

## Problem 8

Evaluate the following limit:

$$\lim_{x \rightarrow +\infty} e^{-x} \sin(x)$$

## Solution

We analyze the given function step by step:

1. \*\*Behavior of the individual terms:\*\*

- The exponential term  $e^{-x}$  decays to zero as  $x \rightarrow +\infty$ :

$$e^{-x} \rightarrow 0^+ \quad \text{as } x \rightarrow +\infty.$$

- The sine term  $\sin(x)$  oscillates between  $-1$  and  $1$  for all  $x$ :

$$-1 \leq \sin(x) \leq 1 \quad \text{for all } x.$$

2. \*\*Behavior of the product  $e^{-x} \sin(x)$ :

- As  $x \rightarrow +\infty$ , the term  $e^{-x}$  approaches  $0^+$ , causing the entire product to decay to 0.
- While  $\sin(x)$  oscillates indefinitely, the exponential decay of  $e^{-x}$  dominates and forces the product  $e^{-x} \sin(x) \rightarrow 0$ .

3. \*\*Conclusion:\*\*

$$\lim_{x \rightarrow +\infty} e^{-x} \sin(x) = 0.$$

## Final Answer

$\lim_{x \rightarrow +\infty} e^{-x} \sin(x) = 0$
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