The Philosophy Of Probability

The classical Interpretation

- **Fraction** of the **total number** of **possible ways** in which the event can occur.

- **Probability = Number of favorable cases / Number of possible cases**

- Assumption : all possible outcomes are **equally** likely.

- Objection (Example) :

    - A biased coin / dice.
    - Possibility of someone fell in love with another person
    - Picking up the real number (2.35) from the set of real numbers between 2 and 3

- Solution to objection : it is always **possible** to **divide the set of cases** so as to all possible outcomes are equally likely.

- Conforms to the axioms of probability.

The frequency interpretation

- **Ratio** between the number of **times** the event **has occurred** divided by the **total number** of **observed cases**.

- **Probability = Total number of positive instances / Total number of trials**

- There should **always** be a **reference class**.

- Objection :

    - It is not always possible to find a correct reference class and the size of reference class

- Venn argued that the frequency interpretation make sense **only if** the reference class is taken to be **infinitely large**.

- More precisely, Venn argued that one should distinguish sharply between the underlying *limiting* frequency of an event and the frequency *observed* so far.

- *Limiting* frequency is best though of as the proportion of successful outcomes one *would* get if one were to repeat *one and the same* experiment infinitely many times.

- To examine if this theory conforms to the axioms of probability.

The propensity interpretation

- Probabilities refer to **certain features** of the **external world**.
- Propensity / Disposition / **Tendency** of an object to give rise to a certain effect.
- **Hybrid version**: The propensity of an object is taken into account when there is **no access** to **long-run frequencies**.
- **Pure version**: **Propensity** is the main issue in all the cases
- Objection :
    - Problem of clarifying the concepts such as disposition and propensity.
- Particular objection : inverted possibility.
    - Propensities have a **temporal direction**
        - temporal : relating to practical matters or physical things.
    - If A has a propensity to give rise to B, A cannot occur after B
    - Propensities function very much like **causality**
        - **causality** (因果关系) : the principle that there is a cause for everything that happens.
        - one event, process, state, or object contributes to the production of another event, process, state, or object.
    - If A causes B, then A cannot occur after B.
    - However, probabilities lack this temporal direction.

Logical View

- Probability is the logical relation between a hypothesis and the evidence that supports it
- Probability is the degree of support and confirmation that a piece of evident provides for a hypothesis.
- Deductive logic : deterministic
- Inductive logic : indeterministic / probabilistic
- Objection : It is too dependent on **evidence**
    - It can lead to pure guesses that are irrational.

- However, this can be verified using probability calculus to check whether a set of guesses is coherent or not.

Subjective Views

- Probability is the measure of an agent's **degrees of belief** in a statement regarding an event.
- For a partial belief to qualify as a **probability**, one's **degrees of belief** must be **rational**.
- Since subjective probabilities may vary from person to person
    - **Disagreement** concerning the probability of an event is normal.
- People can use Bayes' theorem to **update** their beliefs.
- Issues
    - The utility function is not **linear** for all people.
    - Not all people form their preferences in accordance with the principle of maximizing expected monetary value.

Savage's Method

- A set of structural conditions on preferences over uncertain options
    - Apply restriction on what combinations of preferences are legitimate to have.
- Proves that if a decision maker's preferences over those options satisfy the axioms
- Behaves as if she were forming her preferences by first
    - assigning subjective probabilities
    - utilities to each option
    - maximizing expected utility

The Dutch Book Method

- A Dutch Book is a combination of bets that is **certain to lead to a loss**.
- Violate the axioms of the probability calculus.
    - G V ~ G = 1

- The Dutch Book theorem states that a decision maker's degrees of belief **satisfy** the **probability axioms** if and only if **no Dutch Book** can be **made against** her.

- Assumption : decision maker's utility of money and other goods is linear.

- A *fair price* for a bet, is an amount such that the decision maker is equally willing to act

    - as a player
    - as a bookie

- Exactly one fair price for every bet :

    - equally willing to buy or sell a bet costing $100.
    - preference must be altered if the price is changed to $101.

- Ratio between the fair price of a bet and the absolute value of the amount at stake is called the betting quotient.

    - stake: (share involvement in something such as a business)

- Decision maker has to announce his betting quotients for a fairly large number of bets. (detailed 164).

Case Study

- You believe to degree 0.55 that at least one person from India will win a gold medal (event G).

- Your subjective degree of belief is 0.52 that no Indian will win a gold medal (event ~G).

- Also suppose that a bookie offers you to bet on both these events.

- The bookie promises to pay you $1 for each event that actually take place.

- Now, you will pay up

    - $1 * 0.55 = $0.55 for event G
    - $1 * 0.52 = $52 for event ~G.

- Now, you have paid $1.07 for taking on two bets that certain give you a payoff of $1 no matter what happen.

- Regardless of what happen, you certainly loss $0.07.