

Problem 11

Evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{(1+x)^5 - (1+5x)}{x^2 + x^5}$$

Solution

We start by expanding the numerator using the binomial expansion.

Binomial Expansion of $(1+x)^5$

The binomial expansion of $(1+x)^5$ is:

$$(1+x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$$

Thus, the numerator becomes:

$$(1+x)^5 - (1+5x) = (1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5) - (1 + 5x)$$

Simplifying:

$$(1+x)^5 - (1+5x) = 10x^2 + 10x^3 + 5x^4 + x^5$$

Simplify the Expression

Now, we simplify the entire expression:

$$\frac{10x^2 + 10x^3 + 5x^4 + x^5}{x^2 + x^5}$$

Factor x^2 from Numerator and Denominator

Factor x^2 from both the numerator and the denominator:

$$\frac{10x^2 + 10x^3 + 5x^4 + x^5}{x^2 + x^5} = \frac{x^2(10 + 10x + 5x^2 + x^3)}{x^2(1 + x^3)}$$

Cancel the common factor of x^2 :

$$\frac{10 + 10x + 5x^2 + x^3}{1 + x^3}$$

Evaluate the Limit as $x \rightarrow 0$

Now, substitute $x = 0$ into the simplified expression:

$$\frac{10 + 10(0) + 5(0)^2 + (0)^3}{1 + (0)^3} = \frac{10}{1} = 10$$

Conclusion

Therefore, the limit is:

$$\lim_{x \rightarrow 0} \frac{(1+x)^5 - (1+5x)}{x^2 + x^5} = 10$$