

Problem 8

Evaluate the following limit:

$$\lim_{x \rightarrow +\infty} e^{-x} \sin(x)$$

Solution

We analyze the given function step by step:

1. **Behavior of the individual terms:**

- The exponential term e^{-x} decays to zero as $x \rightarrow +\infty$:

$$e^{-x} \rightarrow 0^+ \quad \text{as } x \rightarrow +\infty.$$

- The sine term $\sin(x)$ oscillates between -1 and 1 for all x :

$$-1 \leq \sin(x) \leq 1 \quad \text{for all } x.$$

2. **Behavior of the product $e^{-x} \sin(x)$:**

- As $x \rightarrow +\infty$, the term e^{-x} approaches 0^+ , causing the entire product to decay to 0.
- While $\sin(x)$ oscillates indefinitely, the exponential decay of e^{-x} dominates and forces the product $e^{-x} \sin(x) \rightarrow 0$.

3. **Conclusion:**

$$\lim_{x \rightarrow +\infty} e^{-x} \sin(x) = 0.$$

Final Answer

$$\boxed{\lim_{x \rightarrow +\infty} e^{-x} \sin(x) = 0}$$