Decisions Under Risk

- In decisions under risk, the decision maker knows the probabilities of each state.
- The main rule to apply is the principle of maximizing expected utility.
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    - EU = $p_1 \cdot u_1 + p_2 \cdot u_2 + ... + p_n \cdot u_n$

Principle of Maximizing Expected

- The main rule to apply is the principle of maximizing expected:
    - monetary value
    - value
    - utility
- EMV = $p_1 \cdot m_1 + p_2 \cdot m_2 + ... + p_n \cdot m_n$
- EV = $p_1 \cdot v_1 + p_2 \cdot v_2 + ... + p_n \cdot v_n$
- EU = $p_1 \cdot u_1 + p_2 \cdot u_2 + ... + p_n \cdot u_n$
- It is not necessary that **it is irrational** if a person does not follow / a person violates the principle of maximizing expected monetary value if there are other values a person need to consider and evaluate.
    - **Not just money**

Marginal Value

- Winning more is always better than winning less. However, the more one wins, the lower is the value of winning yet another million.

Utility

- Not all concepts of value are reliable guides to rational decision making.
- The *utility* of an outcome depends on *how valuable* the outcome is from the decision maker's point of view.

Maximizing Expected Utility

The Law Of Large Numbers

- A mathematical theorem : everyone who maximizes expected utility will almost certainly be better off ***in the long run***.

- If a random experiment is repeated ***n*** times, and each experiment has a probability ***p*** of leading to predetermined outcome,
- then the probability that the percentage of such outcome ***differs*** from ***p*** by more than a ***very small amount*** ε ***converges*** to ***0*** as the number trails ***n approaches infinity***.

- This holds true for every $\varepsilon > 0$, no matter how small.
- Hence, by performing the random experiment ***sufficiently many times***, the probability that the average outcome differs from the expected outcome can be ***rendered arbitrarily small***.
- In the ***long run***, the actual outcome approaches the expected number.

Example

- Example: You are offered 1 unit of utility for sure or a lottery ticket that will yield either 10 units with a probability of 0.2, or nothing with a probability of 0.8.

	(0.2)	**(0.8)**
Lottery A	1	1

	(0.2)	(0.8)
Lottery B	10	10

- EU (A) = (0.2 * 1) + (0.8 * 1) = 1
- EU (B) = (0.2 * 10) + (0.8 * 0) = 2

Problem of The law Of Large Numbers

- No real life decision maker will ever face any decision an infinite number of times.
- It is not acceptable that a decision
- maker ever faces the very same decision problem several times.
- Many decisions under risk are unique
    - Marriage
    - Presidential election
    - Attending a graduate school

The Axiomatic Approach

Indirect Axiomatization

- Propose a set of axioms (structural constraints), such as transitivity and asymmetry
- **Show** that if a decision maker's preferences over a set of risky acts are **consistent with these axioms**, a decision maker **behaves as** if that decision maker **tries to maximize expected utility**.

Direct Axiomatization

- There are four direct axioms.
- By applying four direct axioms, it can prove the principle of maximize expected utility.

- However, this cannot prove that a decision maker is actually following it.

Axiom 1

- If all outcomes of an act have utility u, then the utility of the act is u.

	s_1	s_2	s_3	s_4
a_1	9	9	9	9
a_2	4	4	4	4

- U (a_1) = 9
- U (a_2) = 4

Axiom 2

- If one act is certain to lead to better outcomes **under all states** than another, then the utility of the first act exceeds that of the latter;
- If both acts lead to equal outcomes they have the same utility.

Axiom 3

- Every decision problem can be **transformed** into a decision problem with equally probable states, in which the **utility of all acts is preserved**.

	0.6	**0.4**
a_1	7	5
a_2	4	8

	(0.2)	(0.2)	(0.2)	(0.2)	(0.2)
a_1	7	7	7	5	5
a_2	4	4	4	8	8

Axiom 4

- If two outcomes are **equally probable**, and if the **better** outcome is made **slightly worse**
- then this can be **compensated** for by **adding some amount of utility** to the other outcome, such that the overall utility of the act is preserved.
- Trade-off principle

	(0.5)	(0.5)
a_1	5	5
a_2	2	10

	(0.5)	(0.5)
a_1	5	5
a_2	$2+\varepsilon_2$	$10-\varepsilon_1$

- There is some number $\delta > 0$, such that for all ϵ , $0 \le \epsilon_1 \le \delta$, there is a number ϵ_2 such that the suggested trade off is unimportant to you, i.e. the utility of the original and the modified acts is the same.

Paradox

- Rare and extreme cases that exploit the weak spots of the principle.
    - Principle is still useful in usual case.

Ellsberg's Paradox

The St. Petersburg Paradox

- The are several other games with infinite or arbitrary expected utilities.
- In general, they point to two problems:
    - That their expected utilities are infinite or arbitrary.
    - That they are not comparable.

The Two-Envelope Paradox

- You are offered a choice between two envelopes, A and B, each of which contains some money. One of them contains twice as much as the other.
- Since you don't know which envelope contains more money, you decide to pick one at random, say A. Just before you open it, you are offered to swap and take

B instead

- What would you like to do?
- At first, you picked A containing x. Now, B has to contain either 2x or x/2. Since the probabilities of both possibilities are equal, the expected monetary value of swapping to B is $(1/2) \cdot 2x + (1/2) \cdot (x/2) = 5x/4$.
- So, the principle recommends that you should switch to B
- What about swapping to A again?!