Problem 50

Evaluate the following limit:

$$\lim_{x \to 0} \left(\sqrt{1-2x} - \sqrt[3]{1-3x} \right).$$

Solution

We need to compute the limit:

$$\lim_{x \to 0} \left(\sqrt{1-2x} - \sqrt[3]{1-3x} \right).$$

Step 1: Use Binomial Expansions

We use the binomial expansion for small values of x to approximate both terms.
- For $\sqrt{1-2x}$, we use the binomial expansion for $(1+z)^{1/2}$ around $z=0$:

$$\sqrt{1-2x} = 1 - x + O(x^2).$$

- For $\sqrt[3]{1-3x}$, we use the binomial expansion for $(1+z)^{1/3}$ around $z=0$:

$$\sqrt[3]{1-3x} = 1 - x + O(x^2).$$

Step 2: Substitute the Expansions

Substitute these expansions into the original expression:

$$\sqrt{1-2x} - \sqrt[3]{1-3x} = \left(1 - x + O(x^2)\right) - \left(1 - x + O(x^2)\right).$$

Simplifying:

$$\sqrt{1-2x} - \sqrt[3]{1-3x} = 0 + O(x^2).$$

Step 3: Take the Limit

As $x \to 0$, the higher-order terms $O(x^2)$ vanish, so we have:

$$\lim_{x \to 0} \left(\sqrt{1-2x} - \sqrt[3]{1-3x} \right) = 0.$$

Final Answer

Thus, the value of the limit is:

$$\lim_{x \to 0} \left(\sqrt{1-2x} - \sqrt[3]{1-3x} \right) = 0.$$