

Bayesianism and Pragmatic Arguments

Bayesianism

- **Bayesian decision theories** are formal theories of rational agency. They aim to tell us both:
 - what the properties of a rational state of mind are ...
 - *epistemic* component
 - what your state of mind ought to be like
 - what action it is rational for an agent to perform, given his / her state of mind.
 - *deliberative* component
 - how you ought to act given that state of mind
 - The axiomatic conditions on preferences employed in Bayesian theories merely restrict which combinations of preferences are legitimate.
 - Asymmetry
 - Transitivity
 - Completeness
 - etc.
 - Preferences are revealed in one's choice behavior.
 - If the agent is offered a choice between two uncertain prospects and chooses one of them,
 - we may conclude that he / she preferred the chosen one.
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The epistemic component

- a claim about
 - what rational agents ought to believe
 - which combinations of beliefs and desires are rationally permissible
- The theory holds that one is free to believe whatever one wishes *as long as* one's beliefs can be
 - represented by a **subjective probability function**
 - and those beliefs are updated in accordance with *Bayes' theorem*

- Virtually, it offers no substantial advice on *how* one ought to go about when exploring the world.
 - merely provides a set of structural restrictions on
 - what it's permissible to believe
 - how one is permitted to revise those beliefs with new info.
 - A major issue of disagreement : how to determine ***prior probabilities***
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The deliberative component

- Tells the agent what it's rational to do given his / her present state of mind.
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Principles

1. (Chapter 5) Subjective degrees of belief can be represented by
 - a ***probability function*** defined in terms of the decision maker's preferences over uncertain prospects
 2. (Chapter 7) The decision maker's desires can be represented by
 - a ***utility function*** defined in terms of preferences over uncertain prospects
 3. Rational decision makers act ***as if*** they maximize ***subjective expected utility***.
 - a. Why it's "***as if***" instead of "***according to***"
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- Third principle : decision maker does *not* prefer an uncertain prospect to another *because*
 - he / she judges the utilities and probabilities of the outcomes to be more favorable than those of another.
 - The probability and utility functions are established by reasoning backwards :
 - since the agent preferred some uncertain prospects to others,
 - and the preferences over uncertain prospects satisfy a number of structural axioms,
 - the agent behaves as if he / she had acted from
 - a subjective probability function and a utility function
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Probability, Utility And Preferences

Example Of Eliciting Subjective Probabilities (Page 176)

Example Of Eliciting Utility (Page 177)

Establish equality between events

- Suppose that the agent strictly prefers one object
 - a fancy designer watch to another object.
 - Then, if the agent is **indifferent** between the prospect in
 - a. she wins the first object if Event 1 occurs and the second object if Event 2 occurs
 - b. she wins the second object if Event 1 occurs and the first object if Event 2 occurs
 - c. two events are by definition equally probable.
 - Examples : the agent considers the mutually exclusive events ***R*** and ***~ R*** to be equally probable.
 - Then she will be indifferent between winning, say,
 - 200 units of utility if ***R*** occurs and 100 units if ***~R*** occurs
 - 100 units of utility if ***R*** occurs and 200 units if ***~R*** occurs
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Pragmatic Arguments

- Bayesian decision theory are the transitivity, completeness and independence axioms.
 - Transitivity : If $x > y$ and $y > z$, then $x > z$
 - Completeness : $x > y$ or $y > x$ or $x \sim y$
 - Independence : If $x > y$, then $xpz \succ ypz$

- where xpz is a lottery that gives you x with probability p and z with probability $1 - p$
 - **Why Axioms ?**
 - One might claim they are intuitively true
 - If it was so, no disagreements could exist.
 - Logical or mathematical proofs cannot be provided.
 - The only resort is ***pragmatic justification***.
 - If your preferences don't satisfy the axioms, then you get into trouble.
 - ***Pragmatic arguments*** seek to show that decision makers who **violate** certain principles of rationality may face a decision problem in which it is ***certain*** that they will **stand to lost, come what may**
 - Need not always be formulated in monetary terms
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Money-pump Argument

- Preference between two objects is not ***asymmetric***
 - The arguments shows that if a decision maker doesn't follow the principles of rational decision making,
 - This person can be pumped all the money without getting anything back.
 - At the end of each stage, that person is back the beginning, the only difference being that that person lost some money.
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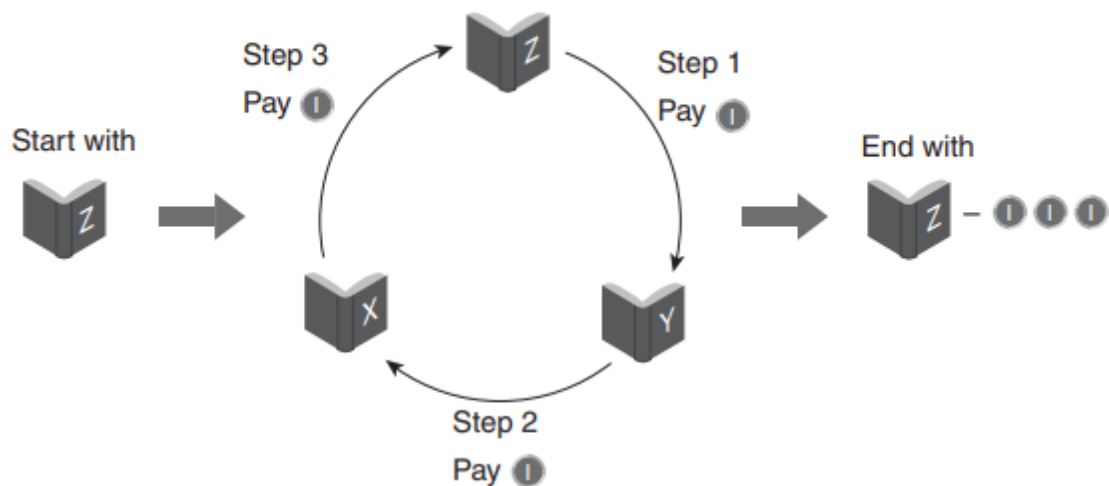
Must a Rational Preference be Transitive ?

- The pragmatic argument for transitivity seeks to show that
 - anyone whose preferences are **cyclic** (and hence not transitive)
 - may end up in a situation in which it is **certain** that she will lose an infinite amount of money.
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Argument

- Imagine that your preference ordering over the three novels x , y and z is cyclic.
- You prefer x to y , and y to z , and z to x

- $x > y > z > x$.
- Now suppose that you are in possession of z , and that you are invited to swap z for y .
- Because you prefer y to z , it is rational for you to swap even if you have to pay a *small* amount, say one cent, for swapping.
- So you swap, and temporarily get y .
- You are then invited to pay one cent for swapping y for x , which you do, because you prefer x to y .
- Finally, you are offered to pay one cent for swapping x for z .
- Because z is strictly better than x , even after you have paid the fee for swapping, rationality tells you that you should accept the offer.
- This means that you end up where you started, the only difference being that you now have three cents less.
- This procedure is thereafter iterated over and over again. After a billion cycles you go bankrupts.
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- Conclusion is **not** that a rational preference must be transitive.
- The conclusion is rather that if we permit cyclic preference orderings, then the contradiction in the **money pump argument** is unavoidable.
- What if $x > y$ and $y > z$, then x and z to be incommensurable
 - ordering is not cyclic
 - it is not certain that that person can be money pumped.
 - Hence, it's difficult to see how the money pump argument could support the transitivity axiom.
- What makes a cyclic preference irrational is not that the agent can be exploited
 - rather the fact that she is **acting against her own preference**
 - "What is irrational about being money pumped is that one chooses against one's preference ... Whether someone else thereby gets rich at

your expense is irrelevant for whether you are rational" (Johan Gustafsson)

Must a Rational Preference be Complete ?

- The **completeness axiom** - holding that a rational decision maker must either
 - prefer one object to another
 - or be indifferent between the two
 - **Completeness axiom** entails that any pair of objects, no matter how disparate, can be compared.
 - You must either prefer one to the other, or be indifferent between the two.
 - The axiom can be violated if one holds that there are pair of options that are **incommensurable**
 - **Incommensurability** is irrational.
 - If it is rationally permissible to swap between two incommensurable objects, then it is easy to construct a money pump.
 - If one denies that it is permissible to swap, it would no longer be possible to construct this kind of money pump.
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Small Improvement Argument

- When an agent A doesn't prefer x to y nor y to x
 - It follows from the completeness axiom that A is indifferent between x and y.
 - If x and y are equally valuable, then adding a small amount of value to x would make A ready to prefer x over y.
 - But this not the case in many situations. So indifference is not the only alternative to $x > y$ and $y > x$. (Proof Page 185.)
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Paul Samuelson's Revealed Preference Theory

- A forceful objection to the small improvement argument.
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