

Problem Set 1

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40.316 Game Theory - Term 8

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Question 1: [Declining Industry.] Consider two competing firms in a declining industry that cannot support both firms profitably. Each firm has three possible choices as it must decide whether or not to exit the industry immediately, at the end of this quarter, or at the end of the next quarter. If a firm chooses to exit then its payoff is 0 from that point onward. Every quarter that both firms operate yields each a loss equal to -1, and each quarter that a firm operates alone yields a payoff of 2. For example, if firm 1 plans to exit at the end of this quarter while firm 2 plans to exit at the end of the next quarter, then the payoffs are (-1, 1) because both firms lose -1 in the first quarter and firm 2 gains 2 in the second. The payoff for each firm is the sum of its quarterly payoffs.

- (a) Write down this game in matrix form
- (b) Is there any pure strategy that is dominated by some mixed strategy? Why?
- (c) Find the pure strategy Nash equilibria.
- (d) Find the unique mixed strategy Nash equilibrium (hint: you can use your answer to (b) to make things easier).

Solution

- (a)
- (b)
- (c)
- (d)

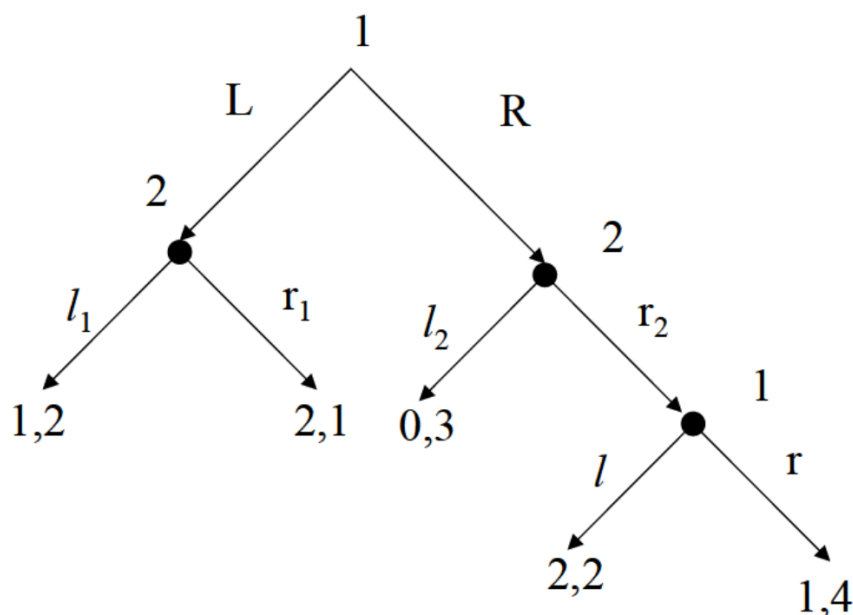
Question 2: Consider the zero-sum game known as the “matching-pennies” game. Here and elsewhere we refer to the row player as player 1 and the column player as player 2.

		2	
		L	R
1	T	1, -1	-1, 1
	B	-1, 1	1, -1

Table 1: Matching-pennies

This game has no equilibrium in pure strategies. It has a unique equilibrium in mixed strategies, where each player randomizes with equal probability yielding the equilibrium payoffs $(0, 0)$. Can you design a correlated equilibrium to improve both players’ payoffs? Hint: the mediator can design an event such that players assess with distinct subjective probabilities.

Question 3: Consider the following game in extensive form:



Solution

- Apply backwards induction in this game and find the subgame perfect equilibrium of the game
- Write the game in normal-form and find the set of pure strategy Nash equilibria.

Solution

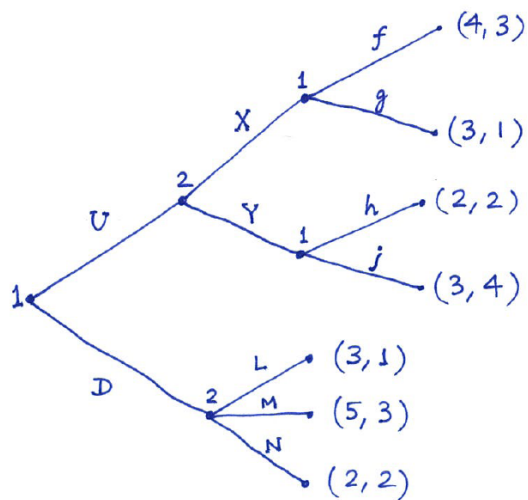
(a)

(b)

Question 4: A firm's output is $L(100 - L)$ when it uses $L \leq 50$ units of labour, and 2500 when it uses $L > 50$ units of labour. The price of output is 1. A union that represents workers presents a wage demand (a nonnegative number w), which the firm either accepts or rejects. If the firm accepts the demand, it chooses the number L of workers to employ (which you should take to be a continuous variable, not an integer); if it rejects the demand, no production takes place ($L = 0$). The firm's preferences are represented by its profit; the union's preferences are represented by the value of wL . Formulate this situation as an extensive form game and find the subgame perfect equilibria of the game. Is there an outcome of the game that both parties prefer to any subgame perfect equilibrium outcome?

Solution

Question 5: Consider the following game in extensive form:



The nodes are labeled with a 1 or a 2 depending on which player moves at that decision node.

(a) Find a subgame perfect equilibrium (in pure strategies).

- (b) Is there a Nash equilibrium for the game that is not SPE? If yes, write down one such equilibrium.

Solution

(a)

(b)

Question 6: Consider the Stackelberg game of sequential quantity choice where firm 1 moves first and firm 2 is the follower. After firm 1 sets a quantity, firm 2 chooses its quantity after observing firm 1's choice. The inverse demand function in the market is given by:

$$p = 1000 - 2q_1 - 2q_2$$

Each firm incurs a cost of 200 for each unit of output it produces. Find the subgame perfect equilibrium of the game. What is the market price and profit for each firm in the SPE?

Solution