Problem 15

Evaluate the following limit:

$$\lim_{x \to 8} \frac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2}$$

Solution

We begin by substituting $x = 8$ into both the numerator and the denominator:

Substitute $x = 8$

- Numerator:

$$\sqrt{9+2x}-5 = \sqrt{9+2(8)}-5 = \sqrt{9+16}-5 = \sqrt{25}-5 = 5-5 = 0$$

- Denominator:

$$\sqrt[3]{x}-2 = \sqrt[3]{8}-2 = 2-2 = 0$$

Since both the numerator and denominator are 0, we have an indeterminate form $\frac{0}{0}$. Therefore, we will apply L'Hôpital's Rule.

Apply L'Hôpital's Rule

Differentiate the numerator and denominator:

- Numerator:

$$\frac{d}{dx}\left(\sqrt{9+2x}\right) = \frac{1}{2}(9+2x)^{-1/2} \cdot 2 = \frac{2}{2\sqrt{9+2x}} = \frac{1}{\sqrt{9+2x}}$$

- Denominator:

$$\frac{d}{dx}\left(\sqrt[3]{x}\right) = \frac{1}{3}x^{-2/3}$$

Evaluate the New Limit

Now, we evaluate the limit of the new expression:

$$\lim_{x \to 8} \frac{\frac{1}{\sqrt{9+2x}}}{\frac{1}{3}x^{-2/3}} = \frac{3x^{2/3}}{\sqrt{9+2x}}$$

Substitute $x = 8$

Now substitute $x = 8$ into the simplified expression: **- Numerator:**

$$3 \cdot 8^{2/3} = 3 \cdot 4 = 12$$

- Denominator:

$$\sqrt{9+2(8)} = \sqrt{9+16} = \sqrt{25} = 5$$

Thus, the limit is:

$$\frac{12}{5}$$

Conclusion

Therefore, the limit is:

$$\lim_{x \to 8} \frac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2} = \frac{12}{5}$$