Problem 9

Analyze the behavior of the function $y = x\sqrt{\frac{x-1}{x+1}}$ as x approaches positive infinity.

Solution

We start by simplifying the expression inside the square root:

$$\frac{x-1}{x+1}$$

Dividing the numerator and denominator by x:

$$\frac{\frac{x}{x}-\frac{1}{x}}{\frac{x}{x}+\frac{1}{x}} = \frac{1-\frac{1}{x}}{1+\frac{1}{x}}$$

As $x \to +\infty$, $\frac{1}{x} \to 0$, so:

$$\frac{1-\frac{1}{x}}{1+\frac{1}{x}} \to \frac{1-0}{1+0} = 1$$

Taking the square root:

$$\sqrt{\frac{1-\frac{1}{x}}{1+\frac{1}{x}}} \to \sqrt{1} = 1$$

Therefore, the function y can be approximated as:

$$y = x\sqrt{\frac{x-1}{x+1}} \approx x \cdot 1 = x \quad \text{as} \quad x \to +\infty$$

Thus, as $x \to +\infty$, $y \to +\infty$.

Conclusion

$$\lim_{x \to +\infty} x\sqrt{\frac{x-1}{x+1}} = +\infty$$