50.021 Artificial Intelligence
Theory Homework 4

[Q1]. Consider a layer in CNN that takes in a single channel input of 64×64, and has 96 filters. In each of the following cases, compute the number of parameters that are learned in this layer. We assume that bias is present for each weight.

1. A convolution layer with filters of same size as the input.
2. A convolution layer with 8×8 filters with stride of 4
3. A convolution layer with 1×1 filter and a stride of 1

[Q2].

Suppose you would have a neuron which has an RBF kernel as activation function (remember the evil wolf? Drop your linear style of thoughts. Circumferential thoughts can be nice too.)

$$y = \exp(-(x_1^2 + x_2^2)) + b$$

with parameter b. What would be the shapes realized by the set of points $\{(x_1, x_2) : y((x_1, x_2)) = 0\}$ as a function of b ? Explain in at most 2 sentences and/or a little math.

Suppose now we add weights:

$$y = \exp(-(w_1 x_1^2 + w_2 x_2^2)) + b$$

What shapes could you realize now? Explain in at most 5 sentences and/or a little math. You can make references to publicly available in the internet materials to explain.

[Q3].

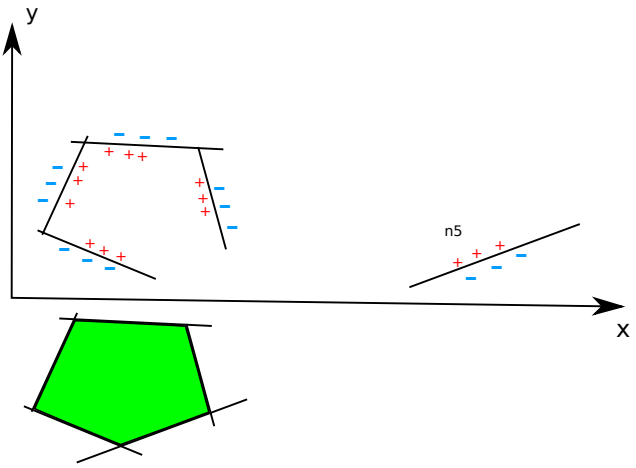

Figure 1: shapes

Suppose you have five linear neurons neurons $n_1, \ldots, n_5$, realizing above decision boundaries as shown in Figure 1. That is: for every decision boundary we have outputs are $= 0.5$ in the zones marked with red plusses, and $= 0.2$ in the zones marked with the blue minuses. As you know, each neuron is realized by:

$$n_i = 0.3H(w_1^{(i)}x_1 + w_2^{(i)}x_2 + b^{(i)}) + 0.2, H(z) \in \{0, 1\}$$

where H is the threshold activation function.

You want to predict positive values in a shape marked in green in Figure 1. You want to achieve this prediction by combining these neurons using a threshold neuron H:

$$y = H(\sum_i v_i^* n_i + b^*)$$

1. what do you have to do with the weights of n_5 so that you can move the decision boundary of n_5 so that you can realize the shape in green shown above (in the sense of having positive values inside and negative values outside.)? Give a qualitative description. Note: Give a qualitative description in 3 sentences at most. Note that there is an x- and an y-axis, which helps you to express vectors qualitatively.

2. after moving the decision boundary of n_5 appropriately, the green shape looks a bit like an logical AND-combination of the +-zones for every neuron. How to choose the weights v_i^* and the bias b^* in

   $$y = H(\sum_i v_i^* n_i + b^*)$$

   so that you can realize the green shape (in the sense of having positive values inside and negative values outside that shape)?

   Note: n_i gives out values either 0.5 or 0.2.

   I hope that exercise explains you more what neural networks with 2 layers can achieve as shapes. With 3 layers you can realize an OR-combination of green shapes as above.