

The Philosophy Of Probability

The classical Interpretation

- **Fraction** of the **total number** of **possible ways** in which the event can occur.
 - **Probability** = **Number of favorable cases** / **Number of possible cases**
 - Assumption : all possible outcomes are **equally** likely.
 - Objection (Example) :
 - A biased coin / dice.
 - Possibility of someone fell in love with another person
 - Picking up the real number (2.35) from the set of real numbers between 2 and 3
 - Solution to objection : it is always **possible** to **divide the set of cases** so as to all possible outcomes are equally likely.
 - Conforms to the axioms of probability.
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The frequency interpretation

- **Ratio** between the number of **times** the event **has occurred** divided by the **total number** of **observed cases**.
- **Probability** = **Total number of positive instances** / **Total number of trials**
- There should **always** be a **reference class**.
- Objection :
 - It is not always possible to find a correct reference class and the size of reference class
- Venn argued that the frequency interpretation make sense **only if** the reference class is taken to be **infinitely large**.
- More precisely, Venn argued that one should distinguish sharply between the underlying **limiting** frequency of an event and the frequency **observed** so far.
- **Limiting** frequency is best thought of as the proportion of successful outcomes one **would** get if one were to repeat **one and the same** experiment infinitely many times.

- To examine if this theory conforms to the axioms of probability.
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The propensity interpretation

- Probabilities refer to **certain features** of the **external world**.
 - Propensity / Disposition / **Tendency** of an object to give rise to a certain effect.
 - **Hybrid version**: The propensity of an object is taken into account when there is **no access to long-run frequencies**.
 - **Pure version**: **Propensity** is the main issue in all the cases
 - Objection :
 - Problem of clarifying the concepts such as disposition and propensity.
 - Particular objection : inverted possibility.
 - Propensities have a **temporal direction**
 - temporal : relating to practical matters or physical things.
 - If A has a propensity to give rise to B, A cannot occur after B
 - Propensities function very much like **causality**
 - **causality** (因果关系) : the principle that there is a cause for everything that happens.
 - one event, process, state, or object contributes to the production of another event, process, state, or object.
 - If A causes B, then A cannot occur after B.
 - However, probabilities lack this temporal direction.
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Subjective Views

- Probability is the measure of an agent's **degrees of belief** in a statement regarding an event.
- For a partial belief to qualify as a **probability**, one's **degrees of belief** must be **rational**.
- Since subjective probabilities may vary from person to person
 - **Disagreement** concerning the probability of an event is normal.
- People can use Bayes' theorem to **update** their beliefs.
- Issues
 - The utility function is not **linear** for all people.

- Not all people form their preferences in accordance with the principle of maximizing expected monetary value.
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Savage's Method

- A set of structural conditions on preferences over uncertain options
 - Apply restriction on what combinations of preferences are legitimate to have.
 - Proves that if a decision maker's preferences over those options satisfy the axioms
 - Behaves as if she were forming her preferences by first
 - assigning subjective probabilities
 - utilities to each option
 - maximizing expected utility
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The Dutch Book Method

- A Dutch Book is a combination of bets that is **certain to lead to a loss**.
- Violate the axioms of the probability calculus.
 - $G \sim V \sim G = 1$
- The Dutch Book theorem states that a decision maker's degrees of belief **satisfy** the **probability axioms** if and only if **no Dutch Book** can be **made against** her.
- Assumption : decision maker's utility of money and other goods is linear.
- A *fair price* for a bet, is an amount such that the decision maker is equally willing to act
 - as a player
 - as a bookie
- Exactly one fair price for every bet :
 - equally willing to buy or sell a bet costing \$100.
 - preference must be altered if the price is changed to \$101.
- Ratio between the fair price of a bet and the absolute value of the amount at stake is called the betting quotient.
 - stake: (share involvement in something such as a business)
- Decision maker has to announce his betting quotients for a fairly large number of bets. (detailed 164).