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/* Geometry: Complex Arithmetic -----
// These two values are used in most of the geometry algorithms.
double PI = 2*acos(0.0);
double EPS = 1E-8:
struct pol {
 double r, t;
 pol(double R = 0, double T = 0) : r(R), t(T) {}
 };
struct point {
 double x, y;
 point (double X = 0, double Y = 0) : x(X), y(Y) {}
  point(const pol &P) : x(P.r*cos(P.t)), y(P.r*sin(P.t)) {}
  point conj() const { return point(x, -y); }
  double mag2() const { return x*x + y*y; }
  double mag() const { return sqrt(mag2()); }
  double arg() const { return atan2(y, x); }
  point operator-() const { return point(-x, -y); }
  point& operator+=(const point &a) { x += a.x; y += a.y; return *this; }
  point& operator = (const point &s) { x -= s.x; y -= s.y; return *this; }
  point& operator*=(const point &m) {
   double tx = x*m.x - y*m.y, ty = x*m.y + y*m.x;
   x = tx; y = ty; return *this;
   }
  point& operator/=(const point &d) {
   double tx = y*d.y + x*d.x, ty = y*d.x - x*d.y, t = d.mag2();
   x = tx/t; y = ty/t; return *this;
  bool operator<(const point &g) const {
   if (fabs(y-q.y) < EPS) return x < q.x;
   return y < q.y;
  bool operator==(const point &q) const {
   return (fabs(x-q.x) < EPS) && (fabs(y-q.y) < EPS);
  bool operator!=(const point &q) const { return !operator==(q); }
  };
point operator+(point a, const point &b) { return a += b; }
point operator-(point a, const point &b) { return a -= b; }
point operator*(point a, const point &b) { return a *= b; }
point operator/(point a, const point &b) { return a /= b; }
/* Geometry: Area of a polygon (positive <-> CCW orientation) ------*/
double areaPoly(vector<point> &p) {
 double sum = 0; int n = p.size();
  for (int i = n-1, j = 0; j < n; i = j++)
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sum += (p[i].conj()*p[j]).v;
 return sum/2;
/* Geometry: Heron's formula for triangle area -----*/
// Given side lengths a, b, c, returns area or -1 if triangle is impossible
double area heron(double a, double b, double c) {
 if (a < b) swap(a, b);
 if (a < c) swap(a, c);
 if (b < c) swap(b, c);
 if ((c-(a-b)) < 0) return -1;
 return sqrt((a+(b+c))*(c-(a-b))*(c+(a-b))*(a+(b-c)))/4.0;
point closest pt lineseg(point a, point b, point c) {
 b -= a; c -= a; if (b == 0) return a;
 double d = (c/b).x;
 if (d < 0) d = 0; if (d > 1) d = 1;
 return a + d*b;
/* Geometry: Rectangle in rectangle test -----
// Checks if rectangle of sides x,y fits inside one of sides X,Y
// Not tested with doubles but should work fine :)
// Code as written rejects rectangles that just touch.
bool rect in rect(int X, int Y, int x, int y) {
 if (Y > X) swap(Y, X);
 if (y > x) swap(y, x);
 double diagonal = sqrt(double(X)*X + double(Y)*Y);
 if (x < X \&\& y < Y)
  return true;
 else if (y \ge Y \mid | x \ge diagonal)
  return false;
   double w, theta, tMin = PI/4, tMax = PI/2;
   while (tMax - tMin > EPS) {
     theta = (tMax + tMin)/2.0;
     w = (Y-x*\cos(theta))/\sin(theta);
     if (w < 0 \mid \mid x * sin(theta) + w * cos(theta) < X)
       tMin = theta;
```

```
else tMax = theta;
     }
   return (w > v);
/* Geometry: Centroid of a simple polygon [O(N)] -----
// Points must be oriented (either CW or CCW), and non-convex is OK
point centroid(point p[], int n) {
 double sum = 0; point c;
 for (int i = n-1, j = 0; j < n; i = j++) {
   double area = (p[i].conj()*p[j]).y;
   sum += area; c += (p[i]+p[j]) *area;
 sum *= 3.0; c /= sum;
 return c;
/* Geometry: Convex Hull -----
struct polar cmp {
 point P0;
 polar cmp(point p = 0) : P0(p) {}
 double turn(const point &p1, const point &p2) const {
   return ((p2-P0)*(p1-P0).conj()).y;
 bool operator()(const point &p1, const point &p2) const {
   double d = turn(p1, p2);
   if (fabs(d) < EPS)
    return (p1-P0).mag2() < (p2-P0).mag2();
   else return d > 0;
vector<point> convex hull(vector<point> p) {
 sort(p.begin(), p.end());
 int n = unique(p.begin(), p.end()) - p.begin();
 sort (p.begin()+1, p.begin()+n, polar cmp(p[0]));
 if (n <= 2) return vector<point>(p.begin(), p.begin()+n);
 vector<point> hull(p.begin(), p.begin()+2); int h = 2;
 for (int i = 2; i < n; ++i) {
   while ((h > 1) & (polar cmp(hull[h-2]).turn(hull[h-1], p[i]) < EPS)) {
```

hull.pop back(); --h;

hull.push back(p[i]); ++h;

```
return hull;
/* Geometry: Area of intersection of two circles -----*/
struct circle {
 point c; double r;
double CIArea(circle &a, circle &b) {
 double d = (b.c-a.c).mag();
 if (d <= (b.r - a.r)) return a.r*a.r*PI;</pre>
 if (d <= (a.r - b.r)) return b.r*b.r*PI;</pre>
 if (d \ge a.r + b.r) return 0;
 double alpha = acos((a.r*a.r+d*d-b.r*b.r)/(2*a.r*d));
 double beta = acos((b.r*b.r+d*d-a.r*a.r)/(2*b.r*d));
 return a.r*a.r*(alpha-0.5*sin(2*alpha))+b.r*b.r*(beta-0.5*sin(2*beta));
/* Geometry: Area of union of rectangles [O(N^2)] -----*/
// Rectangle sides are parallel to the x & v axes
// May be desirable to add a constructor to 'rect' to ensure that the
// coordinates are properly sorted
struct rect {
 double minx, miny, maxx, maxy;
struct edge {
 double x, miny, maxy;
 bool operator<(const edge &e) const {
  return x < e.x;
 };
double area unionrect(vector<rect> R) {
 int n = R.size();
 vector<double> ys(2*n);
 vector<edge> e(2*n);
 for (int i = 0; i < n; ++i) {
   e[2*i].miny = e[2*i+1].miny = ys[2*i] = r[i].miny;
   e[2*i].maxy = e[2*i+1].maxy = ys[2*i+1] = r[i].maxy;
   e[2*i].x = r[i].minx;
   e[2*i].m = 1;
   e[2*i+1].x = r[i].maxx;
   e[2*i+1].m = -1;
```

```
sort(ys.begin(), ys.end());
  sort(e.begin(), e.end());
  double sum = 0, cur = 0;
  for (int i = 0; i < 2*n; ++i) {
   if (i) sum += (ys[i]-ys[i-1])*cur;
   int flag = 0; double sx = cur = 0;
   for (int j = 0; j < 2*n; ++j) {
     if (e[j].miny <= ys[i] && ys[i] < e[j].maxy) {</pre>
       if (!flag) sx = e[j].x;
       flag += e[j].m;
       if (!flag) curr += e[j].x-sx;
  return sum;
/* Geometry: Line segment a-b vs. c-d intersection (IP returned in p) -----*/
// returns 1 if intersect, 0 if not, -1 if coincident
int intersect line(point a, point b, point c, point d, point &p) {
 double num1 = ((a-c)*(d-c).conj()).v, num2 = ((a-c)*(b-a).conj()).v;
 double denom = ((d-c)*(b-a).conj()).y;
  if (fabs(denom) > EPS) {
   double r = num1/denom, s = num2/denom;
   if ((0 <= r) && (r <= 1) && (0 <= s) && (s <= 1)) {
     p = a+r*(b-a);
     return 1;
     }
    return 0;
  if (fabs(num1) > EPS) return 0;
  if (b < a) swap(a, b); if (d < c) swap(c, d);
  if (a.y == b.y) {
   if (b.x == c.x) \{ p = b; return 1; \}
   else if (a.x == d.x) { p = a; return 1; }
   else if ((b.x < c.x) \mid | (d.x < a.x)) return 0;
   }
  else {
   if (b.y == c.y) { p = b; return 1; }
   else if (a.y == d.y) { p = a; return 1; }
   else if ((b.y < c.y) \mid (d.y < a.y)) return 0;
```

return -1;

```
I/* Geometry: Area of intersection of two general polygons [O(N^2)] ------*/
int ORDER = -1; // CCW ordering, 1 for CW
struct triangle {
 point p[3];
double cross(point a, point b, point c, point d) {
 d -= c; b -= a;
 return (d*b.conj()).y;
int leftRight(const point &a, const point &b, const point &p) {
 // -1: p left of a->b, +1: p right of a->b, 0: p on a->b
 double d = cross(a, b, a, p);
  if (d > EPS) return -1;
 if (d < -EPS) return 1;
 return 0;
bool isConcave(point &a, point &b, point &c) {
  // tests if b in a->b->c is concave/flat
  return ORDER*leftRight(a, b, c) <= 0;</pre>
bool isInsideTriangle(point &a, point &b, point &c, point &p) {
  int r1 = leftRight(a,b,p), r2 = leftRight(b,c,p), r3 = leftRight(c,a,p);
 return (ORDER*r1 >= 0) && (ORDER*r2 >= 0) && (ORDER*r3 >= 0);
vector<triangle> triangulate(vector<point> &orig) {
  // Accepts a vector of n ordered vertices, returns triangulation.
 // No triangles if n < 3.
  vector<triangle> T;
  if (orig.size() < 3) return T;</pre>
  list<point> P(orig.begin(), orig.end());
  list<point>::iterator a, b, c, q;
  for (a = b = P.beqin(), c = ++b, ++c; c != P.end(); a = b, c = ++b, ++c)
   if (!isConcave(*a, *b, *c)) {
      q = P.begin(); if (q == a) { ++q; ++q; ++q; }
      while ((q != P.end()) && !isInsideTriangle(*a, *b, *c, *q)) {
        ++q; if (q == a) \{ ++q; ++q; ++q; \}
       }
      if (q == P.end()) {
        triangle t; t.p[0] = *a; t.p[1] = *b; t.p[2] = *c; T.push back(t);
       P.erase(b); b = a;
       if (b != P.begin()) --b;
  return T;
 oool isectLineSegs(point &a, point &b, point &c, point &d, point &p) {
```

```
return inside;
/* Geometry: Polygon midpoints -> vertices (n odd) ---
vector<point> midpts2vert(vector<point> &midpts) {
 int n = midpts.size(); vector<point> poly(n);
 poly[0] = midpts[0];
 for (int i = 1; i < n-1; i += 2) {
   poly[0].x += midpts[i+1].x - midpts[i].x;
   poly[0].y += midpts[i+1].y - midpts[i].y;
  for (int i = 1; i < n; i++) {</pre>
   polv[i].x = 2.0*midpts[i-1].x - polv[i-1].x;
   polv[i].v = 2.0*midpts[i-1].v - polv[i-1].v;
 return poly;
/* Geometry: 3D Primitives -----
struct point3 {
 double x, y, z;
 point3(double X=0, double Y=0, double Z=0) : x(X), y(Y), z(Z) {}
 point3 operator+(point3 p) { return point3(x + p.x, y + p.y, z + p.z); }
 point3 operator*(double k) { return point3(k*x, k*y, k*z); }
 point3 operator-(point3 p) { return *this + (p*-1.0); }
 point3 operator/(double k) { return *this*(1.0/k); }
 double mag2() { return x*x + y*y + z*z; }
 double mag() { return sqrt(mag2()); }
 point3 norm() { return *this/this->mag(); }
double dot(point3 a, point3 b) {
 return a.x*b.x + a.y*b.y + a.z*b.z;
```

//* Geometry: Point in polygon -----

inside = !inside;

int n = p.size(); bool inside = false;

for (int i = 0, j = n-1; i < n; j = i++) {

bool pt in poly(vector<point> &p, const point &a) {

if ((a-p[i]).mag()+(a-p[i]).mag()-(p[i]-p[i]).mag() < EPS)

return true; // Boundary case (pt on edge), you may want false here

if $(((p[i].y \le a.y) \&\& (a.y < p[i].y)) || ((p[j].y \le a.y) \&\& (a.y < p[i].y)))$

if (a.x-p[i].x < (p[j].x-p[i].x)*(a.y-p[i].y) / (p[j].y-p[i].y))

```
// Finds intersection p of segments a-b and c-d (returns 0 if none/inf)
 double n1 = cross(c, d, c, a), n2 = -cross(a, b, a, c);
  double dn = cross(a, b, c, d);
  if (fabs(dn) > EPS) {
   double r = n1/dn, s = n2/dn;
   if ((0 <= r) && (r <= 1) && (0 <= s) && (s <= 1)) {
     p = a+r*(b-a);
     return true;
   }
  return false:
struct radialLessThan {
 point P0;
 radialLessThan(point p = 0) : PO(p) {}
 bool operator()(const point &a, const point &b) const {
   return (ORDER == leftRight(P0, a, b));
   }
 };
double isectAreaTriangles(triangle &a, triangle &b) {
 vector<point> P:
 point p; triangle T[2] = \{a, b\};
 for (int r = 1, t = 0; t < 2; r = t++)
   for (int i = 2, j = 0; j < 3; i = j++) {
     if (isInsideTriangle(T[r].p[0],T[r].p[1],T[r].p[2],T[t].p[i]))
      P.push back(T[t].p[i]);
     for (int u = 2, v = 0; v < 3; u = v++)
       if (isectLineSegs(T[t].p[i],T[t].p[j],T[r].p[u],T[r].p[v],p))
         P.push back(p);
 if (P.emptv()) return 0;
 sort(P.begin(), P.end());
 vector<point> U; unique copy(P.begin(), P.end(), back inserter(U));
  if (U.size() >= 3) {
   sort(++U.begin(), U.end(), radialLessThan(U[0]));
   return areaPoly(U);
   }
  return 0;
double isectAreaGpoly(vector<point> &P, vector<point> &Q) {
 vector<triangle> S = triangulate(P), T = triangulate(Q);
 double area = 0;
 for (vector<triangle>::iterator s = S.begin(); s != S.end(); ++s)
   for (vector<triangle>::iterator t = T.begin(); t != T.end(); ++t)
      area += isectAreaTriangles(*s, *t);
  return -ORDER*area;
```

```
return point3(a.y*b.z - b.y*a.z, b.x*a.z - a.x*b.z, a.x*b.y - b.x*a.y);
struct line {
 point3 a, b;
 line(point3 A=point3(), point3 B=point3()) : a(A), b(B) {}
 // Direction unit vector a -> b
 point3 dir() { return (b - a).norm(); }
 };
point3 cpoint iline(line u, point3 p) {
  // Closest point on an infinite line u to a given point p
 point3 ud = u.dir();
 return u.a - ud*dot(u.a - p, ud);
double dist ilines(line u, line v) {
  // Shortest distance between two infinite lines u and v
 return dot(v.a - u.a, cross(u.dir(), v.dir()).norm());
point3 cpoint ilines(line u, line v) {
  // Finds the closest point on infinite line u to infinite line v.
 // Note: if (uv*uv - uu*vv) is zero then the lines are parallel and such a
 // single closest point does not exist. Check for this if needed.
 point3 ud = u.dir(); point3 vd = v.dir();
 double uu = dot(ud, ud), vv = dot(vd, vd), uv = dot(ud, vd);
 double t = dot(u.a, ud) - dot(v.a, ud); t *= vv;
  t = uv*(dot(u.a, vd) - dot(v.a, vd));
  t /= (uv*uv - uu*vv);
  return u.a + ud*t;
point3 cpoint lineseg(line u, point3 p) {
 // Closest point on a line segment u to a given point p
 point3 ud = u.b - u.a; double s = dot(u.a - p, ud)/ud.mag2();
 if (s < -1.0) return u.b;
  if (s > 0.0) return u.a;
  return u.a - ud*s;
  }
struct plane {
 point3 n, p;
 plane(point3 ni = point3(), point3 pi = point3()) : n(ni), p(pi) {}
 plane(point3 a, point3 b, point3 c) : n(cross(b-a, c-a).norm()), p(a) {}
  //Value of d for the equation ax + by + cz + d = 0
 double d() { return -dot(n, p); }
point3 cpoint plane (plane u, point3 p) {
  //Closest point on a plane u to a given point p
  return p - u.n*(dot(u.n, p) + u.d());
```

point3 cross(point3 a, point3 b) {

```
point3 iline isect plane(plane u, line v) {
  //Point of intersection between an infinite line v and a plane u.
  //Note: if dot(u.n. vd) == 0 then the line and plane do not intersect at
 //a single point. Check for this case if it is needed.
 point3 vd = v.dir();
  return v.a - vd*((dot(u.n, v.a) + u.d())/dot(u.n, vd));
line isect planes(plane u, plane v) {
 //Infinite line of intersection between two planes u and v.
 //Note: if dot(v.n, uvu) == 0 then the planes do not intersect at a line.
 //Check for this case if it is needed.
 point3 o = u.n*-u.d(), uv = cross(u.n, v.n);
 point3 uvu = cross(uv, u.n);
  point3 a = o - uvu*((dot(v.n, o) + v.d())/(dot(v.n, uvu)*uvu.mag2()));
  return line(a, a + uv);
/* Geometry: Great Circle distance (lat[-90,90], long[-180,180])------*/
double greatcircle(double lt1, double lo1, double lt2, double lo2, double r) {
 double a = PI*(1t1/180.0), b = PI*(1t2/180.0);
 double c = PI*((102-101)/180.0);
  return r*acos(sin(a)*sin(b) + cos(a)*cos(b)*cos(c));
/* Geometry: Circle described by three points ------*/
bool circle(point p1, point p2, point p3, point &center, double &r) {
 double G = 2*((p2-p1).conj()*(p3-p2)).y;
 if (fabs(G) < EPS) return false;
 center = p1*(p3.mag2()-p2.mag2());
 center += p2*(p1.mag2()-p3.mag2());
  center += p3*(p2.mag2()-p1.mag2());
  center /= point(0, G); r = (p1-center).mag();
  return true;
/* Arithmetic: Discrete Logarithm solver [O(sqrt(P)] ------*/
// Given prime P, B, and N, finds least L such that B^L == N \pmod{P}
typedef unsigned int UI;
typedef unsigned long long ULL;
map<UI,UI> M;
```

UI times(UI a, UI b, UI m) {

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```
return (ULL) a * b % m;
UI power(UI val, UI power, UI m) {
 UI res = 1;
 for (UI p = power; p; p >>= 1) {
   if (p & 1)
    res = times(res, val, m);
   val = times(val, val, m);
   }
 return res;
UI discrete log(UI p, UI b, UI n) {
 UI jump = sqrt(double(p)); M.clear();
 for (UI i = 0; i < jump && i < p-1; ++i)
   M[power(b,i,p)] = i+1;
 for (UI i = 0, j; i < p-1; i += jump)
  if (j = M[times(n, power(b, p-1-i, p), p)])
     return (i+j-1)%(p-1);
 return -1;
/* Arithmetic: Cubic equation solver -----
struct Result {
 int n:
         // Number of solutions
 double x[3]; // Solutions
 };
Result solve cubic (double a, double b, double c, double d) {
 long double a1 = b/a, a2 = c/a, a3 = d/a;
 long double q = (a1*a1 - 3*a2)/9.0, sq = -2*sqrt(q);
 long double r = (2*a1*a1*a1 - 9*a1*a2 + 27*a3)/54.0;
 double z = r*r-q*q*q, theta;
 Result s;
 if(z \le 0)
   s.n = 3; theta = acos(r/sqrt(q*q*q));
   s.x[0] = sq*cos(theta/3.0) - a1/3.0;
   s.x[1] = sq*cos((theta+2.0*PI)/3.0) - a1/3.0;
   s.x[2] = sg*cos((theta+4.0*PI)/3.0) - a1/3.0;
   }
  else {
   s.n = 1; s.x[0] = pow(sqrt(z) + fabs(r), 1/3.0);
   s.x[0] += q/s.x[0]; s.x[0] *= (r < 0) ? 1 : -1;
   s.x[0] = a1/3.0;
  return s;
```

```
/* Combinatorics: Digit Occurrence count ------
// Given digit d and value N, returns # of times d occurs from 1..N
long long digit count(int digit, int max) {
 long long res = 0; char buff[15];
 int i, count;
 if(max <= 0) return 0;</pre>
 res += \max/10 + ((\max % 10) >= digit ? 1 : 0);
 if(digit == 0) res--;
 res += digit count(digit, max/10 - 1) * 10;
 sprintf(buff, "%d", max/10);
 for(i = 0, count = 0; i < strlen(buff); i++)
   if(buff[i] == digit+'0') count++;
 res += (1 + max%10) * count;
 return res;
/* Combinatorics: Josephus Ring Survivor (n people, dismiss every m'th) ----*/
int survive[MAXN];
void josephus(int n, int m) {
 survive[1] = 0;
  for (int i = 2; i \le n; i++)
    survive[i] = (survive[i-1]+(m%i))%i;
/* Combinatorics: Permutation index on distinct characters -----*/
// Returns perm. index of a string according to lex. ordering.
// Warning: does not work with repeated chars.
int permdex (char *s) {
 int size = strlen(s), index = 0;
 for (int i = 1; i < size; ++i) {
  for (int j = i; j < size; ++j)
    if (s[i-1] > s[j]) ++index;
   index *= size - i;
 return index;
/* Dynamic Programming: Longest Ascending Subsequence -----
int asc seg(int *A, int n, int *S) {
 int *m, *seq, i, k, low, up, mid, start;
```

m = malloc((n+1) * sizeof(int));

```
seg = malloc(n * sizeof(int));
for (i = 0; i < n; i++) seq[i] = -1;
m[1] = start = 0;
for (k = i = 1; i < n; i++) {
 if (A[i] >= A[m[k]]) {
   seq[i] = m[k++]; start = m[k] = i;
 else if (A[i] < A[m[1]])</pre>
   m[1] = i;
 else {
   low = 1; up = k;
   while (low != up-1) {
    mid = (low+up)/2;
     if (A[m[mid]] <= A[i]) low = mid;</pre>
     else up = mid;
    seq[i] = m[low]; m[up] = i;
   }
for (i = k-1; i >= 0; i--) {
 S[i] = A[start]; start = seg[start];
free(m); free(seq);
return k;
```

```
/* Dynamic Programming: Longest Strictly Ascending Subsequence
```

```
int sasc seq(int *A, int n, int *S) {
 int *m, *seq, i, k, low, up, mid, start;
 m = malloc((n+1) * sizeof(int));
 seq = malloc(n * sizeof(int));
 for (i = 0; i < n; i++) seq[i] = -1;
 m[1] = start = 0;
  for (k = i = 1; i < n; i++) {
   if (A[i] > A[m[k]]) {
   seq[i] = m[k++]; start = m[k] = i;
   }
   else if (A[i] < A[m[1]])
    m[1] = i;
   else if (A[i] < A[m[k]]) {
     low = 1; up = k;
     while (low != up-1) {
       mid = (low+up)/2;
       if (A[m[mid]] <= A[i]) low = mid;</pre>
       else up = mid;
```

```
if (A[i] > A[m[low]]) {
       seq[i] = m[low]; m[up] = i;
  for (i = k-1; i >= 0; i--) {
   S[i] = A[start]; start = seq[start];
  free (m); free (seq);
  return k;
/* Dynamic Programming: Integer Partitioning -----
// Computes number of ways to partition integer N into M parts.
// Equivalent to # of partitions with largest element of size M.
typedef unsigned long long ULL;
ULL A[MAXN+1][MAXN+1];
void Build() {
 memset(A, 0, sizeof(A));
 for(int i = 1; i <= MAXN; ++i) {</pre>
   A[i][1] = 1;
   for (int j = 2; j \le i; ++j)
      A[i][j] = A[i-1][j-1] + A[i-j][j];
/* Generators: Gray Code -----
// All you need is that the i'th Gray Code is i^(i>>1)
char* pbits(char *s, int n, int b) {
 // This function is just printing the last 'b' bits of 'n' into buffer 's'
 char *t = s;
 for (unsigned int i = 1 << (b-1); i != 0; i >>= 1)
   *s++ = n&i ? '1' : '0';
 *s++ = 0;
  return t;
int main() {
 // Prints entire (2^n) Gray code for an input number of bits.
 unsigned int i; int n;
 char s[512];
  scanf("%d", &n);
  for(i = 0; i < (1 << n); i++)
```

```
Page 9 of 25
```

```
printf("%s\n", pbits(s, i^{(i>>1)}, n));
/* Generators: Catalan Numbers -----
long long int cat[33];
void getcat() {
 cat[0] = cat[1] = 1;
 for (int i = 2; i < 33; ++i)
   cat[i] = cat[i-1]*(4*i-6)/i;
/* Generators: Binary Strings generator (cardinal order) -----*/
char bit[MAXN];
void recurse(int n, int curr, int left) {
 if(curr == n)
   Process(n);
 else {
   if(curr+left < n) {</pre>
     bit[curr] = 0; recurse(n, curr+1, left);
   if(left) {
     bit[curr] = 1; recurse(n, curr+1, left-1);
void gen bin card(int n) {
 for(int i = 0; i <= n; i++) {
   printf("Cardinality %d:\n", i);
   recurse(n, 0, i);
/* Graph Theory: Maximum Bipartite Matching -----
/* How to use (sample at bottom):
  For vertex i of set U:
 match[i] = -1 means i is not matched
  match[i] = x means the edge i \rightarrow (x-|U|) is selected
  For simplicity, use addEdge(i,j,n) to add edges, where
  0 \le i \le |U| and 0 \le j \le |V| and |U| = n.
  If there is an edge from vertex i of U to vertex
```

```
i 	ext{ of } V 	ext{ then: } e[i][i+|U|] = e[i+|U|][i] = 1.
                    *****
  - If |U| = n and |V| = m, then vertices are assumed
   to be from [0,n-1] in set U and [0,m-1] in set V.
 - Remember that match[i]-n gives the edge from i, not just match[i].
const int MAXN 300
                           // How many vertices in U+V (in total)
char e[MAXN][MAXN];
                           // MODIFIED Adj. matrix (see note)
int match[MAXN], back[MAXN], q[MAXN], tail;
void addEdge(int x, int y, int n) {
 e[x][y+n] = e[y+n][x] = 1;
int find(int x, int n, int m) {
 int i, j, r;
 if(match[x] != -1) return 0;
  memset(back, -1, sizeof(back));
  for (q[i=0]=x, tail = 1; i < tail; i++)
   for (j = 0; j < n+m; j++) {
     if(!e[q[i]][j]) continue;
     if (match[j] != -1) {
      if(back[j] == -1) {
         back[j] = q[i];
         back[q[tail++] = match[j]] = j;
       }
       match[match[q[i]] = j] = q[i];
       for (r = back[g[i]]; r != -1; r = back[back[r]])
        match[match[r] = back[r]] = r;
       return 1;
  return 0;
void bipmatch(int n, int m) {
 memset(match, -1, sizeof(match));
  for (int i = 0; i < n+m; i++) if (find (i, n, m)) i = 0;
int main() {
 int n, m, esize, x, y;
 int i, count;
 // Read size of set U into n, size of set V into m
 while(scanf("%d %d", &n, &m) == 2) {
   memset(e, 0, sizeof(e));
                                   // Clear edges
   scanf("%d", &esize);
                                      // get # of edges
   while(esize--) {
      scanf("%d %d", &x, &y);
                                      // add edges
```

```
addEdge(x, y, n);
                                    // Edges [0,n-1]->[0,m-1]
     }
   bipmatch(n, m);
                                    // Perform matching
   for (count = i = 0; i < n; i++) { // Print results
     if (match[i] != -1) {
       printf("%d->%d\n", i, match[i]-n);
       count++;
       }
   printf("Matching size: %d\n", count);
  return 0;
/* Graph Theory: Eulerian Graphs -----
/* Before adding edges, call Init() to initialize all
  necessary data structures.
  Use the provided function addEdge(x, v, c) which
  adds c number of edges between x and y.
  isEulerian(int n, int *start, int *end) returns:
    0 if the graph is not Eulerian
    1 if the graph has a Euler cycle
    2 if the graph a path, from start to end
  with n being the number of nodes in the graph
                                                                           */
const int MAXN 105  // Number of nodes
const int MAXM 505
                    // Maximum number of edges
#define min(a,b) (((a)<(b))?(a):(b))
\#define \max(a,b) (((a)>(b))?(a):(b))
#define DEC(a,b) g[a][b]--;g[b][a]--;deg[a]--;deg[b]--
int sets[MAXN], deg[MAXN];
int g[MAXN][MAXN];
int seq[MAXM], seqsize;
// Uncomment if you need copy of graph
// int g2[MAXN][MAXN], deg2[MAXN];
int getRoot(int x) {
 if (sets[x] < 0) return x;</pre>
 return sets[x] = getRoot(sets[x]);
void Union(int a, int b) {
 int ra = getRoot(a), rb = getRoot(b);
 if (ra != rb) {
   sets[ra] += sets[rb];
   sets[rb] = ra;
```

```
void Init() {
  memset(sets, -1, sizeof(sets));
  memset(q, 0, sizeof(q));
  memset(deg, 0, sizeof(deg));
void addEdge(int x, int y, int count) {
 g[x][y] += count; deg[x] += count;
 g[y][x] += count; deg[y] += count;
 Union(x,y);
int isEulerian(int n, int *start, int *end) {
 int odd = 0, i, count = 0, x;
  for (i = 0; i < n; i++)
   if (deg[i]) {
      x = i; count++;
  if (sets[getRoot(x)] != -count) return 0;
  for (i = 0; i < n; i++) {
    if (deg[i] & 1) {
      odd++;
      if(odd == 1) *start = i;
      else if (odd == 2) *end = i;
      else return 0;
  return odd ? 2 : 1;
void getPath(int n, int start, int end) {
  int temp[MAXM], tsize = 1, i, j;
  temp[0] = start;
  while(1) {
    j = temp[tsize-1];
    for (i = 0; i < n; i++) {
     if (i == end) continue;
     if (g[i][j]) {
       temp[tsize++] = i;
        DEC(i,j); break;
    if (i == n) {
     if (g[end][j]) {
       temp[tsize++] = end;
        DEC(j,end);
      }
      break;
```

```
for (i = 0; i < tsize; i++)</pre>
   if (!deg[temp[i]])
      seq[seqsize++] = temp[i];
   else getPath(n, temp[i], temp[i]);
void buildPath(int n, int start, int end) {
  segsize = 0;
 // Uncomment if you need copy of graph
 // memcpy(g, g2, sizeof(g));
 // memcpy(deg, deg2, sizeof(deg));
 getPath(n, start, end);
int main() {
 int i, x, y, start, end, n, m;
 while (scanf("%d %d", &n, &m) == 2) {
   Init();
   for (i = 0; i < m; i++) {
     scanf("%d %d", &x, &y);
     addEdge(x, y, 1);
   // Uncomment if you need copy of graph
    // memcpy(g2, g, sizeof(g2));
   // memcpy(deg2, deg, sizeof(deg2));
   int res = isEulerian(n, &start, &end);
   }
```

/* Graph Theory: Maximum Flow in a directed graph -----

```
/* - Multiple edges from u to v may be added. They are converted into a
    single edge with a capacity equal to their sum
  - Vertices are assumed to be numbered from 0..n-1
  - The graph is supplied as the number of nodes (n), the zero-based
    indexes of the source (s) and the sink (t), and a vector of edges u->v
    with capacity c (M).
const int MAXN 200
struct Edge {
 //Edge u->v with capacity c
 int u, v, c;
int F[MAXN][MAXN]; //Flow of the graph
int maxFlow(int n, int s, int t, vector<Edge> &M) {
 int u, v, c, oh, min, df, flow, H[n], E[n], T[n], C[n][n];
 vector<Edge>::iterator m;
 list<int> N; list<int>::iterator cur;
 vector<int> R[n]; vector<int>::iterator r;
```

```
for (u = 0; u < n; u++) {
  E[u] = H[u] = T[u] = 0;
  R[u].clear();
  for (v = 0; v < n; v++)
    C[u][v] = F[u][v] = 0;
for (m = M.begin(); m != M.end(); m++) {
  u = m->u; v = m->v; c = m->c;
  if (c && !C[u][v] && !C[v][u]) {
    R[u].push back(v);
    R[v].push back(u);
    }
  C[u][v] += c;
H[s] = n;
for (r = R[s].begin(); r != R[s].end(); r++) {
 F[s][v] = C[s][v]; F[v][s] = -C[s][v];
  E[v] = C[s][v]; E[s] -= C[s][v];
N.clear();
for (u = 0; u < n; u++)
 if ((u != s) && (u != t))
    N.push back(u);
for (cur = N.begin(); cur != N.end(); cur++) {
  u = *cur; oh = H[u];
  while (E[u] > 0)
    if (T[u] >= (int)R[u].size()) {
      min = 10000000;
      for (r = R[u].begin(); r != R[u].end(); r++) {
        if ((C[u][v] - F[u][v] > 0) && (H[v] < min))
          min = H[v];
        }
      H[u] = 1 + min;
      T[11] = 0:
    else {
      v = R[u][T[u]];
      if ((C[u][v] - F[u][v] > 0) && (H[u] == H[v]+1)) {
        df = C[u][v] - F[u][v];
       if (df > E[u])
         df = E[u];
        F[u][v] += df; F[v][u] = -F[u][v];
        E[u] -= df; E[v] += df;
        }
```

else

```
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```

```
T[u]++;
   if (H[u] > oh)
     N.splice(N.begin(), N, cur);
   }
 flow = 0;
 for (r = R[s].begin(); r != R[s].end(); r++)
   flow += F[s][*r];
 return flow;
/* Graph Theory: Chinese Postman Problem -----
// The maximum # of vertices solvable is roughly 20
#define MAXN 20
#define DISCONNECT -1
int q[MAXN][MAXN];
                      // Adj matrix (keep lowest cost if multiedge)
int deg[MAXN];
                      // Degree count
int A[MAXN+1];
                      // Used by perfect matching generator
                      // Sum of costs
int sum;
int odd, best;
void floyd(int n) {
 int i, j, k;
 for(k = 0; k < n; k++)
   for(i = 0; i < n; i++)
     for(j = 0; j < n; j++)
       if(g[i][k] != -1 \&\& g[k][j] != -1) {
         int temp = g[i][k] + g[k][j];
         if(g[i][j] == -1 || g[i][j] > temp)
           g[i][j] = temp;
         16/11/0/
 for (i = 0; i < n; i++) g[i][i] = 0;
void checkSum() {
 int i, temp;
 for(i = temp = 0; i < odd/2; i++)
  temp += g[A[2*i]][A[2*i+1]];
 if(best == -1 || best > temp) best = temp;
void perfmatch(int x) {
 int i, t;
 if(x == 2) checkSum();
 else {
   perfmatch (x-2);
```

for (i = x-3; i >= 0; i--) {

```
t = A[i]; A[i] = A[x-2];
      A[x-2] = t; perfmatch (x-2);
   t = A[x-2];
   for (i = x-2; i >= 1; i--) A[i] = A[i-1];
   A[0] = t;
int postman(int n) {
  int i; floyd(n);
  for (odd = i = 0; i < n; i++)
   if(deg[i]%2) A[odd++] = i;
  if (!odd) return sum;
 best = -1;
  perfmatch (odd);
  return sum+best;
int main() {
  int i, u, v, c, n, m;
  while(scanf("%d %d", &n, &m) == 2){
   // Clear graph and degree count
   memset(q, -1, sizeof(q));
   memset(deg, 0, sizeof(deg));
   for (sum = i = 0; i < m; i++) {
      scanf("%d %d %d", &u, &v, &c);
     u--; v--; deg[u]++; deg[v]++;
     if(g[u][v] == -1 \mid \mid g[u][v] > c) g[u][v] = c;
     if(g[v][u] == -1 \mid | g[v][u] > c) g[v][u] = c;
      sum += c;
    printf("Best cost: %d\n", postman(n));
/* Graph Theory: Strongly Connected Components -----
#define VI vector<int>
#define MAXN 1000
VI g[MAXN], curr;
vector< VI > scc;
int dfsnum[MAXN], low[MAXN], id;
char done[MAXN];
void visit(int x) {
 curr.push back(x);
 dfsnum[x] = low[x] = id++;
  for(size t i = 0; i < g[x].size(); i++)
```

```
low[x] <?= low[q[x][i]];
   else if(!done[q[x][i]])
     low[x] <?= dfsnum[g[x][i]];
  if(low[x] == dfsnum[x]) {
   VI c; int y;
   do {
     done[y = curr[curr.size()-1]] = 1;
     c.push back(y);
     curr.pop back();
   while (y != x);
    scc.push back(c);
void strong conn(int n) {
 memset(dfsnum, -1, n*sizeof(int));
 memset(done, 0, sizeof(done));
  scc.clear(); curr.clear();
  for (int i = id = 0; i < n; i++)
   if(dfsnum[i] == -1) visit(i);
/* Graph Theory: Min Cost Max Flow (Edmonds-Karp & Dijkstra) -----
/* Takes a directed graph where each edge has a capacity ('cap') and a
   cost per unit of flow ('cost') and returns a maximum flow network
  of minimal cost ('fcost') from s to t. USE THIS CODE FOR (MODERATELY)
   DENSE GRAPHS; FOR VERY SPARSE GRAPHS, USE mcmf4 (next)
   PARAMETERS:
     - cap (global): adjacency matrix where cap[u][v] is the capacity
      of the edge u->v. cap[u][v] is 0 for non-existent edges.
    - cost (global): a matrix where cost[u][v] is the cost per unit
     of flow along the edge u \rightarrow v. If cap[u][v] == 0, cost[u][v] is
      ignored. ALL COSTS MUST BE NON-NEGATIVE!
   - n: the number of vertices ([0, n-1] are considered as vertices).
    - s: source vertex.
    + t: sink.
   RETURNS:
     - the flow
     - the total cost through 'fcost'
     - fnet contains the flow network. Careful: both fnet[u][v] and
      fnet[v][u] could be positive. Take the difference.
```

if(dfsnum[q[x][i]] == -1)

visit(g[x][i]);

```
- Worst case: O(n^2*flow <? n^3*fcost)
// Watch for commas when typing this in!
#define NN 1024 // the maximum number of vertices + 1
int cap[NN][NN]; // adjacency matrix (fill this up)
int cost[NN][NN]; // cost per unit of flow matrix (fill this up)
int fnet[NN][NN], adj[NN][NN], deg[NN]; // flow network and adjacency list
int par[NN], d[NN];
                          // par[source] = source;
int pi[NN]; // Labelling function
#define CLR(a, x) memset(a, x, sizeof(a))
#define Inf (INT MAX/2)
#define Pot(u,v) (d[u] + pi[u] - pi[v])
bool dijkstra(int n, int s, int t) {
 // Dijkstra's using non-negative edge weights (cost + potential)
 for (int i = 0; i < n; i++)
   d[i] = Inf, par[i] = -1;
 d[s] = 0; par[s] = -n - 1;
 while (1) {
   int u = -1, bestD = Inf;
   for (int i = 0; i < n; i++)
     if (par[i] < 0 && d[i] < bestD)</pre>
       bestD = d[u = i];
    if(bestD == Inf) break;
    par[u] = -par[u] - 1;
    for (int i = 0; i < deg[u]; i++) {
      int v = adj[u][i];
      if (par[v] >= 0) continue;
      if (fnet[v][u] && d[v] > Pot(u,v) - cost[v][u])
        d[v] = Pot(u,v) - cost[v][u], par[v] = -u-1;
      if (fnet[u][v] < cap[u][v] && d[v] > Pot(u,v) + cost[u][v])
        d[v] = Pot(u,v) + cost[u][v], par[v] = -u - 1;
  for (int i = 0; i < n; i++)
   if (pi[i] < Inf)</pre>
      pi[i] += d[i];
  return par[t] >= 0;
#undef Pot.
int mcmf3(int n, int s, int t, int &fcost) {
 CLR(deg, 0); CLR(fnet, 0); CLR(pi, 0);
 for (int i = 0; i < n; i++)
   for (int j = 0; j < n; j++)
     if (cap[i][j] || cap[j][i])
        adj[i][deg[i]++] = j;
  int flow = fcost = 0;
  while (dijkstra(n, s, t)) {
    int bot = INT MAX;
```

```
bot <?= fnet[v][u] ? fnet[v][u] : (cap[u][v] - fnet[u][v]);
   for (int v = t, u = par[v]; v != s; u = par[v = u])
     if (fnet[v][u]) {
        fnet[v][u] -= bot; fcost -= bot * cost[v][u];
       }
     else {
       fnet[u][v] += bot; fcost += bot * cost[u][v];
       }
      flow += bot;
  return flow;
int main() {
  int numV; cin >> numV;
  memset(cap, 0, sizeof(cap));
  int m, a, b, c, cp, s, t;
  cin >> m >> s >> t;
  // fill up cap with existing capacities.
  // if the edge u \rightarrow v has capacity 6, set cap[u][v] = 6.
  // for each cap[u][v] > 0, set cost[u][v] to the
  // cost per unit of flow along the edge i->v
  // Uncomment the commented statements if caps/costs are bidirectional
  for (int i=0; i<m; i++) {
   cin >> a >> b >> cp >> c;
   cost[a][b] = c; // cost[b][a] = c;
   cap[a][b] = cp; // cap[b][a] = cp;
  int fcost, flow = mcmf3(numV, s, t, fcost);
  cout << "flow: " << flow << endl;
  cout << "cost: " << fcost << endl;
/* Graph Theory: Min Cost Max Flow (Edmonds-Karp & fast heap Dijkstra) -----*/
// Same as above, but better for sparse graphs
#define NN 1024 // the maximum number of vertices + 1
int cap[NN][NN]; // adjacency matrix (fill this up)
int cost[NN][NN]; // cost per unit of flow matrix (fill this up)
int fnet[NN][NN], adj[NN][NN], deg[NN]; // flow network and adjacency list
int par[NN], d[NN], q[NN], inq[NN], qs; // Dijkstra's variables
int pi[NN]; // Labelling function
#define CLR(a, x) memset(a, x, sizeof(a))
#define Inf (INT MAX/2)
#define BUBL {
 t = q[i]; q[i] = q[j]; q[j] = t; \
```

for (int v = t, u = par[v]; v != s; u = par[v = u])

```
t = inq[q[i]]; inq[q[i]] = inq[q[j]]; inq[q[j]] = t; }
#define Pot(u,v) (d[u] + pi[u] - pi[v])
bool dijkstra(int n, int s, int t) {
 // Dijkstra's using non-negative edge weights (cost + potential)
 CLR(d, 0x3F); CLR(par, -1); CLR(inq, -1);
  d[s] = qs = 0;
  inq[q[qs++] = s] = 0;
  par[s] = n;
  while (qs) {
   int u = q[0]; inq[u] = -1;
   q[0] = q[--qs];
   if (qs) inq[q[0]] = 0;
   for (int i = 0, j = 2*i + 1, t; j < qs; i = j, j = 2*i + 1) {
     if (j + 1 < qs \&\& d[q[j + 1]] < d[q[j]]) j++;
     if (d[q[j]] >= d[q[i]]) break;
      BUBL;
    for (int k = 0, v = adj[u][k]; k < deg[u]; v = adj[u][++k]) {
     if (fnet[v][u] && d[v] > Pot(u,v) - cost[v][u])
        d[v] = Pot(u,v) - cost[v][par[v] = u];
      if (fnet[u][v] < cap[u][v] && d[v] > Pot(u,v) + cost[u][v])
        d[v] = Pot(u,v) + cost[par[v] = u][v];
      if (par[v] == u) {
        if (inq[v] < 0) { inq[q[qs] = v] = qs; qs++; }
        for (int i=inq[v], j=(i-1)/2, t; d[q[i]] < d[q[j]]; i=j, j=(i-1)/2)
         BUBL;
  for (int i = 0; i < n; i++)
   if (pi[i] < Inf)</pre>
     pi[i] += d[i];
  return par[t] >= 0;
int mcmf4(int n, int s, int t, int &fcost) {
  CLR(deg, 0); CLR(fnet, 0); CLR(pi, 0);
  for (int i = 0; i < n; i++)
   for (int j = 0; j < n; j++)
     if (cap[i][j] || cap[j][i])
        adj[i][deg[i]++] = j;
  int flow = fcost = 0;
  while (dijkstra(n,s,t)) {
   int bot = INT_MAX;
   for (int v = t, u = par[v]; v != s; u = par[v = u])
      bot <?= fnet[v][u] ? fnet[v][u] : (cap[u][v] - fnet[u][v]);
```

for (int v = t, u = par[v]; v != s; u = par[v = u])

```
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```

```
if (fnet[v][u]) {
       fnet[v][u] -= bot; fcost -= bot * cost[v][u];
     else {
      fnet[u][v] += bot; fcost += bot * cost[u][v];
       }
     flow += bot;
   }
  return flow;
/* Graph Theory: Articulation Points & Bridges (adi list) [O(V+E)] ------/*/
/* - array entry art[v] is true iff vertex v is an articulation point
  - array entries bridge[i][0] and bridge[i][1] are the endpoints of a bridge
    in the graph. If bridge (u,v) is represented in the array, (v,u) is not.
  - 'bridges' is the number of bridges in the graph
  - index vertices from 0 to n-1
#define MAX N 200
#define min(a,b) (((a)<(b))?(a):(b))
// Why typedef struct, when struct will do? I didn't change this one.
typedef struct {
int deg;
 int adj[MAX_N];
 } Node;
Node alist[MAX N];
bool art[MAX N], seen[MAX N];
int df num[MAX N], low[MAX N], father[MAX N], cnt;
int bridge[MAX N*MAX N][2], bridges;
void add edge(int v1, int v2) {
 alist[v1].adj[alist[v1].deg++] = v2;
 alist[v2].adj[alist[v2].deg++] = v1;
void add bridge(int v1, int v2) {
 bridge[bridges][0] = v1;
 bridge[bridges][1] = v2;
 ++bridges;
 }
void clear() {
 for (int i = 0; i < MAX N; ++i)
   alist[i].deg = 0;
void search(int v, bool root) {
 int w, child = 0;
 seen[v] = true;
 low[v] = df num[v] = cnt++;
```

```
for (int i = 0; i < alist[v].deg; ++i) {
   w = alist[v].adi[i];
   if (df num[w] == -1) {
     father[w] = v; ++child;
     search(w, false);
     if (low[w] > df num[v]) add bridge(v, w);
     if (low[w] >= df num[v] && !root)
      art[v] = true;
     low[v] = min(low[v], low[w]);
   else if (w != father[v]) {
     low[v] = min(low[v], df num[w]);
 if (root && child > 1) art[v] = true;
void articulate(int n) {
 int child = 0;
 for (int i = 0; i < n; ++i) {</pre>
   art[i] = false;
   df num[i] = father[i] = -1;
 cnt = bridges = 0;
 memset(seen, false, sizeof(seen));
 for (int i = 0; i < n; ++i)</pre>
   if (!seen[i])
     search(i, true);
int main() {
int n, m, v1, v2, c = 0;
 while (true) {
   scanf("%d %d", &n, &m);
   if (!n && !m) break;
   clear();
   for (int i = 0; i < m; ++i) {
     scanf("%d %d", &v1, &v2);
     add edge(v1 - 1, v2 - 1);
     }
   articulate(n);
   printf("Articulation Points:");
   for (int i = 0; i < n; ++i)
    if (art[i]) printf(" %d", i + 1);
   printf("\n");
   printf("Bridges:");
   for (int i = 0; i < bridges; ++i)</pre>
     printf(" (%d,%d)", bridge[i][0] + 1, bridge[i][1] + 1);
   printf("\n\n");
```

```
/* Graph Theory: Maximum Weighted Bipartite Matching [O(n^3)] -
/st Given N workers and N jobs to complete, where each worker has a
   certain compatibility (weight) to each job, find an assignment
   (perfect matching) of workers to jobs which maximizes the
   compatibility (weight).
   - W is a 2 dimensional array where W[i][j] is the weight of
     worker i doing job j. Weights must be non-negative. If
     there is no weight assigned to a particular worker and job
     pair, set it to zero. If there is a different number of
     workers than jobs, create dummy workers or jobs accordingly
     with zero weight edges.
   - M is a 1 dimensional array populated by the algorithm where
     M[i] is the index of the job matched to worker i.
   - This algorithm could be used on non-negative floating point
                                                                             */
     weights as well.
#define MAX_N 100 // Max number of workers/jobs
int W[MAX_N][MAX_N], U[MAX_N], V[MAX_N], Y[MAX_N]; // weight vars
int M[MAX_N], N[MAX_N], P[MAX_N], Q[MAX_N], R[MAX_N], S[MAX_N], T[MAX_N];
int Assign(int n) {
// Returns max weight, corresponding matching inside global M
 int w, y; // weight vars
  int i, j, m, p, q, s, t, v;
  for (i = 0; i < n; i++) {
   M[i] = N[i] = -1; U[i] = V[i] = 0;
   for (j = 0; j < n; j++)
     if (W[i][j] > U[i])
       U[i] = W[i][j];
  for (m = 0; m < n; m++) {
    for (p = i = 0; i < n; i++)
    T[i] = 0; Y[i] = -1;
     if (M[i] == -1) {
       S[i] = 1; P[p++] = i;
    else S[i] = 0;
    1}
   while (1) {
     for (q = s = 0; s < p; s++)
      i = P[s];
       for (j = 0; j < n; j++)
```

if (!T[j]) {

y = U[i] + V[j] - W[i][j];

```
if (y == 0) {
          R[j] = i;
          if (N[j] == -1)
            goto end_phase; // I hate goto's!
          T[j] = 1; Q[q++] = j;
        else if ((Y[j] == -1) \mid | (y < Y[j])) {
          Y[j] = y; R[j] = i;
  if (q == 0) {
    y = -1;
    for (j = 0; j < n; j++)
      if (!T[j] \&\& ((y == -1) || (Y[j] < y)))
        y = Y[j];
    for (j = 0; j < n; j++) {
      if (T[j])
        V[j] += y;
      if (S[j])
        U[j] -= y;
    for (j = 0; j < n; j++)
     if (!T[j]) {
        Y[j] = y;
        if (Y[j] == 0) {
          if (N[j] == -1)
            goto end_phase; // again!
          T[j] = 1; Q[q++] = j;
  for (p = t = 0; t < q; t++) {
   i = N[Q[t]];
    S[i] = 1; P[p++] = i;
end_phase:
i = R[j]; v = M[i];
M[i] = j; N[j] = i;
while (v != -1) {
  j = v; i = R[j];
  v = M[i];
  M[i] = j; N[j] = i;
```

for (i = w = 0; i < n; i++)

```
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```

```
return w;
int main() {
  int w; // weight var
 int n, i, j;
 while ((scanf("%d", &n) == 1) && (n != 0)) {
   for (i = 0; i < n; i++)
     for (j = 0; j < n; j++)
       scanf("%d", &W[i][j]);
   w = Assign(n);
   printf("Optimum weight: %d\n", w);
   printf("Matchings:\n");
   for (i = 0; i < n; i++)
     printf("%d matched to %d\n", i, M[i]);
/* Linear Programming: Simplex Method-----
/* m - number of (less than) inequalities
   n - number of variables
   C - (m+1) by (n+1) array of coefficients:
       row 0
                    - objective function coefficients
                    - less-than inequalities
       column 0:n-1 - inequality coefficients
       column n - inequality constants (0 for objective function)
   X[n] - result variables
    return value - maximum value of objective function
                   (-inf for infeasible, inf for unbounded)
#define MAXM 400 // leave one extra
#define MAXN 400 // leave one extra
#define EPS 1e-9
#define INF 1.0/0.0
double A[MAXM][MAXN];
int basis[MAXM], out[MAXN];
void pivot(int m, int n, int a, int b)
 for (i=0;i<=m;i++)
   if (i!=a)
     for (j=0;j<=n;j++)
       if (j!=b)
        A[i][j] = A[a][j] * A[i][b] / A[a][b];
  for (j=0;j<=n;j++)
   if (j!=b) A[a][j] /= A[a][b];
  for (i=0;i<=m;i++)</pre>
```

w += W[i][M[i]];

```
if (i!=a) A[i][b] = -A[i][b]/A[a][b];
 A[a][b] = 1/A[a][b];
 i = basis[a];
 basis[a] = out[b];
 out[b] = i;
double simplex(int m, int n, double C[][MAXN], double X[]) {
 int i,j,ii,jj; // i,ii are row indexes; j,jj are column indexes
 for (i=1;i<=m;i++)</pre>
   for (j=0;j<=n;j++)
     A[i][j] = C[i][j];
 for (j=0; j<=n; j++)</pre>
   A[0][j] = -C[0][j];
 for (i=0;i<=m;i++)</pre>
   basis[i] = -i;
 for (j=0; j<=n; j++)</pre>
   out[j] = j;
 for(;;) {
   for (i=ii=1;i<=m;i++)</pre>
     if (A[i][n]<A[ii][n] || (A[i][n]==A[ii][n] && basis[i]<basis[ii]))
       ii=i;
   if (A[ii][n] >= -EPS) break;
   for (j=jj=0;j<n;j++)</pre>
     if (A[ii][j]<A[ii][jj]-EPS || (A[ii][j]<A[ii][jj]-EPS && out[i]<out[j]))</pre>
       jj=j;
   if (A[ii][jj] >= -EPS) return -INF;
   pivot(m,n,ii,jj);
   }
 for(;;) {
   for (j=jj=0;j<n;j++)</pre>
     if (A[0][j]<A[0][jj] || (A[0][j]==A[0][jj] && out[j]<out[jj]))</pre>
        jj=j;
   if (A[0][jj] > -EPS) break;
   for (i=1, ii=0; i<=m; i++)</pre>
     if (A[i][jj]>EPS &&
        (!ii | A[i][n]/A[i][jj]<A[ii][n]/A[ii][jj]-EPS |
        (A[i][n]/A[i][jj]< A[ii][n]/A[ii][jj]+EPS
        && basis[i] <basis[ii])))
        ii=i;
   if (A[ii][jj] <= EPS) return INF;</pre>
   pivot(m,n,ii,jj);
 for (j=0;j<n;j++)</pre>
   X[j] = 0;
 for (i=1;i<=m;i++)</pre>
   if (basis[i] >= 0)
```

X[basis[i]] = A[i][n];

```
/* Java Template: IO Reference -----
                                                                               /* Description: This document is a reference for the use of the
               java for regular IO purposes. It covers stdin and
                                                                               /* Description: This document is a reference for the use of the
               stdout as well as file IO. It also shows how to use
                                                                                              BigInteger class in Java. It contains sample code
               StringTokenizer for parsing.
                                                                          * /
                                                                                              that computes GCDs of pairs of integers.
import java.util.*;
                                                                                  Constants:
import java.io.*;
class IO {
                                                                                   BigInteger.ONE - The BigInteger constant one.
 public static void main(String[] args) {
                                                                                   BigInteger.ZERO - The BigInteger constant zero.
   try {
                                                                                  Creating BigIntegers
                                                                                  _____
     // For file IO, use:
     // BufferedReader in=new BufferedReader(new FileReader("probl.dat"));
                                                                                  1. From Strings
     // PrintWriter out=new PrintWriter(
                                                                                    a) BigInteger(String val);
     // new BufferedWriter(new FileWriter("prob1.out")));
                                                                                    b) BigInteger(String val, int radix);
     // For stdin/stdout IO, use:
                                                                                 2. From byte arrays
     PrintStream out = System.out;
                                                                                    a) BigInteger(byte[] val);
     BufferedReader in = new BufferedReader(new InputStreamReader(System.in));
                                                                                    b) BigInteger(int signum, byte[] magnitude)
                                                                                 3. From a long integer
     String line;
     int num=0;
                                                                                    a) static BigInteger BigInteger.valueOf(long val)
     StringTokenizer st;
                                                                                 Math operations:
     while(true) {
       // Newlines are removed by readLine()
                                                                                                       --> C = A.add(B);
                                                                                  A + B = C
       line = in.readLine();
                                                                                                        --> C = A.subtract(B);
       if(line == null) break;
                                                                                                        --> C = A.multiply(B);
                                                                                 A * B = C
       num++;
                                                                                 A / B = C
                                                                                                        --> C = A.divide(B);
       out.println("Line #" + num);
                                                                                                        --> C = A.remainder(B);
                                                                                 A % B = C
       // Split on whitespace
                                                                                 A % B = C where C > 0 --> C = A.mod(B);
       st = new StringTokenizer(line);
                                                                                 A / B = Q & A % B = R \longrightarrow C = A.divideAndRemainder(B);
       while(st.hasMoreTokens()) {
                                                                                                                  (Q = C[0], R = C[1])
                                                                                 A ^ b = C
        out.print("Token: ");
                                                                                                        --> C = A.pow(B);
        out.println(st.nextToken());
                                                                                 abs(A) = C
                                                                                                        --> C = A.abs();
                                                                                 -(A) = C
                                                                                                        --> C = A.negate();
       // To split on something else, use:
       // st = new StringTokenizer(line, delim);
                                                                                                        --> C = A.gcd(B);
                                                                                 gcd(A,B) = C
       // Or use this to change in the middle of parsing:
                                                                                  (A ^ B) % M
                                                                                                       --> C = A.modPow(B,M);
                                                                                 C = inverse of A mod M --> C = A.modInverse(M);
                                                                                 max(A,B) = C
                                                                                                        --> C = A.max(B);
      // You must flush for files!
                                                                                 min(A,B) = C
                                                                                                        --> C = A.min(B);
     out.flush();
                                                                                 Bit Operations
   catch (Exception e) {
                                                                                  \sim A = C
                                                                                                        --> C = A.not();
     System.err.println(e.toString());
                                                                                  A \& B = C
                                                                                              (AND)
                                                                                                        --> C = A.and(B);
```

return A[0][n];

```
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```

```
A \mid B = C (OR)
                         --> C = A.or(B);
                                                                                       if(line == null) break;
  A ^ B = C  (XOR)
                         --> C = A.xor(B);
                                                                                       st = new StringTokenizer(line);
  A & \sim B = C (ANDNOT) --> C = A.andNot(B);
                                                                                       a = new BigInteger(st.nextToken());
  A \ll n = C (LSHIFT) --> C = A.shiftLeft(n);
                                                                                       b = new BigInteger(st.nextToken());
  A >> n = C (RSHIFT) --> C = A.shiftRight(n);
  Clear n'th bit of A --> C = A.clearBit(n);
  Set n'th bit of A --> C = A.setBit(n);
  Flip n'th bit of A --> C = A.flipBit(n);
                                                                                   catch (Exception e) {
  Test n'th bit of A \longrightarrow C = A.testBit(n);
  Bitcount of A = n
                         --> n = A.bitCount();
  Bitlength of A = n
                        --> n = A.bitLength();
  Lowest set bit of A --> n = A.getLowestSetBit();
  Comparison Operations
                         --> A.compareTo(B) == -1;
  A < B
                                                                                // Converts from base b1 to base b2
  A == B
                         --> A.compareTo(B) == 0
                                                                                import java.math.*;
                          or A.equals(B);
                                                                                import java.io.*;
  A > B
                         --> A.compareTo(B) == 1;
                                                                                import java.util.*;
  A < 0
                         --> A.signum() == -1;
                                                                                class base convert {
  A == 0
                         --> A.signum() == 0;
  A > 0
                         --> A.signum() == 1;
  Conversion:
                                                                                   int i, x;
  double
                         --> A.doubleValue();
                                                                                   String n2 = "", n3 = "";
                        --> A.floatValue();
                                                                                   BigInteger a = BigInteger.ZERO,
                         --> A.intValue();
                         --> A.longValue();
  byte[]
                         --> A.toByteArray();
                         --> A.toString();
                                                                                     a = a.multiply(b1);
  String (base b) --> A.toString(b);
                                                                                     x = key.indexOf(n.charAt(i));
  Reads in lines of input until EOF is encountered. For each line
  of input it will extract two integers and then print out their GCD.
import java.math.*;
                                                                                   while (a.signum() == 1) {
import java.io.*;
import java.util.*;
class BigIntegers {
                                                                                     a = r[0];
 public static void main(String[] args) {
   BufferedReader in = new BufferedReader(new InputStreamReader(System.in));
   String line;
   StringTokenizer st;
                                                                                   return n3;
   BigInteger a;
   BigInteger b;
   try {
     while(true) {
                                                                                     String line, n;
       line = in.readLine();
```

```
System.out.println(a.gcd(b));
     System.err.println(e.toString());
/* Number Theory: Converting between bases (Java, arb. precision) -------
 // invalid is the string that is returned if the N is not valid
 static String invalid = new String("Number is not valid");
 private static String convert base(int b1, int b2, String n, String key) {
              b1 = BigInteger.valueOf(base1),
              b2 = BigInteger.valueOf(base2);
   for (i = 0; i < n.length(); i++) {</pre>
     if (x == -1 \mid \mid x >= base1) return invalid;
     a = a.add(BigInteger.valueOf(x));
     BigInteger r[] = a.divideAndRemainder(b2);
     n2 += key.charAt(r[1].intValue());
   for (i = n2.length()-1; i >= 0; i--) n3 += n2.charAt(i);
   if (n3.length() == 0) n3 += '0';
 public static void main(String[] args) {
```

```
int tnum, base1, base2;
     StringTokenizer st;
     // key is the base system that you may change as needed
     String key = new
   String("0123456789ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopgrstuvwxyz");
     BufferedReader in = new BufferedReader(new InputStreamReader(System.in));
     PrintStream out = System.out;
     // File IO
     // BufferedReader in = new BufferedReader(new FileReader("prob1.dat"));
      // PrintWriter out = new BufferedWriter(new FileWriter("prob1.out"));
     line = in.readLine();
                                           // Get number of test cases
     st = new StringTokenizer(line);
      tnum = Integer.parseInt(st.nextToken());
     for (int t = 0; t < tnum; t++) {</pre>
       line = in.readLine();
       st = new StringTokenizer(line);
       base1 = Integer.parseInt(st.nextToken());
       base2 = Integer.parseInt(st.nextToken());
       n = st.nextToken();
       String result = convert base(base1, base2, n, key2);
       out.println(result);
   catch (Exception e) {
     System.err.println(e.toString());
/* Number Theory: Primality Testing -----
bool isPrime(int x) {
 if (x == 1) return ONEPRIME;
 if (x == 2) return true;
 if(!(x & 1)) return false;
 for (int i = 3; i*i <= x; i += 2)
  if (!(x % i)) return false;
 return true;
/* Number Theory: Number of Divisors [O(sqrt(N))] -----
int num_divisors(int n) {
 int i, count, res = 1;
```

```
for(i = 2; i*i <= n; i++) {
   count = 0:
   while(!(n%i)) {
     n /= i; count++;
   if(count) res *= (count+1);
 if(n > 1) res *= 2;
  return res;
/* Number Theory: Prime Factorization ------
int primes[MAXP]; int psize;
void getPrimes() {
 int i, j, isprime;
 psize = 0; primes[psize++] = 2;
 for (i = 3; i \le MAXN; i+= 2) {
   for (isprime = j = 1; j < psize; j++) {</pre>
     if (i % primes[j] == 0) {
       isprime = 0;
       break;
       }
     if (1.0*primes[j]*primes[j] > i) break;
   if(isprime) primes[psize++] = i;
struct Factors {
 int size;
 int f[32];
 };
Factors getPFactor(int n) {
 Factors x;
 int i:
 x.size = 0;
 for (i = 0; i < psize; i++) {</pre>
   while (n % primes[i] == 0) {
     x.f[x.size++] = primes[i];
     n /= primes[i];
   if(1.0*primes[i]*primes[i] > n) break;
 if(n > 1)
   x.f[x.size++] = n;
  return x;
```

```
/* Given n relatively prime modular in m[0], ..., m[n-1], and right-hand
                                                                                     sides a[0], ..., a[n-1], the routine solves for the unique solution
/* Mumber Theory: Primality testing with a sieve -----*/
                                                                                     in the range 0 \le x \le m[0]*m[1]*...*m[n-1] such that x = a[i] \mod m[i]
                                                                                     for all 0 \le i \le n. The algorithm used is Garner's algorithm, which
// Consider using typedefs and functions instead of defines...
                                                                                     is not the same as the one usually used in number theory textbooks.
#define TEST(f,x) (*(f+(x)/16)&(1<<(((x)%16L)/2)))
#define SET(f,x) * (f+(x)/16) |=1 << (((x) %16L)/2)
                                                                                     It is assumed that m[i] are positive and pairwise relatively prime.
#define ONEPRIME 0 // whether or not 1 is considered to be prime
                                                                                     a[i] can be any integer.
#define UL unsigned long
#define UC unsigned char
                                                                                     If the system of equations is
UC *primes = NULL;
                                                                                      x = a[0] \mod m[0]
UL getPrimes(UL maxn) {
                                                                                      x = a[1] \mod m[1]
 UL x, y, psize=1;
 primes = calloc(((maxn)>>4)+1L, sizeof(UC));
                                                                                    then a[i] should be reduced mod m[i] first.
 for (x = 3; x*x \le maxn; x+=2)
                                                                                     Also, if 0 \le a[i] \le m[i] for all i, then the answer will fall
   if (!TEST(primes, x))
                                                                                     in the range 0 \le x \le m[0]*m[1]*...*m[n-1].
      for (y = x*x; y \le maxn; y += x \le 1) SET (primes, y);
                                                                                  int gcd(int a, int b, int *s, int *t) {
 // Comment out if you don't need # of primes <= maxn
                                                                                    int r, r1, r2, a1, a2, b1, b2, q;
  for (x = 3; x \le maxn; x+=2)
                                                                                    a1 = b2 = 1;
   if(!TEST(primes, x)) psize++;
                                                                                    a2 = b1 = 0;
  return psize;
                                                                                    while (b) {
 }
                                                                                      g = a / b; r = a % b;
int isPrime(UL x) {
                                                                                     r1 = a1 - q*b1;
 // Returns whether or not a given POSITIVE number is prime
                                                                                      r2 = a2 - g*b2;
 if(x == 1) return ONEPRIME;
                                                                                      a = b; a1 = b1; a2 = b2;
 if(x == 2) return 1;
                                                                                     b = r; b1 = r1; b2 = r2;
 if(x % 2 == 0) return 0;
  return (!TEST(primes, x));
                                                                                    *s = a1; *t = a2;
                                                                                    return a:
                                                                                   Int cra(int n, int *m, int *a) {
/* Number Theory: Sum of divisors [O(sqrt(N))] -----
                                                                                    int x, i, k, prod, temp;
typedef long long int LL;
                                                                                    int *gamma, *v;
                                                                                    gamma = malloc(n*sizeof(int));
LL sum divisors(LL n) {
                                                                                    v = malloc(n*sizeof(int));
 int i, count; LL res = 1;
                                                                                    for (k = 1; k < n; k++) {
 for (i = 2; i*i <= n; i++)
   count = 0;
                                                                                      prod = m[0] % m[k];
                                                                                      for (i = 1; i < k; i++) {
   while (n \% i == 0) {
   n /= i; count++;
                                                                                       prod = (prod * m[i]) % m[k];
   if (count) res *= (pow(i, count+1)-1)/(i-1);
                                                                                      gcd(prod, m[k], gamma+k, &temp);
                                                                                      gamma[k] %= m[k];
```

if (n > 1) res *= (pow(n, 2)-1)/(n-1);

return res;

if (gamma[k] < 0)

gamma[k] += m[k];

/* Number Theory: Chinese Remainder Theorem ------

```
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```

```
v[0] = a[0];
  for (k = 1; k < n; k++) {
   temp = v[k-1];
   for (i = k-2; i >= 0; i--) {
     temp = (temp * m[i] + v[i]) % m[k];
     if (temp < 0)
       temp += m[k];
     }
   v[k] = ((a[k] - temp) * gamma[k]) % m[k];
   if (v[k] < 0)
     v[k] += m[k];
   }
  x = v[n-1];
  for (k = n-2; k >= 0; k--)
   x = x * m[k] + v[k];
 free (gamma); free (v);
  return x;
  }
int main(void) {
 int n, *m, *a, i, x;
 while (scanf("%d", &n) == 1 && n > 0) {
   m = malloc(n*sizeof(int));
   a = malloc(n*sizeof(int));
   printf("Enter moduli:\n");
   for (i = 0; i < n; i++)</pre>
     scanf("%d", m+i);
   printf("Enter right-hand side:\n");
   for (i = 0; i < n; i++)
     scanf("%d", a+i);
   x = cra(n, m, a);
   printf("x = %d\n", x);
    free(m); free(a);
/* Number Theory: Extended Euclidean Algorithm -----
// Assumes non-negative input. Returns d s.t. d = a*x + b*y
// x,y passed in by reference, #include <algorithm> for swap function
int gcd(int a, int b, int &x, int &y) {
 x = 1; y = 0; int nx = 0, ny = 1;
 while (b) {
   int q = a/b;
   x = q*nx; swap(x, nx);
   y = q*ny; swap(y, ny);
   a = q*b; swap(a, b);
```

```
return a;
/* Number Theory: Generalized Chinese Remaindering -----
/* Given [a 0, ..., a (n-1)] and [m 0, ..., m (n-1)]
  Computes 0 \le x \le lcm(m 0, ..., m (n-1)) such that
 x == a \ 0 \ mod \ m \ 0, \ldots, x == a \ (n-1) \ mod \ m \ (n-1), if
 such an x exists. True is returned iff such an
  x exists. If x does not exist then the value at
  the address of x will not be affected.
 Complexity: O(n \log(MAX(m 0, ..., m (n-1)))
typedef long long int LLI;
LLI safe mod(LLI a, LLI m) {
 if (a < 0) return (a + m + m * (-a/m)) % m;
 else return a % m;
LLI abs(LLI a) {
 return a < 0 ? -a : a;
LLI gcdex(LLI a, LLI b, LLI *ss, LLI *tt) {
 LLI q, r[150], s[150], t[150];
 int num = 2;
 r[0] = a; r[1] = b;
 s[0] = t[1] = 1;
 s[1] = t[0] = 0;
 while (r[num - 1]) {
   q = r[num - 2] / r[num - 1];
   r[num] = r[num - 2] % r[num - 1];
   s[num] = s[num - 2] - q * s[num - 1];
   t[num] = t[num - 2] - q * t[num - 1];
   ++num;
  *ss = s[num - 2]; *tt = t[num - 2];
  return r[num - 2];
bool gen chrem(LLI *a, LLI *m, int n, LLI *x) {
 LLI g, s, t, a tmp, m tmp;
 a \text{ tmp} = \text{safe mod}(a[0], m[0]);
 m tmp = m[0];
  for (int i = 1; i < n; ++i) {
   q = \gcdex(m tmp, m[i], &s, &t);
   if (abs(a tmp - a[i]) % q) return false;
   a tmp = safe mod(a tmp + (a[i] - a tmp) / g * s * m tmp, m tmp/g*m[i]);
    m tmp = m[i];
```

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```
x = a_tmp;
  return true;
int main() {
 int n; LLI a[20], m[20], x;
 while (true) {
   scanf("%lld", &n);
   if (!n) break;
   for (int i = 0; i < n; ++i)
     scanf("%lld %lld", &a[i], &m[i]);
   if (!gen chrem(a, m, n, &x))
     printf("No solution.\n\n");
     printf("X = %11d\n', x);
/* Number Theory: Rational Reconstruction [O(log m)] -----
/* Description: Given integers m, q and k, computes integers 'num' and 'den'
   (if they exist) such that num == g*den mod m where <math>|num| < k and
  0 < \text{den} < g/k. True is returned iff den is invertible mod m. This algorithm
  is useful if computations on rational numbers is to be used when the input
  and output numbers have small numerators and denominators but intermediate
  results can have very large numerators and denominators. To use in this
  fashion, reduce the input rationals modulo some number m (probably a prime),
  perform the operations modulo m and then use rational reconstruction to
  recover the results. m and k must be selected such that |num|, den < k
  and 2*k*k < m for all input and output rational numbers.
typedef long long int LLI;
int gcd table(LLI a, LLI b, LLI *r, LLI *q, LLI *s, LLI *t) {
 int n = 2:
 assert(0 <= a && 0 < b);
 r[0] = a; r[1] = b;
 s[0] = t[1] = 1;
 s[1] = t[0] = 0;
  while (r[n-1]) {
   r[n] = r[n - 2] % r[n - 1];
   q[n-1] = r[n-2] / r[n-1];
   s[n] = s[n-2] - s[n-1] * q[n-1];
   t[n] = t[n - 2] - t[n - 1] * q[n - 1];
   ++n;
```

return n;

```
LLI gcd(LLI a, LLI b) {
  if (a < 0) return gcd(-a, b);
  if (b < 0) return gcd(a, -b);
 if (!b) return a;
 return gcd(b, a % b);
bool rat recon(LLI m, LLI q, LLI k, LLI *num, LLI *den) {
 int n, j;
  LLI r[200], q[200], s[200], t[200], quo, tj, rj;
  assert(0 <= q && q < m && 1 <= k && k <= m);
  n = \gcd table(m, q, r, q, s, t);
  q[0] = q[n - 1] = 0;
  for (j = 0; j < n \&\& r[j] >= k; ++j);
  if (t[j] > 0) {
    *num = r[j]; *den = t[j];
  else {
    *num = -r[j]; *den = -t[j];
  if (gcd(r[j], t[j]) == 1) return true;
  else {
    quo = (j == n - 1 ? 0 : (k - r[j-1]) / r[j] + 1);
    rj = r[j - 1] - quo*r[j];
    tj = t[j - 1] - quo*t[j];
    if (\gcd(rj, tj) != 1 || (tj > 0 ? tj : -tj) * k > m)
      return false;
    if (ti > 0) {
      *num = rj; *den = tj;
    else {
      *num = -rj; *den = -tj;
    return true;
int main() {
  LLI m, g, k, r, t; char c;
  scanf("%lld %lld %lld", &m, &g, &k);
  c = (rat recon(m, g, k, &r, &t) ? 'y' : 'n');
  printf("%c %lld / %lld\n", c, r, t);
/* Search: Golden section search --
```

/* Given an function f(x) with a single local minimum, a lower and upper

bound on x, and a tolerance for convergence, this function finds the

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```
minimizing value of x. f(x) should evaluate globally.
                                                                                    if (P[k + 1] == P[q]) ++k;
#define GOLD 0.381966
                                                                                      pi[q] = k;
#define move(a,b,c)
                   x[a]=x[b];x[b]=x[c];fx[a]=fx[b];fx[b]=fx[c]
double f(double x) { return x*x; } // Just an example
double golden (double xlow, double xhigh, double tol) {
                                                                                  int kmp match(char *T, char *P, int *shift) {
 double x[4], fx[4], L;
                                                                                   int n, m, q = -1, shifts = 0;
 int iter = 0, left = 0, mini, i;
                                                                                    n = strlen(T); m = strlen(P);
                                                                                    compute prefix(P, m, pi);
 fx[0] = f(x[0]=xlow);
 fx[3] = f(x[3]=xhigh);
                                                                                    for (int i = 0; i < n; ++i) {
 while (1) {
                                                                                     while (q > -1 \&\& P[q + 1] != T[i]) q = pi[q];
   L = x[3]-x[0];
                                                                                     if (P[q + 1] == T[i]) ++q;
                                                                                     if (q == m - 1) {
   if (!iter || left) {
     x[1] = x[0] + GOLD*L;
                                                                                       shift[shifts++] = i - m + 1;
     fx[1] = f(x[1]);
                                                                                       q = pi[q];
     }
                                                                                       }
   if (!iter || !left) {
     x[2] = x[3] - GOLD*L;
                                                                                   return shifts;
     fx[2] = f(x[2]);
                                                                                  int main() {
   for (mini = 0, i = 1; i < 4; i++)
                                                                                    char T[MAX LEN + 1], P[MAX LEN + 1];
     if (fx[i] < fx[mini]) mini = i;</pre>
                                                                                   int shift[MAX LEN], shifts;
   if (L < tol) break;
                                                                                   while (scanf("%s %s", T, P) != -1) {
   if (mini < 2) {
                                                                                     shifts = kmp match(T, P, shift);
     left = 1;
                                                                                     if (shifts) {
     move(3,2,1);
                                                                                       printf("Pattern occurs with shifts:");
     }
                                                                                        for (int i = 0; i < shifts; ++i) printf(" %d", shift[i]);</pre>
   else {
                                                                                        printf("\n\n");
     left = 0;
                                                                                       }
     move(0,1,2);
                                                                                      else printf("No matches.\n\n");
     }
   iter++;
  return x[mini];
                                                                                  /* Search: Suffix array [O(N log N)] ------
                                                                                   /* Notes:
                                                                                                 The build sarray routine takes in a string S of n
                                                                                                 characters (null-terminated), and constructs two
/* Search: KMP String Matching -----
                                                                                                 arrays sarray and lcp. The properties are:
/* Given strings T and P, computes the indices of T where P occurs
                                                                                               - If p = sarray[i], then the suffix of str starting at
   as a substring and stores in 'shift'. The return integer is the
                                                                                                p (i.e. S[p..n-1] is the i-th suffix when all the
  number of indices stored in 'shift'
                                                                             */
                                                                                                 suffixes are sorted in lexicographical order
#define MAX LEN 1000
                                                                                               - NOTE: the empty suffix is not included in this list,
int pi[MAX LEN];
                                                                                                       so sarray[0] != n.
void compute prefix(char *P, int m, int *pi) {
                                                                                               - lcp[i] contains the length of the longest common
 int k = pi[0] = -1;
                                                                                                prefix of the suffixes pointed to by sarray[i-1]
 for (int q = 1; q < m; ++q) {
                                                                                                 and sarray[i]. lcp[0] is defined to be 0.
   while (k >= 0 \&\& P[k + 1] != P[q]) k = pi[k];
                                                                                               - To see whether a pattern P occurs in str, you can
```

```
done with a binary search in O(|P| log n) time.
// You probably need to include <climits> here.
#define MAXN 100000
int bucket[CHAR MAX-CHAR MIN+1];
int prm[MAXN], count[MAXN];
char bh[MAXN+1];
void build_sarray(char *str, int* sarray, int *lcp) {
 int n, a, c, d, e, f, h, i, j, x;
 n = strlen(str);
 memset(bucket, -1, sizeof(bucket));
  for (i = 0; i < n; i++) {
   j = str[i] - CHAR_MIN;
   prm[i] = bucket[j];
   bucket[j] = i;
  for (a = c = 0; a \leftarrow CHAR_MAX - CHAR_MIN; a++) {
   for (i = bucket[a]; i != -1; i = j) {
     j = prm[i]; prm[i] = c;
     bh[c++] = (i == bucket[a]);
     }
   }
  bh[n] = 1;
  for (i = 0; i < n; i++)
   sarray[prm[i]] = i;
 x = 0;
  for (h = 1; h < n; h *= 2) {
   for (i = 0; i < n; i++) {
     if (bh[i] & 1) {
       x = i; count[x] = 0;
     prm[sarray[i]] = x;
   d = n - h; e = prm[d];
   prm[d] = e + count[e];
   count[e]++;
   bh[prm[d]] |= 2;
   i = 0;
   while (i < n) {
    for (j = i; (j == i || !(bh[j] & 1)) && j < n; j++) {
    d = sarray[j] - h;
       if (d >= 0) {
         e = prm[d]; prm[d] = e + count[e];
         count[e]++; bh[prm[d]] |= 2;
      for (j = i; (j == i || !(bh[j] & 1)) &  j < n; j++) {
```

look for it as the prefix of a suffix. This can be

```
d = sarray[j] - h;
       if (d >= 0 && (bh[prm[d]] & 2)) {
         for (e = prm[d]+1; bh[e] == 2; e++);
         for (f = prm[d]+1; f < e; f++)
           bh[f] &= 1;
     i = j;
   for (i = 0; i < n; i++) {
     sarray[prm[i]] = i;
     if (bh[i] == 2)
       bh[i] = 3;
 for (i = 0; i < n; i++) {</pre>
   e = prm[i];
   if (e > 0) {
     j = sarray[e-1];
     while (str[i+h] == str[j+h])
      h++;
     lcp[e] = h;
     if (h > 0) h--;
 lcp[0] = 0;
int main() {
 int sarray[MAXN], lcp[MAXN], i, n, j;
 char S[MAXN], T[MAXN];
 while (scanf("%s", S) == 1) {
   n = strlen(S);
   for (i = 0; i < n; i++)
     S[n+i] = S[i];
   S[n+n] = 0;
   build_sarray(S, sarray, lcp);
   for (i = 0; S[i]; i++)
     if (sarray[i] < n) {</pre>
       printf("%3d: %2d [%d]\n", i, lcp[i], n);
       for (j = 0; j < n; j++)
         printf("%c", S[sarray[i]+j]);
       printf("\n");
```