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$\mathcal{MAX}$-$\mathcal{MIN}$ PARACONSISTENT ANT ALGORITHM

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Abstract. *This paper presents a strategy for joining the Paraconsistent Logic (PL) and Ant Colony Optimization (ACO). Due to the possibility to work with situations that are not included in the field of classical logic, PL allows for meta-heuristics of the ants more realistic decisions in choosing their paths that may be interesting in the optimization process. With this, the individuals of the colony along the optimization process will no longer take random decisions of the original strategy of ACO, to follow a path possibly better due to its expertise acquired by the use of PL. In order to show the performance achieved, the results of the strategy with a classical meta-heuristics of ACO, named $\mathcal{MAX}$-$\mathcal{MIN}$, are compared with the proposed hybrid system with the PL in four instances of problems of traveling salesman, featuring attractive results.*
Keywords: Paraconsistent logic, Ant colony, Hybrid intelligent systems, Meta-heuristics

1. **Introduction.** Nowadays, many numerical optimization problems have a great difficulty to be resolved, both in the field of continuous solutions and discrete solutions. It occurs due to the fact that the amount of possible solutions to a problem may be enormous on the search space, being, generally, an exponential amount in relation to the instance of the problem (nondeterministic polynomial time – NP) [1].

So often, the search for the best solution for a particular problem will not be realistic for the time available and an approximate result is perhaps the best choice at the time. On this feature, researchers from around the world offer new methods to solve optimization problems more simply and/or efficiently [2]. A simple proof of this is the new intelligence-based optimization techniques of swarms, where through cooperation between individuals (direct or indirect) can obtain a better adaptation of population to the environment.

An example of an intelligent swarm technique that will be addressed in this paper is the Ant Colony Optimization (ACO), inspired by the behavior of agents (artificial ants) in search of food. The ACO is especially recommended for work in the field of discrete solutions for numerical optimization problems. The ACO can be applied in various issues, being among the best known, for example, the traveling salesman problem, such as generating timetables and vehicle routing, which are difficult computing problems. With this, the ACO has been showing as a promising approach and competitive in relation to other strategies presented in the literature.

Because it is a commonly used technique for solving combinatorial optimization problems, the meta-heuristics ACO has been developing through a few variations of algorithms. Ant System (AS) was the first algorithm developed which followed the ideas of meta-heuristics of ACO.

After this algorithm, other algorithms have been developed in order to improve performance and achieve better results. Among the variations [3,4] we have the following algorithms such as Ant System based on elitist strategy, based on rank (ASrank), $\mathcal{MAX}$-$\mathcal{MIN}$ Ant System (MMAS) and Ant Colony System (ACS).

However, any strategy used, in its original conception, the choice of paths made by each one of the ants in the colony follows the principle of probability, which takes into consideration the pheromone trails and heuristic information of the problem being optimized. With this, the choice of this path can go through situations of inconsistency, inaccuracy and unlimited knowledge, which correctly used can cheapen the process of optimization, because the random choice of a path to be followed by an ant may not be a good option at the end of the process of optimization. It is expected that, at the end of the process of optimization, the knowledge produced by the ants in the colony is sufficient to determine the best solution.

This work is based on applying a new strategy developed in [5], which proposes the use of non-classical logic, named Paraconsistent Logic (PL), able to treat on those situations not foreseen by the original method of ACO. This paper presents a comparison between the strategy of meta-heuristics known as the $\mathcal{MAX}$-$\mathcal{MIN}$ Ant System and a proposed hybrid paraconsistent ant algorithm. These implementations of ACO are applied for solving four instances of the traveling salesman problem, in order to compare their performances.

The paper is organized as follows. Section 2 introduces the concepts of PL that will be used for the adaptation of this on meta-heuristics of ACO. Section 3 presents the idealized algorithm in this work. Section 4 describes the settings used for the experimental results are described and discussed in Section 5. And, finally, Section 6 brings a conclusion of the work.

2. **Paraconsistent Logic.** In real situations, when accurate decisions must be taken, it is difficult to set limits for what is true and what is false, regarding quality of things. In the case of ants, for example, it is difficult to determine, between n possible paths, from the quality of the path determined by pheromone content, which path the ant must follow. The situations of non-definition, uncertainty, ambiguity and contradiction arise naturally when real-world situations are described. To handle real situations, in which classical logic is not able to represent, many non-classical logics have been developed, and among those, the Paraconsistent Logic [6]. Some fundamentals and possible applications of PL can be found in Abe [7].

In this paper, the goal is in developing an algorithm that uses PL to help the ants in their decision making. An interpretation of PL, which admits implementation, was proposed by Abe [7], called Annotated Paraconsistent Logic with annotation of two values (APL2v), as illustrated in the unit square in the Cartesian Plane (USCP), shown in Figure 1. In this interpretation, the decision is taken in accordance with the degree of belief (or degree of favorable evidence), denoted in figure for $\boldsymbol{\mu}$, and degree of disbelief (or degree of unfavorable evidence) of a proposition, denoted by $\boldsymbol{\lambda}$, such that $\boldsymbol{\mu}$, $\boldsymbol{\lambda} \in [0,1] \subset \Re$.

So, given a proposition $P_{(\mu,\lambda)}$, we have four special annotations: $P_{(1,0)}$ which indicates a total favorable evidence and a null unfavorable evidence to proposition P, that is the *truth* of classical logic (T); $P_{(0,1)}$ which indicates a null favorable evidence and a total unfavorable evidence to proposition P, which corresponds to *falsity* of classical logic (F); $P_{(0,0)}$ indicating a null favorable evidence and a null unfavorable evidence to proposition P, which corresponds to the situation of *indeterminacy* (I); and, $P_{(1,1)}$ indicating a total

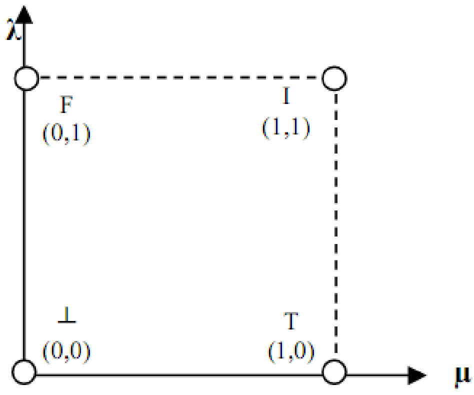


Figure 1. Unit square in the cartesian plane (USCP) with annotation of two values (APL2v)

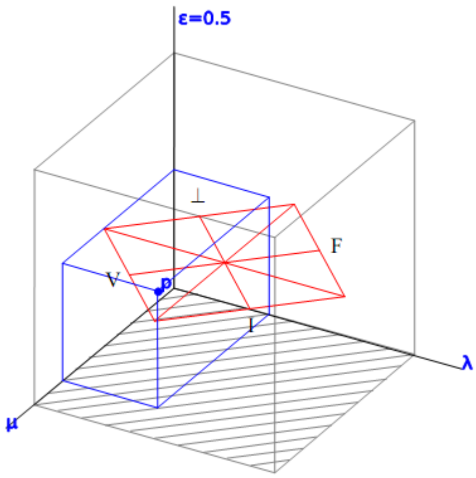


Figure 2. Unit cube in the cartesian plane (UCCP) with annotation of three values (APL3v), $\varepsilon = 0.5$

favorable evidence and a total unfavorable evidence to proposition P, which corresponds to the situation of *inconsistency* ($\perp$).

Given a pair ($\boldsymbol{\mu}$, $\boldsymbol{\lambda}$), representing respectively the degree of belief and disbelief of a proposition, it is possible to calculate the degree of certainty (DC) and the degree of uncertainty (DU) as defined in Equations (1) and (2):

$$DC = \boldsymbol{\mu} + \boldsymbol{\lambda} - 1 \tag{1}$$

$$DU = \boldsymbol{\mu} - \boldsymbol{\lambda} \tag{2}$$

From the degree of certainty and uncertainty of a proposition $P_{(\mu,\lambda)}$, a diagnosis D_P to this proposition can be determined by:

$$D_P = \begin{cases} T, & \text{if } DC \geq 0.5 \\ F, & \text{if } |DC| < 0.5 \\ I, & \text{if } DU \geq 0.5 \\ \perp, & \text{if } |DU| < 0.5 \end{cases} \tag{3}$$

In [8], the degree of specialty is added, denote by ε, in the interpretation of the two original variables, degree of belief, $\boldsymbol{\mu}$, and degree of disbelief, $\boldsymbol{\lambda}$. With this third variable, the problem can be described in a more realistic, considering the degree of specialized knowledge in the evaluation of each diagnosis. The cube for Paraconsistent Logic Analyzer annotated with three variables (APL3v) is set with the inclusion of the axis perpendicular to the plane formed by the degree of belief and disbelief, that varies in the closed interval

[0, 1], representing the degree of specialty, as shown in Figure 2. In this case, the diagnosis D_P to this proposition can be determined by:

$$D_P = \begin{cases} T, & \text{if } DC \geq \varepsilon \\ F, & \text{if } |DC| < |-\varepsilon| \\ I, & \text{if } DU \geq |1-\varepsilon| \\ \perp, & \text{if } |DU| < |\varepsilon - 1| \end{cases} \tag{4}$$

3. **The Proposed MMPAS Algorithm.** The main idea of the algorithm proposed in this paper is to use PL to adapt the concept of learning in the ants in the colony. The ants begin the process of building solutions without knowledge. According to the iterations, the ants become more and more specialists, and decide which way following deterministic way, considering only the amount of pheromone on the trails. As discussed in [5], this proposal serves to adjust the convergence of the ants in the colony. We demonstrate in this work, by applying statistical tests, that the proposed modification in this new algorithm ensures results equal to or better than the $\mathcal{MAX}$-$\mathcal{MIN}$ Ant System algorithm (MMAS).

The decision-making process of the ant colony using PL logic, named $\mathcal{MAX}$-$\mathcal{MIN}$ Paraconsistent Ant System algorithm (MMPAS), is determined by Equations (5) to (8).

$$x = \max_{x \in \{1,2,\ldots,v\}} \{[\tau_{ix}]^{\alpha}[\eta_{ix}]^{\beta}\} \tag{5}$$

$$y = \min_{y \in \{1,2,\ldots,v\}} \{[\tau_{iy}]^{\alpha}[\eta_{iy}]^{\beta}\} \tag{6}$$

$$z = \max_{z \in \{1,2,\ldots,v\} - x} \{[\tau_{iz}]^{\alpha}[\eta_{iz}]^{\beta}\} \tag{7}$$

where $\{1, 2, \ldots, v\}$ represents the not-visited neighborhood by ant k, and x is the neighbor with the greatest value of the product $[\tau_{ix}]^{\alpha}[\eta_{ix}]^{\beta}$ in the not-visited neighborhood by ant k from the city i; y is the neighbor with the smallest value of the product $[\tau_{iy}]^{\alpha}[\eta_{iy}]^{\beta}$ in the not-visited neighborhood by ant k from the city i; where z is nearby with the second largest product value $[\tau_{iz}]^{\alpha}[\eta_{iz}]^{\beta}$ in the not-visited neighborhood by ant k from the city i.

$$\varepsilon = 1 - \left(\frac{t}{N}\right)^{\delta} \tag{8}$$

And specialty ε varies according to the iterations. The variable t in Equation (8) represents the current iteration, the variable N represents the total number of iterations and the parameter δ determines how is specialty variation in function of the iterations. The value of λ is computed by:

$$\lambda = (x - z)/(x - y) \tag{9}$$

4. **System Tests.** For the performance of the tests, the original implementation of the algorithm MMAS and the proposed hybrid MMPAS algorithm were applied to the solution of the traveling salesman problem (TSP). As it has already been mentioned, this problem is an excellent benchmark for assessment of ACO algorithms. The formal description of the problem of TSP to the pheromone of the ACO model can be found in [9].

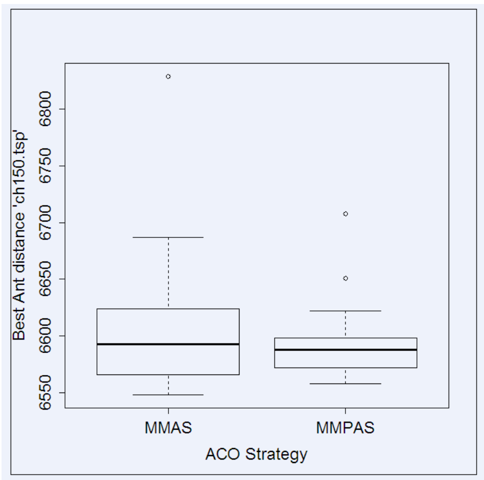
To both methodologies, four instances of TSP were used from TSPLIB repository database [10]. Instances chosen for testing were: ch150.tsp, tsp225.tsp, a280.tsp and pcb442.tsp, representing problems with 150, 225, 280 and 442 towns, respectively. For each of the instances, a value of 2000 repetitions per experiment was set, being these repetitions divided into four steps of 500 iterations, keeping the best solution found between all ants. The other parameters used by both techniques were: the number of ants is equal to the number of cities the problem instance, $\alpha = 1$, $\beta = 2$, $\rho = 0.02$. According to the reference [4], these parameters are good for optimization process. In addition, in all experiments the parameter $Q_0 = 0$ and no local search operation by the ACO algorithms were used.

5. **Results and Discussions.** In this section, the results found in experiments performed to the methodologies of the strategy of the ACO, the original MMAS and the proposed hybrid MMAPS, are presented. The effect of the introduction of PL in the MMAS are presented and discussed.

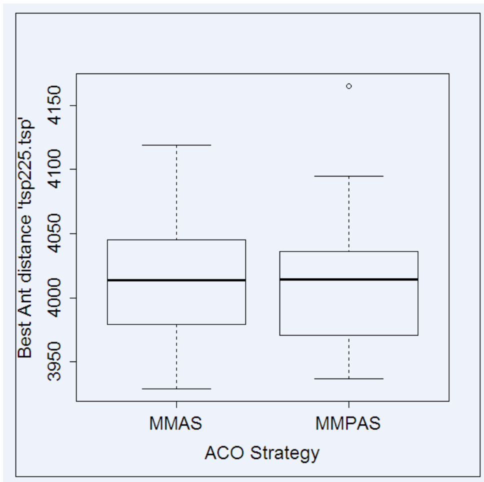
Table 1 presents the results obtained for both the strategies already discussed. This table shows for each approach, the used instance, the average of the solutions, the standard deviation and the best result obtained by the respective approach. In addition to these information, for each instance are shown the number of cities and the known best solution, available in [10].

Table 1. Analysis of the path for the better ant

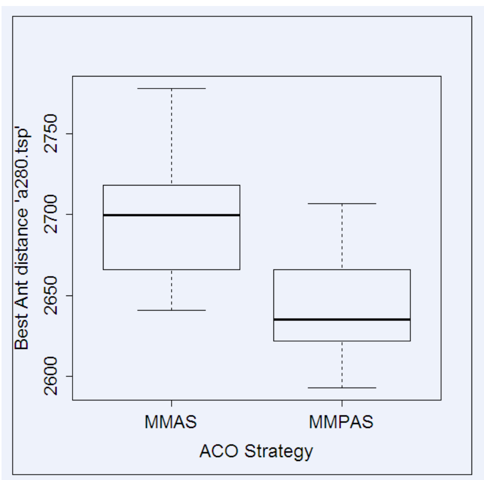
Name of the Problem	Number of Cities	Optimal Solution	Original MMAS			Proposed MMPAS		
			Mean	Standard Deviation	Best Result	Mean	Standard Deviation	Best Result
ch150	150	6528	6602,77	54,86	**6548**	6591,53	30,38	6558
tsp225	225	3919	4013,9	47,98	**3929**	4012,07	47,48	3937
a280	280	2579	2695,7	33,47	2641	2642,77	28,94	**2593**
pcb442	442	50778	53671,73	618,38	52805	53290,1	598,72	**52129**



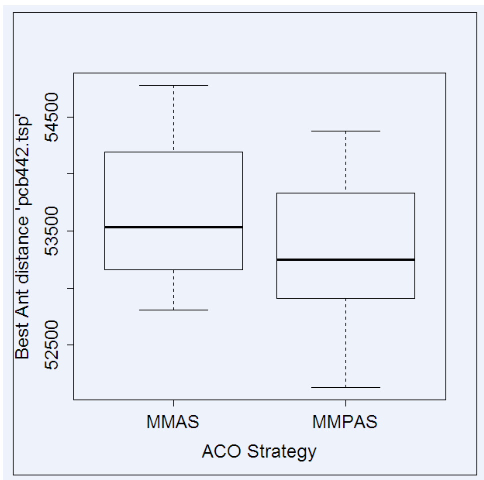
(a) ch150



(b) tsp225



(c) a280



(d) pcb442

Figure 3. Boxplot chart for each instance test

Figure 3 shows the boxplot chart for the samples taken from each of the instances used in the study. With this, an interesting statistical analysis of the data is carried out, being possible to evaluate symmetry and the dispersion of samples and data to infer some conclusions about each of the approaches used.

From Table 1, comparing the mean and standard deviation of both approaches, one can observe that the strategy MMPAS logic was always more consistent results, showing their results with values closer to the better known. However, despite this feature, during the implementation of other statistical tests for comparison between averages of samples, these being the Student's t-test (for samples that showed normality of data) and the Mann Whitney U test (for samples that did not show normal between your data), only the instances a280 and pcb442 presented a result with 95% confidence, that the averages are statistically different. In other words, the strategy presented in this work obtained a paraconsistent best result.

For the other two instances, in spite of the best average performance, the MMPAS algorithm can find a better result is not supported. However, despite this, it can be observed that the standard deviation of paraconsistent approach was, in all cases, to a lesser extent, a differentiator against the standard approach of MMAS.

6. **Conclusion.** In this paper, a new approach to working with ACO was discussed. It is applied to non-classical logic, named Paraconsistent Logic, during the decision making of the path to be followed by the ants in the colony to build a solution. Increasingly, the ants will no longer use the default strategy of ACO probability-based, adopting so deterministic decisions based on Paraconsistent Logic that takes into account the experience gained during the process of optimization. So, as seen in tests conducted for the four instances of TSP, this strategy applied in addressing MMAS can bring benefits for the optimization of combinatorial problems.

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REFERENCES

[1] G. Lambert-Torres, L. E. Borges da Silva, B. Valiquette, H. Greiss and D. Mukhedkar, A fuzzy knowledge-based system for bus load forecasting, in *Fuzzy Logic Technology and Applications*, Robert J. Marks II (ed.), IEEE Press, 1994.

[2] G. Lambert-Torres, A. P. Alves da Silva, V. H. Quintana and L. E. Borges da Silva, Classification of power system operation point using rough sets techniques, *Proc. of IEEE International Conference on Systems, Man and Cybernetics – IEEE SMC'96*, Beijing, China, vol.3/4, pp.1898-1903, 1996.

[3] M. Dorigo, V. Maniezzo and A. Colorni, Ant system: Optimization by a colony of cooperating agents, *IEEE Trans. on Systems, Man, and Cybernetics – Part B*, vol.26, no.1, pp.29-41, 1996.

[4] M. Dorigo, M. Birattari and T. Stützle, *Ant Colony Optimization – Artificial Ants as a Computational Intelligence Technique*, IRIDIA – Institut de Recherches Interdisciplinaires et de Développements en Intelligence Artificielle, Université Libre de Bruxelles, Belgium, 2006.

[5] L. E. Silva, G. Lambert-Torres, R. M. Salgado and H. C. B. Oliveira, A hybrid system based in ant colony and paraconsistent logic, in *Ant Colonies: Behavior in Insects and Computer Applications*, E. C. Sun (ed.), Nova Science Publishers, 2011.

[6] N. C. A. da Costa, On the theory of inconsistent formal systems, *Notre Dame Journal of Formal Logic*, vol.11, 1974.

[7] J. M. Abe, *Fundamentals of Annotated Logic*, Ph.D. Thesis, FFLCH/USP, SP, Brazil, 1992.

[8] H. G. Martins, G. Lambert-Torres and L. F. Pontin, *Annotated Paraconsistent Logic*, 2007 (in Portuguese).

[9] M. Dorigo and K. Socha, *An Introduction to Ant Colony Optimization*, IRIDIA – Institut de Recherches Interdisciplinaires et de Développements en Intelligence Artificielle, Université Libre de Bruxelles, Belgium, 2006.

[10] G. Reinelt, TSPlib: A traveling salesman problem library, *ORSA Journal on Computing*, vol.3, pp.376-384, 1991.