Problem 13

Evaluate the following limit:

$$\lim_{x \to -1} \frac{x^3 - 2x - 1}{x^5 - 2x - 1}$$

Solution

We begin by substituting $x = -1$ into both the numerator and the denominator:

Substitute $x = -1$

- Numerator:

$$x^3 - 2x - 1 = (-1)^3 - 2(-1) - 1 = -1 + 2 - 1 = 0$$

- Denominator:

$$x^5 - 2x - 1 = (-1)^5 - 2(-1) - 1 = -1 + 2 - 1 = 0$$

Since both the numerator and denominator are 0, we have an indeterminate form $\frac{0}{0}$. We will apply L'Hôpital's Rule.

Apply L'Hôpital's Rule

Differentiate the numerator and denominator:

- Numerator:

$$\frac{d}{dx}(x^3 - 2x - 1) = 3x^2 - 2$$

- Denominator:

$$\frac{d}{dx}(x^5 - 2x - 1) = 5x^4 - 2$$

Evaluate the New Limit

Now, we evaluate the limit of the new expression:

$$\lim_{x \to -1} \frac{3x^2 - 2}{5x^4 - 2}$$

Substitute $x = -1$:

- Numerator:

$$3(-1)^2 - 2 = 3(1) - 2 = 1$$

- Denominator:

$$5(-1)^4 - 2 = 5(1) - 2 = 3$$

Therefore, the new limit is:

$$\frac{1}{3}$$

Conclusion

Thus, the limit is:

$$\lim_{x \to -1} \frac{x^3 - 2x - 1}{x^5 - 2x - 1} = \frac{1}{3}$$