Problem 17

Evaluate the following limit:

$$\lim_{x \to +\infty} \frac{\ln\left(2 + e^{3x}\right)}{\ln\left(3 + e^{2x}\right)}$$

Solution

As $x \to +\infty$, the exponential terms e^{3x} and e^{2x} grow very large, and the constants 2 and 3 in the expressions become negligible. Thus, for large x, we have:

$$2 + e^{3x} \approx e^{3x}, \quad 3 + e^{2x} \approx e^{2x}.$$

This allows us to simplify the logarithms:

$$\ln\left(2 + e^{3x}\right) \approx \ln\left(e^{3x}\right) = 3x$$

and

$$\ln\left(3 + e^{2x}\right) \approx \ln\left(e^{2x}\right) = 2x.$$

Substituting these approximations into the limit expression:

$$\frac{\ln\left(2 + e^{3x}\right)}{\ln\left(3 + e^{2x}\right)} \approx \frac{3x}{2x} = \frac{3}{2}.$$

Conclusion

Therefore, the value of the limit is:

$$\lim_{x \to +\infty} \frac{\ln\left(2 + e^{3x}\right)}{\ln\left(3 + e^{2x}\right)} = \frac{3}{2}.$$