ELEC2208:
Power Electronics and **Motor Drives**

Chapter 2: Power Electronic Control of DC Motor Drives

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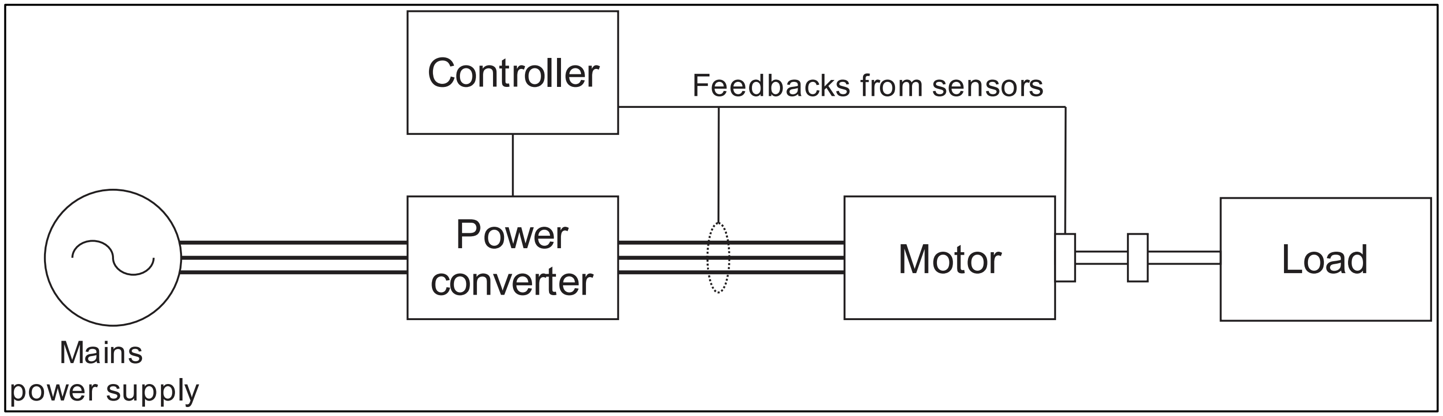
Overview

- In this lecture, we will learn about:
  - Fundamentals of DC machines
  - Speed control of separately excited DC motors
  - Rectifier control of separately excited DC motors
  - Quadrants versus drives converter topology
  - PWM-controlled power converters (DC choppers) for DC machines
  - Regenerative braking and a simplified DC motor drive hardware schematic

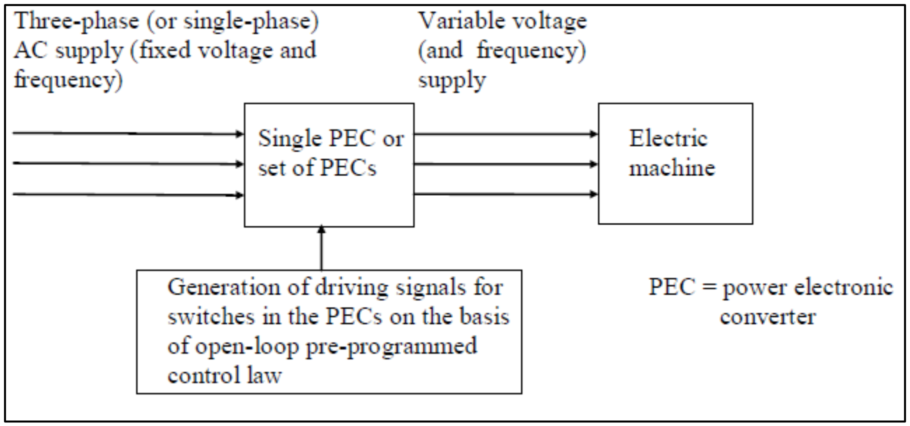
Basic components of an electric drive



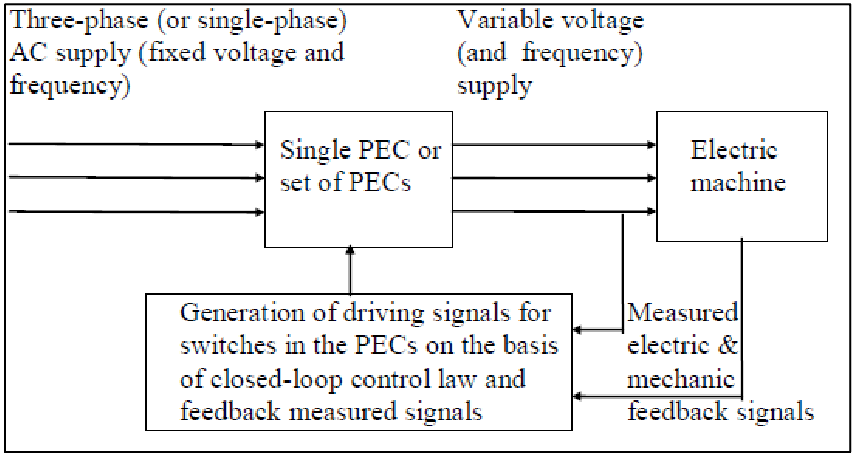
- A basic electric drive system consisting an electric source, a power-electronic converter (and auxiliary instrumentation and control components), and a load.
- Control of electrical machines can be performed in an <u>open-loop</u> manner or in a <u>closed loop</u> manner
  - **Open loop**: simple control algorithm, the speed at steady state may have small error
  - **Closed loop**: control algorithm could be very complicated, but precise control of position, speed and/or torque in steady state as well as in transient is possible.

Introduction

Open-loop low-performance electric drive



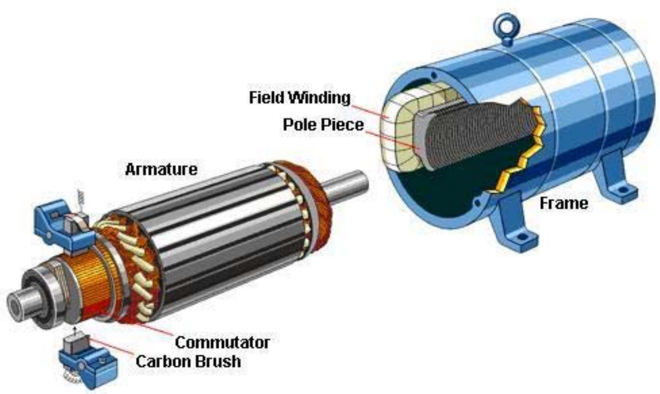
Closed-loop high-performance electric drive

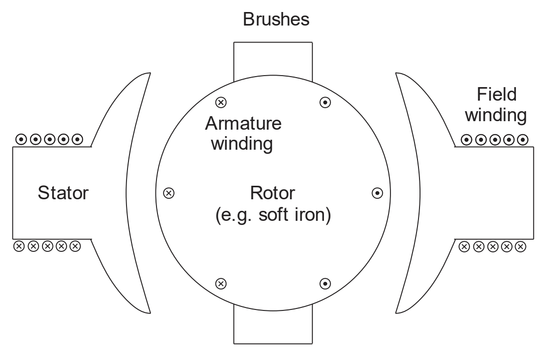


- Numerous applications require very precise position control: these applications require drives that are usually termed servo-drives and the typical examples are robotics and machine tools.
  - The drive will be realized as a closed-loop drive, with appropriate measurement and feedback.
  - The structure of the control system will in several cases be very complicated.
  - Unique feature of servo-drives is the requirement that the given variable (position, speed or torque) is controlled precisely not only in steady-state but in transient operation as well. Such control can be realized rather easily with DC motors and it explains why DC machines were the standard choice for servo, or high-performance, applications in the past.
- The application of AC machines for high performance operation has become possible only recently: so-called vector control (or field-oriented control) principles must be applied if an AC machine is to yield good dynamic response.

Basic structure of a DC motor (ideal)

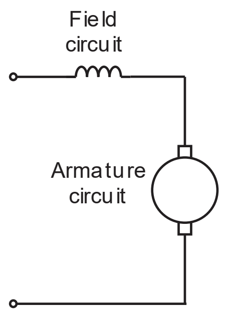
- DC motors have two main circuits: armature and field.
- This armature circuit is supplied from an external voltage supply connected through brushes at commutators placed along the ideal magnetic neutral axis. This ideal axis is located at the mid-point between the stator field poles and is perpendicular to the field flux.
- The field winding the field winding is supplied from a DC voltage source (which may or may not be the same source as the armature's supply) that produces field current with which the field flux is set up. The presence of armature current in the field set up by the field winding creates the driving torque that rotates the rotor.
- Due to the rotor/armature movement, a voltage known as back EMF is induced in the armature, as described by the Faraday's law. This back EMF somewhat affects the current flow in the armature winding. This interaction continues to happen until the electrical steady state is achieved. This briefly summarizes the torque and back EMF creation in DC motors.

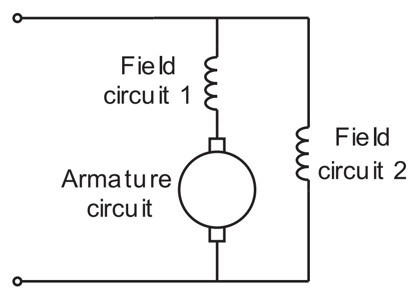


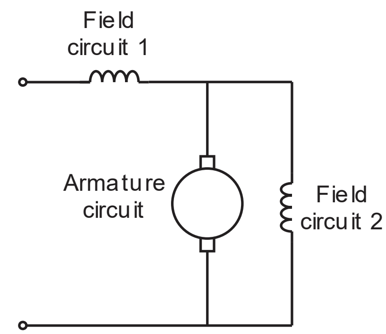


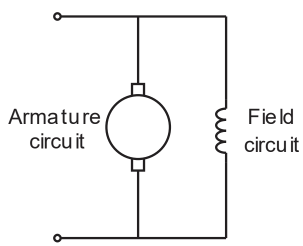
Various connections of armature and field windings

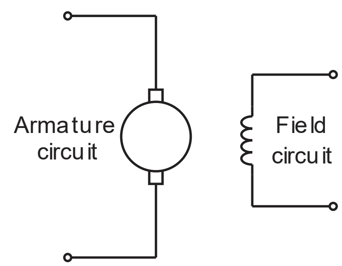
- A basic feature of DC motors is the possibility to connect the field and armature windings (assuming non-permanent-magnet DC motor) in different ways:







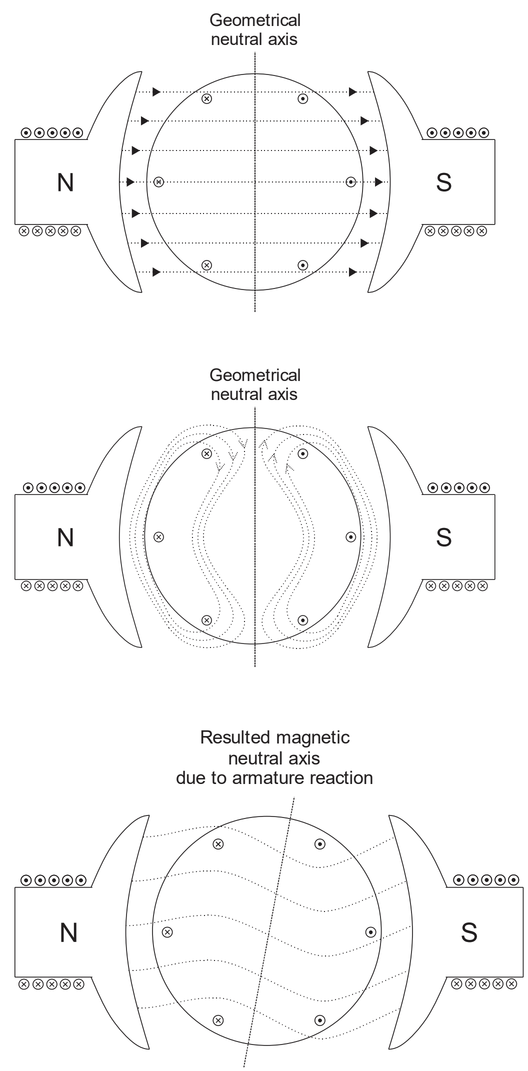




- Separately excited DC machine
- Shunt DC machine
- Series DC machine
- Long shunt compound motor
- Short shunt compound motor

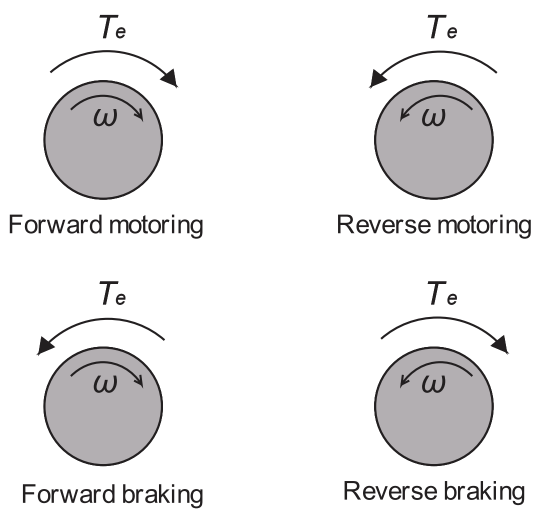
Armature reaction (basic)

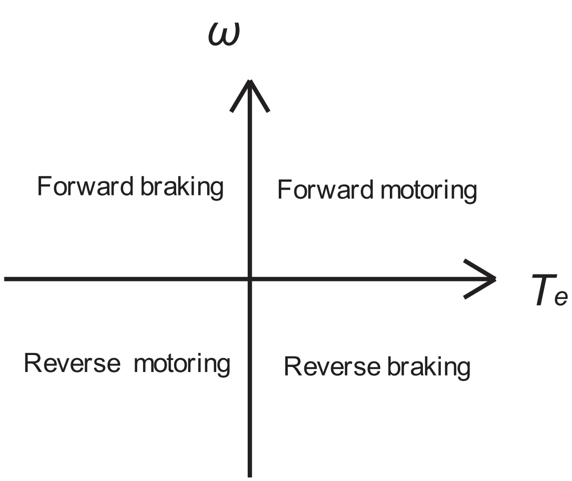
- Armature reaction is a non-ideal phenomenon that is present in a DC motor.
- Following the Ampere's law, the armature current also creates a magnetic field around the armature winding.
- This armature field interacts with the field flux and subsequently deviates the magnetic neutral axis from the mid-point between the stator field poles.
- This has negative consequences such as unaligned commutation which leads electric spark at the brushes (especially in DC generator) and delayed commutations.



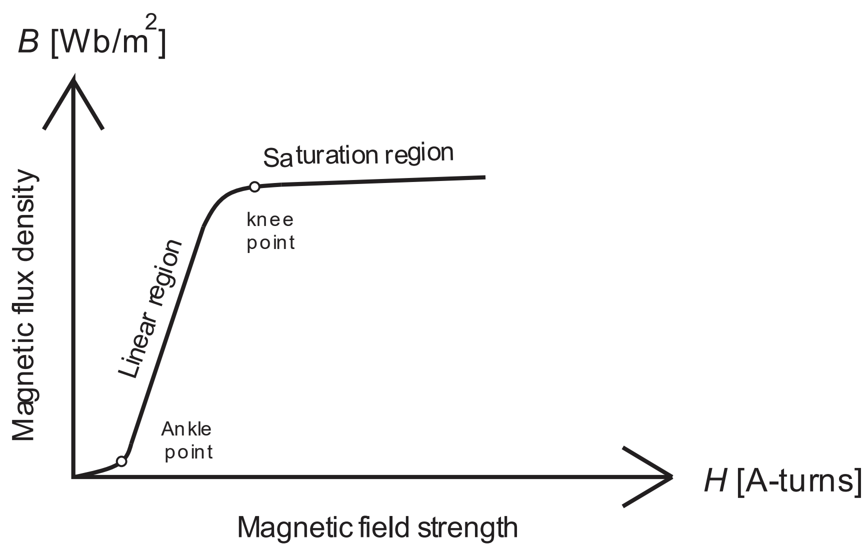
Four-quadrant operation

- There are four basic operating regions of DC machines, known as forward motoring, forward braking, reverse motoring and reverse braking.
- E.g. of forward motoring and reverse motoring are fans and pumps rotating in forward and reverse direction.
- E.g. of forward braking is a forward-moving train going downhill.
- A DC drive can operate in single-quadrant (forward or reverse motoring), two-quadrant (e.g. forward-motoring-forward-braking combination in electric vehicle, forward-motoring-reverse-braking combination in crane), or four-quadrant (train) operations.



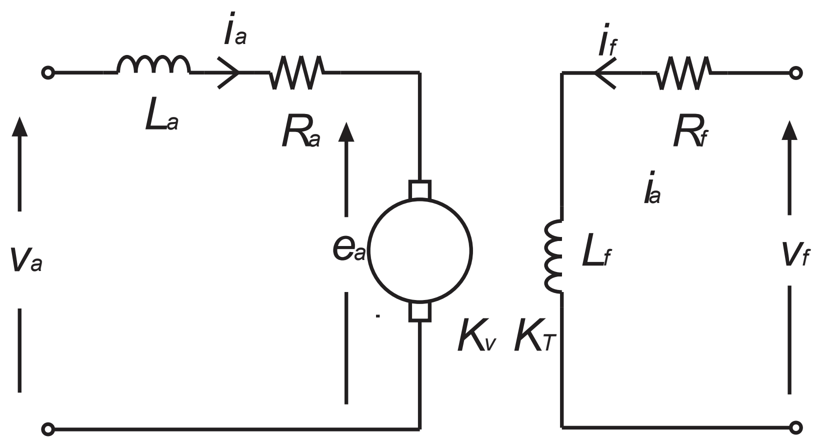


B-H curve and speed regions



- For rated speed operation, the magnetic field will be established by the field current to attain a designed rated value, located at the knee point or saturation point of the magnetizing curve (or B-H curve). By controlling the armature current, the motor can operate upto the rated speed.
- In the field weakening region, the magnetic flux in the machine is reduced by reducing the field current and therefore the magnetic field strength *H*. The machine then operates at a point in between the ankle point and the knee point.

Principle of speed control of DC motor



- Voltage balanced equation at the field circuit:

$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

- Voltage balanced equation at the field circuit:

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a$$

- The induced back-EMF and the developed torque are:

$$e_a = K_v \omega i_f$$

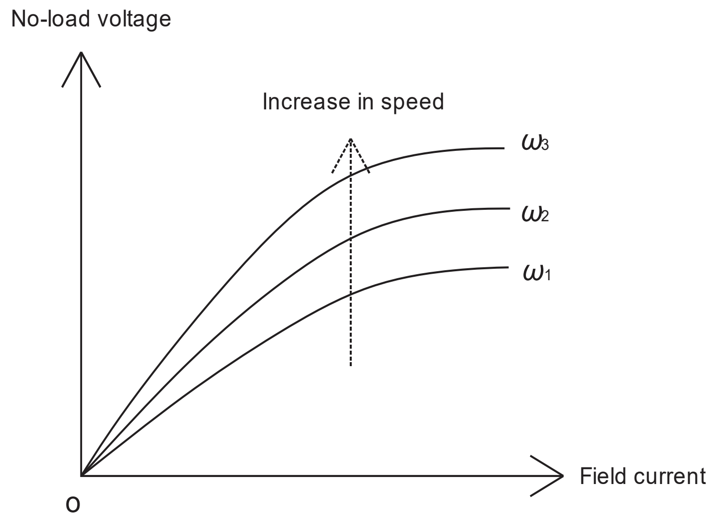
$$T_e = K_T i_f i_a$$

where

K_v and K_T are the back-EMF/voltage constant and torque constant

Effect of magnetic saturation leads to non-linearity

- The relationship between the field current i_f and the back EMF e_a is in practice nonlinear due to magnetic saturation



Note:

- Low-dynamic speed regulation can be achieved through open-loop drives control algorithm;
- Dynamical equation is required when fast-dynamic torque/ speed/position regulation/tracking is of interest.

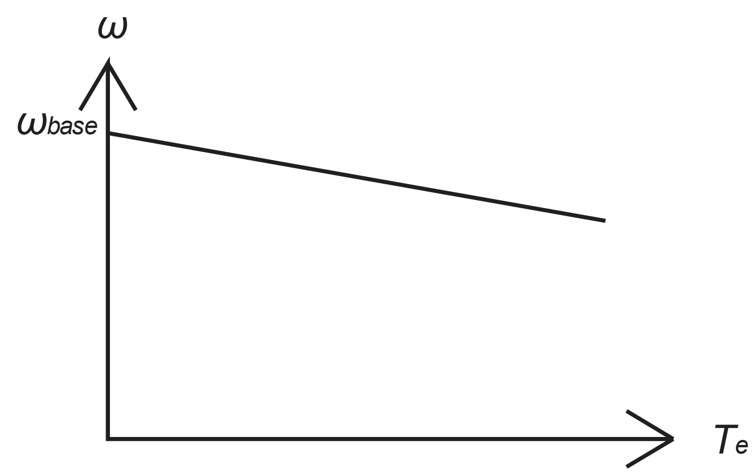
Principle of speed control of DC motor

- For analysis purpose or if one only deals with open-loop control (i.e. low-dynamic regulation/control of speed), the following can be assumed: given a DC motor under electrical steady state, all time-derivative terms will be nullified:

$$V_f = R_f I_f$$
$$E_a = K_v \omega I_f$$
$$V_a = R_a I_a + E_a = R_a I_a + K_v \omega I_f$$
$$T_e = K_T I_f I_a = B\omega + T_L$$

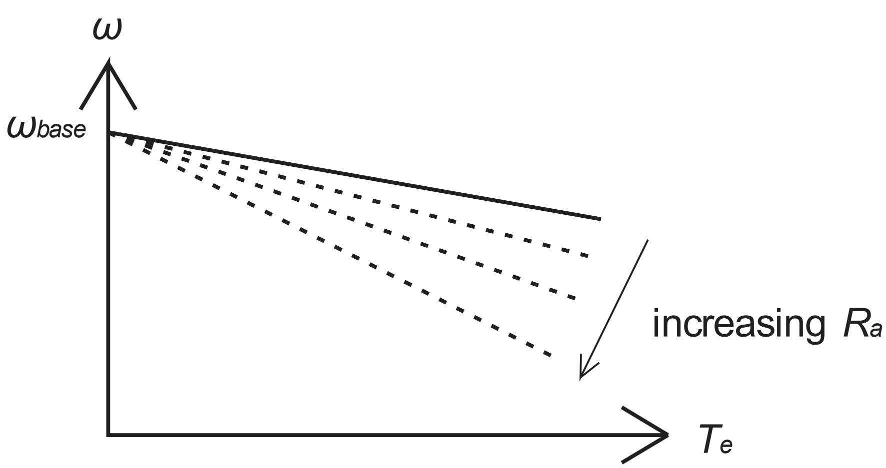
- The steady-state motor speed can be derived from:

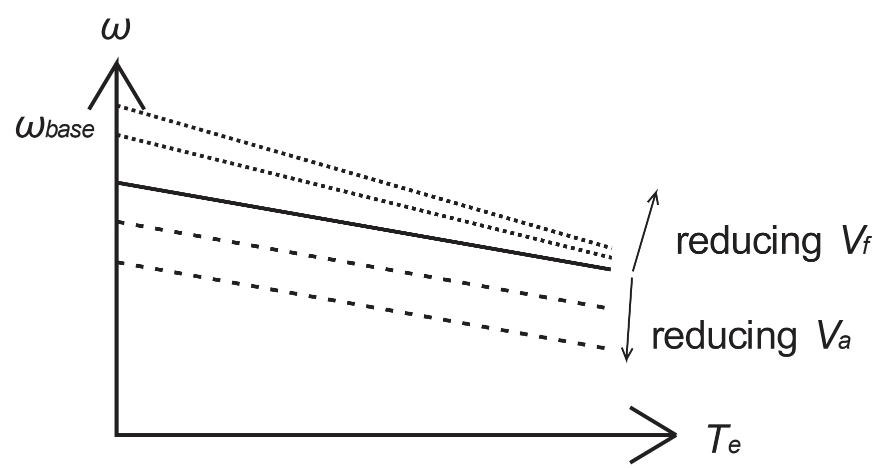
$$\omega = \frac{V_a - R_a I_a}{K_v V_f / R_f}$$



Effects of varying R_a, V_a, and V_f on the speed-torque curves

- Varying R_a (i.e. by connecting a variable resistor in series to the armature winding) is a conventional speed-control method which results in much higher copper losses.
- In modern drives with power electronic control, the speed regulation can be achieved by adjusting V_a or I_f





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Speed regulation

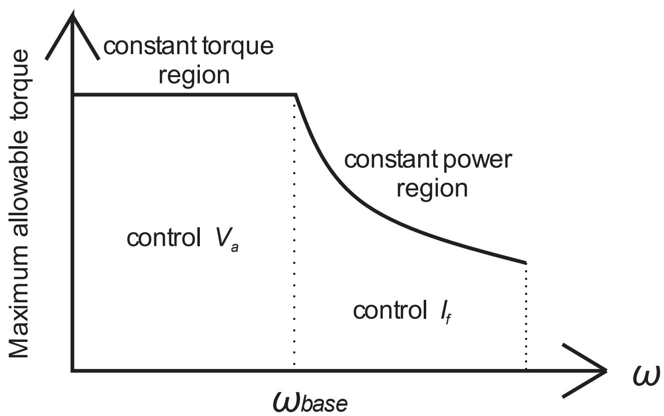
- Another important parameter is the speed regulation of the DC motor, which is defined as:

$$\text{speed regulation} = \frac{\omega_{no-load} - \omega_{full-load}}{\omega_{full-load}} \times 100\%$$

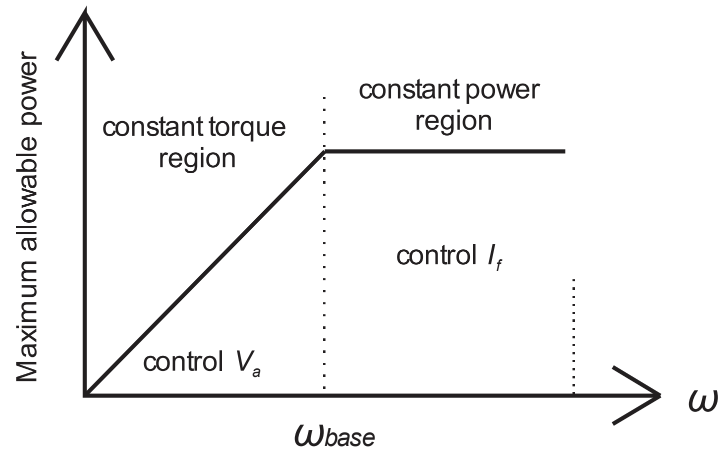
- Other speed unit can be used.
- It represents the range of natural speed change from no-load to full-load.

Base speed and field weakening regions

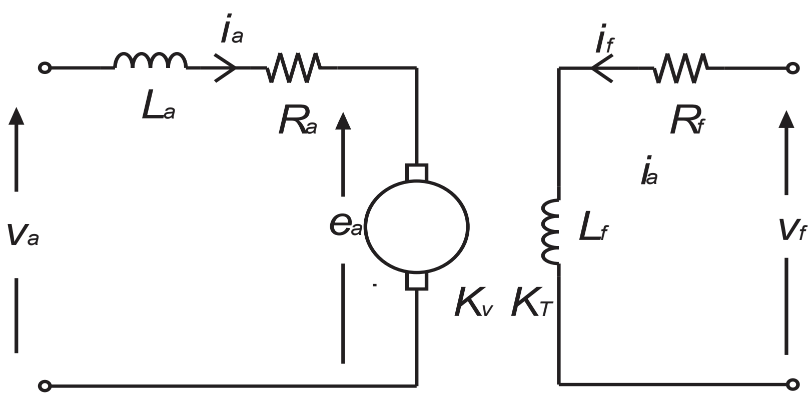
- Fig. below shows the limits of the torque and power of DC motors under the V_a and I_f control.
- In the base speed region, V_a control is the dominant technique as the I_f is usually kept at the rated value to produce a rated flux in the DC machines and maximizes the torque production.
- In the field-weakening region, I_f is reduced as the operating speed increases. V_a is usually kept at the maximum value available to maximize the torque production.

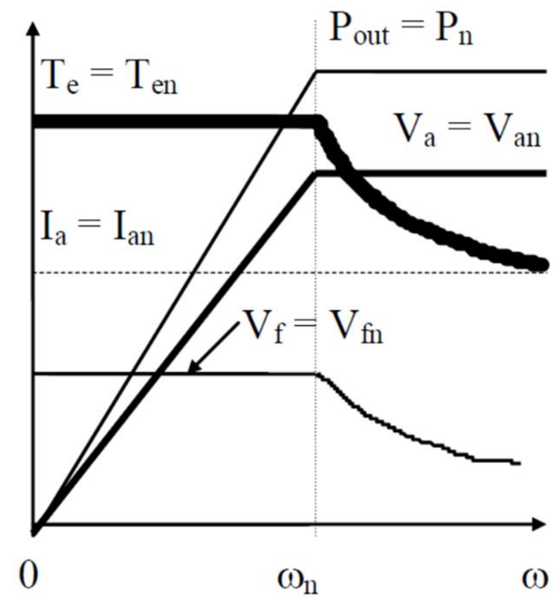


- The maximum power deliverable by the DC motor can be illustrated through



Max. armature and field voltage, power, and torque versus operating speed





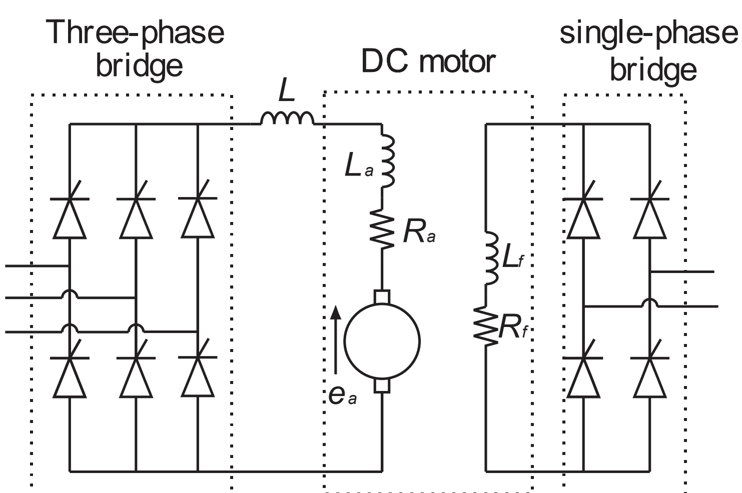
- Field voltage is usually kept constant, and decreases exponentially after the rated speed.
- Max torque is constant in the base speed region, and decreases exponentially after the rated speed.

Exercise

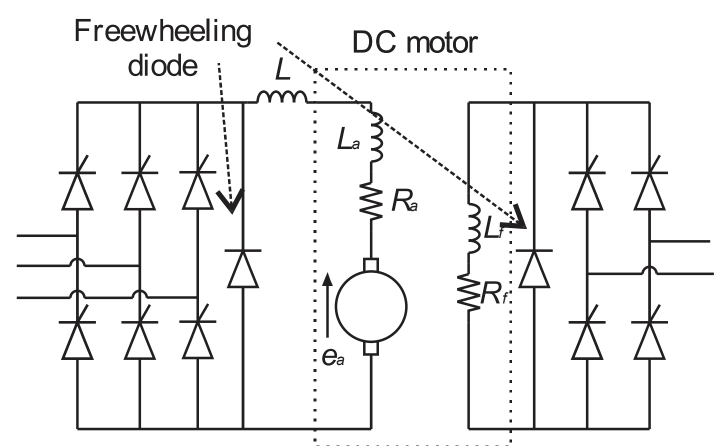
- A 30 hp 300 V 1500 rpm separately excited DC motor is powered by an ideal 300V DC source at its armature while its field winding is being current-controlled to 1.5A to produce the rated flux. The motor has the following parameters: armature resistance being 1Ω, voltage constant K_v being 1.2V/(A-rad/s).
  - Calculate the rated armature current.
  - Calculate the rated torque
  - What is the torque constant of the machine?
  - If the machine is loaded with a half the rated torque while keeping the armature voltage supply and field current the same, what is the motor speed?
  - If the machine is loaded with a quarter of the rated torque while the field current is reduced to a half of its rated, what is the motor speed?
  - If the machine is not loaded, describe how the motor speed will change when the field current is changed from its rated to a half of its rated.

Rectifier control of separately excited DC motors

- Depending on the application requirements such as the quadrants required, one can choose from various SCR-device-based topologies, e.g. half-/full-wave rectifier/converter, bidirectional half-/full-wave converters, as the actuator for speed control.
- The emphasis here is placed on a controllable dual-converter drive suppling a separately excited DC motor:
  - The armature winding is supplied by a controllable single-/three-phase rectifier
  - The field winding is supplied by a controllable single-/three-phase rectifier.

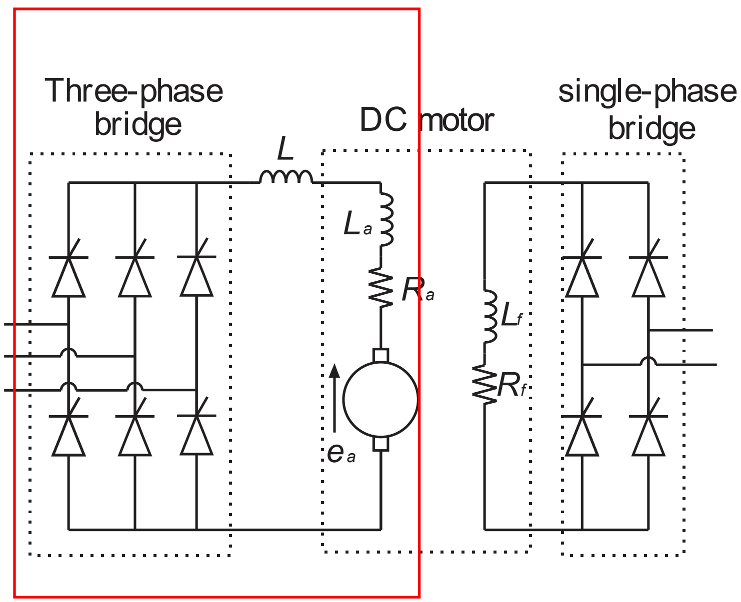


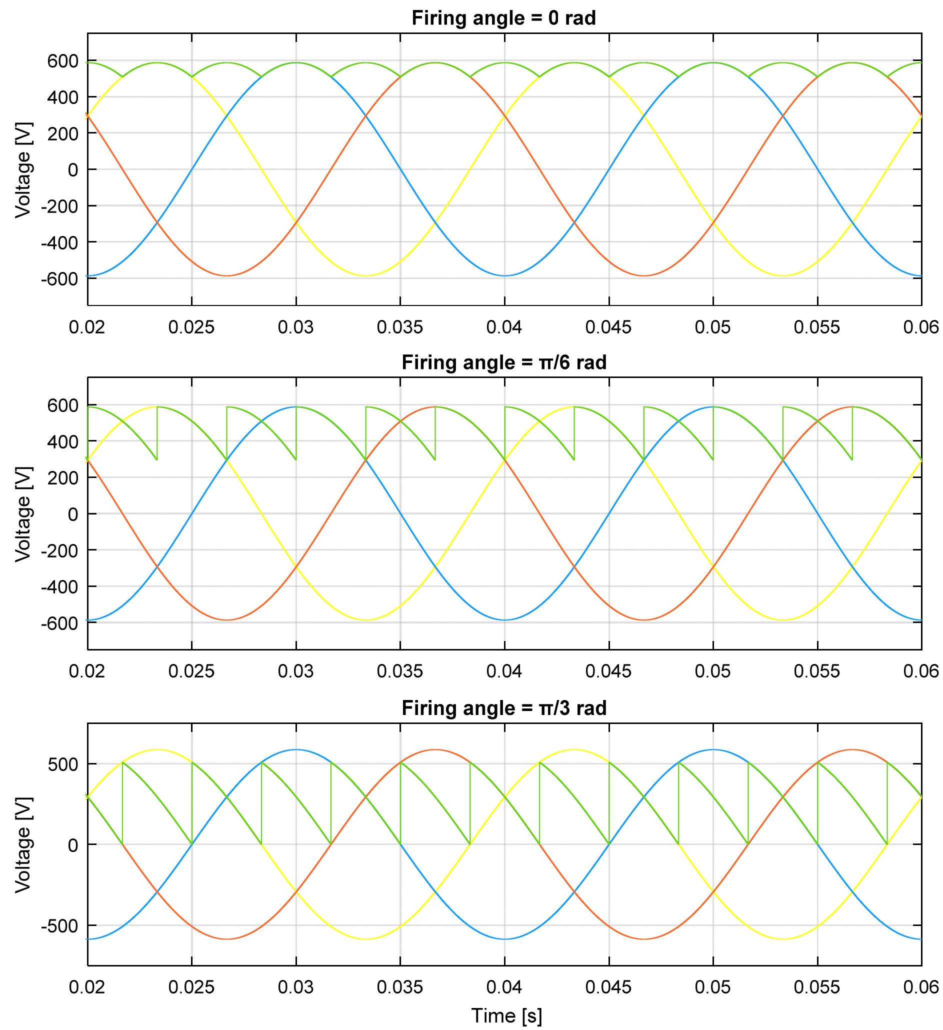
DC motor drive schematic – an SCR-based example



Practical – with free-wheeling diode (important in forced commutated devices)

Three-phase full-wave converter waveform





Armature voltage produced by the three-phase full-wave rectifier (a.k.a full-converter)

$$v_{an} = V_{pk}\sin(\omega t)$$
$$v_{bn} = V_{pk}\sin(\omega t - 2\pi/3)$$
$$v_{cn} = V_{pk}\sin(\omega t + 2\pi/3)$$

$$v_{ab} = v_{an} - v_{bn} = \sqrt{3}V_{pk}\sin(\omega t + \pi/6)$$
$$v_{bn} = v_{bn} - v_{cn} = \sqrt{3}V_{pk}\sin(\omega t - \pi/2)$$
$$v_{cn} = v_{cn} - v_{an} = \sqrt{3}V_{pk}\sin(\omega t + \pi/2)$$

$$V_a = \frac{1}{2\pi}\int_{\frac{\pi}{6}+\alpha_a}^{2\pi+\frac{\pi}{6}+\alpha_a} (\text{voltage ripple waveform})\cdot d(\omega t)$$
$$= 6\left(\frac{1}{2\pi}\int_{\frac{\pi}{6}+\alpha_a}^{\frac{\pi}{2}+\alpha_a} \sqrt{3}V_{pk}\sin(\omega t + \pi/6)\cdot d(\omega t)\right)$$
$$= \frac{3\sqrt{3}V_{pk}}{\pi}\left[-\cos(\omega t + \pi/6)\right]_{\frac{\pi}{6}+\alpha_a}^{\frac{\pi}{2}+\alpha_a}$$
$$= \frac{3\sqrt{3}V_{pk}}{\pi}\left[-\cos(\frac{2\pi}{3}+\alpha_a) + \cos(\frac{\pi}{3}+\alpha_a)\right]$$
$$= \frac{3\sqrt{3}V_{pk}}{\pi}\left[-\cos(\frac{2\pi}{3})\cos\alpha_a + \sin(\frac{2\pi}{3})\sin\alpha_a + \cos(\frac{\pi}{3})\cos\alpha_a - \sin(\frac{\pi}{3})\sin\alpha_a\right]$$
$$= \frac{3\sqrt{3}V_{pk}}{\pi}\cos\alpha_a$$

- Where V_{pk} is the peak value of the sinusoidal phase voltage; and $0 < \alpha_a < \pi$ rad, measured from $\pi/3$ rad point from the zero crossing of line voltage V_{ab}.

Field voltage produced by the single-phase converter

- It may be of interest to know the rms value instead of the average value of the three-phase full converter/rectifier output voltage:

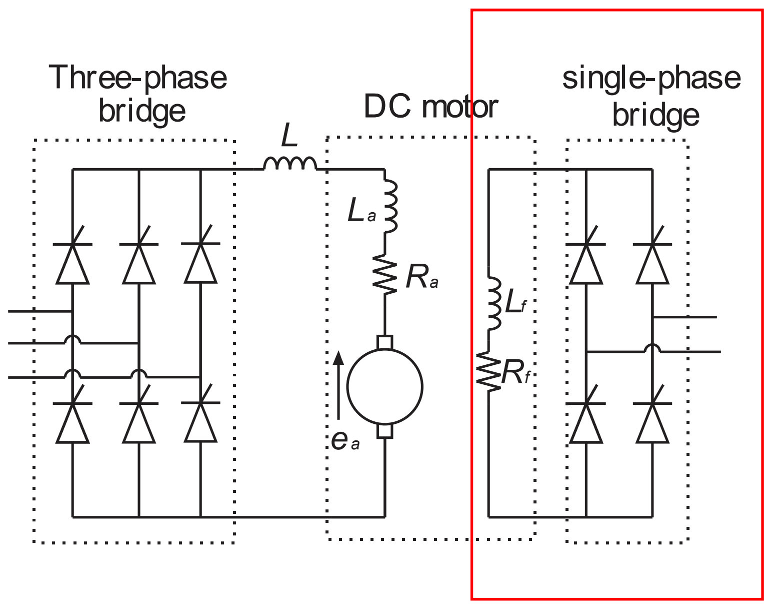
$$V = \sqrt{\frac{6}{2\pi} \int_{\frac{\pi}{6}+\alpha_a}^{\frac{\pi}{2}+\alpha_a} 3V_{pk}^2 \sin^2(\omega t + \pi/6) \cdot d(\omega t)}$$

$$= \sqrt{3} V_{pk} \sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha_a}$$

- This rms value gives a more accurate representation of the actual voltage that is imposed at the armature terminal.
  - However, the averaging value derived earlier is simpler.

Field voltage produced by the single-phase half-wave rectifier (a.k.a. single-phase converter) - Exercise

- Let's try to obtain the field voltage produced by the **single-phase converter.**
  - Is there any assumption required? Will you be able to sketch the waveform?

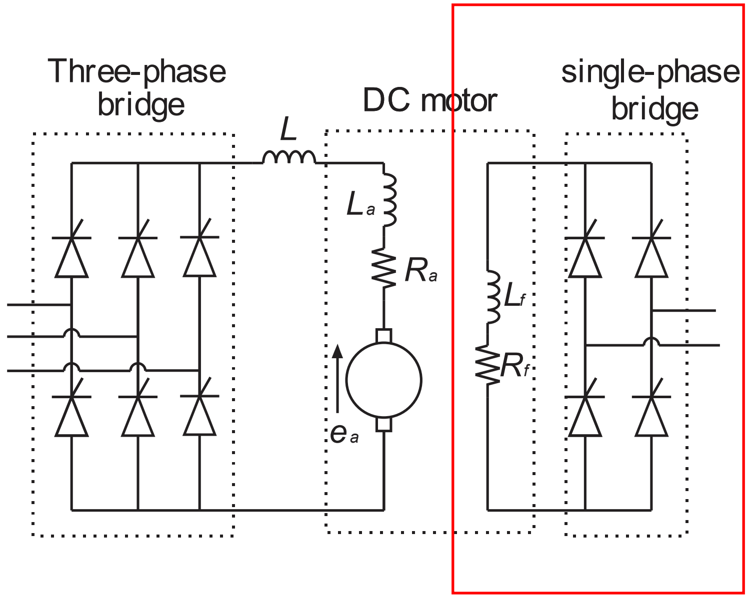


Field voltage produced by the single-phase half-wave rectifier (a.k.a. single-phase converter) - Exercise

- The voltage produced by the **single-phase converter** is:

$$v_{an} = V_{pk}\sin(\omega t)$$

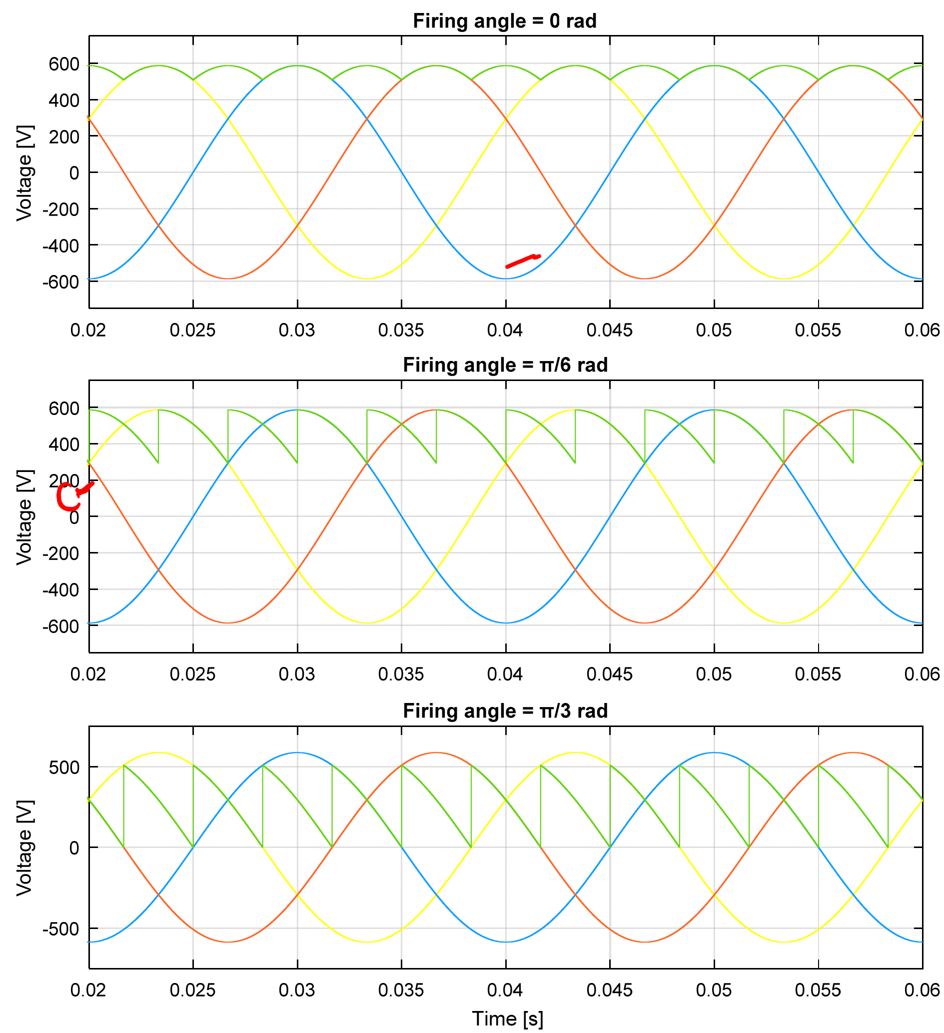
$$\begin{aligned}
V_a &= \frac{2}{2\pi}\int_{\alpha_a}^{\pi+\alpha_a} V_{pk}\sin(\omega t)\cdot d(\omega t) \\
&= \frac{V_{pk}}{\pi}\left[-\cos(\omega t)\right]_{\alpha_a}^{\pi+\alpha_a} \\
&= \frac{V_{pk}}{\pi}\left[-\cos(\pi+\alpha_a)+\cos(\alpha_a)\right] \\
&= \frac{V_{pk}}{\pi}\left[-\cos(\pi)\cos\alpha_a+\sin(\pi)\sin\alpha_a+\cos(\alpha_a)\right] \\
&= \frac{2V_{pk}}{\pi}\cos\alpha_a
\end{aligned}$$



$$V_f = \frac{2V_{pk}}{\pi}\cos\alpha_f$$

Sixth-order harmonics

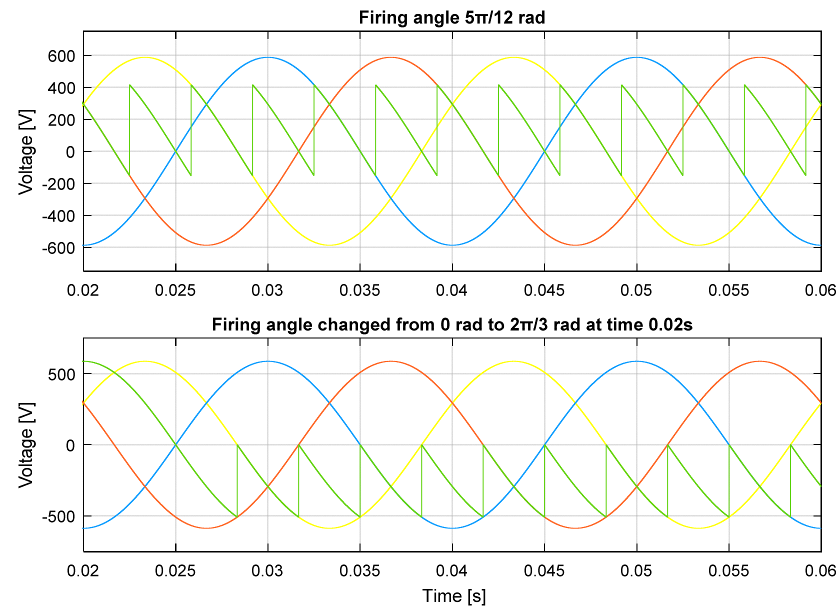
- As shown by the figure, the full-wave rectifier's DC output voltage contains significant sixth-order harmonics.
  - Armature winding is highly inductive. Hence the current is natural smoother than the voltage waveform. However, sixth-order harmonics still prevalent.
  - These voltage and current ripples would result in torque ripple, and hence the speed and position ripples. A large portion of the voltage and current harmonics do not lead to usable torque.
  - Depending on the smoothness requirement, a large, external inductor can be introduced in series to the armature winding.

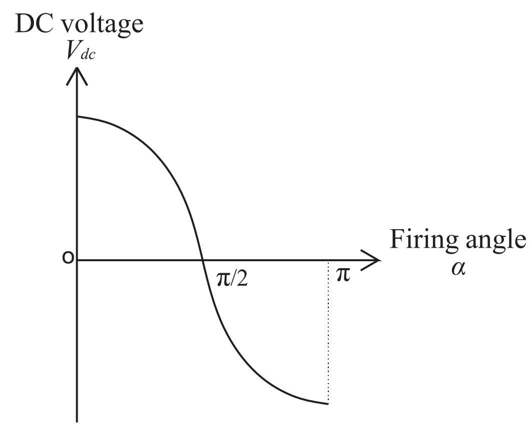


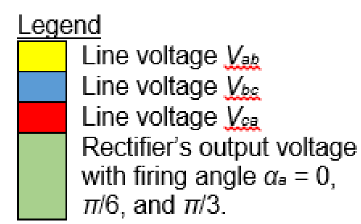
Legend
- Line voltage V_{ab}
- Line voltage V_{bc}
- Line voltage V_{ca}
- Rectifier's output voltage with firing angle $\alpha_a = 0$, $\pi/6$, and $\pi/3$.

Continuous conduction mode allows "inversion" to happen

- For a highly inductive load (such as this case with the large inductance L) where the armature current does not rise/fall across zero within the 50Hz's period, the first half of the full α range that can be utilized is: $0 \le \alpha < \pi/2$ rad. The average output voltage is positive.
    - Can you deduce what will happen to the output voltage if the assumption of large L does not hold?
- For the second half of α range: $\pi/2 \le \alpha < \pi$ rad, the average output voltage is effectively negative (while current continues to conduct in the positive direction; recall also that thyristors can only conduct unidirectional currents).
- This mode of operation is known as "inversion" mode. Therefore, a negative terminal voltage with a positive terminal current signifies a regeneration of power from the motor to the AC source.

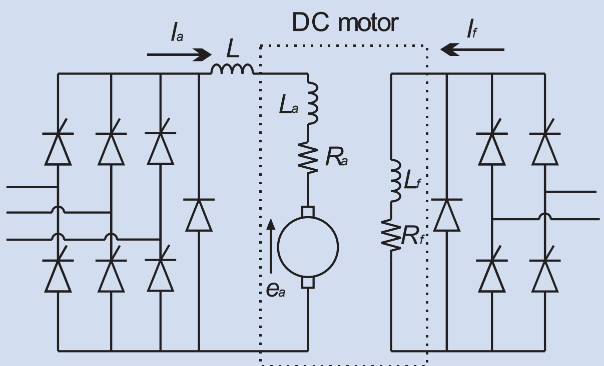
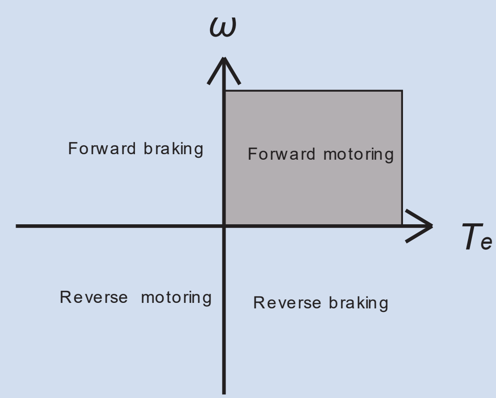
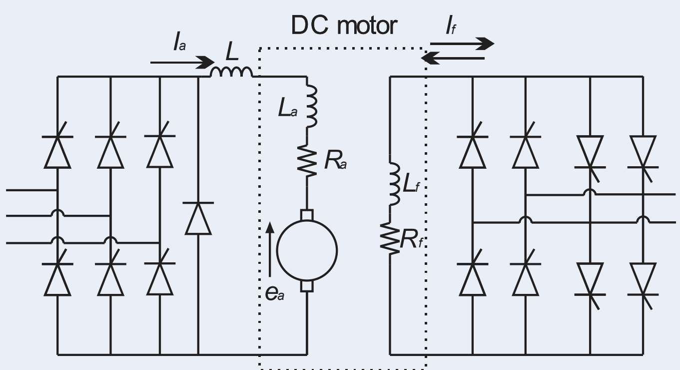
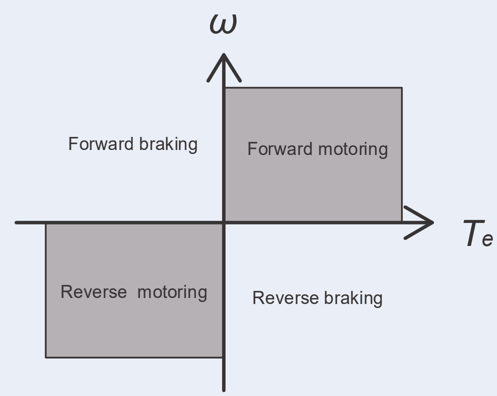




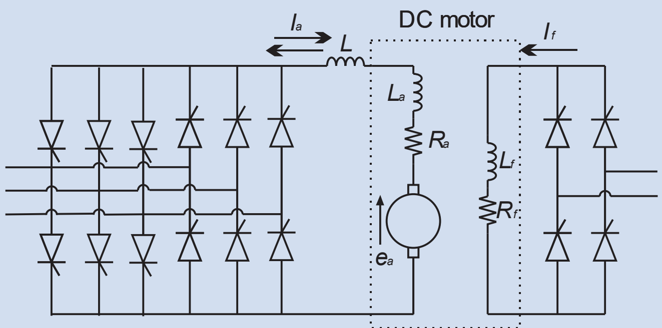
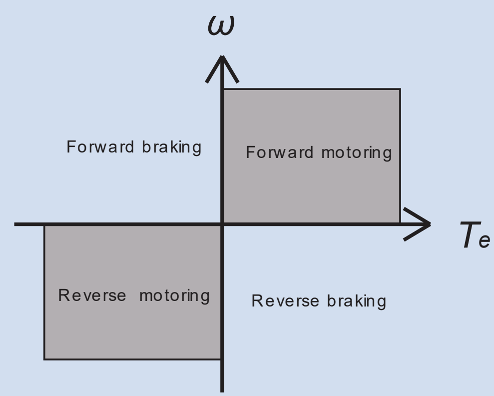
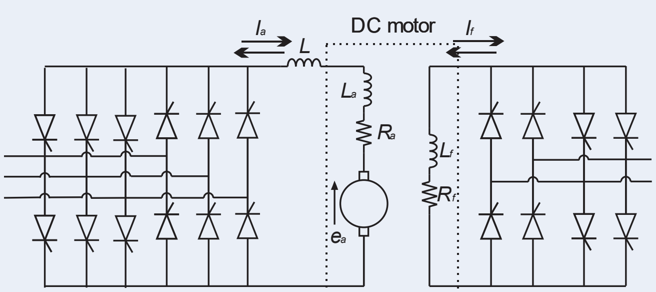
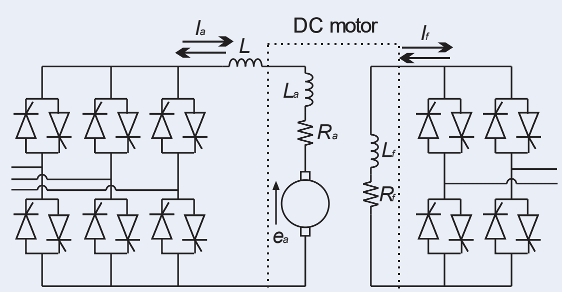
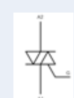
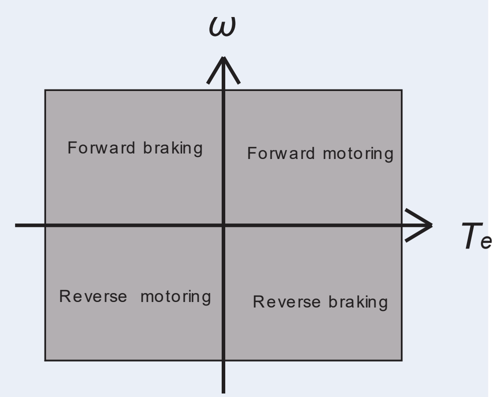


Quadrants of SCR-based DC motor drives

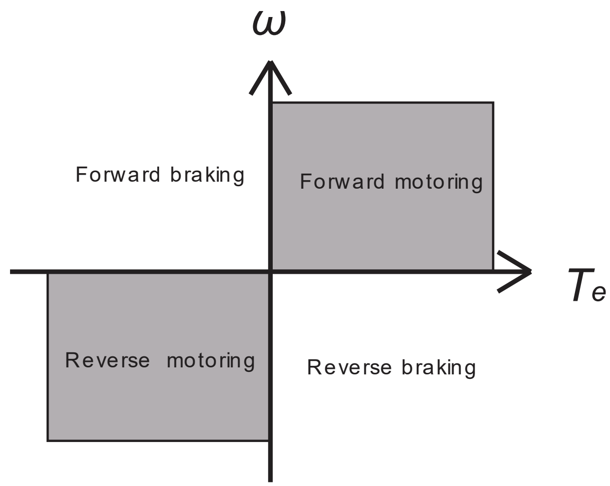
- Quadrant of operation of a DC motor drive depends on the topology of the power converter.

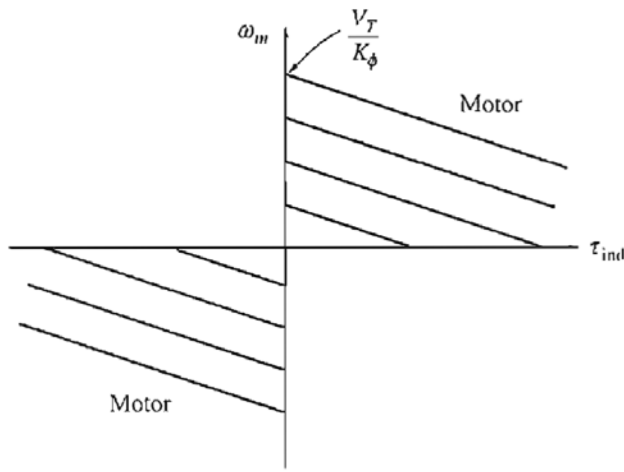
DC motor drives topology (SCR-based)	Quadrants
 Single-quadrant drive	
 Two-quadrant drive	

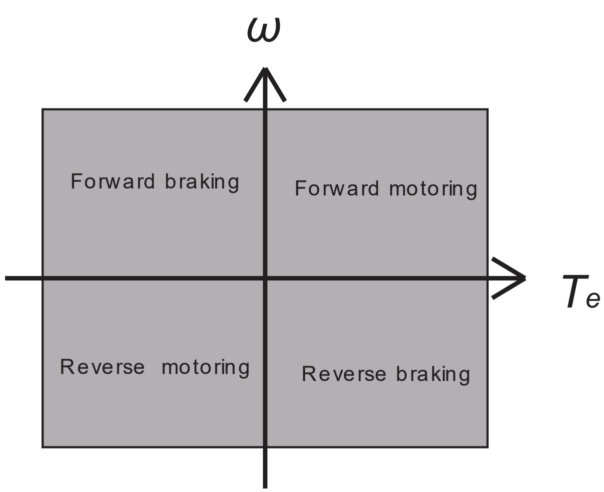
Quadrants of SCR-based DC motor drives

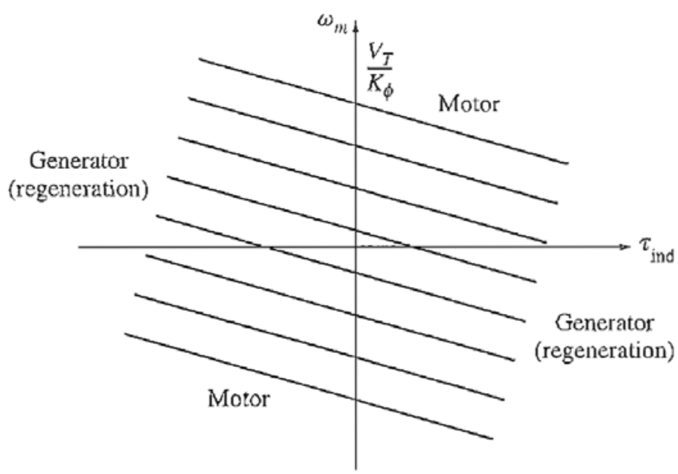
DC motor drives topology (SCR-based)	Quadrants
 Two-quadrant drive	
 Thyristor-based  TRIAC-based  Four-quadrant drive	

Torque-speed curves









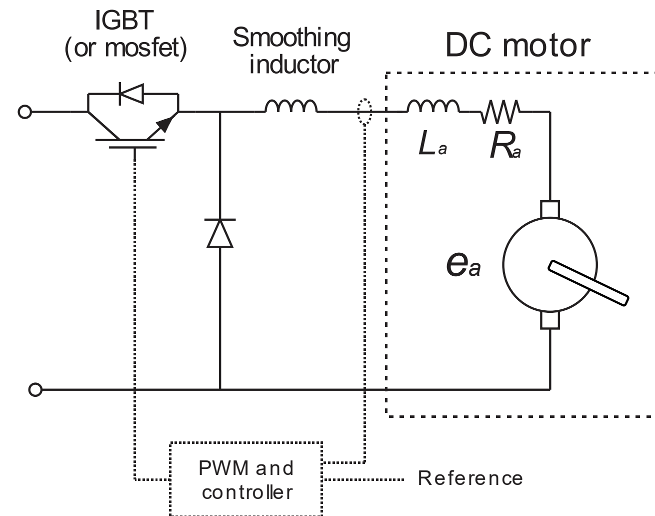
- Power-electronic-based drives eliminate the needs of classical Ward-Leonard system (an old four-quadrant variable-speed DC drive topology that couples a three-phase rectified generator to a DC motor armature)

Exercise

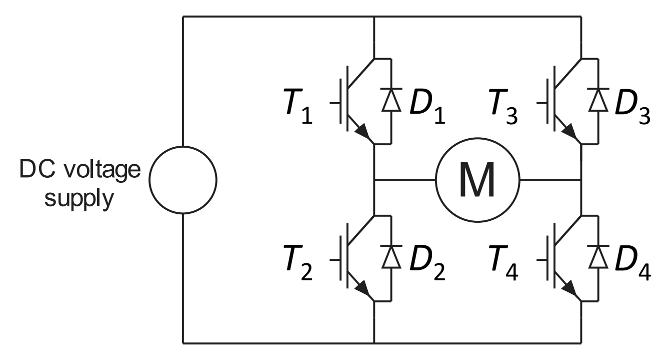
- The speed of a 20 hp 300 V 1800 rpm separately excited DC motor is controlled by a three-phase full converter drive. The field current is controlled by a single-phase full converter supplied by one pair of lines from the three-phase supply and is set to the maximum possible value. The AC input is a three-phase star-connected 208 V 60 Hz supply. The armature resistance R_a and field resistances R_f are 0.25 Ω and 163.33 Ω respectively, and the motor voltage constant is K_v = 1.2 V/(A-rad/s). The armature and field currents can be assumed to be continuous and ripple-free. The viscous/windage friction is negligible.
  - (a) Determine the delay angle of the armature converter, α_a, if this converter supplies the motor that operates at rated speed with rated current.
  - (b) Determine the no-load speed if the delay angles are the same as in part (a) and the armature current at no-load is 10% of the rated value.
  - (c) Determine the speed regulation.

IGBT/MOSFET-based DC motor drives

- In the previous rectifier-fed DC drives based on thyristors, the firing angle and hence then armature voltage can only be altered only twice in a cycle of the input voltage (i.e. 10 ms for 50Hz AC supply). This limits the achievable dynamic response.
  - Moreover, the machine losses become higher due to the presence of significant low-order harmonics in the armature voltage and current waveforms.
- DC choppers generally have higher efficiency compared to the thyristor counterparts.
  - The fundamental operating principle is about changing the output voltage by varying the fraction of the time that the DC source is connected to its load.
- In the past, the same mechanism is achieved through Gate-turn-off thyristor. The switching frequency was low hence the smoothing inductor required was often bulky.
- Modern DC drives with a minimum of two-quadrant requirement often makes use of full/half H-bridge.

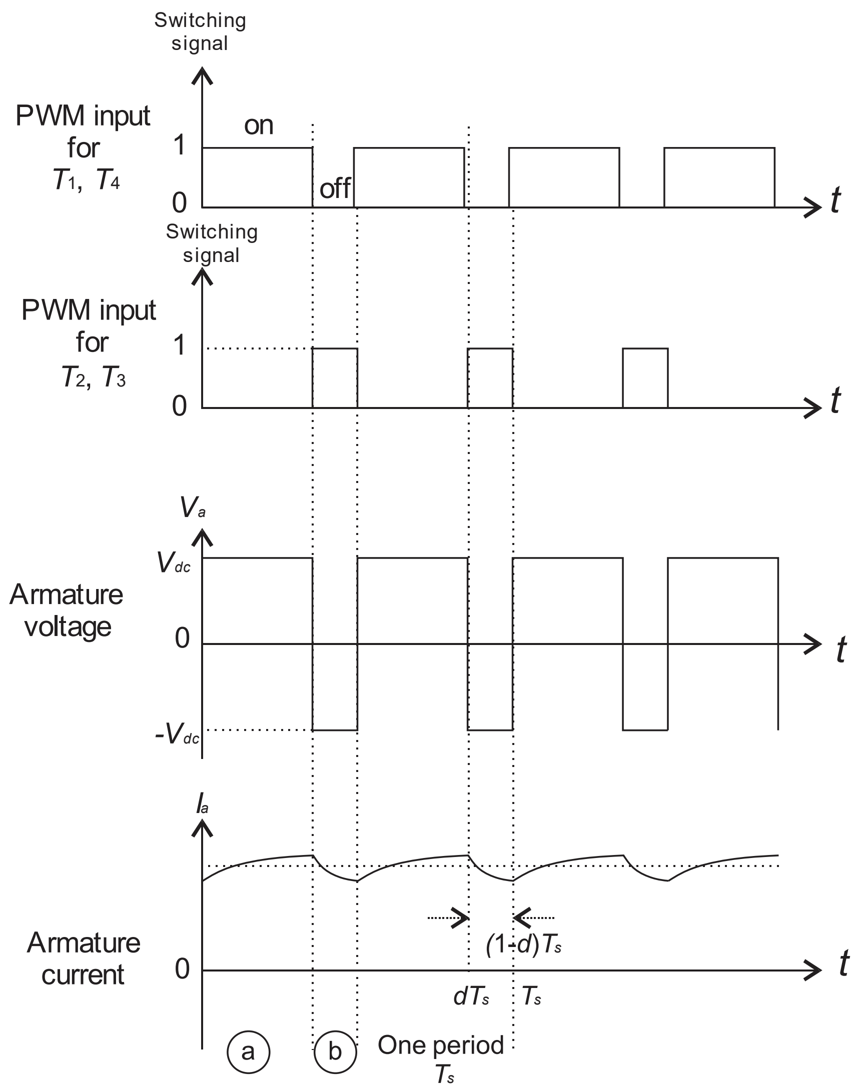


(Unidirectional) buck-converter-based armature voltage control (DC chopper)



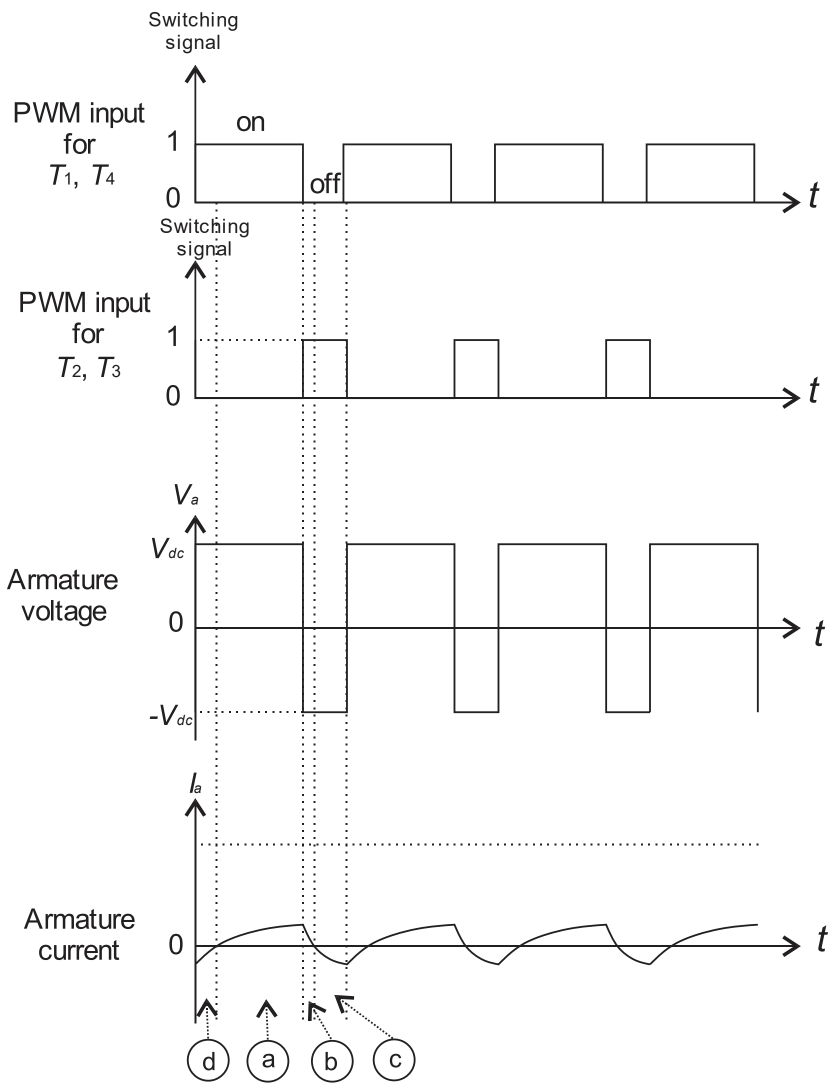
H/Full-bridge-based armature voltage control

Continuous conduction mode (Full-bridge)



- In the continuous conduction mode, within each of the switching period, there are two operational sub-cycles:
  - T_1 and T_4 are turned on, and the armature current flows in the positive direction;
  - T_2 and T_3 are turned on but not conducting. Instead, freewheeling diodes D_2 and D_3 are latched on, and the armature current flows in the positive direction;
- The same applies to negative armature current.

Discontinuous conduction mode (Full-bridge)



- In the discontinuous conduction mode, within each of the switching period, there are four operational sub-cycles:
  - T_1 and T_4 are turned on, and the armature current flows in the positive direction;
  - T_2 and T_3 are turned on but not conducting. Instead, freewheeling diodes D_2 and D_3 are latched on, and the armature current flows in the positive direction;
  - T_2 and T_3 are turned on and conducting, and the armature flows in the negative direction;
  - T_1 and T_4 are turned on but not conducting. Instead, freewheeling diodes D_1 and D_4 are latched on, and the armature current flows in the negative direction;

Effective armature current due to PE switching (continuous)

- Assume that upon steady state, the instantaneous current in the motor is I_1 in the beginning of each switching period.
- ***Sub-cycle a with T_1* and *T_4* ON**: current flows in the positive direction through the *RL*, the back-EMF *E*, the switches T_1 and T_4, and the source V_{dc}:

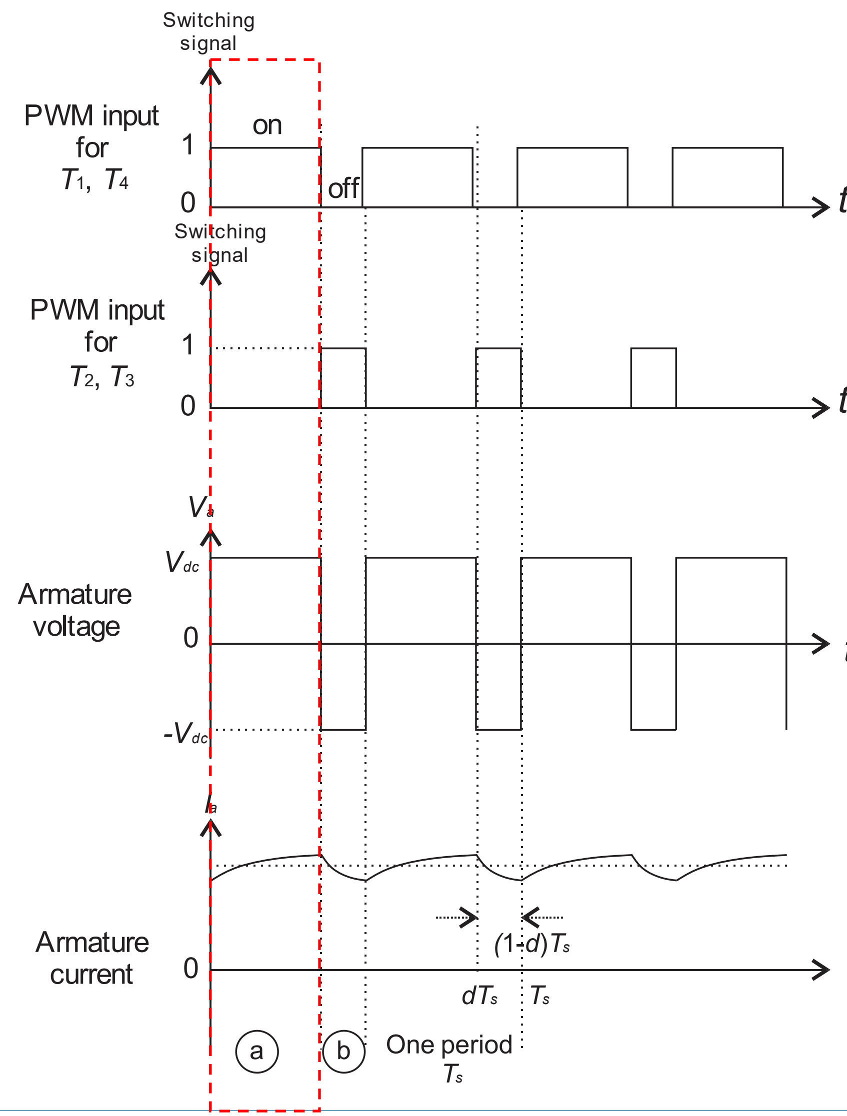
$$V_{dc} = L\frac{di}{dt} + Ri + E$$

- For $0 < t < kT$, the current waveform takes the form of:

$$i_1(t) = I_1 e^{-\frac{R}{L}t} - \frac{(E - V_{dc})}{R}\left[1 - e^{-\frac{R}{L}t}\right]$$

- The current rises to I_2 at the end of ON period of T_1 and T_4, i.e. at time $t = kT$:

$$I_2 = I_1 e^{-\frac{R}{L}kT} - \frac{(E - V_{dc})}{R}\left[1 - e^{-\frac{R}{L}kT}\right]$$



Effective armature current due to PE switching

- ***Sub-cycle b with T_2* and *T_3* ON**: current flows in the positive direction through the *RL*, the back-EMF *E*, the anti-parallel diodes of T_2 and T_3, and the source V_{dc}:

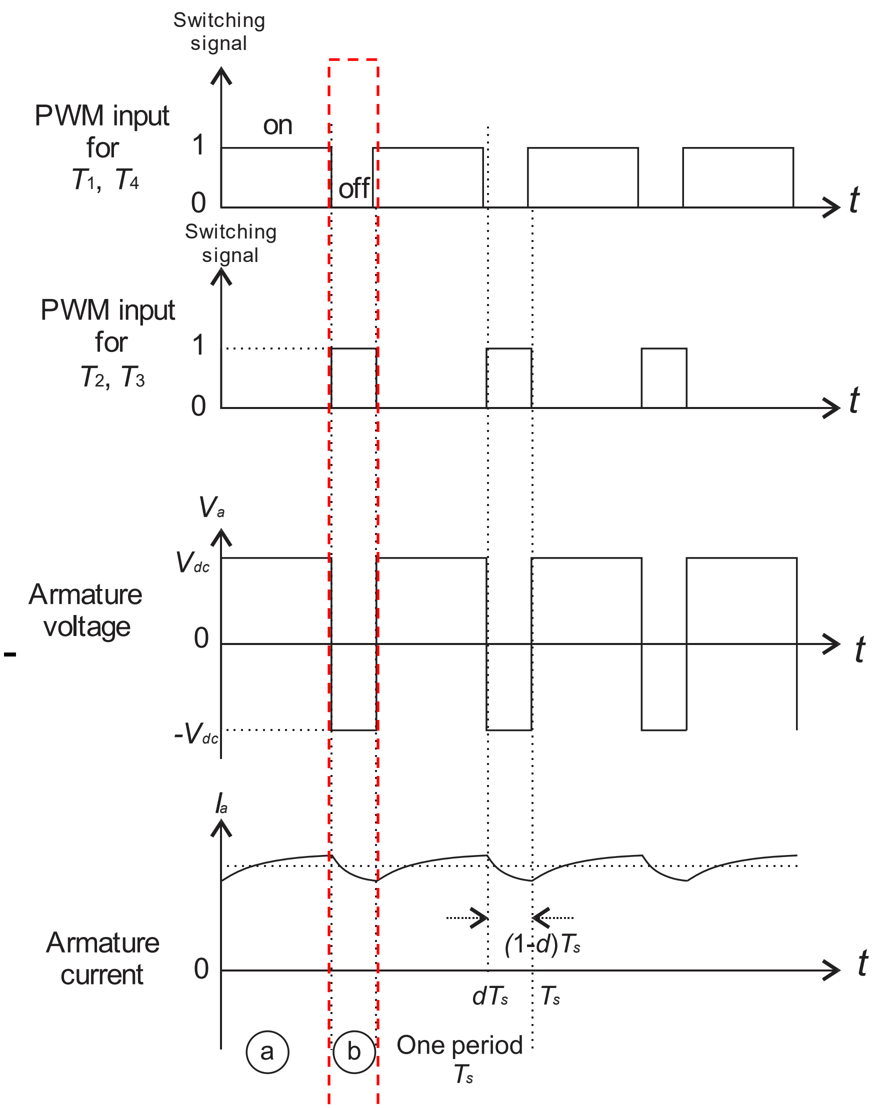
$$-V_{dc} = L\frac{di}{dt} + Ri + E$$

- For $0 < t' < (1-k)T$, the current waveform is:

$$i_2(t) = I_2 e^{-\frac{R}{L}t} - \frac{(E+V_{dc})}{R}\left[1 - e^{-\frac{R}{L}t}\right]$$

- The current falls back to I_1 (assumed that the non-linear circuit has attained steady state) at the end of ON period of T_2 and T_3, i.e. at time $t' = (1-k)T$:

$$I_1 = I_2 e^{-\frac{R}{L}(1-k)T} - \frac{(E+V_{dc})}{R}\left[1 - e^{-\frac{R}{L}(1-k)T}\right]$$

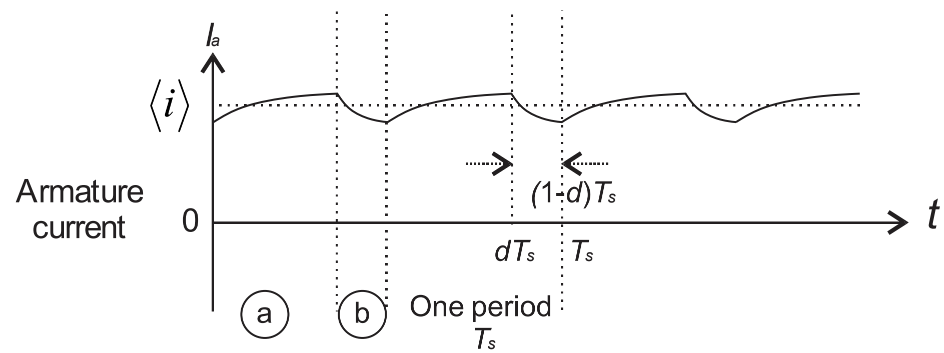


Effective armature current due to PE switching

- I_1 and I_2 can be re-expressed in terms of circuit parameters, the switching period T, and duty cycle k:

$$I_1 = -\frac{E}{R} + \frac{V_{dc}}{R}\left[\frac{2e^{-\frac{R}{L}(1-k)T} - e^{-\frac{R}{L}T} - 1}{1 - e^{-\frac{R}{L}T}}\right]$$

$$I_2 = -\frac{E}{R} + \frac{V_{dc}}{R}\left[\frac{-2e^{-\frac{R}{L}kT} + e^{-\frac{R}{L}T} + 1}{1 - e^{-\frac{R}{L}T}}\right]$$



- Upon steady state, the average voltage and current across/through the load are calculated from:

$$\langle i \rangle = \frac{1}{T}\left[\int_0^{kT}\left\{I_1 e^{-\frac{R}{L}t} - \frac{(E - V_{dc})}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right\}.dt + \int_0^{(1-k)T}\left\{I_2 e^{-\frac{R}{L}t} - \frac{(E + V_{dc})}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right\}.dt\right]$$

Effective armature current due to PE switching

- The H-bridge topology shown earlier can be used for four-quadrant DC drives or a single-phase AC drive.

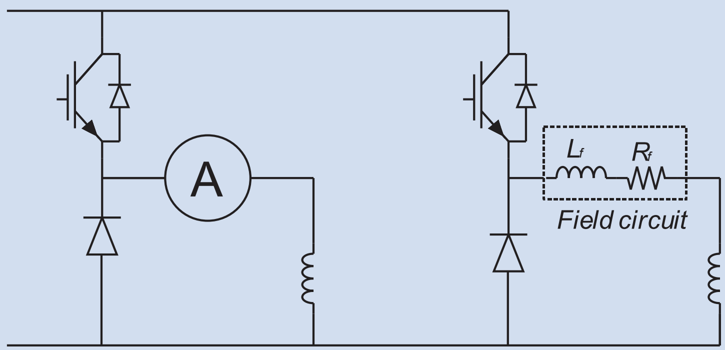
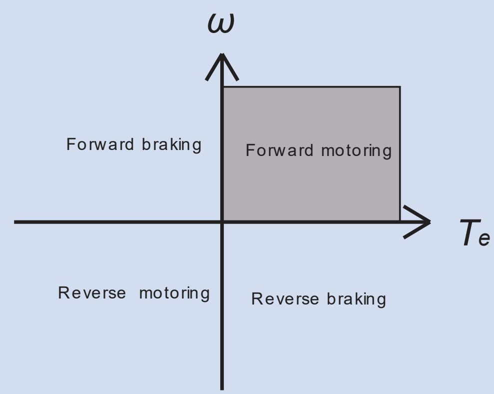
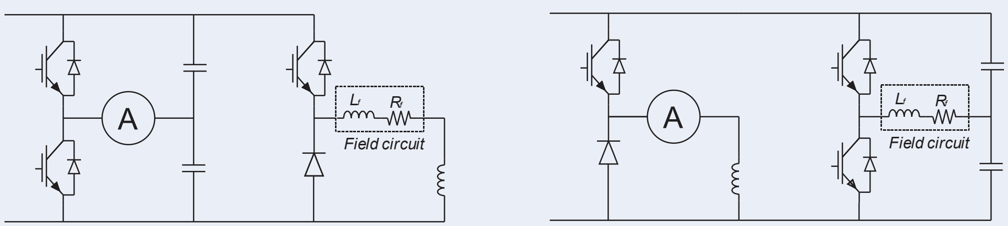
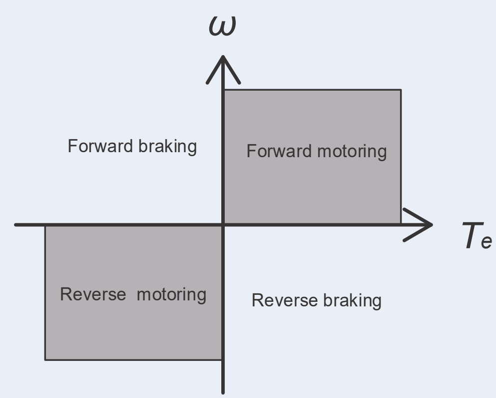
- For DC drives, the average voltage model of a H-bridge without taking into account the dead-time effect is:

$$\langle V \rangle = \frac{1}{T}\left[\int_0^{kT} V_{dc} \cdot dt + \int_{kT}^{T} -V_{dc} \cdot dt\right] = (2k-1)V_{dc}$$

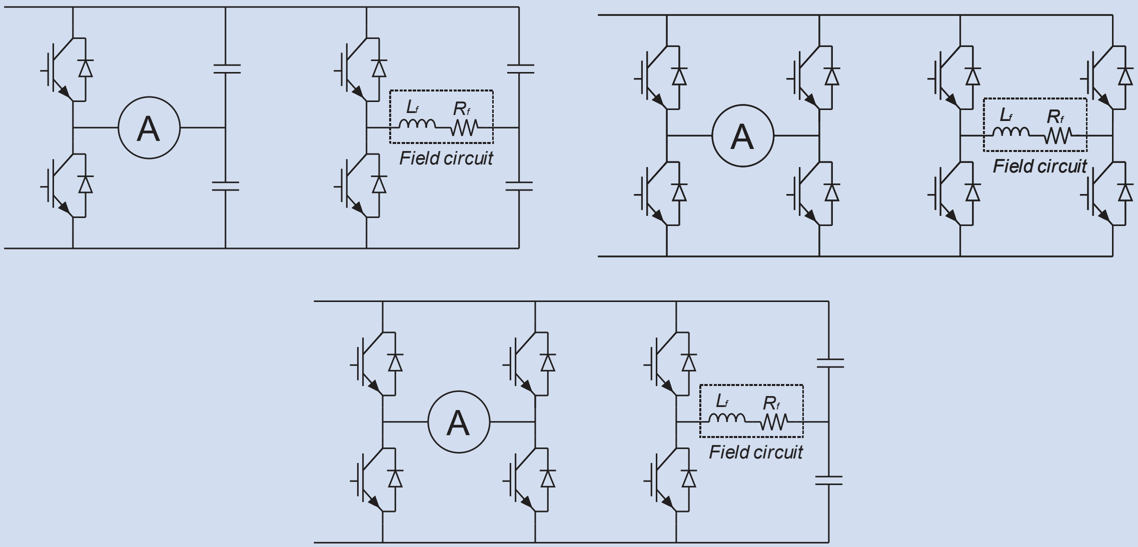
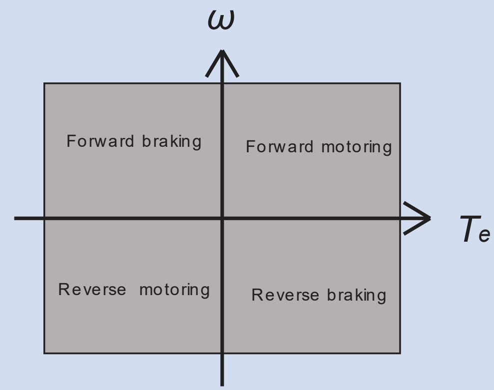
- i.e.

$$V_o = (2k-1)V_{dc}$$

Quadrants of MOSFET/IGBT-based DC motor drives

DC motor drives topology (examples)	Quadrants
	
 Two-quadrant drives	

Quadrants of MOSFET/IGBT-based DC motor drives

DC motor drives topology	Quadrants
 Four-quadrant drives	

- Various DC drives topology can be achieved (not limited to buck converter and H-bridge).
- Criteria for final selection: number of PE switches, cost, efficiency of the whole circuit, etc.

Advantages of MOSFET/IGBT-based drives

- MOSFET/IGBT-based drives carry several advantages that are not found in the conventional SCR-based drives:
  - Fast response
  - Flexible in topology
  - Less number of PE switches
  - Much wider selection of ratings of PE switches

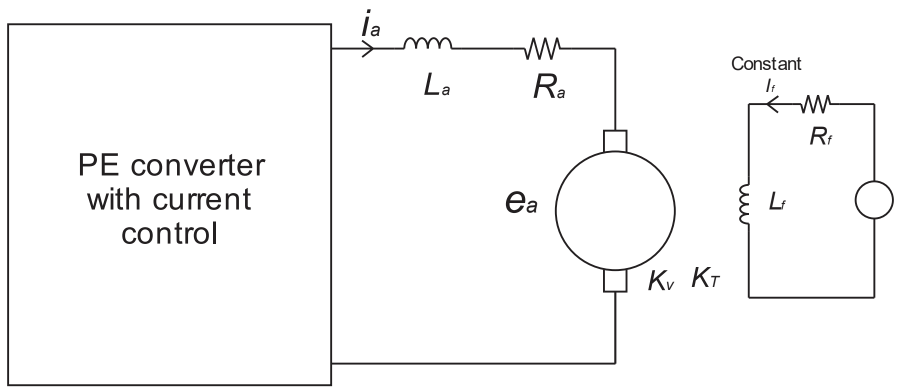
- However, the complexity rises:
  - Fast dynamical controller/control algorithm
  - Parameter estimation

- SCR-based drives are still in use for very high power drives. However, today's trend is to use AC motor and drives instead of the DC counterpart.

Other DC motors

- All previous DC drives is based entirely on the separately excited DC motor. The main reason is that the separate excitations produces a linear system that facilitates the control means (made available by controllable PE switches).
- Another option is the permanent magnet DC motor, which is only common nowadays for small motor (hence drives is seldom needed).
- Series, shunt, and compounded DC motors have non-linear relationship between the current and motor torque.
  - The classical speed control methods is mostly based on resistance adjustment, which would then result in poor efficiency of overall system.
  - The use of PE-based drives in conjunction of these DC motors is rare.

Regenerative braking of a DC motor



$$v_m = K_e \omega_m + i_a R_a + L_a \frac{di_a}{dt}$$

$$T = K_t i_a$$

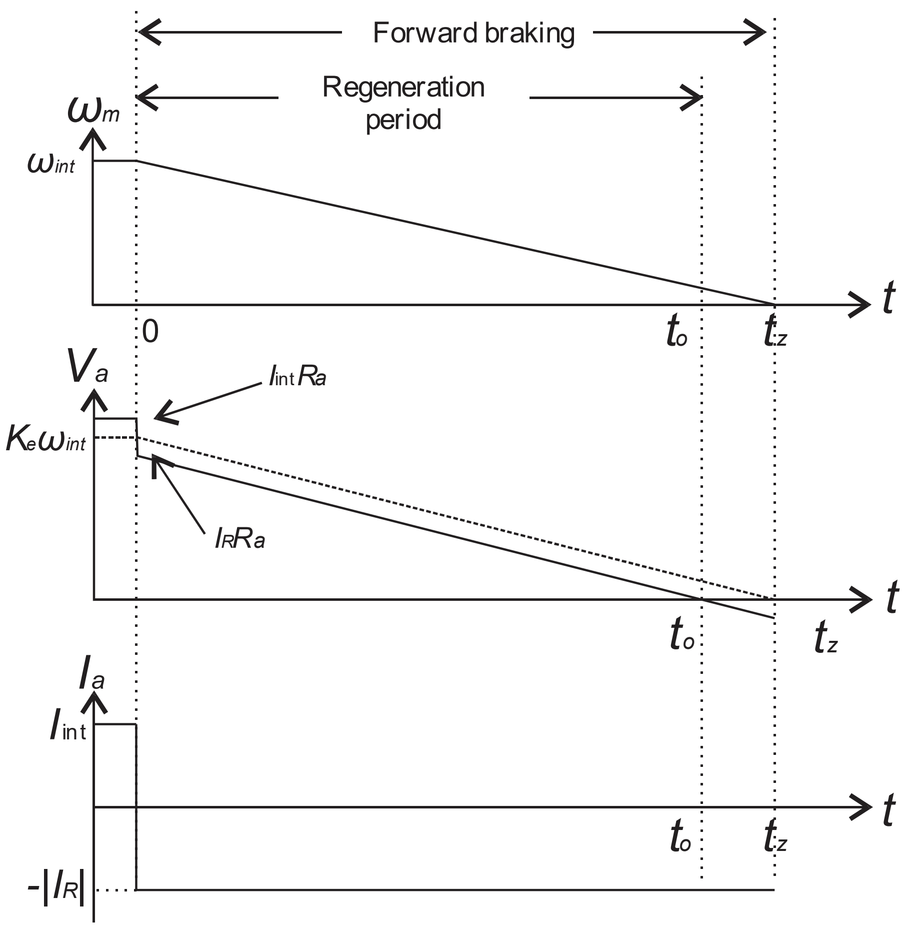
$$T = J \frac{d\omega_m}{dt} + B\omega_m$$

where
ω_m is the speed of rotation, in rad/s;
K_e is the motor's speed constant, in V/(rad/s); K_t is the motor torque constant, in Nm/A;
L_a is the armature inductance, in H;
T is the electromagnetic torque, in Nm.

- During regenerative braking, the kinetic energy from the rotational motion of the rotor is recaptured by the power-electronic converter.
- In order to realize regenerative braking, a closed-loop current control is required. This allows the assumption of constant regenerative current (hence $di_a/dt = 0$) to be made and thus simplifies the derivation.

- Assumption: bidirectional converter (current).

Regenerative braking of a DC motor



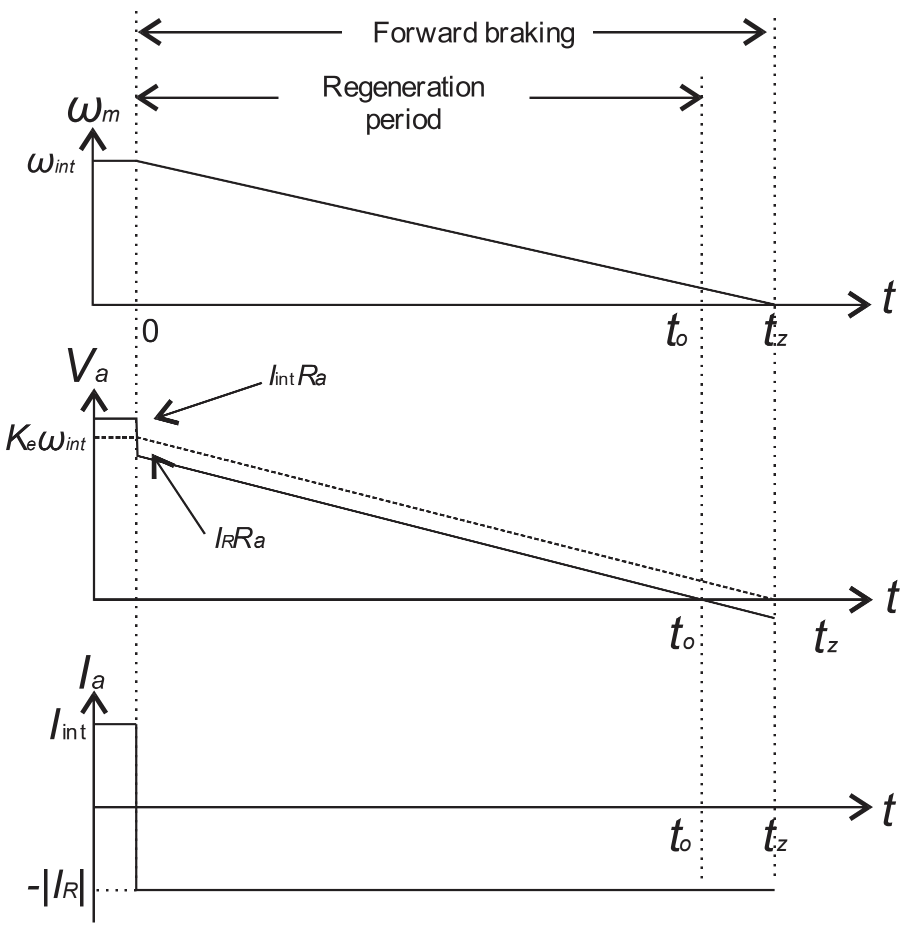
- The braking current is controlled by the DC drive to remain constant I_R (negative) during the braking period, therefore the voltage balanced equation is simplified to:

$$V_m(t) = K_e \omega_m(t) + I_a R_a + 0$$

- Assume that the windage friction is zero, no load is connected, and with constant braking armature current, the braking torque therefore remain constant too during the braking period. The deceleration is therefore a constant and the speed profile is a straight line

$$\omega_m(t) = \left(1 - \frac{t}{t_z}\right)\omega_{\text{int}}$$

Regenerative braking of a DC motor



- The time t_o can be determined by equating the terminal voltage to zero, giving:

$$0 = -\frac{K_e\omega_{int}}{t_z}t_o + K_e\omega_{int} + I_R R_a$$

- Solve for t_o:

$$t_o = t_z\left(1 + \frac{I_R R_a}{K_e\omega_{int}}\right)$$

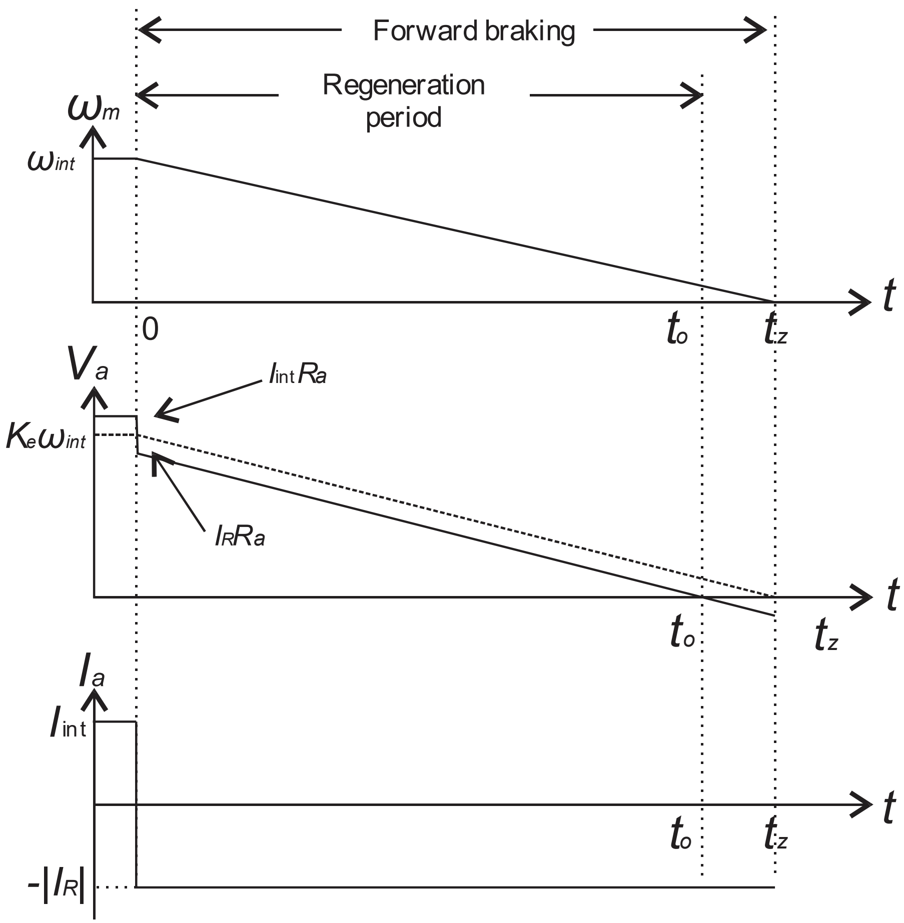
- Note I_R is negative value.

- Braking torque developed during the deceleration/regeneration (assume no load and no windage friction):

$$T_{braking} = K_t I_R = J\alpha$$

$$\alpha = \frac{K_t I_R}{J}$$

Regenerative braking of a DC motor



- Since $\dfrac{K_t I_R}{J} = \dfrac{-\omega_{\text{int}}}{t_z}$

$$t_z = \frac{-J\omega_{\text{int}}}{K_t I_R}$$

- From the relationship between t_o and t_z:

$$t_o = \frac{-J\omega_{\text{int}}}{K_t I_R}\left(1 + \frac{I_R R_a}{K_e \omega_{\text{int}}}\right)$$

- Lastly, the regenerated energy is:

$$E_{regen} = \int_0^{t_o} V_m(t) I_R \cdot dt$$

$$= \int_0^{t_o} \left[-\frac{K_e \omega_{\text{int}} t}{t_z} + K_e \omega_{\text{int}} + I_R R_a \right] I_R \cdot dt$$

$$= \frac{t_o}{2}\left[K_e \omega_{\text{int}} I_R + I_R^2 R_a \right]$$

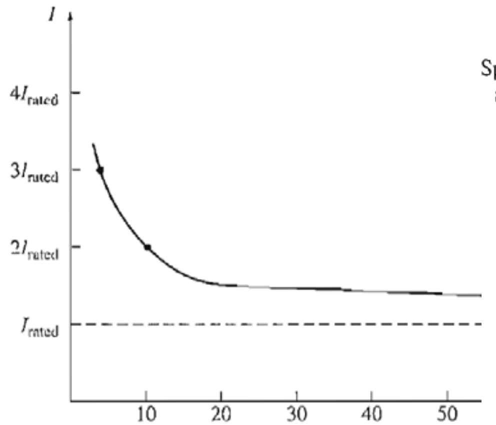
Exercise

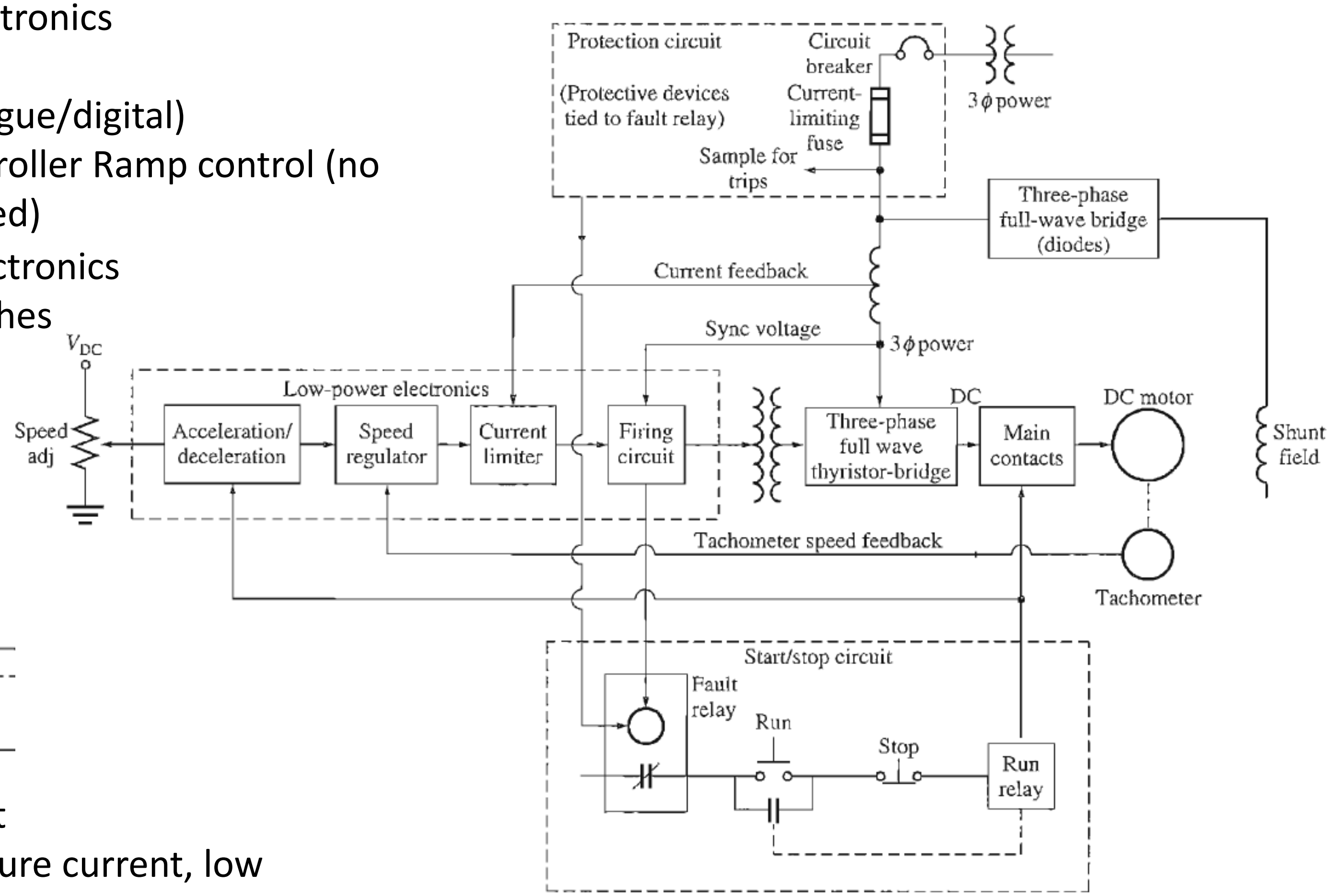
- A rotating mass having an inertia of 0.1 kgm^2 and is spun at 1000 rpm by a directly coupled separately excited DC motor (having these characteristics: K_e = 0.5 V/(rad.s^{-1}), K_T = 0.5 Nm.A^{-1}, inertia 0.05kgm^2, armature resistance of 1 Ω). Assume that a four-quadrant DC drive is used.
  - What is the stored kinetic energy?
  - What is the regenerative current required to stop the rotating mass in 10 s? Assume an ideal, constant current/torque braking.
  - How long does it take for the regenerative energy to be completely returned to the source?
  - The four-quadrant drive is a H-bridge that has 339 V as the operating DC bus voltage, what is the minimum size of capacitor at the DC bus to keep the maximum voltage rise to a value below 400 V?

A simplified DC motor drive hardware schematic

Section 1: Low-power electronics
- Firing circuits
- Speed regulator (analogue/digital)
- Torque regulation controller Ramp control (no starting resistor required)

Section 2: High-power electronics
- Power-electronic switches





Section 3: Protection circuit
- Against excessive armature current, low terminal voltage, and loss of field current
- Fuses, instantaneous/ overload/undervoltage relay, temperature/field loss tripping

Section 4: Start/stop circuit
- connect/disconnect from the mains
- Start/stop button