Problem 27

Given the function:

$$f(x) = x^2,$$

and the value $x_0 = 1$, find the derivative of the function $f(x)$ at $x_0 = 1$.

Solution

To find the derivative of the function $f(x) = x^2$ at $x_0 = 1$, we use the definition of the derivative. The derivative of $f(x)$ at $x = x_0$ is given by:

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}.$$

Substitute the values into the definition

For $x_0 = 1$, we have:

$$f'(1) = \lim_{h \to 0} \frac{f(1 + h) - f(1)}{h}.$$

Since $f(x) = x^2$, we know:

$$f(1 + h) = (1 + h)^2 = 1 + 2h + h^2,$$

and

$$f(1) = 1^2 = 1.$$

Substitute into the formula for the derivative

$$f'(1) = \lim_{h \to 0} \frac{(1 + 2h + h^2) - 1}{h} = \lim_{h \to 0} \frac{2h + h^2}{h}.$$

Simplify the expression

$$f'(1) = \lim_{h \to 0} \frac{h(2 + h)}{h} = \lim_{h \to 0} (2 + h).$$

Evaluate the limit

As $h \to 0$, the expression $2 + h$ approaches 2. Therefore:

$$f'(1) = 2.$$

Conclusion

The derivative of $f(x) = x^2$ at $x_0 = 1$ is:

$$f'(1) = 2.$$