Problem 6

Evaluate the integral:

$$I = \int \frac{x^2}{1-x^2}\,dx.$$

Solution

To solve the given integral, we decompose the integrand and use substitution techniques.

Step 1: Simplify the integrand

We rewrite the fraction $\frac{x^2}{1-x^2}$ by splitting it into simpler terms:

$$\frac{x^2}{1-x^2} = -1 + \frac{1}{1-x^2}.$$

Thus, the integral becomes:

$$I = \int \frac{x^2}{1-x^2}\,dx = \int \left(-1 + \frac{1}{1-x^2}\right)\,dx.$$

Step 2: Separate the integral

We now split the integral into two parts:

$$I = \int -1\,dx + \int \frac{1}{1-x^2}\,dx.$$

1. **First term:** The integral of -1 is:

$$\int -1\,dx = -x.$$

2. **Second term:** The integral of $\frac{1}{1-x^2}$ is a standard integral:

$$\int \frac{1}{1-x^2}\,dx = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|.$$

Step 3: Combine the results

Combining the results of both terms, we have:

$$I = -x + \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C,$$

where C is the constant of integration.

Final Answer

The solution to the integral is:

$$I = -x + \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C.$$