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1.

$$
\begin{aligned}
&NOR\\
&(\lambda xy|xx(yy))(\lambda x|xy)(\lambda x|x)\\
&\rightarrow_\beta (\lambda x|xy)(\lambda x|xy)((\lambda x|x)(\lambda x|x))\\
&\rightarrow_\beta (\lambda x|xy)y((\lambda x|x)(\lambda x|x))\\
&\rightarrow_\beta yy((\lambda x|x)(\lambda x|x))\\
&\rightarrow_\beta yy(\lambda x|x)
\end{aligned}
$$

$$
\begin{aligned}
&AOR\\
&(\lambda xy|xx(yy))(\lambda x|xy)(\lambda x|x)\\
&\rightarrow_\beta (\lambda x|xy)(\lambda x|xy)((\lambda x|x)(\lambda x|x))\\
&\rightarrow_\beta (\lambda x|xy)(\lambda x|xy)(\lambda x|x)\\
&\rightarrow_\beta (\lambda x|xy)y(\lambda x|x)\\
&\rightarrow_\beta yy(\lambda x|x)
\end{aligned}
$$

2. (a)

$$
\begin{aligned}
NOT &= (\lambda x|xFT)\\
OR &= (\lambda xy|xTy)\\
T &= (\lambda xy|x)\\
F &= (\lambda xy|y)\\
x \subset y &\equiv \neg x \vee y\\
\text{IMP} &= (\lambda xy|NOTxTy)\\
&\rightarrow_\alpha (\lambda xy|(\lambda x|xFT)xTy)\\
&\rightarrow_\beta (\lambda xy|xFTTy)\\
&\rightarrow_\alpha (\lambda xy|x(\lambda xy|y)(\lambda xy|x)(\lambda xy|x)y)\\
&\rightarrow_\alpha (\lambda xu|x(\lambda xy|y)(\lambda xy|x)(\lambda xy|x)u)
\end{aligned}
$$

2. (b)

$$
\begin{aligned}
&\textbf{IMP T F}\\
&\rightarrow_\alpha (\lambda xu|x(\lambda xy|y)(\lambda xy|x)(\lambda xy|x)u)\ \text{T F}\\
&\rightarrow_\beta \text{T}(\lambda xy|y)(\lambda xy|x)(\lambda xy|x)\text{F}\\
&\rightarrow_\alpha (\lambda xy|x)(\lambda xy|y)(\lambda xy|x)(\lambda xy|x)(\lambda xy|y)\\
&\rightarrow_\beta (\lambda xy|y)(\lambda xy|x)(\lambda xy|y)\\
&\rightarrow_\beta (\lambda xy|y)\\
&\rightarrow_\alpha \text{F}
\end{aligned}
$$

$$
\begin{aligned}
&\textbf{IMP F T}\\
&\rightarrow_\alpha (\lambda xu|x(\lambda xy|y)(\lambda xy|x)(\lambda xy|x)u)\ \text{F T}\\
&\rightarrow_\beta \text{F}(\lambda xy|y)(\lambda xy|x)(\lambda xy|x)\text{T}\\
&\rightarrow_\alpha (\lambda xy|y)(\lambda xy|y)(\lambda xy|x)(\lambda xy|x)(\lambda xy|x)\\
&\rightarrow_\beta (\lambda xy|x)(\lambda xy|x)(\lambda xy|x)\\
&\rightarrow_\beta (\lambda xy|x)\\
&\rightarrow_\alpha \text{T}
\end{aligned}
$$

3.

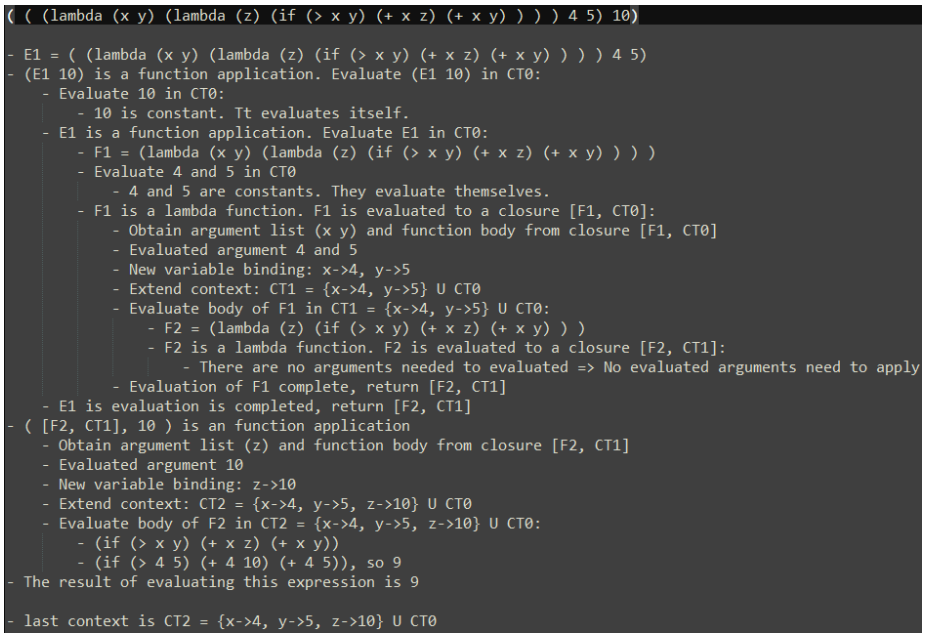

Figure 1: solution