- This is the table schema from which we have to eliminate all redundant left hand side attributes and all redundant functional dependencies, and which we must normalize
  - Attributes: PNAME, PNUMBER, PLOCATION, DNUMBER, SSN, HOURS
  - Functional dependencies:
    - PNAME -> PNUMBER, PLOCATION, DNUMBER
    - PNUMBER -> PNAME, PLOCATION, DNUMBER
    - PNAME, SSN -> PNUMBER, HOURS
    - PNUMBER, SSN -> PNAME HOURS

•The computation is much simpler if we give each attribute a single-character alias. So, we'll do the job using

  •Attributes: A, B, C, D, E, F, where:

    •A stands for PNAME

    •B stands for PNUMBER

    •C stands for PLOCATION

    •D stands for DNUMBER

    •E stands for SSN

    •F stands for HOURS

  •Functional dependencies:

    •A -> BCD

    •B -> ACD

    •AE -> BF

    •BE -> AF

- Results of unbundling:
  - A -> B
  - A -> C
  - A -> D
  - B -> A
  - B -> C
  - B -> D
  - AE -> B
  - AE -> F
  - BE -> A
  - BE -> F

Is A redundant in AE->B?

F1
- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- AE -> B
- AE -> F
- BE -> A
- BE -> F

F2
- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- E -> B
- AE -> F
- BE -> A
- BE -> F

Is A redundant in AE->B?
Closures to Compute

F1

- $A^{+F1} = ABCD$

  $A^{+F2} = ABCD$

- $B^{+F1} = BACD$

  $B^{+F2} = BACD$

- $(AE)^{+F1} = AEBCDF$

  $(AE)^{+F2} = AEBCDF$

- $(BE)^{+F1} = BEACDF$

  $(BE)^{+F2} = BEACDF$

F2

- $A^{+F2} =$

  $A^{+F1} =$

- $B^{+F2} =$

  $B^{+F1} =$

- $E^{+F2} = EBACDF$

  $E^{+F1} = E$

- $(AE)^{+F2} =$

  $(AE)^{+F1} =$

- $(BE)^{+F2} =$

- $(BE)^{+F1} =$

Answer is NO

Is E redundant in AE->B?

F1

- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- AE -> B
- AE -> F
- BE -> A
- BE -> F

F2

- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- A -> B
- AE -> F
- BE -> A
- BE -> F

Is E redundant in AE->B?
Closures to Compute

F1

- $A^{+F1} = ABCD$

  $A^{+F2} = ABCD$

- $B^{+F1} = BACD$

  $B^{+F2} = BACD$

- $(AE)^{+F1} = AEBCDF$

  $(AE)^{+F2} = AEBCDF$

- $(BE)^{+F1} = BEACDF$

  $(BE)^{+F2} = BEACDF$

F2

- $A^{+F2} =$

  $A^{+F1} =$

- $B^{+F2} =$

  $B^{+F1} =$

- $(AE)^{+F2} =$

  $(AE)^{+F1} =$

- $(BE)^{+F2} =$

- $(BE)^{+F1} =$

Answer is YES

New Set of FD's

F2
- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- A -> B
- AE -> F
- BE -> A
- BE -> F

Is A redundant in AE->F?

F2

- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- A -> B
- AE -> F
- BE -> A
- BE -> F

F3

- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- A -> B
- E -> F
- BE -> A
- BE -> F

Is A redundant in AE->F?
Closures to Compute

F2

- $A^{+F2} = ABCD$

  $A^{+F3} = ABCD$

- $B^{+F2} = BACD$

  $B^{+F3} = BACD$

- $(AE)^{+F2} = AEBCDF$

  $(AE)^{+F3} = AEBCDF$

- $(BE)^{+F2} = BEACDF$

  $(BE)^{+F3} = BEACDF$

F3

- $A^{+F3} =$

  $A^{+F2} =$

- $B^{+F3} =$

  $B^{+F2} =$

- $E^{+F3} = EF$

  $E^{+F2} = E$

- $(AE)^{+F3} =$

  $(AE)^{+F2} =$

- $(BE)^{+F3} =$

  $(BE)^{+F2} =$

Answer is NO

Is E redundant in AE->F?

F2

- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- A -> B
- AE -> F
- BE -> A
- BE -> F

F4

- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- A -> B
- A -> F
- BE -> A
- BE -> F

Is E redundant in AE->F? Closures to Compute

F2

- $A^{+F2} = ABCD$

  $A^{+F4} = ABCD$

- $B^{+F2} = BACD$

  $B^{+F4} = BACD$

- $(AE)^{+F2} = AEBCDF$

  $(AE)^{+F4} = AEBCDF$

- $(BE)^{+F2} = BEACDF$

  $(BE)^{+F4} = BEACDF$

F4

- $A^{+F4} =$

  $A^{+F2} =$

- $B^{+F4} =$

  $B^{+F2} =$

- $E^{+F4} = EF$

  $E^{+F2} = E$

- $(BE)^{+F4} =$

  $(BE)^{+F2} =$

Answer is NO

Is B redundant in BE->A?

F2

- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- A -> B
- AE -> F
- BE -> A
- BE -> F

F5

- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- A -> B
- AE -> F
- E -> A
- BE -> F

Is B redundant in BE->A?
Closures to Compute

F2

- $A^{+F2} = ABCD$

  $A^{+F5} = ABCD$

- $B^{+F2} = BACD$

  $B^{+F5} = BACD$

- $(AE)^{+F2} = AEBCDF$

  $(AE)^{+F5} = AEBCDF$

- $(BE)^{+F2} = BEACDF$

  $(BE)^{+F5} = BEACDF$

F5

- $A^{+F5} =$

  $A^{+F2} =$

- $B^{+F5} =$

  $B^{+F2} =$

- $E^{+F5} = EABCDF$

  $E^{+F2} = E$

- $(BE)^{+F5} =$

  $(BE)^{+F2} =$

Answer is NO

Is E redundant in BE->A?

F2
- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- A -> B
- AE -> F
- BE -> A
- BE -> F

F6
- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- A -> B
- AE -> F
- B -> A
- BE -> F

Is E redundant in BE->A?
Closures to Compute

F2

- $A^{+F2} = ABCD$

  $A^{+F6} = ABCD$

- $B^{+F2} = BACD$

  $B^{+F6} = BACD$

- $(AE)^{+F2} = AEBCDF$

  $(AE)^{+F6} = AEBCDF$

- $(BE)^{+F2} = BEACDF$

  $(BE)^{+F6} = BEACDF$

F6

- $A^{+F6} =$

  $A^{+F2} =$

- $B^{+F6} =$

  $B^{+F2} =$

- $E^{+F6} = E$

  $E^{+F2} = E$

- $(BE)^{+F6} =$

  $(BE)^{+F2} =$

Answer is YES

New Set of FD's

F6

- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- A -> B
- AE -> F
- B -> A
- BE -> F

It's pretty clear that one copy of
A->B is redundant and one copy
Of B->A is redundant, so

New Set of FD's

F7

- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

Is A->B Redundant in F7?

F7

- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

F8

- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

Is A->B Redundant in F7?
Closures to Compute

F7
- $A^{+F7} = ABCD$
  $A^{+F8} = ACD$
- $B^{+F7} =$
  $B^{+F8} =$
- $(AE)^{+F7} =$
  $(AE)^{+F8} =$
- $(BE)^{+F7} =$
  $(BE)^{+F8} =$

F8
- $A^{+F8} =$
  $A^{+F7} =$
- $B^{+F8} =$
  $B^{+F7} =$
- $(AE)^{+F8} =$
  $(AE)^{+F7} =$
- $(BE)^{+F8} =$
  $(BE)^{+F7} =$

Answer is NO

Is A->C Redundant in F7?

F7

- A -> B
- A -> C
- A -> D
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

F9

- A -> B
- A -> D
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

Is A->C Redundant in F7?
Closures to Compute

F7

- $A^{+F7} = ABCD$

  $A^{+F9} = ABDA$

- $B^{+F7} = BACD$

  $B^{+F9} = BACD$

- $(AE)^{+F7} = AEBCDF$

  $(AE)^{+F9} = AEBCDF$

- $(BE)^{+F7} = BEACDF$

  $(BE)^{+F9} = BEACDF$

F9

- $A^{+F9} =$

  $A^{+F7} =$

- $B^{+F9} =$

  $B^{+F7} =$

- $(AE)^{+F9} =$

  $(AE)^{+F7} =$

- $(BE)^{+F9} =$

  $(BE)^{+F7} =$

Answer is YES

New Set of FD's

F9

- A -> B
- A -> D
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

Is A->D Redundant in F9?

F9

- A -> B
- A -> D
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

F10

- A -> B
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

Is A->D Redundant in F9?
Closures to Compute

F9

- $A^{+F9} = ABDC$
  $A^{+F10} = ABCD$
- $B^{+F9} = BACD$
  $B^{+F10} = BACD$
- $(AE)^{+F9} = AEBDCF$
  $(AE)^{+F10} = AEBCDF$
- $(BE)^{+F9} = BEACDF$
  $(BE)^{+F10} = BEACDF$

F10

- $A^{+F10} =$
  $A^{+F9} =$
- $B^{+F10} =$
  $B^{+F9} =$
- $(AE)^{+F10} =$
  $(AE)^{+F9} =$
- $(BE)^{+F10} =$
  $(BE)^{+F9} =$

Answer is YES

New Set of FD's

F10

- A -> B
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

Is B->A Redundant in F10?

F10
- A -> B
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

F11
- A -> B
- B -> C
- B -> D
- AE -> F
- BE -> F

Is B->A Redundant in F10?
Closures to Compute

F10
- $A^{+F10} = ABCD$

  $A^{+F11} = ABCD$
- $B^{+F10} = BACD$

  $B^{+F11} = BCD$
- $(AE)^{+F10} =$

  $(AE)^{+F11} =$
- $(BE)^{+F10} =$

  $(BE)^{+F11} =$

F11
- $A^{+F11} =$

  $A^{+F10} =$
- $B^{+F11} =$

  $B^{+F10} =$
- $(AE)^{+F11} =$

  $(AE)^{+F10} =$
- $(BE)^{+F11} =$

  $(BE)^{+F10} =$

Answer is NO

Is B->C Redundant in F10?

F10

- A -> B
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

F12

- A -> B
- B -> A
- B -> D
- AE -> F
- BE -> F

Is B->C Redundant in F10?
Closures to Compute

F10

- $A^{+F10} = ABCD$

  $A^{+F12} = ABD$

- $B^{+F10} =$

  $B^{+F12} =$

- $(AE)^{+F10} =$

  $(AE)^{+F12} =$

- $(BE)^{+F10} =$

  $(BE)^{+F12} =$

F12

- $A^{+F12} =$

  $A^{+F10} =$

- $B^{+F12} =$

  $B^{+F10} =$

- $(AE)^{+F12} =$

  $(AE)^{+F10} =$

- $(BE)^{+F12} =$

  $(BE)^{+F10} =$

Answer is NO

Is B->D Redundant in F10?

F10
- A -> B
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

F13
- A -> B
- B -> A
- B -> C
- AE -> F
- BE -> F

Is B->D Redundant in F10?
Closures to Compute

F10

- $A^{+F10} = ABCD$

  $A^{+F13} = ABC$

- $B^{+F10} =$

  $B^{+F13} =$

- $(AE)^{+F10} =$

  $(AE)^{+F13} =$

- $(BE)^{+F10} =$

  $(BE)^{+F13} =$

F13

- $A^{+F13} =$

  $A^{+F10} =$

- $B^{+F13} =$

  $B^{+F10} =$

- $(AE)^{+F13} =$

  $(AE)^{+F10} =$

- $(BE)^{+F13} =$

  $(BE)^{+F10} =$

Answer is NO

Is AE->F Redundant in F10?

F10
- A -> B
- B -> A
- B -> C
- B -> D
- AE -> F
- BE -> F

F14
- A -> B
- B -> A
- B -> C
- B -> D
- BE -> F

Is AE->F Redundant in F10?
Closures to Compute

F10

- $A^{+F10} = ABCD$

  $A^{+F14} = ABCD$

- $B^{+F10} = BACD$

  $B^{+F14} = BACD$

- $(AE)^{+F10} = AEBCDF$

  $(AE)^{+F14} = AEBCDF$

- $(BE)^{+F10} = BEACDF$

  $(BE)^{+F14} = BEACDF$

F14

- $A^{+F14} =$

  $A^{+F10} =$

- $B^{+F14} =$

  $B^{+F10} =$

- $(BE)^{+F14} =$

  $(BE)^{+F10} =$

Answer is YES

New Set of FD's

F14

- A -> B
- B -> A
- B -> C
- B -> D
- BE -> F

Is BE->F Redundant in F14?

F14
- A -> B
- B -> A
- B -> C
- B -> D
- BE -> F

F15
- A -> B
- B -> A
- B -> C
- B -> D

Is BE->F Redundant in F14?
Closures to Compute

F14

- $A^{+F14} = ABCD$

  $A^{+F15} = ABCD$

- $B^{+F14} = BACD$

  $B^{+F15} = BACD$

- $(BE)^{+F14} = BEACDF$

  $(BE)^{+F15} = BEACD$

F15

- $A^{+F15} =$

  $A^{+F14} =$

- $B^{+F15} =$

  $B^{+F14} =$

Answer is NO

Final Set of FD's

F14

- A -> B
- B -> A
- B -> C
- B -> D
- BE -> F

Final Set of FD's

F14

- A -> B
- B -> A
- B -> C
- B -> D
- BE -> F

F14

- PNAME -> PNUMBER
- PNUMBER -> PNAME
- PNUMBER -> PLOCATION
- PNUMBER -> DNUMBER
- PNUMBER, SSN -> HOURS

AFTER BUNDLING:

- PNAME -> PNUMBER
- PNUMBER -> PNAME, PLOCATION, DNUMBER
- PNUMBER, SSN -> HOURS

Normalized Version

- (PNUMBER, PNAME, PLOCATION, DNUMBER), with fd's:
  - PNUMBER -> PNAME,
    PLOCATION, DNUMBER
  - PNUMBER -> PNAME,
- (PNUMBER, SSN, HOURS), with fd:
  - PNUMBER, SSN -> HOURS