Problem 7

Evaluate the following limit:

$$\lim_{x \to +\infty} e^x \sin(x)$$

Solution

We analyze the given function step by step:

1. The behavior of the individual terms:
   - e^x grows without bound as $x \to +\infty$: $e^x \to +\infty$.
   - $\sin(x)$ oscillates between -1 and 1 for all x, so it does not have a limit.
2. Behavior of the product $e^x \sin(x)$:
   - The exponential term e^x dominates and grows arbitrarily large.
   - The sine term $\sin(x)$ oscillates indefinitely without damping, taking both positive and negative values.
   - As a result, $e^x \sin(x)$ oscillates wildly with increasing magnitude as $x \to +\infty$.
3. Conclusion:

   $$\lim_{x \to +\infty} e^x \sin(x) \quad \text{does not exist.}$$

   This is because the function is unbounded and oscillates without settling to a single value.

Final Answer

$$\boxed{\text{The limit does not exist.}}$$