50.021 Artificial Intelligence
Theory Homework 5

Due: every Tuesday, 6PM

[Q1]. Consider the following 3×3 filter.

$$w = \begin{bmatrix} -3 & 0 & 3 \\ -4 & 0 & 4 \\ -3 & 0 & 3 \end{bmatrix}$$

This filter w is applied to a grayscale image shown in Figure 1. Assume that the dimension the image in Figure 1 is way larger than 3×3. We can express the image in terms of matrix M, each element is numbered between 0 to 1 (0 being completely black and 1 being completely white). We are applying convolution of the filter w to M : $(M * w)$. Answer the following questions,

1. For which part of the image will the filter return a number thats furthest possible from zero (very positive or very negative)? (ignore the arrow and the words, thats for the next question) Give a max of 3 sentences explanation.
2. Will the convolution output at the location indicated in Figure 1 be positive, negative, or zero in value? Give a max of 3 sentences explanation.

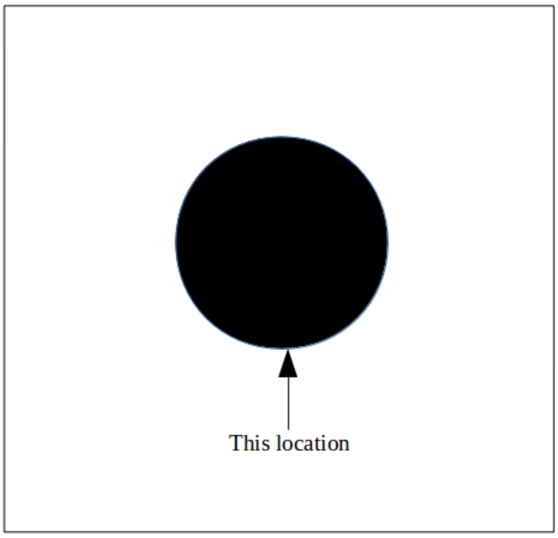

Figure 1: Figure for Q1

[Q2]. Figure 2 illustrates the before and after effect of a blurring filter when applied on a greyscale image. Blurring is an operation that makes strong edges weaker by some kind of averaging. Black pixels take a value of 0 and white pixes take a value of 1. Assume that the blurring effect is done using the following 5 by 5 filters with stride 1, and there's enough zero padding on the image before processing so that the filter will fit on the edges.
Determine whether each of the filters below can or cannot give a blurring effect and give your reason in not more than 2 sentences :

1. $\frac{1}{10}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

2. $\frac{1}{4}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 0 & 5 & 0 & 5 & 0 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & -2 & 0 \\ 0 & -5 & 0 & -5 & 0 \end{bmatrix}$

4. $\frac{1}{150}\begin{bmatrix} 3 & 4 & 8 & 4 & 3 \\ 4 & 7 & 9 & 7 & 4 \\ 8 & 9 & 10 & 9 & 8 \\ 4 & 7 & 9 & 7 & 4 \\ 3 & 4 & 8 & 4 & 3 \end{bmatrix}$

Figure 2: Blurring filter application

[Q3] Does the following neural networks suffer strongly from the vanishing gradient problem? Give your reason in not more than 2 sentences.

1. 1-Layer Feed-Foward NN
2. Very Deep Feed-Forward NN
3. Recurrent NN
4. LSTM NN
5. ResNet