Problem 10

Evaluate the limit:

$$\lim_{x \to 0} \frac{x^2 - 1}{2x^2 - x - 1}$$

Solution

We start by factoring both the numerator and the denominator.

Factoring

- **Numerator:** The numerator $x^2 - 1$ is a difference of squares:

$$x^2 - 1 = (x - 1)(x + 1)$$

- **Denominator:** To factor the denominator $2x^2 - x - 1$, we look for two numbers that multiply to $2 \times (-1) = -2$ and add to -1. These numbers are -2 and 1:

$$2x^2 - x - 1 = (2x + 1)(x - 1)$$

Simplification

Substituting the factored forms into the original expression:

$$\frac{(x - 1)(x + 1)}{(2x + 1)(x - 1)}$$

We can cancel the common factor $(x - 1)$ in the numerator and denominator (for $x \neq 1$):

$$\frac{x + 1}{2x + 1}$$

Evaluating the Limit

Now, we evaluate the limit as x approaches 0:

$$\lim_{x \to 0} \frac{x + 1}{2x + 1} = \frac{0 + 1}{2 \times 0 + 1} = \frac{1}{1} = 1$$

Conclusion

Therefore,

$$\lim_{x \to 0} \frac{x^2 - 1}{2x^2 - x - 1} = 1$$