- The first guess that the system chose is 5461, as we can see here there is 0-bull and 1-cow
- Hence, common numbers between guess and current possibility should be exactly 1 and no current numbers in guess should be in same place as all possibilities. So, the rest is all removed which doesn't match this condition.
- After the above elimination, the probability count will be reduced to 1440 and entropy is reduced to 10.49
- The above steps will continue till the secret number is guessed correctly. For each set of bulls and cows the elimination conditions change.
- Finally, the entropy will reach 0 and possibility count will also be reduced to 1 since this is the only remaining possibility and there is no uncertainty left since there is only one possibility of occurrence.
- Let's see a working example in the coming slide, where the secret number is 1862 and how entropy is calculated.





Code Explaination

- https://drive.google.com/file/d/1buCWQdasIeNQlA3tiD24kTiVLwpb0lV6/view?usp=drive_link

Formal Presentation

- https://drive.google.com/file/d/1yKjp8UmLWawIgJwtjDhAQC3u1qtw5oy3/view?usp=drive_link