UNIVERSITÀ DEGLI STUDI DI TRENTO

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THESIS

Phase-matching method by temperature control of partially degenerate Four-Wave Mixing fenomena

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Abstract: studying degenerate Four-Wave Mixing fenomena in a rectangular silicon-based waveguide. Analyze its behaviour at different temperatures to maximize phase-match. Developing a project to build an actual prototype.

Contents

1 Introduction

1.1 Motivation

In my scholastic career I approached the study of optics in an elective course of my third year of undergraduate academic degree. I became very interested in the subject, in particular because of the potential developement of photonics for communication and computing. I discussed different topics with the professors and some researchers. One of the researcher, who would later become my advisor, suggested me a few interesting topics in guided-wave optics which could be studied as a undergraduate thesis.

I chose to study the temperature behaviour of silicon waveguides. The reason of the choice is that there are always discrepancies between the design of silicon objects and their actual production by the factory. The aim of the study is to exploit behaviour of silicon to correct these possible production defects of the materials.

The study is focused on a silicon-based waveguide with a silicon (Si) core of rectangular section and a silica ($Si0_2$) cladding. The purpose of the waveguide is to generate light through a partial degenerate Four-Wave Mixing phenomenon.

1.2 Partially degenerate Four-Wave Mixing

Four-Wave Mixing (FWM) is a nonlinear optical effect due to the susceptibility of silicon. Silicon is an isotropic material and therefore second order susceptibility $\chi^{(2)}$ (almost) vanishes. Third order susceptibility $\chi^{(3)}$, on the other hand, is non-zero and is responsible among the other effects for FWM, which is indeed a third order phenomenon.

In general FWM implies the mix of four waves of different frequency. Partial degenerate FWM, instead, is a specific case in which two waves have the same frequency and are considered as one (pump). Therefore it is about the pump wave and a second weaker one (signal) which generate a third wave (idler) of frequency different from that of the former two.

2 Electromagnetic waves in dielectric media

To understand electromagnetic phenomena the starting point is the well known Maxwell's equations:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1a}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{1b}$$

$$\nabla \cdot \mathbf{D} = \rho_\mathrm{f} \tag{1c}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{1d}$$

where $\mathbf{E}$ and $\mathbf{H}$ are electric and magnetic field vectors and $\mathbf{D}$ and $\mathbf{B}$ are corresponding electric and magnetic flux densities. $\mathbf{J}$ is the current density vector and ρ_f is the free charge density. In dielectric media, such as silicon, we can also consider $\mathbf{J} = 0$ and $\rho_\mathrm{f} = 0$.

$\mathbf{D}$ and $\mathbf{B}$ are related to the electromagnetic fields through the relations:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \tag{2a}$$

$$\mathbf{H} = \mu_0 \mathbf{H} + \mathbf{M} \tag{2b}$$

where ε_0 is the vacuum permittivity, μ_0 is the vacuum permeability, and $\mathbf{P}$ and $\mathbf{M}$ are the induced electric and magnetic polarizations repectively. For a nonmagnetic medium, as in our case, $\mathbf{M} = 0$.

Solution in linear dielectric medium

A linear dielectric medium is characterized by a linear relation between the polarization density and the electric field:

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E} \tag{3}$$

where χ is the susceptibility of the medium and $\varepsilon_r \equiv 1 + \chi$ is the relative dielectric constant.

Taking the curl of Eq. (1a) and substituting in it Eq. (1c) it leads to the Helmholtz equation:

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} \tag{4}$$

where $c = 1/\sqrt{\mu_0 \varepsilon_0}$ is the speed of light in vacuum.

Defining the refractive index $n \equiv \sqrt{\varepsilon_r}$ of the material the previous equation (4) and using the definition of $\mathbf{P}$ as in Eq. (3)

$$\nabla^2 \mathbf{E} - \frac{n^2}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \tag{5}$$

Moreover, n represent the ratio between the speed of light in vacuum and in the material $n = c/v$ and it depends on the frequency of the EM wave.

The general solution of Eq. (4) is a transverse electromagnetic (TEM) wave, whose electric and magnetic fields are perpendicular to the propagation direction. Without losing generality, we could assume $\hat{z}$ as the propagation direction and electric field of the form:

$$\mathbf{E} = \frac{1}{2} \hat{x}\, E \exp[\mathrm{i}(kz - \omega t)] + c.c. \tag{6}$$

with E amplitude of the electric field, $k = 2\pi/\lambda$ the wavevector and $\omega = 2\pi f$ the angular frequency. λ and f are relatively the wavelength and the frequency of the electromagnetic wave. The magnetic field has a similar equation.

2.1 Silicon waveguides

In linear optics the technology for trasmitting a light wave by confining it in a finite space is the guided-wave optics. The instruments employed for achieving such purpose are called waveguides.

In a ray-optics picture, silicon waveguides are dielectric waveguides and make use of the interface between two media with different refractive index. Precisely they exploit the phenomenon of total internal reflection: if a propagating wave reaches a boundary between two mediums, one with higher refractive index (n_H) and another with lower refractive index (n_L), with an angle of incidence greater than the *critical angle*, then the wave is completely reflected in the first medium. Angles are defined relative to the normal vector of the interface and the critical angle can be obtained by the Snell law:

$$\theta_C = \arcsin\left(\frac{n_L}{n_H}\right)$$

Silicon waveguides are composed by core and cladding **(?)**. The former is the higher refractive index medium in which the wave is confined, the latter is the lower refractive index medium which surrounds the core, thus creating the interface. Theoretically the cladding could also be vacuum.

Wave equation and modes

Considering an ideal dielectric waveguide (no absorption) with the purpose of transmitting EM waves in the $\hat{z}$ direction, the total field distribution is a sum of continous TEM plane waves. Defining the amplitude of the fields as $E(\mathbf{r}, t) = F(x, y)A(z, t)$ and $k = \beta$ and substituting them in Eq. (6) leads to the following equation:

$$\mathbf{E} = \frac{1}{2}\hat{x}\, F(x, y)A(z, t)\exp[\mathrm{i}(\beta z - \omega t)] + c.c.$$

The previous equation represents what is called a "transverse electric" (TE) wave. A second family of solution could be acquired by defining the sum of TEM wave with the magnetic field always perpendicular to the propagation direction. This is called a "transverse magnetic" (TM) wave.

Is then possible to study the propagation of the wave along the waveguide using the Helmholtz equation (4) or (5), obtaining the next equations:

$$\frac{\partial^2 F(x, y)}{\partial x^2} + \frac{\partial^2 F(x, y)}{\partial y^2} + \left[k_0^2 n^2(x, y) - \beta^2\right] F(x, y) = 0 \tag{7a}$$

$$\frac{\partial^2 \tilde{A}(z, \omega)}{\partial z^2} + 2\mathrm{i}\beta\frac{\partial \tilde{A}(z, \omega)}{\partial z} + 3n^2 k_0^2 \tilde{A}(z, \omega) = 0 \quad ! \neq ! \quad 2\mathrm{i}\beta_0\frac{\partial \tilde{A}(z, \omega)}{\partial z} + (\tilde{\beta}^2 - \beta_0^2)\tilde{A}(z, \omega) = 0 \tag{7b}$$

where k_0 is the wavevector in vacuum and $n = n(x, y)$ is a function of the position (mainly x and y for a straight waveguide).

The first equation, once solved, leads to the field distribution $F(x, y)$ in the plane perpendicular to the waveguide axis ($\hat{z}$). Its solution is strictly linked to the geometry of the waveguide and to the dependence of the refractive index n to the position in the plane.

The second equation (7b), on the other hand, define the change of field distribution along the $\hat{z}$ axis, or the waveguide axis. Imposing to this equation to be zero means that we force the field distribution F not to change. Indeed not all field distributions are transmitted through the waveguide without energy loss **(?)** and thus without changing F. Those field distributions which maintain the same transverse distribution and polarization at all locations along the waveguide axis are called modes. These modes can be categorized in two main groups, depending on the polarization of the wave: transverse electric (TE) modes and transverse magnetic (TM) modes, which have respectively the electric and magnetic field transverse to the waveguide axis.

Solving for β, we obtain that only a discrete set of values β_m verify the equation. Replacing those values in Eq. (7a), leads to the field distribution F(x,y) to be defined. Therefore both TE and TM modes can be again classified by their order, that is proportional to the number of maxima of the modulus of the electric and magnetic fields in the core section. The first order has only one maximum at the center of the core, the second order has two maxima placed simmetrically from the center and so on.

Moreover it is noteworthy that the field distributions of even orders are even functions and those of odd orders are odd functions.

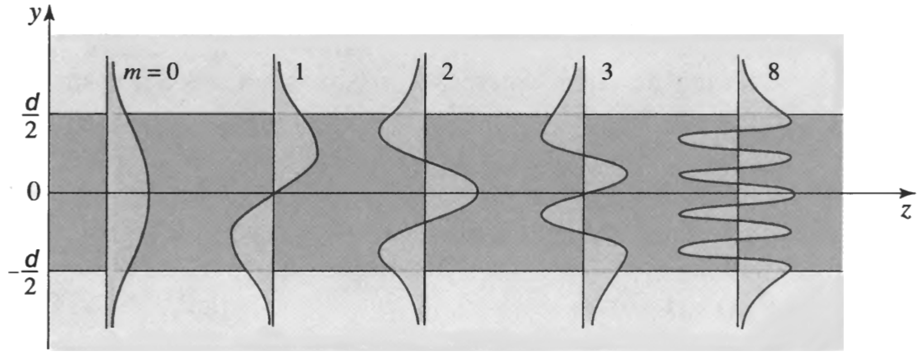

Figure 1: graphical rapresentation of field distribution of 1D modes

Propagation constant and effective index

A very important parameter of each mode is its *propagation constant* β, defined as the component of the wavevector k in the waveguide axis:

$$\beta \equiv k|_z \tag{8}$$

Higher-order modes travel with smaller propagation constants. For a 1D slab waveguide β has a simple relation with the bounce angles θ_m, which are quantized between 0 and $\bar{\theta}_C = \pi - \theta_C$:

$$\beta_m = n_H k_0 \cos\theta_m$$

From the propagation constant, we can also define the refractive index relative to the propagation mode, called *effective index*:

$$\beta = k_0 n_{eff} \quad \Rightarrow \quad n_{eff} \equiv \frac{\beta}{k_0} \tag{9}$$

Its value is between the higher index of the core and lower one of the cladding. Again, for the 1D slab waveguide case, the effective index can be rewritten as in the following formula:

$$n_{eff} \equiv n_H \cos\theta_m$$

2.2 Rectangular dielectric waveguides

In general, in a rectangular dielectric waveguide the classification of modes need two indexes because we have two degrees of freedom. In Figure (2.2) is *** a graphical representation of the distribution of the fields in modulus.

For our purpose, due to the thin geometry of the core, modes with the higher effective index are fundamental modes in the height axis. The first row is of our interest. Only one significant index is then left to describe the order of modes.

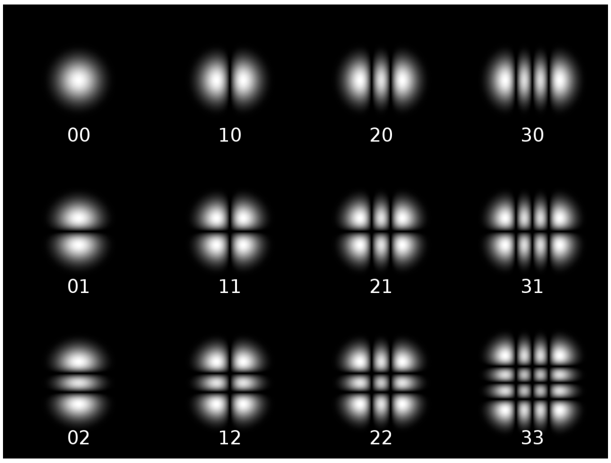

Figure 2: graphical rapresentation of field distribution of 2D modes

3 Non-linear optics

A nonlinear dielectric medium is characterized by a nonlinear relation between $\mathbf{P}$ and $\mathbf{E}$, such as

$$\mathbf{P} = \varepsilon_0 \left(\chi^{(1)} \cdot \mathbf{E} + \chi^{(2)} : \mathbf{E}^2 + \chi^{(3)} \vdots \mathbf{E}^3 + \cdots \right) = \mathbf{P}_L + \mathbf{P}_{NL} \tag{10}$$

which originates from the behaviour of bound electrons.

In everyday condition linear effects are much larger than nonlinear ones. The relation between $\mathbf{P}$ and $\mathbf{E}$ becomes nonlinear when E has a value comparable to interatomic electric fields, which are typically $\sim 10^5 - 10^8$ V m^{-1}.

For an isotropic medium the second order term is zero (in the dipole approximation) and this is our scenario. Thus the dominant nonlinearity is the third order and the material is called a Kerr medium.

3.1 Third order effect: Four-Wave Mixing

Many processes are result of third order nonlinearities: Third-Harmonic Generation (THG), Kerr Effect, Cross-phase modulation (XPM), Self-phase modulation (SPM) and Four-Wave Mixing (FWM).

The parametric process of Four-Wave Mixing originates from the third order nonlinear response of a material to an electromagnetic field and involves interaction among four optical waves.

The nonlinear component of the induced electric polarization vector, considering significant only the third-order term, is

$$\mathbf{P}_{NL} = \varepsilon_0 \chi^{(3)} \vdots \mathbf{E}^3 \tag{11}$$

Considering a silicon waveguide with a rectangular core section, to study the propagation of four wave, the total electric field and be expressed as

$$\mathbf{E} = \frac{1}{2} \hat{x} \sum_{j=1}^{4} E_j \exp[i(\beta_j z - \omega_j t)] + c.c. \tag{12}$$

Substituting Eq. (11) in Eq. (12) we obtain $8^3 = 512$ terms. We can then write the non linear terms P_{NL} of the induced electric polarization as

$$\mathbf{P} = \frac{1}{2}\hat{x}\sum_{j=1}^{4} P_j \exp[i(\beta_j z - \omega_j t)] + c.c. \tag{13}$$

Replacing **??** in Eq. (4) leads to eight coupled equation: four of them are equivalent of Eq. (7a) and describe the field distribution of the mode of each wave. The latter four are the equivalent of Eq. (7b) and describe how the field change while travelling through the waveguide.

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considering $E_j(\mathbf{r}) = F_j(x, y)A_j(z)$ where F_j is the spatial distribution of the mode

$$\frac{dA_1}{dz} = \frac{\mathrm{i}n_L\omega_1}{c}\left[\left(f_{11}|A_1|^2 + 2\sum_{k\neq 1} f_{1k}|A_k|^2\right)A_1 + 2f_{1234}A_2^*A_3A_4\mathrm{e}^{+\mathrm{i}\Delta k_z}\right] \tag{14a}$$

$$\frac{dA_2}{dz} = \frac{\mathrm{i}n_L\omega_2}{c}\left[\left(f_{22}|A_2|^2 + 2\sum_{k\neq 2} f_{2k}|A_k|^2\right)A_2 + 2f_{2134}A_1^*A_3A_4\mathrm{e}^{+\mathrm{i}\Delta k_z}\right] \tag{14b}$$

$$\frac{dA_3}{dz} = \frac{\mathrm{i}n_L\omega_3}{c}\left[\left(f_{33}|A_3|^2 + 2\sum_{k\neq 3} f_{3k}|A_k|^2\right)A_3 + 2f_{3412}A_1A_2A_4^*\mathrm{e}^{-\mathrm{i}\Delta k_z}\right] \tag{14c}$$

$$\frac{dA_4}{dz} = \frac{\mathrm{i}n_L\omega_4}{c}\left[\left(f_{44}|A_4|^2 + 2\sum_{k\neq 4} f_{4k}|A_k|^2\right)A_4 + 2f_{4321}A_1A_2A_3^*\mathrm{e}^{-\mathrm{i}\Delta k_z}\right] \tag{14d}$$

where

$$f_{ij} = \frac{\langle |F_i|^2|F_j|^2\rangle}{\sqrt{\langle |F_i|^2\rangle\langle |F_j|^2\rangle}} \tag{15}$$

and

$$f_{ijkl} = \frac{\langle F_i^*F_j^*F_kF_l\rangle}{\sqrt{\langle |F_i|^2\rangle\langle |F_j|^2\rangle\langle |F_k|^2\rangle\langle |F_l|^2\rangle}} \tag{16}$$

Approximate solution

The previous equations are quite general (they include SPM, XPM in addition to FWM) and require a numerical approach to be solved exactly. For this reason, we propose an approximate solution to understand the physical implication. The first approximation is called "undepleted pump" and consist of assuming the pump waves to be much more intense than the other waves. As a further simplification, we assume that all overlap integrals are nearly the same

$$f_{ijkl} \approx f_{ij} \approx \frac{1}{A_{eff}} \qquad i, j, k, l = 1, 2, 3, 4 \tag{17}$$

From this definition follows naturally the nonlinear parameter definition

$$\gamma_j = \frac{n_2\omega_j}{cA_{eff}} \approx \gamma \tag{18}$$

where γ is the average value (because frequencies have relatively small differencies).

Using the last two equations and the undepleted pump approximation, (14a) and (14b) are easily solved to obtain the pump fields

$$A_1(z) = A_1(0)\exp[\mathrm{i}\gamma(P_1 + 2P_2)z] \tag{19a}$$
$$A_2(z) = A_2(0)\exp[\mathrm{i}\gamma(P_2 + 2P_1)z] \tag{19b}$$

where $P_j = |A_j(z = 0)|^2$.

Using the same approximations on (14c) and (14d) and replacing Eqs (19) where needed we obtain two linear coupled equations for signal and idler:

$$\frac{dA_3}{dz} = 2\mathrm{i}\gamma[(P_1 + P_2)A_3 + A_1(0)A_2(0)\mathrm{e}^{-\mathrm{i}\phi}A_4^*] \tag{20a}$$
$$\frac{dA_4}{dz} = 2\mathrm{i}\gamma[(P_1 + P_2)A_4 + A_1(0)A_2(0)\mathrm{e}^{-\mathrm{i}\phi}A_3^*] \tag{20b}$$

where $\phi = [\Delta k - 3\gamma(P_1 + P_2)]z$.

These equations can be solved introducing $B_j = A_j \exp[-2\mathrm{i}\gamma(P_1 + P_2)z]$ for $j = 3, 4$ giving their general solution as

$$B_j(z) = (a_j\mathrm{e}^{gz} + b_j\mathrm{e}^{-gz})\exp(-i\kappa z/2)$$

with $\kappa = \Delta + \gamma(P_1 + P_2)$ *effective phase mismatch* and $g = \sqrt{4\gamma^2P_1P_2 - \kappa^2/4}$ the *parametric gain*.

The solutions of Eqs (20a) and (20b) are therefore

$$A_3(z) = \left(a_3\mathrm{e}^{gz} + b_3\mathrm{e}^{-gz}\right)\exp\left[-\frac{\mathrm{i}}{2}(\Delta k - 3\gamma(P_1 + P_2))z\right] \tag{21a}$$
$$A_4(z) = \left(a_4\mathrm{e}^{gz} + b_4\mathrm{e}^{-gz}\right)\exp\left[-\frac{\mathrm{i}}{2}(\Delta k - 3\gamma(P_1 + P_2))z\right] \tag{21b}$$

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**** è necessario verificare conservazione di energia e momento

In order to be efficient, the parametric process needs to verify both the conservation of energy and momentum.

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

$$\Delta k = \beta_3 + \beta_4 - \beta_1 - \beta_2$$

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$$\Omega_S = \omega_1 + \omega_3 = \omega_4 - \omega_1$$

where we assume $\omega_3 < \omega_4$.

$$\Omega_S = |\omega_P - \omega_I| = |\omega_S - \omega_P|$$

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parlare di L_{coh}

4 Computational models

4.1 Simulation

To understand the behaviour of the waveguide, it is necessary to first know how waves of different frequency are transmitted through the waveguide. We chose to simulate the modes with temperature and frequency as a parameters.

Our aim was to develop a model to work with an infrared laser pump with $\lambda_p = 1.55\,\mu\text{m}$. We narrowed our simulations to a frequency gap centered in $\omega_p = 2\pi\nu_p = 2\pi c/\lambda_p$. The gap chosen was precisely [1.45 µm, 1.65 µm] or the equivalent [182 THz, 207 THz].

The reason for this choice is that silicon is transparent at these wavelength and also because of the availability of lasers that emit at the specific wavelength.

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Our data output was therefore the field distribution and effective index for a number of modes for each parametric configuration.

The next step was to classify these data depending on the order and type (TE or TM) of the modes. We achieved this by analyzing the maxima of the field distributions. We obtained then a well organized data matrix filled with the value of effective index for each combination of parameters.

4.2 Data processing

Starting from raw data consisting of the effective index for each parametric configuration, we fitted them with a polinomial function of frequency $\omega = 2\pi\nu$ and temperature T, obtaining:

$$n_{eff} = f_{mt,\,mo}\,(\omega, T)$$

where *mt* and *mo* means respectively *mode type* (TE or TM) and *mode order* (1, 2, 3, ...)

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Order combination

Due to the intrinsic symmetry of the problem (and thus of the integrals) not all combinations of orders can produce FWM. Only those in which the sum of the orders is an even number are interesting. To narrow the possible sets, we chose to permit only full TE or full TM combinations.

We eventually obtained a list of combinations and for each one we verified the conservation of energy and momentum.

Conservation of energy and momentum

Starting from the data of the effective index of each order at different wavelengths and temperatures, we evaluated the phase mismatch of all sets. The evaluation was made on a grid of temperature and signal wavelength.

Each point was evaluated with the next system:

$$\begin{cases} \omega_I & = 2\omega_P - \omega_S \\ \\ \Delta k & = \beta_S + \beta_I - 2\beta_P \\ & = \frac{2\pi}{\lambda_S} n_{eff}\left(\omega_s, T\right) + \frac{2\pi}{\lambda_I} n_{eff}\left(\omega_I, T\right) - 2\frac{2\pi}{\lambda_P} n_{eff}\left(\omega_P, T\right) \end{cases}$$

Imposing the conservation of energy, we obtained the only frequency permitted for ω_I. We replaced its value in the conservation of momentum formula, giving us the phase mismatch as a function of signal wavelength and temperature, for each set of orders.

Selection of sets

Once evaluated all the points on the grid for all the sets available, we selected only the interesting combinations. The critical parameter on which the choice was based on is the coherence length: only those sets which had a maximum value of coherence length greater than a supposed sample length L_{sam} =1 cm was chosen.

$$L_{coh} = \frac{2\pi}{|\Delta k|} \geq L_{sam}$$

Again, we had a list of the only combination interesting for our purpose.

It is important to note that, in general, each combination could have more that one maximum of coherence length L_{coh} in wavelength for each temperature value.

Data aggregation

After the selection of sets, we evaluated the dependence of each maximum in temperature.

5 Prototyping

5.1 Heater configurations

In order to achieve a greater effect from the different temperature, we tried to move the heater from over the center of the core to the side. This operation was intended to increase the difference of behaviour between low and high orders.

5.2 Different geometries

6 Conclusion

References

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[2] B. E. A. Saleh, M. C. Teich. Fundamentals of photonics, 2nd edition. 2007