Decisions Under Ignorance

- In a decision under ignorance, the decision maker :
  - **knows** what the alternative *acts* and possible *states* are, and what outcomes they may result it, but
  - **unable** to **assign** any *probabilities* to the states corresponding to the outcomes
- Ordinal scale is sufficient when we are going to apply the dominance.
- There are no difference between *principle* and *rule*.

Notation

- The notations represent the rationality relationship between two acts instead of numbers.
- $\succ$ - succeed
- $\succcurlyeq$ - equal to
- $\sim$ - similar
- $a_i \succ a_j$: it is **more rational** to perform the act a_i rather than the act a_j.
- $a_i \succcurlyeq a_j$: the act a_i is **at least as rational as** the act a_j.
- $a_i \sim a_j$: the two acts are **equally rational**.

Dominance Principle

- Dominated acts must not chosen.
- Dominated acts : *acts*
  - whose all outcomes under every state are **at least as rational as** other outcomes of other acts under every state
  - **and** there are some outcomes under some states are **less rational** than outcomes of other acts under those states
- Let $v(a_i, s_j)$ be the **value of the outcome** corresponding to the act a_i and the state s_j.
- Notice if one act **does not dominate** another act in **one state** while it can **dominate another act** in **others state**,
  - the **domination process** terminate

Weak Dominance

- ***Weak Dominance*** : $a_i \succcurlyeq a_j$ if and only if $v(a_i, s) \geq v(a_j, s)$ for every state s.

Strong Dominance

- ***Strong Dominance*** : $a_i \succ a_j$ if and only if v(a_i, s_m) $\geq$ v(a_j, s_m) for every state s_m, and there is some state s_n such that v(a_i, s_n) > v(a_j, s_n).

Advantage and Disadvantage

- It can also be applied to decision under risk.
- It cannot always single out an act as the most rational.

Example

- Neither a3 nor a4 are dominated each other because $v(a_4, s_2) >$

Maximin Principle

- When decision makers cannot choose an act that is more rational among other acts by using *Dominance Principle*,
- The maximin principle focuses on the ***worst*** possible outcome of each act.
- One should ***maximize*** the ***minimal*** value obtainable with each act.
- If the **worst possible outcome** of an act is **better than** the **worst possible outcome** of another act, the **first** act should be chosen.
- This should not be used in ***decision under risk***.

	s_1	s_2	s_3	s_4	s_5
a_1	3	1	-2	0	7
a_2	11	4	5	-3	1
a_3	1	2	5	6	8
a_4	2	0	7	3	12

- Formally speaking : $a_i \succeq a_j$ if an only if min(a_i) $\geq$ min(a_j).
- If the minimum value corresponding to the outcomes of two or more acts are the same, the maximin principle ranks the acts as equally rational.

Leximin Rule

- Extended version of *Maximin Principle*
- Let $min^1(a_i)$ be the value of the worst outcome of act a_i,
- $min^2(a_i)$ be the value of its second worst outcome,
- In general,$min^n(a_i)$ be the value of its n-th worst outcome

- a_i a_j if and only if there is some positive integer n such that
  - $min^n(a_i) > min^n(a_j)$
  - regardless $min^m(a_i) = min^m(a_j)$ for all m < n

Example

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	s_1	s_2	s_3	s_4	s_5	s_6
a_1	7	-1	7	3	2	7
a_2	15	2	-1	4	7	10
a_3	-1	10	8	5	5	1
a_4	10	4	2	12	7	-1

- The worst outcomes

	s_1	s_2	s_3	s_4	s_5	s_6
a_1	7	-1	7	3	2	7
a_2	15	2	-1	4	7	10
a_3	-1	10	8	5	5	1
a_4	10	4	2	12	7	-1

- The second worst outcomes

	s_1	s_2	s_3	s_4	s_5	s_6
a_1	7	-1	7	3	2	7
a_2	15	2	-1	4	7	10
a_3	-1	10	8	5	5	1
a_4	10	4	2	12	7	-1

- The third worst outcomes

	s_1	s_2	s_3	s_4	s_5	s_6
a_1	7	-1	7	3	2	7
a_2	15	2	-1	4	7	10
a_3	-1	10	8	5	5	1
a_4	10	4	2	12	7	-1

- The fourth worst outcomes

	s_1	s_2	s_3	s_4	s_5	s_6
a_1	7	-1	7	3	2	7
a_2	15	2	-1	4	7	10
a_3	-1	10	8	5	5	1
a_4	10	4	2	12	7	-1

- The fifth worst outcomes

	s_1	s_2	s_3	s_4	s_5	s_6
a_1	7	-1	7	3	2	7
a_2	15	2	-1	4	7	10
a_3	-1	10	8	5	5	1
a_4	10	4	2	12	7	-1

- The sixth worst outcomes

Maximax principle

- The maximax principle focuses on the best outcomes.
- Rationality requires us to prefer alternatives in which the best possible outcome is as good as possible.
- You should maximize the maximal value obtainable with an act.

Optimism-pessimism principle

- A decision maker's degree of optimism can be represented by a **real number** α between 0 and 1.
- α = 1: maximal optimism
- α = 0: maximal pessimism
- max (a_i) : the best outcome of act a_i
- min (a_i) : the worst outcome of act a_i

- The value of act a_i = [α · max(a_i)] + [(1 - α) · min(a_i)]
  - $\alpha \cdot max(a_i)$ is optimism part on act a_i
  - $(1-\alpha) \cdot min(a_i)$ is pessimism part on act a_i
- α is a subjective interpretation

- α : an agent's optimism index
- [α · max(a_i)] + [(1 - α) · min(a_i)] : an act's α-index (subjective)

- Principle definition : $a_i > a_j$ if and only if α-index (a_i) > α-index (a_j)

$$[\alpha \cdot (a_i)] + [(1-\alpha) \cdot min(a_i)] > [\alpha \cdot (a_j)] + [(1-\alpha) \cdot min(a_j)]$$

- If $\alpha = 1$, evaluate α-index $\rightarrow max(a_i)$ which means the optimism pessimism rule
  collapses to the maximax rule.
- If $\alpha = 0$, evaluate α-index $\rightarrow min(a_i)$ which means the optimism pessimism rule collapses to the maximin rule.

- Note that, in order for this rule to work, the value of outcomes should be measured on an interval scale.

- Is it rational to focus just on the best and the worst cases?
- Determine a series of α, α_1, α_2, ..., α_n, such that $\alpha_1 + \alpha_2 + ... + \alpha_n = 1$, then define α-index of an act a_i as

$$\alpha_1 \cdot max^1(a_i) + \alpha_2 \cdot max^2(a_i) + ... + \alpha_n \cdot min(a_i)$$

- then choose the act with the **greatest** α-index

- Note that α_1, α_2, ..., α_n are ***not probabilities***
  - similar to (subjective) **probabilities**
  - but not ***equivalent to*** **probabilities**
- They are chosen only according to how **much importance** the decision maker attaches to the best, the second best, ..., and the worst outcome of each act.
- The **relative importance** of an outcome **doesn't necessarily correspond** to its probability.

Example

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	s_1	s_2	s_3	s_4	s_5
a_1	10	75	5	2	10
a_2	10	21	43	71	16

- α = 0.5
- α-index(a_1) = (0.5 · 75) + [(1-0.5)· 2] = 38.5
- α-index(a_2) = (0.5 · 71) + [(1-0.5)· 10] = 40.5
- Therefore, $a_2 \succ a_1$

	s_1	s_2	s_3	s_4	s_5
a_1	10	75	5	2	10
a_2	10	21	43	71	16

- α = 0.7
- α-index(a_1) = (0.7 · 75) + [(1-0.7)· 2] = 53.1
- α-index(a_2) = (0.7 · 71) + [(1-0.7)· 10] = 52.7
- Therefore, $a_1 \succ a_2$

	s_1	s_2	s_3	s_4	s_5
a_1	10	75	5	2	10
a_2	10	21	43	71	16

- α = 0.2
- α-index(a_1) = (0.2 · 75) + [(1-0.2)· 2] = 16.6
- α-index(a_2) = (0.2 · 71) + [(1-0.2)· 10] = 22.2
- Therefore, $a_2 \succ a_1$

Minimax Regret

- The best alternative is one that **minimizes** the **maximum** amount of regret.
- $a_i \succ a_j$ if and only if the maximum regret of a_i is less than the maximum regret of a_j, or to put it formally :

$$max\{(v(a_i, s_1) - max(s_1), (v(a_i, s_2) - max(s_2)), \dots\} < max\{(a_j, s_1) - max(s_1), (v(a_j, s_2) - max$$

Procedure

- The value of regret for each outcome is calculated by subtracting the value of the **best outcome** of **each state** from the value of the outcome in question.
  - This obtains the ***regret matrix***
- The act chosen based on Minimax Regret principle is an act whose maximum regret for all states is minimum among other acts for all states.
  - Find the maximum regret for each act.
  - Choose the act with the least maximum regret.

	s_1	s_2	s_3	s_4
a_1	15	13	1	5
a_2	**21**	10	9	9
a_3	9	10	0	12

	s_1	s_2	s_3	s_4
a_1	-6	13	1	5
a_2	**0**	10	9	9
a_3	-12	10	0	12

	s_1	s_2	s_3	s_4
a_1	-6	**13**	1	5
a_2	**0**	10	9	9
a_3	-12	10	0	12

Irrelevant Alternatives

- The ranking of the alternatives cannot be changed by adding a non optimal alternative.
- In other words, the **addition** of a new act, which is **not regarded as better** than the original ones, should **not change** a rational agent's ranking of the **old acts**.
- Minimax regret violates the axiom of *Irrelevant Alternatives

	s_1	s_2	s_3
a_1	8	-1	**3**
a_2	**15**	7	0
a_3	12	**8**	-3

	s_1	s_2	s_3
a_1	-7	**-9**	0
a_2	0	-1	**-3**
a_3	-3	0	**-6**

	s_1	s_2	s_3
a_1	8	-1	3
a_2	**15**	7	0
a_3	12	**8**	-3
a_4	-2	6	**15**

	s_1	s_2	s_3
a_1	-7	-9	**-12**
a_2	0	-1	**-15**
a_3	-3	0	**-18**
a_4	**-17**	-2	0

- The ranking of the old acts is changed. a_1 is the best act rather a_2 in the original one after a_4 is added.

The principle Of Insufficient Reason

- If one has no reason to think that one state of the world is **more probable than another**, then all states should be assigned **equal** probability.
- By applying the principle of insufficient reason, an initial decision problem under **ignorance** is transformed into a decision problem under **risk**.
- If one advocates the principle of maximizing expected value as the best rule for decisions under risk, then the principle of insufficient reason can be stated as:
- $a_i \succ a_j$ if and only if $\sum_{x=1}^{n} \frac{1}{n} v(a_i, s_x) > \sum_{x=1}^{n} \frac{1}{n} v(a_j, s_x)$

	s_1	s_2	s_3
a_1	17	6	12
a_2	8	19	7
a_3	4	15	10

- $EV(a_1)=(0.33 \cdot 17)+(0.33 \cdot 6)+(0.33 \cdot 12)=11.67$
- $EV(a_2)=(0.33 \cdot 8)+(0.33 \cdot 19)+(0.33 \cdot 7)=11.33$
- $EV(a_3)=(0.33 \cdot 4)+(0.33 \cdot 15)+(0.33 \cdot 10)=9.67$

- Problem: it makes a decision problem very sensitive to how the states are individuated.
- Problem: if one has no reason to think that one state is more probable than another, does it follow that the probabilities are equal?