Lab 3

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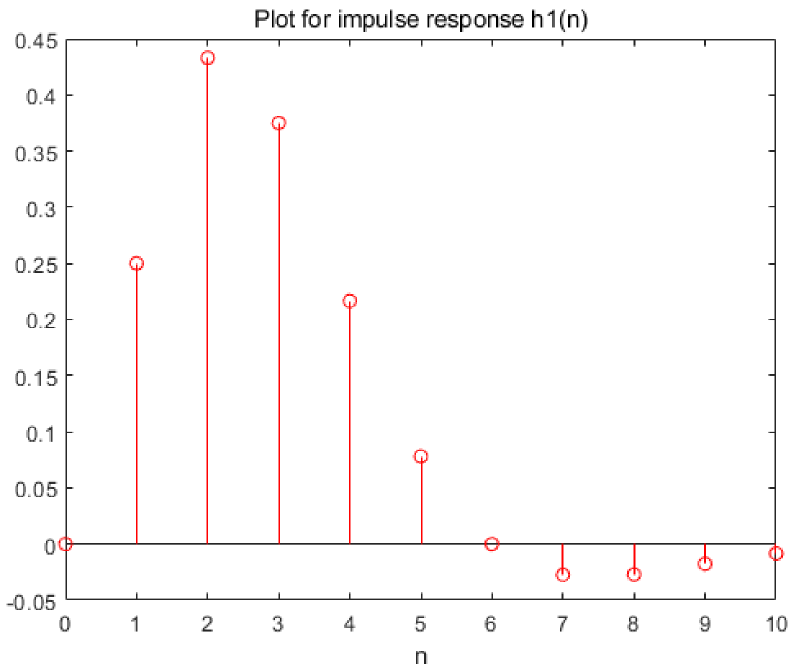
Section 801
Nov 3rd, 2021

Question 1

(a) Code and plot for the impulse response $h_1[n]$

```
n = (0:1:10);
h1 = n .* 0.5 .^ n .* sin(pi / 6 .* n);

stem(n, h1, 'r-');
title('Plot for impulse response h1(n)');
xlabel('n');
```

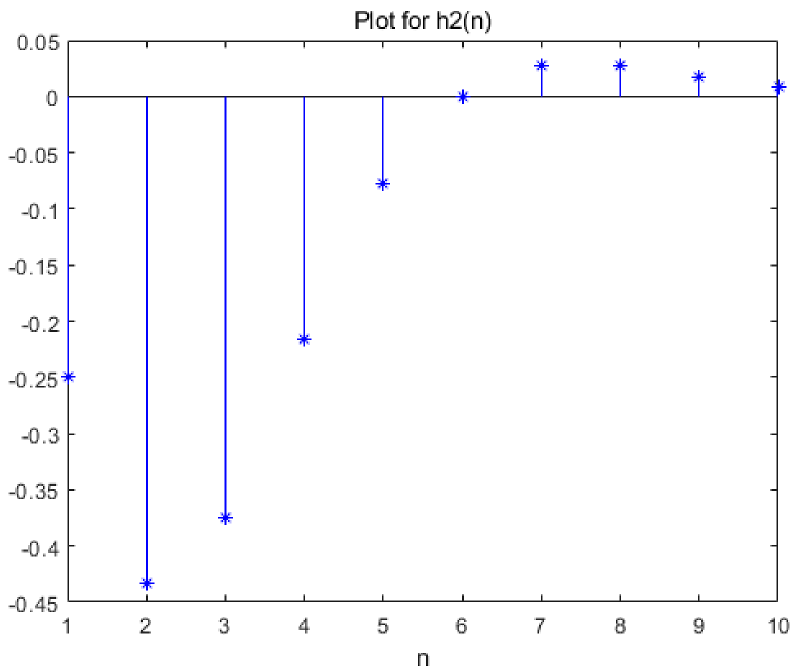


(b) Using $b^n \sin(an) = \frac{\sin(a)bz}{z^2 - 2cos(a)bz + b^2}$ and $nx[n] = -z(\frac{dX(z)}{dz})$, it yields

$$
\begin{aligned}
a &= \frac{\pi}{6} \\
b &= 0.5 \\
X(z) &= \frac{0.5\sin(\frac{\pi}{6})z}{z^2 - 2\cos(\frac{\pi}{6})0.5z + 0.25} \\
&= \frac{z}{4z^2 - 2\sqrt{3}z + 1} \\
z\frac{dX(z)}{dz} &= z\frac{-4z^2 + 1}{(4z^2 - 2\sqrt{3}z + 1)^2} \\
&= \frac{-4z^3 + z}{(4z^2 - 2\sqrt{3}z + 1)^2} \\
H(z) &= \frac{N(z^{-1})}{D(z^{-1})} = \frac{-4z^3 + 0z^2 + z + 0}{16z^4 - 16\sqrt{3}z^3 + 20z^2 - 4\sqrt{3}z + 1} \\
H(z) &= \frac{N(z)}{D(z)} = \frac{z^{-4}(0 - 4z^{-1} + 0z^{-2} + z^{-3} + 0z^{-4})}{z^{-4}(16 - 16\sqrt{3}z^{-1} + 20z^{-2} - 4\sqrt{3}z^{-3} + z^{-4})} \\
&= \frac{0 - 4z^{-1} + 0z^{-2} + z^{-3} + 0z^{-4}}{16 - 16\sqrt{3}z^{-1} + 20z^{-2} - 4\sqrt{3}z^{-3} + z^{-4}}
\end{aligned}
$$

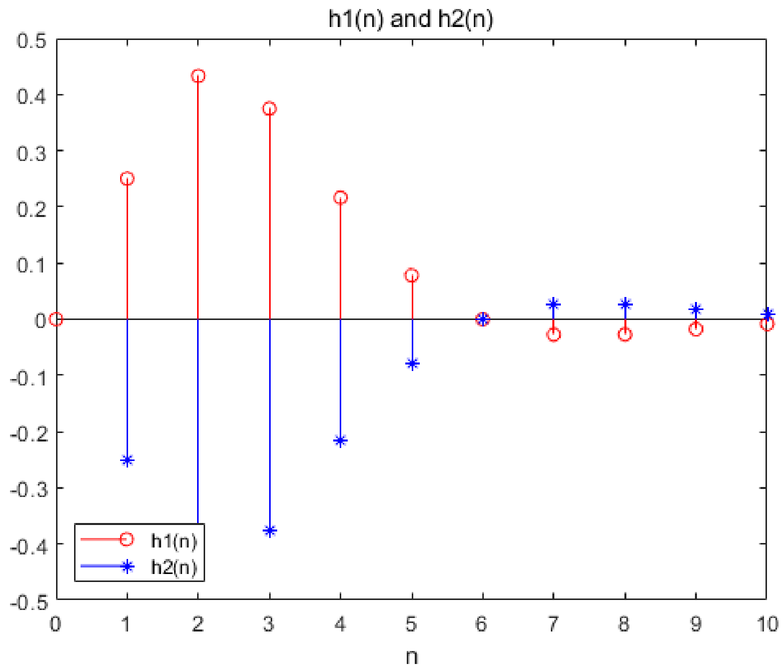
(c) Code and plot for $h_2[n]$

```
x = zeros(1, 10);
x(1) = 1;
N = [-4 0 1];
D = [16 -16*sqrt(3) 20 -4*sqrt(3) 1];
h2 = filter(N, D, x);

stem(h2, 'b*');
title('Plot for h2(n)');
xlabel('n');
```



(d) The value of $h_2[n]$ for each n has the same magnitude to $h_1[n]$ but with inverted sign.

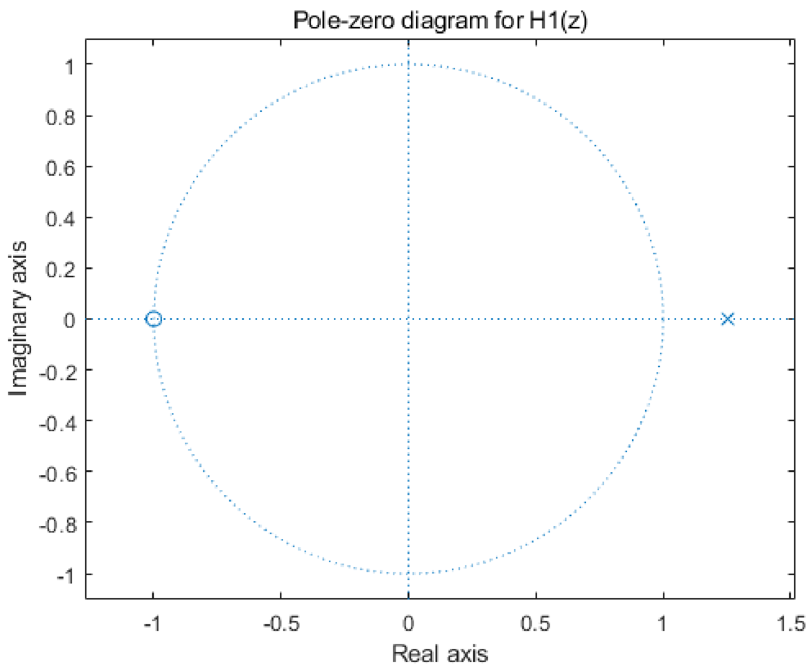
```
n = (0:1:10);
h1 = n .* 0.5 .^ n .* sin(pi / 6 .* n);

x = zeros(1, 10);
x(1) = 1;
N = [-4 0 1];
D = [16 -16*sqrt(3) 20 -4*sqrt(3) 1];
h2 = filter(N, D, x);

stem(h2, 'b*');
title('Plot for h2(n)');
xlabel('n');

stem(n, h1, 'r-');
hold on
stem(h2, 'b*');
title('h1(n) and h2(n)');
xlabel('n');
legend({'h1(n)', 'h2(n)'}, 'Location', 'southwest');
```

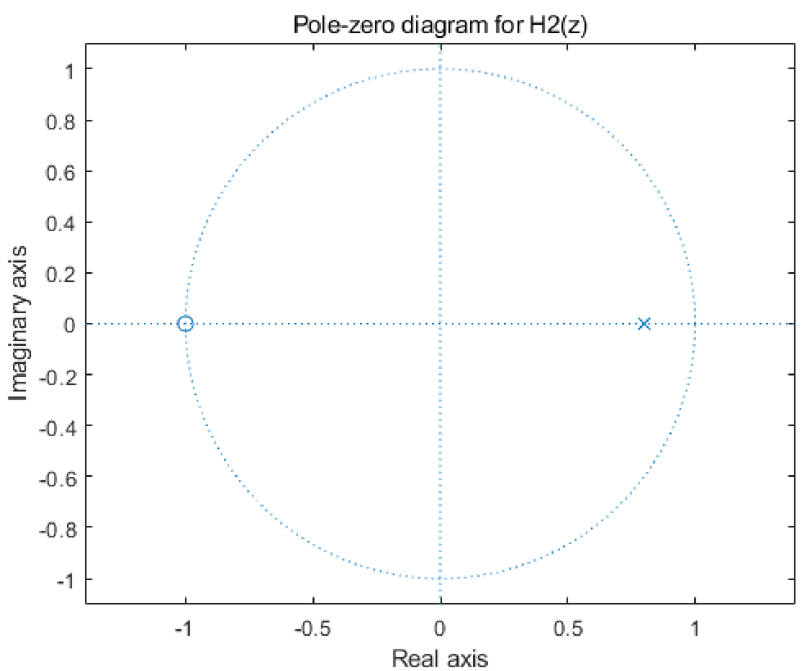


Question 2

(a) $H_1(z)$ is unstable because the only pole of $H_1(z)$, which is $z = 1.25$, is outside of the unit circle $|z| = 1$. $H_2(z)$ is stable because the only pole of $H_2(z)$, which is $z = 0.8$, is inside of the unit circle $|z| = 1$.

```
b1 = [2 2];
a1 = [1 -1.25];
[z1, p1, k1] = tf2zpk(b1, a1);

b2 = [2 2];
a2 = [1 -0.8];
[z2, p2, k2] = tf2zpk(b2, a2);

zplane(z1, p1);
title('Pole-zero diagram for H1(z)');
xlabel('Real axis');
ylabel('Imaginary axis');

zplane(z2, p2);
title('Pole-zero diagram for H2(z)');
xlabel('Real axis');
ylabel('Imaginary axis');
```

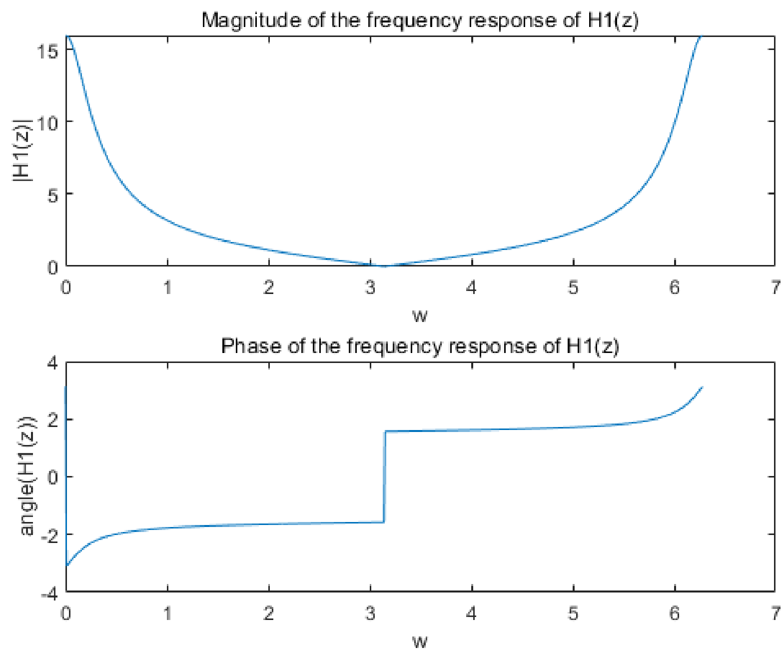

Pole-zero diagram for H1(z)
Imaginary axis
Real axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1
1.5

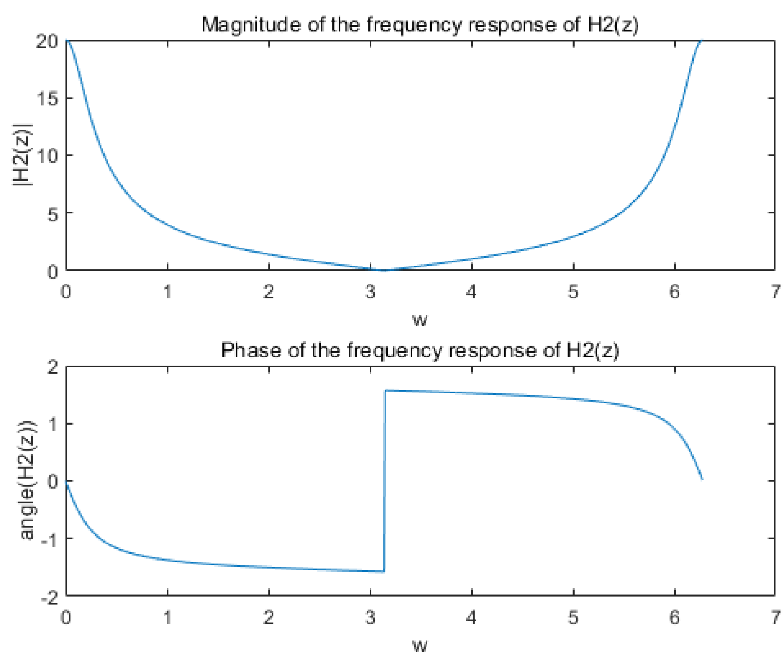

Pole-zero diagram for H2(z)
Imaginary axis
Real axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1

(b)

```
w = (0:0.01:2*pi());
```

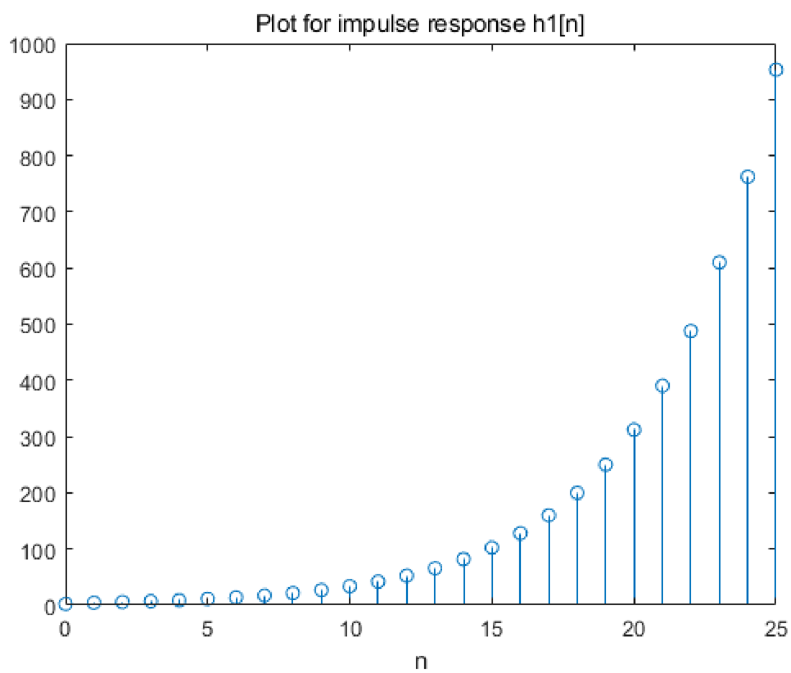
```
H1 = (2 + 2 .* exp(-1j .* w)) ./ (1 - 1.25 .* exp(-1j .* w));
H2 = (2 + 2 .* exp(-1j .* w)) ./ (1 - 0.8 .* exp(-1j .* w));

magH1 = abs(H1);
magH2 = abs(H2);

phaseH1 = angle(H1);
phaseH2 = angle(H2);

%subplot(2, 1, 1);
%plot(w, magH1);
%title('Magnitude of the frequency response of H1(z)');
%xlabel('w');
%ylabel('|H1(z)|');

%subplot(2, 1, 2);
%plot(w, phaseH1);
%title('Phase of the frequency response of H1(z)');
%xlabel('w');
%ylabel('angle(H1(z))');

subplot(2, 1, 1);
plot(w, magH2);
title('Magnitude of the frequency response of H2(z)');
xlabel('w');
ylabel('|H2(z)|');

subplot(2, 1, 2);
plot(w, phaseH2);
title('Phase of the frequency response of H2(z)');
xlabel('w');
ylabel('angle(H2(z))');
```

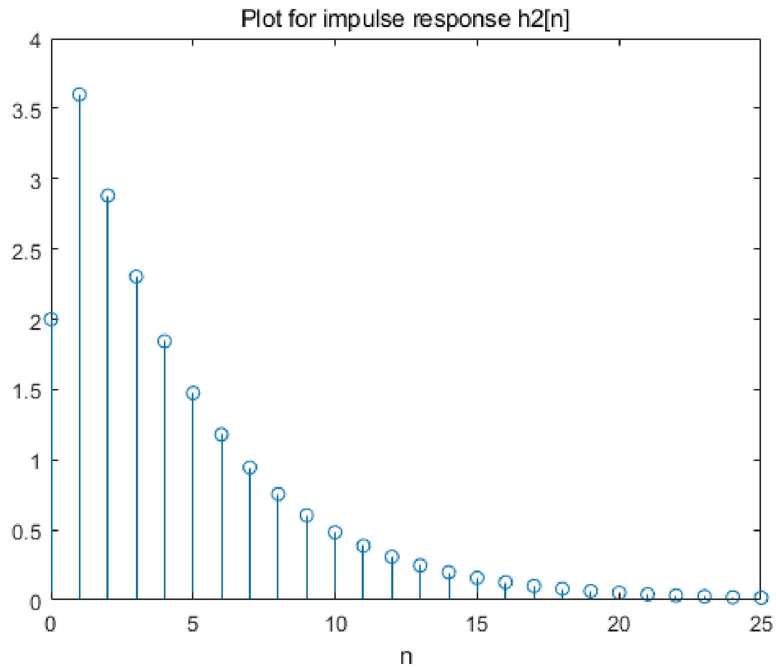




(c) $h_1[n] = -1.6\delta[n] + 3.6 \cdot 1.25^n u[n]$ and $h_2[n] = -2.5\delta[n] + 4.5 \cdot 0.8^n u[n]$.The plots in (c) matches the stability stated in part (a). $h_1[n]$ is unstable. The impulse response grows exponentially to infinity when n approaces inf.$h_2[n]$ is stable. The impulse response decreases exponentially to 0 when n approaces inf.

```
n = (0:25);
h1 = 3.6 * 1.25 .^ n;
h1(1) = h1(1) + -1.6;

h2 = 4.5 * 0.8 .^ n;
h2(1) = h2(1) + -2.5;

stem(n, h1);
title('Plot for impulse response h1[n]');
xlabel('n');

stem(n, h2);
title('Plot for impulse response h2[n]');
xlabel('n');
```




Plot for impulse response h2[n]
4
3.5
3
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25
n