Summary

Dominance Principle

- Dominated acts must not chosen.
- Dominated acts
    - whose all outcomes under every state are **at least as rational as** other outcomes of other acts under every state
    - **and** there are some outcomes under some states are **less rational** than outcomes of other acts under those states
- Let $v(a_i, s_j)$ be the **value of the outcome** corresponding to the act a_i and the state s_j.

Weak Dominance

- ***Weak Dominance*** : $a_i \succcurlyeq a_j$ if and only if $v(a_i, s) \geq v(a_j, s)$ for every state s.

Strong Dominance

- ***Strong Dominance*** : $a_i \succ a_j$ if and only if v(a_i, s_m) $\geq$ v(a_j, s_m) for every state s_m, and there is some state s_n such that v(a_i, s_n) > v(a_j, s_n).

Maximin Principle

- The maximin principle focuses on the ***worst*** possible outcome of each act.
- One should ***maximize*** the ***minimal*** value obtainable with each act.
- If the **worst possible outcome** of an act is **better than** the **worst possible outcome** of another act, the **first** act should be chosen.
- Formally speaking : $a_i \succeq a_j$ if an only if min(a_i) $\geq$ min(a_j).
- If the minimum value corresponding to the outcomes of two or more acts are the same, the maximin principle ranks the acts as equally rational.

Leximin Rule

- Extended version of *Maximin Principle*
- Let $min^1(a_i)$ be the value of the worst outcome of act a_i,
- $min^2(a_i)$ be the value of its second worst outcome,
- In general,$min^n(a_i)$ be the value of its n-th worst outcome

- a_i a_j if and only if there is some positive integer n such that
    - $min^n(a_i) > min^n(a_j)$
    - regardless $min^m(a_i) = min^m(a_j)$ for all m < n

Maximax principle

- The maximax principle focuses on the best outcomes.
- Rationality requires us to prefer alternatives in which the best possible outcome is as good as possible.
- You should maximize the maximal value obtainable with an act.

Optimism-pessimism principle

- A decision maker's degree of optimism can be represented by a **real number** α between 0 and 1.
- α = 1: maximal optimism
- α = 0: maximal pessimism
- max (a_i) : the best outcome of act a_i
- min (a_i) : the worst outcome of act a_i

- The value of act a_i = [α · max(a_i)] + [(1 - α) · min(a_i)]
    - $\alpha \cdot max(a_i)$ is optimism part on act a_i
    - $(1-\alpha) \cdot min(a_i)$ is pessimism part on act a_i
- α is a subjective interpretation

- α : an agent's optimism index
- [α · max(a_i)] + [(1 - α) · min(a_i)] : an act's α-index (subjective)

- Principle definition : $a_i > a_j$ if and only if α-index (a_i) > α-index (a_j)

$$[\alpha \cdot (a_i)] + [(1-\alpha) \cdot min(a_i)] > [\alpha \cdot (a_j)] + [(1-\alpha) \cdot min(a_j)]$$

- If $\alpha = 1$, evaluate α-index $\rightarrow max(a_i)$ which means the optimism pessimism rule
  collapses to the maximax rule.
- If $\alpha = 0$, evaluate α-index $\rightarrow min(a_i)$ which means the optimism pessimism rule collapses to the maximin rule.

- Note that, in order for this rule to work, the value of outcomes should be measured on an interval scale.

- Is it rational to focus just on the best and the worst cases?
- Determine a series of α, α_1, α_2, ..., α_n, such that $\alpha_1 + \alpha_2 + \ldots + \alpha_n = 1$, then define α-index of an act a_i as

$$\alpha_1 \cdot max^1(a_i) + \alpha_2 \cdot max^2(a_i) + \ldots + \alpha_n \cdot min(a_i)$$

- then choose the act with the **greatest** α-index

- Note that α_1, α_2, ..., α_n are ***not probabilities***
    - similar to (subjective) **probabilities**
    - but not ***equivalent to*** **probabilities**
- They are chosen only according to how **much importance** the decision maker attaches to the best, the second best, ..., and the worst outcome of each act.
- The **relative importance** of an outcome **doesn't necessarily correspond** to its probability.

Minimax Regret

- The best alternative is one that **minimizes** the **maximum** amount of regret.
- $a_i \succ a_j$ if and only if the maximum regret of a_i is less than the maximum regret of a_j, or to put it formally :

$$max\{(v(a_i, s_1) - max(s_1), (v(a_i, s_2) - max(s_2)), \ldots\} < max\{(a_j, s_1) - max(s_1), (v(a_j, s_2) - max$$

Procedure

- The value of regret for each outcome is calculated by subtracting the value of the **best outcome** of **each state** from the value of the outcome in question.
    - This obtains the ***regret matrix***
- The act chosen based on Minimax Regret principle is an act whose maximum regret for all states is minimum among other acts for all states.
    - Find the maximum regret for each act.
    - Choose the act with the least maximum regret

The principle Of Insufficient Reason

- If one has no reason to think that one state of the world is **more probable than another**, then all states should be assigned **equal** probability.
- By applying the principle of insufficient reason, an initial decision problem under **ignorance** is transformed into a decision problem under **risk**.

- If one advocates the principle of maximizing expected value as the best rule for decisions under risk, then the principle of insufficient reason can be stated as:
- $a_i \succ a_j$ if and only if $\sum_{x=1}^{n} \frac{1}{n} v(a_i, s_x) > \sum_{x=1}^{n} \frac{1}{n} v(a_j, s_x)$

Relationship Properties

- $x \succeq y$ if and only if $x \succ y$ or $x \sim y$
- $x \sim y$ if and only if $x \succeq y$ and $y \succeq x$
- $x \succ y$ if and only if $x \succeq y$ and not $x \sim y$

- For every x and y, it should hold that :
- **Completeness** $x \succ y$ or $x \sim y$ or $y \succ x$
- **Asymmetry** If $x \succ y$, then it is *false* that $y \succ x$.
- **Transitivity** If $x \succ y$ and $y \succ z$, then $x \succ z$.
- **Negative Transitivity** If it is *false* that $x \succ y$ and *false* that $y \succ z$, then it is false that $x \succ z$.
  - It implies that indifference is **transitive** (if $x \sim y$ and $y \sim z$, then $x \sim z$).
  - Indifference does not follow form the assumption in **Transitivity**

Neumann and Morgenstern Method

- **Z** is a finite set of **basic prizes**
- **L** is a set of lotteries that can be constructed from **Z** by applying the following inductive definition:
  - Every basic prize in **Z** is a lottery
  - If A and B are lotteries, then so is the prospect of getting A with probability p and B with probability (1-p), for every 0 $\leq$ p $\leq$ 1.
  - Nothing else is a lottery.

Detailed Example

- Simpson is going to go to a rock concert. Three bands are playing tonight. He thinks A is better than B, which is better than C.
- Two tickets are available :
  - Ticket 1 entitles him to a 70% chance of watching A and a 30% chance of watching C.
  - Ticket 2 entitles him to watch B with 100% certainty.
- Upon reflection, he finds the two tickets equally attractive.
- Suppose we know that Simpson always acts according to the principle of maximizing utility.

- By reasoning backward, we can find the utilities he attaches to A, B, C. That is have to solve the following equation:

$$0.7 \cdot u(A) + 0.3 \cdot u(C) = 1.0 \cdot u(B)$$

- Stipulate that u(A) = 100 and u(C) = 0, then it turns out that u(B) = 70.
    - u(A) and u(C) are picked arbitrary.

The Mathematics of Probability

Kolmogorov Axioms

- $1 \geq p(A) \geq 0$, probability of every event lies between 0 and 1.
- $p(S) = 1$, probability of the entire sample space is 1.
- If $A \cap B = \emptyset$, then $p(A \cup B) = p(A) + p(B)$ (keyword, sample space).

- If it is more natural to think the question by counting thinking instead of using probability for proposition, use counting instead.
    - For example, choose one red card and one black card without drawback.
- Notice that condition probability need to consider independency of an event / proposition.
- Conditional Probability :
    - p(~B|A) = 1 - p(B|A)
    - same for p(~A|B) = 1 - p(A|B)
- To construct inverse conditional probability :
    - Look for what the question is asking about to

Unknown Priors

- Prior probability / unconditional probability of B : p(B)
- Probability that are given
    - Probability of A happen n times given not B : p(A|~B) = x^n
    - Probability of A happen n times given B : p(A|B) = x'^n
- Find probability of B happen given A : p(B|A)