Solitary waves in ultrafast fiber lasers: from solitons to dissipative solitons

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Abstract:
The development of the connection between ultrafast laser dynamics and solitary waves concepts is presented, with an attempt to emphasize on important research milestones and conceptual advances. At various stages, the concepts of conventional and dissipative solitons are found to echo, oppose, or complement each other. The pivotal role of fiber lasers is highlighted, which multiplied the investigation of ultrafast laser dynamics since the 90s, and lead to the widespread usage of dissipative soliton concepts that were instrumental in understanding original dynamical regimes and related optical waveforms, such as the generation of bright solitons in the normal dispersion regime and the self-assembly of optical soliton molecules within laser cavities.

1. Introduction -- solitons entering laser physics

The first experimental evidence of passive laser mode locking by DeMaria et al. in 1966 [1] opened a new avenue for the generation of short optical pulses. Designed as an analog of the microwave regenerative pulse emitter, the laser architecture incorporated a fast saturable absorber dye material. It was considered as the most promising strategy to progress toward sub-picosecond optical pulse generation. By 1975, when H. Haus published his seminal article "Theory of mode locking with a fast saturable absorber," it was affirmed that *a considerable amount of theoretical and experimental work* had already been done on mode locking by a saturable absorber [2]. Nonetheless, the proposed mode locking model was exclusively hinged on dissipative effects: gain with saturation, spectral filtering, and saturable absorption[1]. It adopted a perturbative approach, considering relatively small pulse changes per cavity roundtrip, and completely disregarded the role of the Kerr nonlinearity and that of chromatic dispersion. As a matter of fact, the theoretical prediction a couple of years earlier of the solitary wave propagation in optical fibers by Hasegawa and Tappert [3] was overlooked as fitting another area of nonlinear science. Indeed, in the absence of convenient infrared pulse sources required to reach the anomalous dispersion domain of silica, where the formation of bright solitons would be observed, and the lack of reliable single mode silica fibers, optical soliton observations and their applicative prospects in optical communications remained uncertain projections[2] [4]. Nevertheless, from the mid 70's, while succeeding in reaching sub-picosecond mode-locked pulse durations, laser physicists were confronted with the impact of chromatic dispersion and self-phase modulation (SPM). In the quest of ultrashort pulses, the influence of chromatic dispersion was first considered as a major nuisance, entailing the subsequent development of dispersion compensation schemes. However, if an optical pulse propagates in the anomalous dispersion regime, the combined effects of SPM and dispersion can lead to its temporal compression, which potentially benefits the mode locking process.

[1] The 1975 ultrafast laser model can thus be considered as an early model for dissipative optical solitons.
[2] H. Haus readily entered the topic of soliton propagation in the 80's and pioneered soliton-related investigations in both optical communication and ultrafast lasers arenas.

Following Hasegawa and Tappert, the propagation of optical pulses in dielectric waveguides, in presence of chromatic dispersion and Kerr nonlinearity, is modeled by the nonlinear Schrödinger (NLS) equation such as [3]:

$$i\,\psi_z - \frac{\beta_2}{2}\,\psi_{tt} + \gamma|\psi|^2\psi = 0, \tag{1}$$

where β_2 is the second-order chromatic dispersion coefficient and γ is the effective nonlinear coefficient. ψ is a scalar function that represents the envelope of the electric field (in $\sqrt{W}$unit) as a function of the local time t in the moving pulse frame and the propagation distance z. In the present model, the propagation medium is assumed uniform, lossless, and the vector nature of the electric field is omitted, noting that subsequent extensions beyond these assumptions have led since to a considerable research activity [5-8]. Equation (1) is integrable. In the anomalous dispersion case ($\beta_2 < 0$), with a focusing nonlinearity ($\gamma > 0$), Eq.(1) admits a bright stationary solitary wave, namely the fundamental (N=1) soliton, which is characterized in the time domain by a hyperbolic-secant profile and a constant phase profile, i.e., chirp-free, such as:

$$\psi_{N=1}(z,t) = \sqrt{\frac{|\beta_2|}{\gamma\, T_0^2}}\;sech\left(\frac{t}{T_0}\right)\;exp\left(i\frac{\beta_2}{2T_0^2}z\right), \tag{2}$$

where T_0 is the pulse duration parameter, which is a free parameter in the equation. The fundamental soliton energy is calculated to be:

$$E = \int_{-\infty}^{+\infty}|\psi_{N=1}(z,t)|^2 dt = \frac{2\,|\beta_2|}{\gamma\,T_0} \tag{3}$$

Equation (2) and its consequence Eqn. (3) highlight the existence of a family of fundamental solitons for a given propagation medium, where short solitons carry more energy than long-duration ones. Taking a standard telecom single-mode optical fiber such as SMF-28 and an ultrashort pulse duration parameter of 200 fs yields a fundamental soliton energy around 100 pJ. Equation (1) also admits higher-order soliton solutions, characterized by an integer N>1, which can be excited by launching an initial condition such as:

$$\psi_N(z=0,t) = N\,\psi_{N=1}(z=0,t), \tag{4}$$

which means the above excitation requires an energy N^2times that of the fundamental soliton. Higher-order solitons (N>1), whose spectral and temporal waveform oscillate periodically, can be viewed as solutions where the chromatic dispersion and the Kerr nonlinearity balance *on average*.

In 1980, progress in both infrared ultrafast laser sources and single-mode optical fibers allowed Mollenauer, Stolen, and Gordon to report the first experimental observation of the optical soliton propagation regime [9]. This soliton experiment employed a cryogenically cooled color-center laser tunable in the 1.35-1.75 µm spectral region. By launching 7-ps pulses into a 700-m long single mode fiber, the physicists obtained a clear signature of fundamental as well as higher-order soliton propagation. Such experimental milestone provided incentive to Mollenauer and Stolen for a subsequent experiment, where they incorporated a piece of optical fiber into the feedback loop of the color-center laser cavity, as sketched in Fig. 1(a). Whereas the color-center laser was then offering a path to the generation of picosecond pulses in the near infrared, it suffered complicated adjustments and maintenance, and was prone to unstable mode locking operation. By adding the nonlinear fiber feedback loop, the experimentalists showed that the mode locked regime could be stabilized with a pulse duration fixed by the fiber length: an N=2 high-order soliton propagation took place when the roundtrip fiber length was equal to the soliton period. Thus, the shorter the fiber, the shorter the laser pulse, within practical limits – mode locking instabilities appeared for the shortest sub-picosecond pulse durations reported in the experiment. By demonstrating that solitonic pulse shaping assisted laser mode locking in such a decisive way, the *soliton laser* brilliantly entered the scene of ultrafast lasers [10]. The soliton laser was at the origin of multiple laser experiments that used coupled cavities to enable what was termed "additive-pulse mode locking", a strategy that considerably eased the generation of shorter pulses [11]. In other words, the effectiveness of nonlinear interferences to create a virtual ultrafast saturable absorber was also established. In a virtual saturable absorber, the phase bias of the nonlinear interferences can be generally adjusted by the experimenter, which procures a useful degree of freedom on the resulting nonlinear transfer function to search the mode locking operation. As a final highlight, the 1984 article from Mollenauer and Stolen predicted that, when a suitable fiber gain medium would become available, *the soliton laser would probably take the form of a single loop of fiber closed upon itself. The simplicity and low cost of such devices would make them most attractive* [9].

Despite the first fiber waveguide laser experiment took place in 1961 [12], the fiber laser revolution anticipated by Mollenauer and Stolen still had to wait till the beginning of the 90's, with the advent of the erbium-doped fiber amplifier that transformed the optical communication industry and generalized the use of fiber-integrated components [13]. Meanwhile, decisive progress in bulk ultrafast laser sources were made in the 1980's. Mode-locked dye lasers met their peak in the mid 80's, after the demonstration that the normal chromatic dispersion of the laser cavity could be efficiently compensated by an intracavity prism sequence, yielding pulses as short as 27 fs by the end of 1984 [14]. In parallel, the development of the titanium-sapphire (Ti:S) laser from 1982 was gaining momentum as a reliable solid-state broadband laser source [15]. By using intracavity prims, passively mode locked Ti:S generated sub-100 fs pulse duration in 1991 [16]. From that date, the usage of the dye lasers, which suffer from a delicate maintenance and generate toxic waste, quickly declined. The possibility to achieve mode locking operation of the Ti:S laser in the absence of specific saturable absorber material, initially dubbed as "magic mode locking", was subsequently understood as arising from the nonlinear Kerr lensing effect inside the crystal, which is a spatial equivalent of the virtual saturable absorber effect introduced above. In the wake of ultrafast dye lasers, most mode-locked Ti:S lasers would be operated in a slightly anomalous path-averaged dispersion regime, to fit a "solitonic" operation characterized by a nearly chirp-free hyperbolic-secant pulse profile. It is worthy to mention that the perturbative modeling approach, considering small pulse changes per cavity roundtrip, would often work well to model dye and Ti:S mode locked lasers, which had de facto become the earliest *practical* soliton lasers. Therefore, beyond the fundamental (N=1) soliton propagation that was the leitmotiv for the laser cavity settings, researchers reported N=2 and N=3 higher-order soliton generation, characterized by a soliton period that was up to three orders of magnitude larger than the cavity length [17-19]. This further confirmed the relevance of an average soliton propagation model for these ultrafast lasers.

Enabled by high-quality single-mode erbium-doped fibers (EDF) and by efficient laser diode pumping in the near infrared, both available from the end of the 80s [20-22], the development of mode-locked fiber lasers literally exploded in the early 90s [23-26]. By operating around the 1.5-μm spectral region, where standard silica fibers have anomalous dispersion, ultrafast EDF lasers immediately prompted soliton dynamics inquiry. Let us point out major differences between fiber lasers and bulk lasers such as Ti:S and dye lasers. In fiber lasers, the pulse propagates within, typically, meters of dense media every cavity roundtrip in a confined propagation mode.

In a passive fiber, Kerr nonlinearity and chromatic dispersion constitute the major propagation effects for short pulses, which indeed reconnects to the soliton dynamics. Besides, a meter long EDF can provide high gain, which enables to run ultrafast fiber lasers with significant cavity losses cumulated from the laser output coupler and other fiber integrated components, as well as from the ultrafast saturable absorber mechanism. Consequently, the physical effects acting over a single roundtrip cannot generally be taken as perturbative. Considering soliton propagation in fiber lasers, the substantial pulse reshaping along the optical fibers generates dispersive waves, which can interfere constructively over successive cavity roundtrip with specific spectral components of the soliton, leading to an enhanced scattering of radiation sidebands: this landmark feature of soliton fiber lasers is recalled and explicated by J.R. Taylor in the present special issue [27].

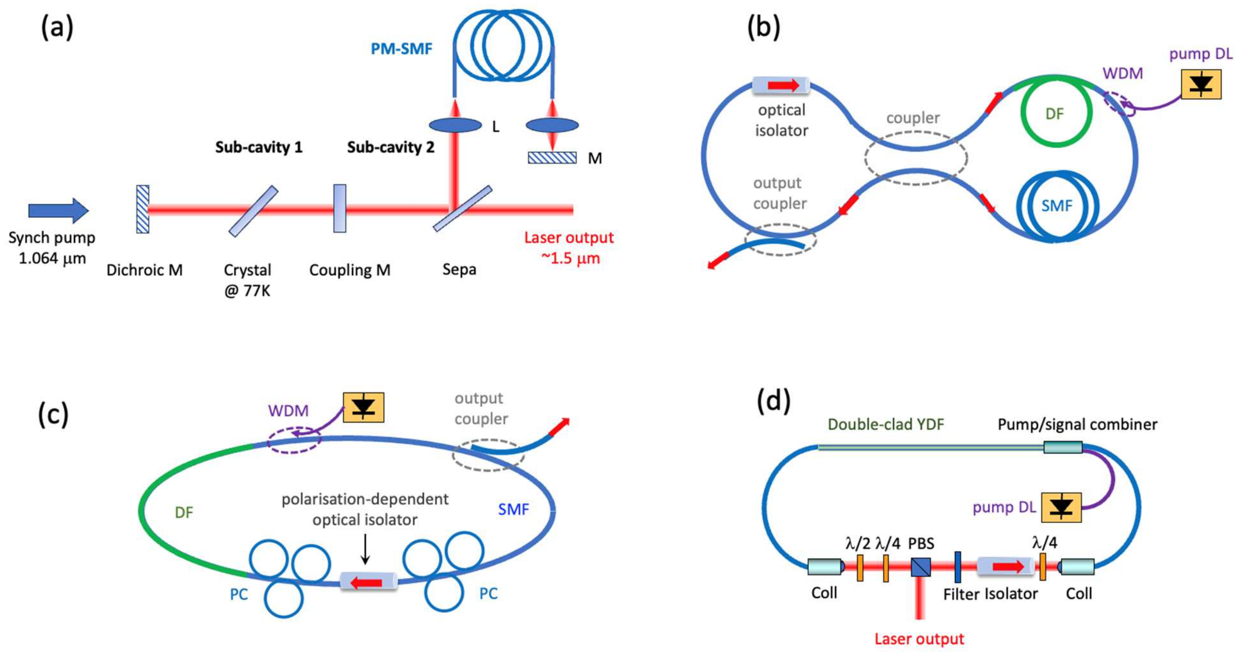

Figure 1. Illustration of the development of mode-locked laser architectures based on soliton and dissipative soliton propagation. (a) The 1984 soliton laser (simplified). M: mirror; L: lens; PM-SMF: polarization-maintaining single mode fiber: Sepa: output coupler mirror. (b) Typical figure-of-eight all-fiber laser based on a nonlinear amplifying loop mirror (NALM). DF: doped fiber; SMF: single-mode fiber; DL: diode laser; WDM: wavelength division multiplexer. (c) Typical all-fiber ring laser mode locked through the nonlinear polarization evolution (NPE) mechanism. PC: polarizer controller. (d) Ytterbium-doped fiber (YDF) laser operating in the all-normal dispersion regime owing to a combination of NPE and spectral filtering. The latter are implemented and adjusted owing to an intracavity open-air section. PBS: polarizing beam splitter; Coll: collimating lens.

The availability of telecom-grade fiber integrated components allowed to test a wide variety of ultrafast fiber laser architectures at shrinking costs. In particular, it was found that fiber laser architectures could easily incorporate various additive-pulse mode locking schemes, or equivalently, several methods enabling an ultrafast virtual saturable absorber effect to take place. For instance, various versions of nonlinear optical loop mirrors – extensions of the Sagnac interferometer – can be implemented. Figure 1(b) displays a fiber laser architecture where the loop mirror includes an optical amplifier, constituting a nonlinear amplifying loop mirror

(NALM) [24,28]. In subsequent versions, by employing polarization-maintaining (PM) fibers, the improved environmental stability of fiber lasers mode locked through nonlinear loop mirrors was conducive to the marketing of rugged turnkey ultrafast fiber lasers [29].

A recurrent and striking observation in ultrafast EDF lasers is their ability to self-generate multiple pulses per cavity roundtrip. This was readily explained by the limitation imposed by the fundamental soliton energy [30]. In a standard telecom fiber (SMF), the (N=1) soliton energy is typically in the range 10-100 pJ for a FWHM pulse duration spanning from 1 ps to 100 fs. For instance, a 50 pJ soliton traveling round a 10-meter-long fiber cavity would make an average intracavity power of 1 mW. Considering a pump laser diode routinely delivering over 100 mW coupled to an efficient intracavity EDF amplifier, the soliton energy limitation soon comes into play. Thus, depending on the mode locking settings, up to tens of coexisting soliton pulses could be observed in early ultrafast EDF laser experiments. Whereas the soliton energy quantization effect provided a convincing explanation, which definitely rooted solitons into ultrafast fiber laser dynamics, several puzzling questions rose in comparison to the original 1984 soliton laser experiment. Firstly, high-order solitons have been reluctant to manifest in ultrafast fiber lasers. High-order solitons could have logically constituted a dynamical possibility when the stored cavity energy exceeded that of the fundamental soliton, but instead, the observation repeatedly confirmed the generation of multiple N=1 soliton pulses. The latter appeared when the pump power was increased beyond a certain bifurcation threshold, triggering a multi-pulsing instability [31]. Secondly, the soliton pulse duration was not obviously related to the cavity length. In the 1984 experiment, the roundtrip length of the fiber feedback loop needed to be equal to the soliton period owing to the high-order soliton propagation, whereas this condition is obviously lifted for N=1 solitons propagating in ultrafast fiber lasers. There are several other remarkable features with solitons generated by ultrafast fiber lasers. Let us recall that the fundamental solitons of the nonlinear Schrödinger (NLS) equation belong to a *family* of solutions, where the soliton pulse duration is not fixed *a priori*. So, if multiple laser pulses are NLS fundamental solitons, why are they identical – same duration, same energy – in the laser cavity, in contrast to what is observed in passive fiber cavities [32]? Why do they tend to form stationary optical patterns in laser experiments? Why do they often feature a residual frequency chirping? These observations lead to the conclusion that the understanding of soliton fiber lasers required to go beyond the NLS model.

The need to surpass the conventional soliton model was also triggered by the development of fiber laser cavities employing dispersion management, which generate stretched pulses whose dynamics is characterized by a sustained temporal breathing of the intracavity pulses [33]. Technically, dispersion management in erbium-doped fiber lasers came up quite naturally. Indeed, to increase the energy transfer efficiency from the pump to the gain medium, doped fibers are routinely designed with a reduced core diameter. This results in an increased waveguide dispersion, shifting the propagation regime within the doped fiber well into the normal dispersion regime. Yet, the implementation of a large dispersion management effect inside a laser was conceptually bold, since it departed from the initial idea of keeping the pulse changes per roundtrip small, as embodied by the early master equations for mode locking, see Section 2. A typical dispersion-managed (DM) fiber ring laser architecture is shown in Fig. 1(c). Within a cavity roundtrip, the laser pulse propagates successively through a normally dispersive doped fiber and an anomalously dispersive passive fiber, which entails pulse stretching and compression twice per roundtrip. Such fiber ring laser architecture is also conducive to the implementation of a virtual saturable absorber effect based on the nonlinear polarization evolution (NPE), which takes place in the fibers, followed by polarization discrimination through a polarizing beam splitter [25].

A DM laser architecture featuring a large stretching factor means that most of the time, the pulse peak intensity is considerably reduced, therefore limiting the accumulated nonlinearity. This way, a pulse of larger energy can be supported without suffering breakup. The DM laser architecture succeeded in enhancing the pulse energy by at least one order of magnitude compared to the previous soliton fiber lasers [33,34]. This was also interpreted in the frame of the *dispersion-managed soliton* concept, which was simultaneously developed to improve optical transmission lines from the mid 90s' [35]. Therefore, stretched pulse dynamics allowed mode locked fiber laser oscillators to step into the nanojoule pulse energy domain, which accelerated the transfer of technology to place compact, energy-efficient, femtosecond fiber laser devices on the market from 1995 [36]. The concept was extended to other laser wavelengths, for instance by using intracavity compression gratings when anomalous dispersion fibers were not available [37].

2. Toward dissipative solitons for ultrafast lasers

In the wake of the 1984 soliton laser experiment, investigations with coupled-cavity mode-locked lasers hinted that stable mode locking could still be reached in the normal dispersion regime of the intracavity fiber feedback loop, ruling out solitonic pulse shaping [38]. In the same year as the soliton laser experiment, Martinez, Fork, and Gordon proposed an extension of Haus's 1975 mode-locked laser model, with the inclusion of self-phase modulation and group-velocity dispersion effects [39]. They analytically predicted chirped hyperbolic-secant pulse solutions, including in the normal dispersion regime, even though the stability of these solutions was not examined. In 1991, Haus et al. performed a major overhaul of the 1975 model, leading to a new master equation for mode locking [40]. In the steady state, this master equation can be written as:

$$\left[i\varphi + (\delta + ix) + g\left(1 + \frac{1}{\Omega_g^2}\frac{d^2}{dt^2}\right) + i\frac{\beta_2 L}{2}\frac{d^2}{dt^2} + (\varepsilon - i\gamma L)|\psi|^2\right]\psi = 0, \tag{5}$$

where $\delta < 0$ represents the cavity linear loss; x and φ, linear phase shifts; $g > 0$, the gain coefficient; Ω_g the gain bandwidth; L is the cavity length. The coefficient $\varepsilon > 0$ represents, at the cubic order, the nonlinear effect of the saturable absorber that promotes mode locking over continuous-wave operation. As in Eqn. (1), β_2 and γ represent the chromatic dispersion and the effective nonlinearity coefficients, respectively.

Solving Eqn. (5) confirmed the existence of pulse solutions in the normal dispersion regime. As a central feature, laser pulse solutions in the normal dispersion regime always require frequency chirping. The authors explained the existence of stable laser pulses within the normal dispersion regime as follows. Propagating in the positive group-velocity dispersion (GVD) regime, the pulse stretches temporally and acquires a large frequency chirp. When the laser gain is applied to the chirped pulse, the limited gain bandwidth turns out to shorten the pulse as it shaves off the high and low frequency pulse wings. In the temporal domain, the pulse is kept in balance by a lengthening coming from the positive GVD. In the optical spectral domain, SPM compensates the spectral narrowing resulting from gain dispersion [40]. Within these few lines, the key points to establish the concept of a dissipative optical soliton are set: the notion of a complex balance where dissipation plays a crucial role is highlighted, which combines with chromatic dispersion and Kerr nonlinearity to enable new types of solitary wave solutions [41,42].

Subsequent theoretical investigations confirmed the profusion of solitary waves in the normal dispersion regime, such as in the frame of the complex cubic-quintic Ginzburg-Landau equation (CGLE) [43], which is exposed below. These findings were strongly echoed a decade later, with thorough experimental and numerical investigations of dissipative solitons in all-normal dispersion fiber lasers, where the needed pass-band effect was purposedly exacerbated by a lumped spectral filter, emphasizing on the energy scalability of the resulting highly chirped pulses [44-47]. The ytterbium-doped fiber (YDF) laser, which operates in the 1-μm band where silica procures normal dispersion, has been instrumental in these investigations. In the laser cavity, the necessary balance between the above physical effects is fulfilled over a cavity roundtrip, as constitutive propagation effects take place sequentially. Exploring dissipative solitons in all-normal dispersion helped to design higher-energy mode locked lasers. This endeavor saw the consecration of the ytterbium-doped fiber laser as the most suitable fiber laser platform for energy scaling so far. Indeed, the YDF amplifier features high gain, low quantum defect, and is therefore highly energy efficient. A typical all-normal YDF laser architecture is sketched on Fig. 1(d) [44]. It features a double-clad YDF, allowing the coupling of a multi-watt pumping power, as well as an open-air section to precisely control the necessary spectral filtering and NPE mode locking in a compact setting. Further energy scaling of YDF lasers operating in the all-normal dispersion regime involved the use of low-nonlinearity large-mode-area (LMA) fibers, reaching the microjoule level from a single laser oscillator [48]. Such energy level, associated with an ultrashort pulse duration, is a key enabler for industrial applications such as precision micromachining, medical surgery, integrated-circuit package cutting, etc.

We can draw a parallel between promoting highly chirped dissipative solitons in fiber lasers for energy scaling and the strategy of chirped-pulse amplification (CPA), which was pioneered in the mid-80's by Strickland and Mourou [49], the main difference being, in the present case, its implementation within laser oscillators. Fiber laser architectures have been able to incorporate modern fiber CPA concepts in a self-consistent way, such as with the self-similar evolution of parabolic pulses [50,51].

To establish the general concept of dissipative solitons for ultrashort laser pulses, the stability of the pulse solutions under perturbations and their behaviors during collisions needed to be investigated, in order to highlight the relevance of such a new solitary wave concept. Theoretical and numerical investigations started in the early 90's and demonstrated that the solitary wave solutions of non-integrable dissipative systems displayed remarkable properties that could find immediate applications in multiple areas in physics and beyond, finally leading to the wide acceptance of the dissipative soliton terminology at the beginning of the new millennium [52-56].

Besides the existence of stable bright laser pulses in the normal dispersion regime discussed above, peculiar features of optical pulses generated by laser cavities in the anomalous dispersion regime cannot be explained in the frame of conventional solitons. We already mentioned the question about understanding the generation of multiple identical soliton pulses. It can be loosely answered by stating that the pulse duration is determined by the available gain bandwidth. Given the fiber dispersion and nonlinearity parameters, knowing the pulse duration then fixes the soliton energy. However, playing with the accessible dissipative cavity parameters such as linear and nonlinear loss, spectral filtering, etc., laser physicists observed that, for a given anomalous dispersion, they could generate chirped pulses spanning over a wide range of pulse duration. Again, the key features of the stable pulse dynamics can only be determined once the dissipative effects are appropriately taken into account, which calls for a unified soliton picture for active optical systems, namely, the dissipative soliton.

3. Dissipative soliton central features

The framework of dissipative soliton cannot operate without the notion of a dynamical attractor. If we contemplate the propagation of laser pulses in a stationary mode locked operation, we realize the extraordinary stability of these solitary waves that can travel the equivalent of billions of kilometers in dense media without being overly affected by various noise sources, a feat inconceivable for NLS solitons. This is due to a restoring "force" that pulls backs the laser pulse to an attracting state, which is enabled by dissipation. Precisely, a dynamical attractor of stable focus type ensures a fixed pulse profile endowed with robustness and stability. Such attractor results from the composite balance involving dispersive and dissipative propagation effects, and defines the dissipative soliton. *In essence, all mode locked laser pulses are dissipative solitons*. Dissipative solitons can be defined as localized formations of the electromagnetic field that are balanced through an energy exchange with the environment in presence of nonlinearity, dispersion and/or diffraction [56]. By allowing a large freedom in handling the balance between the major physical effects at play within the laser cavity, the dissipative soliton concept for ultrafast lasers opens up original dynamical possibilities, as reflected by the diversity of laser architectures and cavity parameter ranges explored, especially since the beginning of this millennium [56-60].

Whereas a stable focus attractor provides a fixed output pulse profile – in both temporal and spectral domains, it is also linked to the existence, at every single location inside the laser cavity, of a fixed attracting profile – namely a local attracting state, which spatially evolves along the cavity according to the propagation dynamics. In other words, in experiments as well as in non-uniform propagation models, the attracting state is always multidimensional. Since the experimental characterization is generally conducted from a single laser output, it misses this important feature that can be investigated through numerical simulations. The existence of an attracting state easily explains why multiple pulses traveling round the laser cavity will generally adopt an identical profile, since all of them will be precisely carved by the same attractor. A strong focus attractor makes an efficient noise filter, bringing stability to the pulsed regime. It also brings robustness, which means that a small amount of system parameter change is tolerated and will not disrupt the pulsed laser operation. This way, even without an active stabilization, mode locked lasers can feature low noise.

The pulse profile of a dissipative soliton is found to vary widely according to the system parameters, spanning for instance from a hyperbolic-secant type to gaussian and even flat-top type profiles. Such diversity can be conveniently explored numerically within distributed master equations. Although the latter do not capture the intracavity dynamics, they are useful in bringing insight about the influence of the major physical effects involved in the laser cavity. This approach, initiated in 1975, was further highlighted by the relevance of Ginzburg-Landau equations – with an emphasis on the complex cubic-quintic Ginzburg-Landau equation (CGLE) – to model universal ultrafast laser dynamics [52-56]. In its standard form, the CGLE can be built by adding the following dissipative terms to the NLS equation: linear loss, nonlinear gain, saturation of the nonlinear gain, and spectral filtering. These dissipative terms model the combined effects of the laser gain and the saturable absorber assuming an instantaneous response. However, the dissipative terms are not perturbative to the NLS. They are allowed to acquire a significant magnitude, which is why original pulse solutions can be found in domains precluded by the NLS – for instance bright solitons in the normal dispersion regime.

The CGLE propagation model reads, for a normalized field amplitude ψ:

$$i\,\psi_z + \frac{D}{2}\,\psi_{tt} + |\psi|^2\psi = i\delta\psi + i\beta\,\psi_{tt} + i\varepsilon|\psi|^2\psi + i\mu|\psi|^4\psi - \nu|\psi|^4\psi. \tag{6}$$

The left-hand side of Eqn.(6) represents the terms of the NLS equation, where D is the dispersion parameter (whose sign is opposite to the group-velocity dispersion parameter β_2, i.e., for normal chromatic dispersion, $D < 0$ while $\beta_2 > 0$). The right-hand side contains, in essence, all the dissipative physical effects required in a mode locked laser system. In addition to the cavity loss ($\delta < 0$) and nonlinear gain ($\varepsilon > 0$) terms present in the 1991 Haus model, the above CGLE features a quintic saturation of the nonlinear gain ($\mu < 0$) that turns out to be essential for the stability of the pulse solutions. Indeed, in the absence of the latter, the cubic gain term may run away, generating a non-physical blowup of the pulse solution [31,43]. Likewise in Eqn. (5), the CGLE features spectral filtering ($\beta > 0$), which is important to stabilize the pulse solution in the frequency domain, taking into account the gain bandwidth as well as other intracavity filtering elements. Finally, the last term is a quintic correction to the nonlinear refractive index. Whereas such a term is usually irrelevant in the case of silica fibers, it may come up from the mathematical averaging procedure that distributes specific propagation effects over the whole cavity roundtrip [53]. It should also be emphasized that the CGLE is not integrable. Whereas for some parameters sets, analytical solutions can be obtained, the general exploration of the CGLE solutions remains essentially numerical.

Figure 2 illustrates the diversity of stable pulse profiles found upon parameter change, here in the normal dispersion regime. By varying the nonlinear gain parameter ε, the pulse amplitude can get close to either a hyperbolic-secant, a gaussian, or a flat-top profile. As anticipated from the above discussions, all pulses bear a significant frequency chirping. This is a general feature of stationary dissipative solitons – in contrast to the fundamental NLS soliton endowed with a constant nonlinear phase shift across its temporal profile – which is understood after recalling that the electromagnetic energy flows through phase gradients. Due to the nonlinear dissipation terms, some parts of the pulse will be amplified during propagation (for instance, the central pulse part), whereas others (for instance, pulse tails) will be attenuated. To maintain a stationary pulse profile, the dissipative soliton requires an internal energy redistribution among its various pulse parts, which is enabled by nonuniform phase gradients, hence the frequency chirping. Whereas this mechanism fits perfectly the explanation provided earlier for the existence of stable bright dissipative solitons in the normal dispersion regime, it is also acting in the anomalous dispersion regime. The frequency chirping in the anomalous dispersion regime has generally a much lower magnitude than in the normal dispersion regime. Nevertheless, it is possible to find domains of laser cavity parameters with anomalous dispersion where dissipative solitons feature an important frequency chirping, such as in the so-called *dissipative soliton resonance* [61,62]. Again, the practical benefit of these highly chirped pulse solution resides in their superior energy scaling.

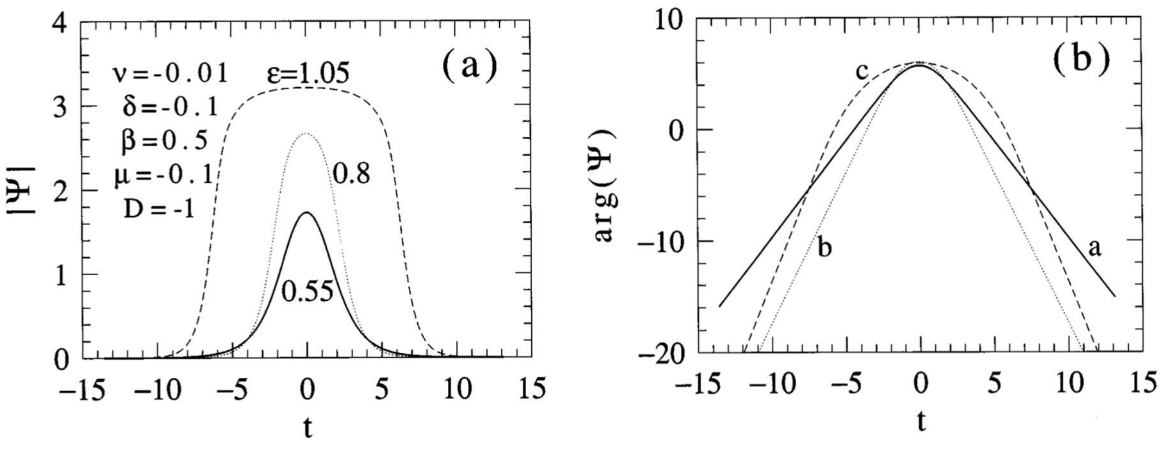

Figure 2. Stable pulse solutions of the CGLE obtained numerically using parameters as indicated, in the normal dispersion regime. (a) Amplitude profile. (b) Phase profile. The three pulse profiles are obtained by varying the nonlinear gain parameter ε. After [43].

Since dissipative solitons exist for both anomalous and normal dispersion regimes without known theoretical limitations, we can find dissipative solitons in the absence of linear dispersion. This is practically the case when the dispersion length is much larger than the cavity length, provided that the dissipative effects are important: this way, the dispersive effects are not cumulated over successive roundtrips [63]. Such situation is also experimentally met in fiber laser cavities endowed with precise dispersion compensation, provided that the length of the dispersion map remains shorter than the dispersion length so that the dynamics experiences the average dispersion [64]. In general, the dissipative soliton paradigm allows to considerably relax the constraints on chromatic dispersion. It is likely to blend with the recently investigated quartic soliton lasers where an original pulse shaping arises from the balance between fourth-order dispersion and the Kerr nonlinearity [65]. The flexibility of the dissipative soliton workbench can be extrapolated to the domain of transverse spatial propagation [66,67]. In multimode fiber lasers, some amount of intermodal dispersion can be balanced, with the help of spatial filtering, enabling spatiotemporal mode locking, which has become an important research topic in recent years [68-73].

4. Dissipative solitons in the light of complexity

4.1 Dissipative soliton molecules

The efficient tool of the dissipative soliton attractor can operate at a larger physical scale, such as for a set of interacting soliton pulses. In a laser cavity, multiple solitons always interact: traveling within a bounded system, they virtually have an infinite time to reveal even faint interaction processes. In the simplest case of two laser solitons interacting through their overlapped pulse tails, such direct interaction can lead to a stable equilibrium where the relative timing of the two pulses will converge to a fixed separation, while their relative phases will self-lock at a fixed phase difference. In this example, the features of the resulting soliton bound state are determined by a multisoliton attractor. The basin of attraction corresponds to the range of the two-pulse initial conditions leading to the formation of the soliton bound state. The distributed CGLE model has proved to be particularly useful to reveal multisoliton attractors [61,62], which frequently manifest within ultrafast lasers [56].

It is important to state that these dissipative multiple-soliton bound states have no direct relationship with the high-order solitons that can propagate in passive optical fibers modeled by the NLS equation. For high-order NLS solitons, the spectral and temporal profiles strongly oscillate, noting also that such oscillation is destabilized by additional propagation effects such as Raman scattering or third-order dispersion, possibly leading to a breakup of the high-order soliton [76]. Whereas, in the frame of the homogeneous NLS equation, optical solitons also interact through their tails, multiple solitons do not reach a stable equilibrium pattern [77]. In contrast, dissipative soliton bound states can be perfectly stable, which is reflected in experiments: the prominent signature of a stationary bound state of two temporal solitons is the presence, within the measured average optical spectrum, of periodic fringes featuring a high contrast [78-80]. Recently, the relative timing jitter between two self-locked pulses was measured down to the attosecond level [81]. Since they display several striking analogies with matter molecules, the bound states of laser pulses have been called optical soliton molecules, a now widely adopted terminology that originated from the study of soliton propagation in dispersion-managed fiber systems [82]. Figures 3 (a-d) present early experimental confirmations of soliton molecules in ultrafast fiber lasers, obtained through the combination of optical autocorrelation and optical spectral measurements, noting that the few-picosecond internal structure of the soliton molecule could not be directly recorded by a photodiode. Figures 3 (e,f) show numerical simulations of the intracavity dynamics of a soliton molecule in the stationary regime, which means that, following a transition time, the intracavity pulse evolution has become identical roundtrip after roundtrip. within a dispersion-managed laser in both path-averaged chromatic dispersion regimes. Whereas each dissipative soliton can undergo an important evolution within the cavity roundtrip, the soliton molecule appears like a frozen entity, where the internal separation and phase difference between the two solitons remains practically constant. This arises in part due to the relatively weak interaction between two pulses separated by a few pulse widths. Nevertheless, the stable stationary soliton molecule is perfectly maintained in the situation of a major pulse stretching and compression displayed in Fig. 3(e), where both pulses strongly overlap in a significant portion of the laser cavity. The two-pulse entity is strongly stabilized by a soliton molecule attractor. As a matter of fact, to demonstrate such property numerically requires to run the propagation model over thousands of cavity roundtrip, the scale at which the interactions among pulses are clearly revealed. Figs. 3(e,f) illustrate the major difference between the dynamics of soliton molecules and that of higher-order NLS solitons [5]. The topic of dissipative optical soliton molecules has driven considerable fundamental interest, with investigations conducted within various optical platforms, from ultrafast lasers to microresonators [57,83,84]. Potential applications of soliton molecules include multi-level data encoding and processing [85-87].

4.2 Pulsating dissipative solitons

Let us first come back to the situation of a single, initially stationary, dissipative soliton pulse. When the control parameters of the ultrafast laser – such as pump power, nonlinear losses, dispersion, etc. – are altered, a stable focus attractor can be subjected to bifurcations. The Hopf bifurcation is a ubiquitous bifurcation that transforms a single focus attractor into a limit cycle. Subsequent to a Hopf bifurcation, the dissipative soliton pulse will oscillate in time, with oscillation characteristics solely determined by the system parameters. Accordingly, in a distributed propagation model such as the CGLE, the pulse profile oscillates with a given period in the spatial propagation coordinate. If we now consider the ultrashort pulse dynamics within a realistic laser cavity model – i.e., a parameter-managed propagation model, then at each location of the laser cavity, the pulse profile will oscillate from one roundtrip to the next. Therefore, the cavity periodicity implies the observation of a discrete oscillation. This pulse oscillation can be N-periodic with respect to the cavity roundtrip – a feature called entrainment, which means here a self-synchronization of the pulsating dynamics to harmonics of the cavity fundamental frequency [88-91]. As a matter of fact, the period-2 pulsation is the most generic pulsation, which can be followed by a cascade of period doubling bifurcations when the system parameters are moved further and lead to chaotic dynamics. Another frequent observation is that the initial bifurcation leads to a long period pulsation (large N value), which is incommensurate with the cavity roundtrip in the absence of entrainment. It is even possible to combine long and short period pulsations, among many other possibilities, as the bifurcation options for nonlinear systems having an infinite number of degrees of freedom appear limitless [88,92]. Let us now ask the question: is there an immediate relationship between pulsating dissipative solitons and higher-order NLS soliton dynamics? The negative answer is supported by the existence of dissipation-enabled attractors. The asymptotic dissipative soliton dynamics does not depend on the initial waveform conditions when the latter belong to the basin of attraction of the attractor, in contrast to NLS soliton dynamics that are particularly sensitive to the initial field characteristics. Nevertheless, some connection between pulsating ultrafast fiber laser dynamics and high-order solitons dynamics can be found. Indeed, both dynamics classically originate from an excess of self-phase modulation taking place in a propagation fiber, which in the fiber laser system triggers a bifurcation being typically, but not exclusively, a period doubling bifurcation [91].

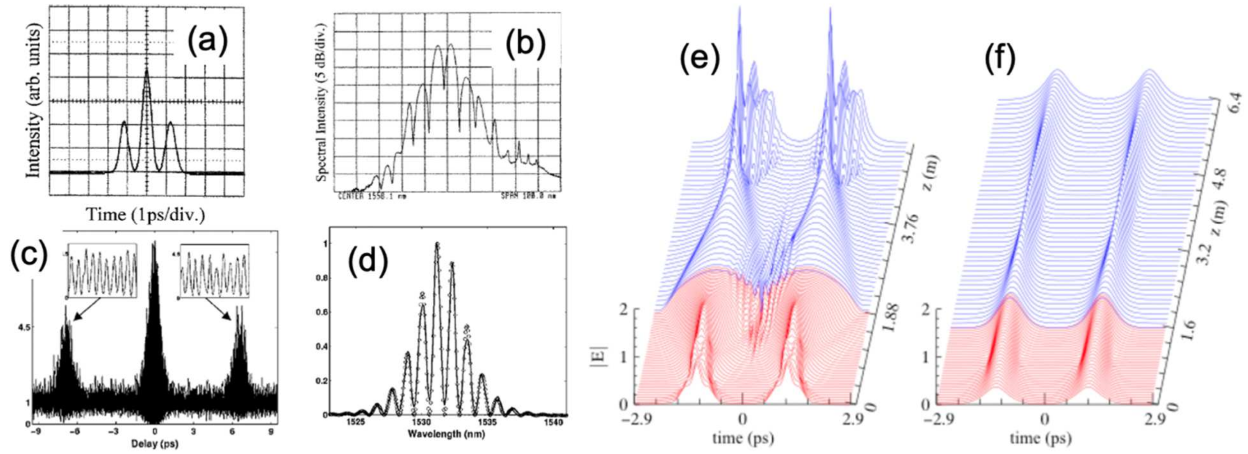

Figure 3. Optical soliton molecules in ultrafast fiber lasers. Early experimental reports of (a,b) out-of-phase and (c,d) quadrature phase-locked soliton pair molecules. (a) Background-free optical autocorrelation ; (b) Optical spectrum (log scale) ; (c) Interferometric autocorrelation ; (d) Optical spectrum (linear scale). (e,f): Numerical simulations showing the propagation of a soliton molecule within a dispersion-managed cavity roundtrip, in the stationary state. The average dispersion is (e) normal ; (f) anomalous. From [78-80].

4.3 Vibrating soliton molecules

We can now move one step further in complexity by combining bifurcations with multiple pulses. Starting from a stationary two-soliton molecule, one can wonder whether there would exist bifurcations leading to a pulsating soliton molecule, which could be similar to an excited vibrating mode of a molecule of matter. We reported the first positive evidence in 2006. By monitoring the output of an ultrafast fiber laser, as the laser parameters were tuned, the blurring of the experimentally recorded spectral fringes and that of second-order cross-correlation peaks were attributed to an oscillation of the relative temporal separation and phase between two bound dissipative solitons, an interpretation supported by detailed numerical simulations of the laser dynamics [93]. The topic of pulsating soliton molecules gained further consideration a decade later, with the generalization of real-time spectral measurements that provided a strong experimental confirmation [94,95]. To this end, the time-stretch dispersive Fourier-transformation (DFT) was used. DFT finds its origin in a groundbreaking technical research article published in 1973 entitled “A new approach to picosecond laser pulse analysis, shaping and coding”, where the principles of spectral analysis at multi-MHz repetition rates as well as those of ultrafast pulse shaping were laid down, an article that was mostly forgotten a generation later [96]. Being a linear technique relatively simple to implement with modern instrumentation, DFT has recently become the most popular real-time spectral measurement method in ultrafast optics. It consists of transmitting each laser output pulse waveform through a highly dispersive medium in a linear and far-field propagation regime. The optical pulse then stretches considerably to adopt an intensity profile that maps its spectral intensity profile, which is directly read out by a fast photodetector and oscilloscope detection system [97]. The optical spectrum $I(\omega)$ of two identical pulses, delayed in time by τ, having a phase difference φ, simply reads:

$$I(\omega) = 2I_0(\omega)[1 + \cos(\omega\tau - \varphi)], \quad (7)$$

where $I_0(\omega)$ is the single pulse spectrum, as a function of the angular frequency ω. Therefore, in the case of a pulsating soliton molecule, by processing consecutively recorded spectra, the evolution of $\tau(n)$ and t $\varphi(n)$ as a function of the roundtrip number n is obtained. This way, several types of pulsating soliton molecules have been characterized experimentally and validated numerically, such as vibrating soliton molecules (featuring both separation and phase oscillation), and phase-only oscillating molecules [94,95]. In some cases, the oscillation of soliton molecules can become strongly anharmonic [98]. In vivid analogy with the dynamics of matter molecules, the formation and dissociation of unstable soliton molecules has been recorded, as well as transient multisoliton states preceding the formation of stable soliton molecules [98,99].

Temporal optical soliton molecules are remarkable physical objects of ultrafast nonlinear science. When their soliton constituents bind at separations significantly larger than their pulse width, most of the dynamics can be captured in a reduced dimensional space featuring, typically, two extra degrees of freedom per additional soliton: the relative separation and phase with its nearest neighbor[3]. Recently, soliton-pair molecules have been shown to exhibit low-dimensional chaotic motions [102]. These dynamics were recorded by the balanced-optical correlation method, a technique which allows the continuous measurement of relative pulse separation with an unprecedented sub-femtosecond resolution [103]. Investigations also demonstrated the possibility to synchronize vibrating soliton molecules to an external oscillator through optical injection [104]. Therefore, the prospect of developing effective dynamical models to predict soliton molecule dynamics in the vicinity of a given configuration is promising, which could for instance become a tool to utilize soliton molecule dynamics to store and process information [87,105].

To close this section, let us remark that the fruitful analogy between soliton with matter molecules has clear limitations, as there are no such linear oscillation modes and quantized excited energy levels. Nevertheless, the formation of soliton molecules entails a change of the stored cavity energy, which yields the equivalent of a quantized binding energy [106]. The major difference remains that dissipative soliton molecules live in a non-Hamiltonian open system sustained by an energy flow, so that they quickly vanish once the laser pump is switched off.

4.4 Incoherent dissipative solitons

Starting from stable mode locking and altering the laser cavity parameters, we typically cross the successive boundaries of stationary and pulsating dissipative solitons, to end up with the complete disruption of the mode-locked pulse operation. Nonetheless, abrupt bifurcations of a different type can occur, leading for instance to peculiar chaotic-pulse dynamics. In the latter, a strong competition between a marked instability and efficient localizing physical effects takes place to maintain a short pulse localized in its average moving reference frame. In this paradoxical situation, despite the persistence of a rather well-defined round-trip time for the traveling chaotic pulse, the laser regime can no longer be qualified as mode locked. Such situation can take various forms and magnitudes, which are encompassed within the notion of incoherent dissipative solitons [107,108]. This recent terminology presents an interesting parallel with the notion of incoherent solitons within conservative systems that involves the self-trapping

[3] The general assumption of identical pulse profiles does not hold true when time-delayed dynamics become important [100], and can also break down for closely interacting soliton molecules [101].

of incoherent light in either the spatial or temporal domain, achieved by using a nonlinear medium characterized by a slow response or by nonlocality [109-111]. For the incoherent solitons observed in ultrafast lasers, dissipation always takes the leading role, enabling the pulse localization in both the temporal and spectral domains – in contrast to the spectrally accelerated temporal incoherent solitons [110]. The meaningfulness of averaged pulse features is the clear signature of an underlying chaotic attractor, which employs dissipative processes to limit the pulse excursions inside of the dynamical system that has otherwise infinite dimensions. Chaotic pulse dynamics in ultrafast lasers include soliton explosions [112,113], dissipative rogue waves [114,115], and noise-like pulse emission [116-124]. Chaotic dissipative soliton attractors can be classified according to the level of incoherence attributed to the laser pulses. Soliton explosions are defined by intermittent periods of pulse instability followed by recovery, the pulse returning to a quasi-stationary coherent mode locking before the next instability develops. Such regime can thus be qualified as partially mode locked.

In contrast, noise-like pulse emission and dissipative rogue waves relate to chaotic pulse dynamics bearing a large degree of incoherence, since the internal field structures behave chaotically at all times, from one cavity roundtrip to the next and even within a given cavity roundtrip. Noise-like pulse (NLP) emission, revealed in 1997 [116], is characterized by the complete loss of mutual coherence between successive pulses [122,123]. This self-starting regime is characterized by a compact waveform, whose shape and duration strongly depend on the laser scaling parameters and a complex fine structure consisting in a large collection of sub-picosecond chaotic pulses. The statistics of the spectral waveforms reveals the presence of spectral rogue waves [123]. Considered initially as a scientific curiosity, NLP dynamics subsequently attracted considerable attention, with attempts to identify its fundamental origin as well as to exploit its potentialities of high-energy and wideband short-pulse generation directly output from a compact fiber laser system [116-121]. According to the fiber laser parameters, NLP energy can reach nanojoule to microjoule levels, scaling with the duration of the incoherent wave packet, from picosecond to nanosecond, as a function of the laser pump power [124]. The physical mechanisms at play seem multifarious, reflecting the diversity of laser cavity architectures and parameters where NLP regimes have been found. More precisely, NLP can build up in either normal or anomalous dispersion regime of the laser, in either short or long cavities. Whereas it is anticipated that in long cavities, NLP dynamics can be strongly affected by higher-order perturbations terms – such as polarization instability and Raman scattering – the onset of these incoherent dissipative soliton dynamics under a reduced set of physical effects – i.e., typically featured in the master Eqn. (5)&(6) – has been demonstrated [108]. This highlights the universality of the phenomenon, which is linked to the existence of strange attractors. Figure 4 displays the self-starting buildup of the NLP regime in a fiber laser cavity in both situations of anomalous and normal dispersion regime that is recorded experimentally through the time stretch DFT technique. In both cases, we can see an explosive spectral broadening transition, and the fading off of pulse precursors. We also clearly see major differences, such as a smoother, less explosive, transition within anomalous dispersion, with a longer decay of the background. Though each case qualifies as the buildup of a generic NLP dynamics, the detailed features strongly depend on the cavity parameters. For instance, an anomalous dispersion is conducive to soliton pulse shaping, which can generate short pulse structures able to maintain over a few cavity roundtrips but nevertheless collide and subsequently disappear within the chaotic pulse bunch. Instead, the normal dispersion regime yields strongly diffusive pulse bursts that quickly pop up and vanish.

By extension, the incoherent dissipative soliton concept is fully applicable to other active optical systems, for instance involving additional field dimensions, such as polarization components or multiple transverse modes [107].

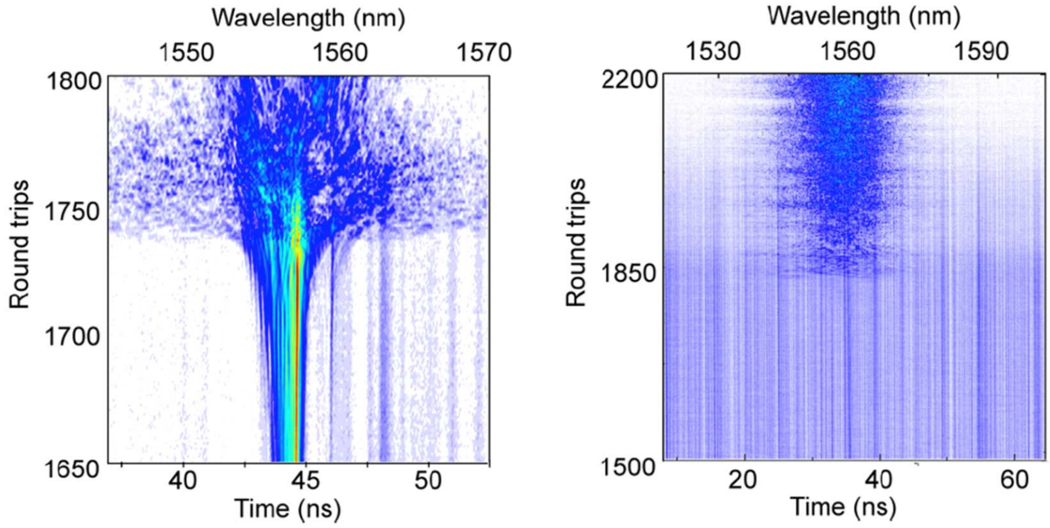

Figure 4. Recording of the buildup of NLP dynamics within an erbium-doped fiber ring laser, using the time stretch DFT method. The cavity dispersion is (a) normal, (b) anomalous. From [108].

5. Conclusion and prospects

In this article, a historical progression was exposed to better appreciate the numerous inputs of soliton concepts in ultrafast laser dynamics, though it was obviously not possible to provide an exhaustive account within the limitations of the present format. Whereas, according to the physical situation, the concepts of conventional (Hamiltonian) and dissipative solitons were found to echo, oppose, or complement each other, *both concepts are definitely indispensable in the culture of laser scientists.* For ultrafast lasers, the concept of dissipative solitons is the broadest and therefore the most applicable one, explaining the generation of seemingly paradoxical regimes such as bright solitons in the normal dispersion regime, highly chirped intracavity pulses, pulsating soliton dynamics, soliton molecules and more complex soliton pattern formations, as well as noise-like pulses and other incoherent and chaotic short pulse formations that abound in ultrafast laser dynamics. Nevertheless, the background of conventional NLS solitons remains essential to apprehend intracavity propagation phenomena, especially when taking place within optical fibers in the anomalous dispersion regime, such as pulse shaping effects, instabilities at high peak intensity, and the resonant scattering of dispersive waves.

The concept of a dissipative soliton is readily generalized to higher dimensions, such as in ultrafast lasers with multiple transverse modes, where it provides a working explanatory picture for spatiotemporal mode locking. Whereas the fiber laser platform has been to date the most prolific one to test and explore dissipative soliton dynamics, most observations have seen an analogue within ultrafast bulk laser systems. The dissipative soliton paradigm tells us that in the most general case, short laser pulses are not mode locked since stationary mode locking merely represents the "tip of the iceberg" of ultrafast dynamics.

Therefore, exploring bifurcations and non-stationary dynamics, beyond being fun fundamental science, also represents a way to develop better control strategies. Indeed, it is important in laser engineering to develop early warning strategies that are able to anticipate the growth of an instability that could lead to the disruption of the expected mode locking regime or to a potential optical damage in industrial processes. For instance, the monitoring in the radiofrequency domain could trace the onset of typical bifurcations, such as the period-doubling bifurcation and ideally use a stabilizing feedback loop to set the laser parameters back into a safe operating domain [91].

In a bigger picture, there is currently an incentive to move beyond the common laser with a single on/off switch and progress toward more flexible multi-purpose ultrafast laser sources. For instance, advanced ultrafast laser oscillators could feature several of the following possibilities accessed through a user-friendly interface: wider spectral tunability, adjustable pulse profile and pulse duration, generation of multi-pulse patterns (soliton molecules, harmonic mode locking, pulse bursts), switching dynamics (coherent and incoherent solitons), and, in the case of lasers with multiple transverse modes, control over the beam profile. Such endeavor is, in general, extremely complex, as laser scientist do not have, in the real experimental world, a fully established correspondence between accessible laser cavity parameters and the laser dynamics. Therefore, one developing paradigm is to get assistance from artificial intelligence (AI). In 2015, we proposed and experimentally demonstrated the first ultrafast laser using AI – with genetic algorithm optimization [125]. Since then, among the growing trend to use AI in all scientific disciplines – and photonics in particular – for tasks as varied as system design optimization, pattern recognition, dynamical prediction, and control, "smart lasers" are being investigated at an increased pace [126-138]. For instance, in the case of complexified laser cavities, such as multimode fiber lasers with spatiotemporal dynamics, AI could bring invaluable services by enabling to navigate within the wealth of accessible dynamics, classify and control them [132,133]. This will likely enable a major evolution of the field of ultrafast lasers and ultrafast photonics in general, from both fundamental and applied perspectives.

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Declaration of competing interests
The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] A.J. DeMaria, D.A. Stetser, H. Heynau, Self modelocking of lasers with saturable absorbers, Appl. Phys. Lett. **8** (1966) 174-176.
[2] H.A. Haus, Theory of mode locking with a fast saturable absorber, J. Appl. Phys. **46**, (1975) 3049.
[3] A. Hasegawa, F. Tappert, Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers I—anomalous dispersion, Appl. Phys. Lett. **23** (1973) 142-1445.
[4] A. Hasegawa, Optical soliton: Review of its discovery and applications in ultra-high-speed communications, Front. Phys. 10:1044845 (2022).

[5] G.P. Agrawal. *Nonlinear fiber optics (5th edition).* Academic Press, New-York (2013).
[6] N. Akhmediev and A. Ankiewicz. *Solitons: nonlinear pulses and beams*. Springer, New-York (1997).
[7] Focus Issue: Optical solitons – Perspectives and applications. Chaos **10**, Issue 3, 471-640 (2000).
[8] Y.S. Kivshar and G.P. Agrawal. *Optical solitons : from fibers to photonic crystals*. Academic Press, San Diego (2003).
[9] L.F. Mollenauer, R.H. Stolen, J.P. Gordon, Experimental observation of picosecond pulse narrowing and solitons in optical fibers, Phys. Rev. Lett. **45** (1980) 1095–1098.
[10] L.F. Mollenauer, R.H. Stolen, The soliton laser, Opt. Lett. **9** 13–5 (1984).
[11] J. Goodberlet, J. Wang, J.G. Fujimoto, P.A. Schultz, Starting dynamics of additive-pulse mode locking in the Ti:Al2O3 laser, Opt. Lett. **15** (1990) 1300-1302.
[12] E. Snitzer, Optical Maser Action of Nd+3 in a Barium Crown Glass, Phys. Rev. Lett. **7**, 444 (1961).
[13] E. Desurvire, The golden age of optical amplifiers, Physics Today **47** (1994) 20-27.
[14] J.A. Valdmanis, R.L. Fork, J.P. Gordon, Generation of optical pulses as short as 27 femtoseconds directly from a laser balancing self-phase modulation, group-velocity dispersion, saturable absorption, and saturable gain, Opt. Lett. **10** (1985) 131-133.
[15] P.F. Moulton, Spectroscopic and laser characteristics of Ti:Al2O3, J. Opt. Soc. Am. B **3** (1986) 125-133.
[16] D.E. Spence, P.N. Kean, W. Sibbett, 60-fsec pulse generation from a self-modelocked Ti: sapphire laser, Opt. Lett. **16** (1991) 42-44.
[17] F. Salin, P. Grangier, G. Roger, A. Brun, Observation of High-Order Solitons Directly Produced by a Femtosecond Ring Laser, Phys. Rev. Lett. **56** (1986) 1132-1135.
[18] F. Salin, P. Grangier, G. Roger, A. Brun, Experimental observation of nonsymmetrical *N*=2 solitons in a femtosecond laser, Phys. Rev. Lett. **60** (1988) 569-572.
[19] T. Tsang, Observation of high-order solitons from a mode-locked Ti:sapphire laser, Opt. Lett. **18** (1993) 293-295.
[20] R.J. Mears, L. Reekie, I.M. Jauncey, D.N. Payne, Low-noise erbium-doped fibre amplifier operating at 1.54 µm, Electron. Lett. **23** (1987) 1026-1028.
[21] E. Desurvire, J.R. Simpson, P.C. Becker, High-gain erbium-doped traveling wave fiber amplifier, Opt. Lett. **12** (1987) 888-890.
[22] M. Nakazawa, Y. Kimura, K. Suzuki, Efficient Er3+-doped optical fiber amplifier pumped by a 1.48 µm InGaAsP laser diode, Appl. Phys. Lett. **54** (1989) 295-297.
[23] D.J. Richardson, R.I. Laming, D.N. Payne, V. Matsas, M.W. Phillips, Self-starting, passively modelocked erbium fiber ring laser based on the amplifying Sagnac switch, Electron. Lett. **27** (1991) 542-544.
[24] M. Nakazawa, E. Yoshida, Y. Kimura, Low threshold, 290-fs erbium-doped fiber laser with a nonlinear amplifying loop mirror pumped by InGaAsP laser-diodes, Appl. Phys. Lett. **59** (1991) 2073-2075.
[25] V.J. Matsas, T.P. Newson, D.J. Richardson, D.N. Payne, Self-starting passively mode-locked fiber ring soliton laser exploiting nonlinear polarization rotation, Electron. Lett. **28** (1992) 1391-1393.
[26] K. Tamura, E.P. Ippen, H.A. Haus, L.E. Nelson, 77-fs pulse generation from a stretched-pulse mode-locked all-fiber ring laser, Opt. Lett. **18** (1993) 1080-1082.
[27] J.R. Taylor, Early optical soliton research at Imperial College London, Opt. Commun. **536** (2023) 129382.

[28] M.E. Fermann, F. Haberl, M. Hofer, H. Hochreiter, Nonlinear amplifying loop mirror, Opt. Lett. 15, 752-754 (1990).
[29] W. Hänsel et al., All polarization-maintaining fiber laser architecture for robust femtosecond pulse generation, Appl. Phys. B, 123:41 (2017).
[30] A.B. Grudinin, D.J. Richardson, D.N. Payne, Energy quantization in figure eight fibre laser Electron. Lett. **28** (1992) 67-68.
[31] J.N. Kutz, Mode-locked soliton lasers, SIAM Review **48** (2006) 629-678.
[32] A. Schwache, F. Mitschke, Properties of an optical soliton gas, Phys. Rev. E **55** (1997) 7720.
[33] K. Tamura, L.E. Nelson, H.A. Haus, E.P. Ippen, Soliton versus nonsoliton operation of fiber ring lasers, Appl. Phys. Lett. **64** (1994) 149-151.
[34] H.A. Haus, K. Tamura, L.E. Nelson, E.P. Ippen, Stretched-pulse additive-pulse mode-locking in fiber ring lasers - theory and experiment, IEEE J. Quantum Electron. **31** (1995) 591-598.
[35] S.K. Turitsyn, B.G. Bale, M.P. Fedoruk, Dispersion-managed solitons in fibre systems and lasers, Physics Reports **521** (2012) 135-203.
[36] *IMRA Femtolite®*, 60 mW-1.2 nJ erbium-doped fiber laser source (1995).
[37] V. Cautaerts, D. J. Richardson, R. Paschotta, D. C. Hanna, Stretched pulse Yb3+: silica fiber laser, Opt. Lett. **30**, 1888 (1997).
[38] P.N. Kean, X. Zhu, D.W. Crust, R.S. Grant, N. Langford, W. Sibbett, Enhanced mode locking of color-center lasers, Opt. Lett. **14** (1989) 39-41.
[39] 0.E. Martinez, R.L. Fork, J.P. Gordon, Theory of passively mode-locked lasers including self-phase modulation and group-velocity dispersion, Opt. Lett. 9, 156-158 (1984).
[40] H.A. Haus, J.G. Fujimoto, E.P. Ippen, Structures for additive pulse mode locking, J. Opt. Soc. Am. B **8** (1991) 2068.
[41] E. Picholle, C. Montes, C. Leycuras, O. Legrand, J. Botineau, Observation of dissipative superluminous solitons in a Brillouin fiber ring laser, Phys. Rev. Lett. **66** (1991) 1454–1457.
[42] E. V. Vanin, A. I. Korytin, A. M. Sergeev, D. Anderson, M. Lisak, L. Vázquez, Dissipative optical solitons. Phys. Rev. A **49** (1994) 2806–2811.
[43] J.M. Soto-Crespo, N.N. Akhmediev, V.V. Afanasjev, S. Wabnitz, Pulse solutions of the CQGLE in case of normal dispersion Phys. Rev. E **55** (1997) 4783.
[44] A. Chong, J. Buckley, W. Renninger, F. Wise, All-normal-dispersion femtosecond fiber laser, Opt. Express **14** (2006) 10095.
[45] B. Ortaç, O. Schmidt, T. Schreiber, J. Limpert, A. Tünnermann, A. Hideur, High-energy femtosecond Yb-doped dispersion compensation free fiber laser, Opt. Express **15** (2007) 10725-10732.
[46] F.W. Wise, A. Chong, W.H. Renninger, High-energy femtosecond fiber lasers based on pulse propagation at normal dispersion, Laser & Photonics Rev. **2** (2008) 58-73.
[47] E.J.R. Kelleher, J.C. Travers, E.P. Ippen, Z. Sun, A.C. Ferrari, S.V. Popov, J.R. Taylor, Generation and direct measurement of giant chirp in a passively mode-locked laser, Opt. Lett. **34** (2009) 3526-3528.
[48] M. Baumgartl, C. Lecaplain, A. Hideur, J. Limpert, A. Tünnermann, 66 W average power from a microjoule-class sub-100 fs fiber oscillator, Opt. Lett. 37, 1640-1642 (2012).
[49] D. Strickland; G. Mourou, Compression of amplified chirped optical pulses, Opt. Commun. **56** (1985) 219.
[50] F. Ö. Ilday, J. R. Buckley, W. G. Clark, F. W. Wise, Self-Similar Evolution of Parabolic Pulses in a Laser, Phys. Rev. Lett. **92** (2004) 213902.
[51] J.M. Dudley, C. Finot, D.J. Richardson, G. Millot, Self-similarity in ultrafast nonlinear optics, Nature Phys. **3** (2007) 597-603.

[52] J.D. Moores, On the Ginzburg-Landau laser mode-locking model with fifth-order saturable absorber term, Opt. Commun. **96** (1993) 65-70.
[53] A. Komarov, H. Leblond, F. Sanchez, Quintic complex Ginzburg-Landau model for ring lasers, Phys. Rev. E **72** (2005) 025604.
[54] N. Akhmediev, A. Ankiewicz (Eds.) Dissipative solitons, Springer, Berlin Heidelberg (2005).
[55] N. Akhmediev, A. Ankiewicz (Eds.) Dissipative solitons: from Optics to Biology and Medicine, Springer Verlag, Berlin Heidelberg (2008).
[56] P. Grelu, N. Akhmediev, Dissipative solitons for mode-locked lasers Nature Photon. **6** (2012) 84-92.
[57] P. Grelu (Ed) Nonlinear optical cavity dynamics: from microresonators to fiber lasers, Wiley-VCH, Weinheim (2016). https://doi.org/10.1002/9783527686476
[58] S.K. Turitsyn, N.N. Rosanov, I.A. Yarutkina, A.E. Bednyakova, S.V. Fedorov, O.V. Shtyrina, M.P. Fedoruk, Dissipative solitons in fiber lasers, Phys.-Usp. **59**, 642-668 (2016).
[59] P. Grelu, "Solitons and dissipative solitons for ultrafast lasers," in Handbook of Laser Technology & Applications, Chapter 39 in Volume II (Laser Design & Laser Systems) 2nd Edition, C. Guo & S.C. Singh (Eds.) Taylor & Francis (2021). https://doi.org/10.1201/9781003127130
[60] M. Ferreira (Ed.), Dissipative Optical Solitons, Springer Series in Optical Sciences (2022). https://doi.org/10.1007/978-3-030-97493-0
[61] W. Chang, A. Ankiewicz, J. M. Soto-Crespo, N. Akhmediev, Dissipative soliton resonances, Phys. Rev. A **78** (2008) 023830.
[62] Ph. Grelu, W. Chang, A. Ankiewicz, J.M. Soto-Crespo, N. Akhmediev, Dissipative soliton resonance as a guideline for high-energy pulse laser oscillators, J. Opt. Soc. Am. B **27** (2010) 2236.
[63] P. Deslandes, J. Saby, D. Sangla, F. Salin, M. Perrin, E. Freysz, Picosecond to femtosecond pulses from high power self mode–locked ytterbium rod-type fiber laser, Opt. Express **21** (2013) 10731.
[64] X. Xue, Ph. Grelu, B. Yang, M. Wang, S. Li, X. Zheng, B. Zhou, Dispersion-less Kerr solitons in spectrally confined optical cavities, Light: Sci. & Appl. **12**, (2023) 19.
[65] A.F. Runge, D.D. Hudson, K. K. Tam, C.M. de Sterke, A. Blanco-Redondo, The pure-quartic soliton laser, Nature Photon. **14** (2020) 492-497.
[66] B.A. Malomed, D. Mihalache, F. Wise, L. Torner, Spatiotemporal solitons, J. Opt. B **7**, (2005) R53.
[67] Ph. Grelu, J. M. Soto-Crespo, N. Akhmediev, Light Bullets and dynamic pattern formation in nonlinear dissipative systems, Opt. Express **13** (2005) 9352-9360.
[68] L.G. Wright, D.N. Christodoulides, F.W. Wise, Spatiotemporal modelocking in multimode fiber lasers, Science **358** (2017) 94–97.
[69] Y. Wang, Y. Tang, S. Yan, J. Xu, High-power mode-locked 2 μm multimode fiber laser, Laser Phys. Lett. 15, 085101 (2018).
[70] K. Krupa, A. Tonello, A. Barthélémy, T. Mansuryan, V. Couderc, G. Millot, P. Grelu, D. Modotto, S.A. Babin, S. Wabnitz, Multimode nonlinear fiber optics, a spatiotemporal avenue, APL Phontonics **4** (2019) 110901.
[71] U. Tegin, E. Kakkava, B. Rahmani, D. Psaltis, C. Moser, Spatiotemporal self-similar fiber laser, Optica **6** (2019) 1412–1415.
[72] L. Kalashnikov, S. Wabnitz, Distributed Kerr-lens mode locking based on spatiotemporal dissipative solitons in multimode fiber lasers, Phys. Rev. A **102** (2020) 023508.

[73] C. Gao, B. Cao, Y. Ding, X. Xiao, D. Yang, H. Fei, C. Yang, C. Bao, All-step-index-fiber spatiotemporally mode-locked laser, Optica 10, 356 (2023).

[74] B. Malomed, Bound solitons in the nonlinear Schrödinger–Ginzburg–Landau equation, Phys. Rev. A **44** (1991) 6954–6957.

[75] N.N. Akhmediev, A. Ankiewicz, J.M. Soto-Crespo, Multisoliton solutions of the complex Ginzburg–Landau equation, Phys. Rev. Lett. **79** (1997) 4047–4051.

[76] A. V. Husakou, J. Herrmann, Supercontinuum Generation of Higher-Order Solitons by Fission in Photonic Crystal Fibers, Phys. Rev. Lett. **87** (2001) 203901.

[77] J.P. Gordon, Interaction forces among solitons in optical fibers, Opt. Lett. **8** (1983) 596-598.

[78] D.Y. Tang, W.S. Man, H.Y. Tam, P.D. Drummond, Observation of bound states of solitons in a passively mode-locked fiber laser, Phys. Rev. A **64** 033814 (2001).

[79] Ph. Grelu, F. Belhache, F. Gutty, J.M. Soto-Crespo, Phase-locked soliton pairs in a stretched-pulse fiber laser, Opt. Lett. **27**, 1321-1323 (2002).

[80] Ph. Grelu, J. Béal, J.M. Soto-Crespo, Soliton pairs in a fiber laser: from anomalous to normal average dispersion regime, Opt. Express **11**, 2238-2243 (2003).

[81] Y. Song, F. Zhou, H. Tian, M. Hu, Attosecond timing jitter within a temporal soliton molecule, Optica **7** (2020) 1531–1534.

[82] M. Stratmann, T. Pagel, F. Mitschke, Experimental observation of temporal soliton molecules, Phys. Rev. Lett. **95** (2005) 143902.

[83] P. Parra-Rivas, D. Gomila, P. Colet, L. Gelens, Interaction of solitons and the formation of bound states in the generalized Lugiato-Lefever equation, Eur. Phys. J. D. **71** (2017) 198.

[84] D.C. Cole, E.S. Lamb, P. Del'Haye, S.A. Diddams, S.B. Papp, Soliton crystals in Kerr resonators, Nature Photon. **11** (2017) 671-676.

[85] P. Rohmann, A. Hause, F. Mitschke, Solitons Beyond Binary: Possibility of Fibre-Optic Transmission of Two Bits per Clock Period, Sci. Reports **2**:866 (2012).

[86] F. Kurtz, C. Ropers, G. Herink, Resonant excitation and all-optical switching of femtosecond soliton molecules," Nat. Photonics **14**, 9–13 (2020).

[87] S. Liu, Y. Cui, E. Karimi, B.A. Malomed, On-demand harnessing of photonic soliton molecules, Optica **9**, 240-250 (2022).

[88] J.M. Soto-Crespo, M. Grapinet, P. Grelu, N. Akhmediev, Bifurcations and multiple-period soliton pulsations in a passively mode-locked fiber laser, Phys. Rev. E **70** (2004) 066612.

[89] D.C. Cole, S.B. Papp, Subharmonic entrainment of Kerr breather solitons, Phys. Rev. Lett. **123**, (2019) 173904.

[90] X. Wu, Y. Zhang, J. Peng, S. Boscolo, C. Finot, H. Zheng, Farey tree and devil's staircase of frequency-locked breathers in ultrafast lasers, Nature Commun. **13** (2022) 5784.

[91] Z. Wang, A. Coillet, S. Hamdi, Z. Zhang, P. Grelu, Spectral pulsations of dissipative solitons in ultrafast fiber lasers: period doubling and beyond, Laser & Photon. Rev. **17** (2023) 2200298.

[92] N. Akhmediev, J.M. Soto-Crespo, G. Town, Pulsating solitons, chaotic solitons, period doubling, and pulse coexistence in mode-locked lasers: Complex Ginzburg–Landau equation approach, Phys. Rev. E **63** (2001) 056602.

[93] M. Grapinet, P. Grelu, Vibrating soliton pairs in a mode-locked laser cavity, Opt. Lett. **31** (2006) 2115–2117.

[94] G. Herink, F. Kurtz F, B. Jalali, D.R. Solli, C. Ropers, Real-time spectral interferometry probes the internal dynamics of femtosecond soliton molecules, Science **356** (2017) 50.

[95] K. Krupa, K. Nithyanandan, U. Andral, P. Tchofo-Dinda, P. Grelu, Real-Time Observation of Internal Motion within Ultrafast Dissipative Optical Soliton Molecules, Phys. Rev. Lett. **118** (2017) 243901.
[96] J. Desbois, F. Gires, P. Tournois, A new approach to picosecond laser pulse analysis shaping and coding, IEEE J. Quantum Electron. **QE-9** (1973) 213.
[97] K. Goda, K.B. Jalali, Dispersive Fourier transformation for fast continuous single-shot measurements, Nat. Photon. **7** (2013) 102.
[98] S. Hamdi, A. Coillet, P. Grelu, Real-time characterization of optical soliton molecule dynamics in an ultrafast thulium fiber laser, Opt. Lett. **43** (2018) 4965-4968.
[99] J. Peng, H. Zeng, Build-up of dissipative optical soliton molecules via diverse soliton interactions, Laser Photon. Rev. **12** (2018) 1800009.
[100] M.J. Lederer, B. Luther-Davies, H.H. Tan, C. Jagadish, N.N. Akhmediev, J.M. Soto-Crespo, Multipulse operation of a Ti:sapphire laser mode locked by an ion-implanted semiconductor saturable-absorber mirror, J. Opt. Soc. Am. B **16** (1999) 895-904.
[101] Y. J. Igbonacho, K. Nithyanandan, K. Krupa, P. Tchofo-Dinda, P. Grelu, A.B. Moubissi, Dynamics of distorted and undistorted soliton molecules in a mode-locked fiber laser, Phys. Rev. A **99** (2019) 063824.
[102] Y. Song, D. Zou, O. Gat, M. Hu, P. Grelu, Chaotic internal dynamics of dissipative soliton molecules, Laser & Photon. Rev. **17** (2023) 2300066.
[103] T. Schibli, J. Kim, O. Kuzucu, J.T. Gopinath, S.N. Tandon, G.S. Petrich, L.A. Kolodziejski, J.G. Fujimoto, E.P. Ippen, F.X. Kaertner, Attosecond active synchronization of passively mode-locked lasers by balanced cross correlation, Opt. Lett. **28** (2003) 947-949.
[104] D. Zou, Y. Song, O. Gat, M. Hu, P. Grelu, Synchronization of the internal dynamics of optical soliton molecules, Optica **9** (2022) 1307-1313.
[105] N. Akhmediev, G. Town, S. Wabnitz, Soliton coding based on shape invariant interacting soliton packets: the three-soliton case, Opt. Commun. **104** (1994) 385–390.
[106] A. Komarov, K. Komarov, F. Sanchez, Quantization of binding energy of structural solitons in passive mode-locked fiber lasers, Phys. Rev. A **79** (2009) 033807.
[107] K. Krupa, K. Nithyanandan, P. Grelu, Vector dynamics of incoherent dissipative optical solitons, Optica **4** (2017) 1239-1244.
[108] Z.Q. Wang, K. Nithyanandan, A. Coillet, P. Tchofo-Dinda, P. Grelu, Buildup of incoherent dissipative solitons in ultrafast fiber lasers, Phys. Rev. Research **2**, 013101 (2020).
[109] M. Mitchell and M. Segev, Self-trapping of incoherent white light, Nature **387** (1997) 880–883.
[110] C. Michel, B. Kibler, J. Garnier, A. Picozzi, Temporal incoherent solitons supported by a defocusing nonlinearity with anomalous dispersion, Phys. Rev. A **86** (2012) 041801(R).
[111] O. Cohen, H. Buljan, T. Schwartz, J. W. Fleischer, M. Segev, Incoherent solitons in instantaneous nonlocal nonlinear media," Phys. Rev. E **73** (2006) 015601(R).
[112] S.T. Cundiff, J.M. Soto-Crespo, N. Akhmediev, Experimental Evidence for Soliton Explosions, Phys. Rev. Lett. **88** (2002) 073903.
[113] A.F.J. Runge, N.G.R. Broderick, M. Erkintalo, Observation of soliton explosions in a passively mode-locked fiber laser, Optica **2** (2015) 36-38.
[114] C. Lecaplain, P. Grelu, J.M. Soto-Crespo, N. Akhmediev, Dissipative Rogue Waves Generated by Chaotic Pulse Bunching in a Mode-Locked Laser, Phys. Rev. Lett. **108** (2012) 233901.
[115] A. Zavyalov, O. Egorov, R. Iliew, F. Lederer, Rogue waves in mode-locked fiber lasers, Phys. Rev. A **85** (2012) 013828.

[116] M. Horowitz, Y. Barad, Y. Silberberg, Noiselike pulses with a broadband spectrum generated from an erbium-doped fiber laser, Opt. Lett. **22**, (1997) 799.
[117] Y. Takushima, K. Yasunaka, Y. Ozeki, K. Kikuchi, 87 nm bandwidth noise-like pulse generation from erbium-doped fiber laser, Electron. Lett. **41**, 399 (2005).
[118] L.M. Zhao, D.Y. Tang, T.H. Cheng, H.Y. Tam, C. Lu, 120 nm bandwidth noise-like pulse generation in an erbium doped fiber laser, Opt. Commun. **281** (2008) 157.
[119] S. Kobtsev, S. Kukarin, S. Smirnov, S. Turitsyn, A. Latkin, Generation of double scale femto/pico-second optical lumps in mode-locked fiber lasers, Opt. Express **17**, 20710 (2009).
[120] J. C. Hernandez-Garcia, O. Pottiez, J. M. Estudillo-Ayalab, Supercontinuum generation in a standard fiber pumped by noise-like pulses from a figure-eight fiber laser, Laser Phys. **22**, 221 (2012).
[121] T. North and M. Rochette, Raman-induced noiselike pulses in a highly nonlinear and dispersive all-fiber ring laser, Opt. Lett. **38** (2013) 890-892.
[122] A. Runge, C. Aguergaray, N.G.R. Broderick, M. Erkintalo, Coherence and shot-to-shot spectral fluctuations in noise-like ultrafast fiber lasers, Opt. Lett. **38** (2013) 4327.
[123] C. Lecaplain, P. Grelu, Rogue waves among noiselike pulse laser emission: An experimental investigation, Phys. Rev. A **90** (2014) 013805.
[124] S. Kobtsev, Noise-like pulses as a source of pump energy, Photonics **10**, 233 (2023).
[125] U. Andral, R. Si Fodil, F. Amrani, F. Billard, E. Hertz, Ph. Grelu, Fiber laser mode locked through an evolutionary algorithm, Optica **2**, 275 (2015).
[126] U. Andral, J. Buguet, R. Si Fodil, F. Amrani, F. Billard, E. Hertz, Ph. Grelu, Toward an autosetting mode-locked fiber laser cavity, J. Opt. Soc. Amer. B **33**, 825–833, (2016).
[127] R.I. Woodward, E.J.R. Kelleher, Toward 'smart lasers': self-optimization of an ultrafast pulse source using a genetic algorithm," Sci. Rep. **6**, 37616 (2016).
[128] D.G. Winters, M.S. Kirchner, S.J. Backus, H.C. Kapteyn, Electronic initiation and optimization of nonlinear polarization evolution modelocking in a fiber laser, Opt. Express **25**, 102–112 (2017).
[129] T. Baumeister, S.L. Brunton, J.N. Kutz, Deep learning and model predictive control for self-tuning mode-locked lasers, J. Opt. Soc. Am. B **35**, 617–626 (2018).
[130] G. Pu, L. Yi, L. Zhang, W.Hu, Intelligent programmable mode-locked fiber laser with a human like algorithm," Optica **6**, 362–369 (2019).
[131] J. Girardot, F. Billard, A. Coillet, E. Hertz, Ph. Grelu, Autosetting mode-locked laser using an evolutionary algorithm and time-stretch spectral characterization, IEEE J. Select. Topics in Quantum Electron. **26**, 1100108 (2020).
[132] X.M. Wei, J.C. Jing, Y. Shen, L.V. Wang, Harnessing a multi-dimensional fibre laser using genetic wavefront shaping. Light. Sci. Appl. **9**, 149 (2020).
[133] Ph. Grelu, Smart lasers tame complex spatiotemporal cavity dynamics, Light. Sci. Appl. **9**:188 (2020).
[134] F.C. Meng, J.M. Dudley, Toward a self-driving ultrafast fiber laser. Light. Sci. Appl. **9**, 26 (2020).
[135] J. Girardot, A. Coillet, M. Nafa, F. Billard, E. Hertz, Ph. Grelu, On-demand generation of soliton molecules through evolutionary algorithm optimization, Opt. Lett. **47**, 134-137 (2022).
[136] S.M. Kobtsev, Perspective paper: Can machine learning become a universal method of laser photonics? Opt. Fib. Technol. **65**, 102626 (2021).

[137] Z.A. Kudyshev , A.V. Kildishev , V.M. Shalaev, A. Boltasseva, Machine learning–assisted global optimization of photonic devices, Chapter in Frontiers in Optics and Photonics (de Gruyter, Berlin, 2021).
[138] G. Genty, L. Salmela, J.M. Dudley, D. Brunner, A. Kokhanovskiy, S. Kobtsev, S.K. Turitsyn, Machine learning and applications in ultrafast photonics, Nat. Photonics **15**, 91-101 (2021).