Learning and Assessment

Learning resources

- Lecture and e-lecture materials
- Tutorial question
- Online material

Assessment

- 1 assignment (27th Mar) – 15%
- 2 question (out of 3, in Part B) in the final exam – 40%
- BLDC lab – 5% (both parts)

References

- C. S. Lim. Course material.
- M. H. Rashid. *Power Electronics – Circuits, Devices, and Applications*, 3rd ed. Pearson. 2003.
- Emil Levi. *Power Electronics, Drives and Systems*. Course material. 2014.
- Richard Crowder. *Electric Drives and Electromechanical Systems.* Elsevier. 2006.

Content

- Chapter 1: **Relevant Electrical and Mechanical Basic Concepts**
  - Introduction, moment of inertia, path control
  - Mechanical power transmission
  - Encoder, power sizing, basic kinematic, ideal transformer

- Chapter 2: **Power Electronic Control of DC Motor Drives**
  - Fundamentals of DC machines
  - Speed control principle, half-/full-wave rectifier and chopper control
  - Regenerative braking
  - Tutorial

- Chapter 3: **Power Electronic Control of AC Motor Drives**
  - Fundamentals of AC machines (equivalent circuits)
  - Speed control methods and principle, scalar control principle
  - Inverter control using sinusoidal PWM
  - High-performance control of AC drives (introduction)
  - Tutorial

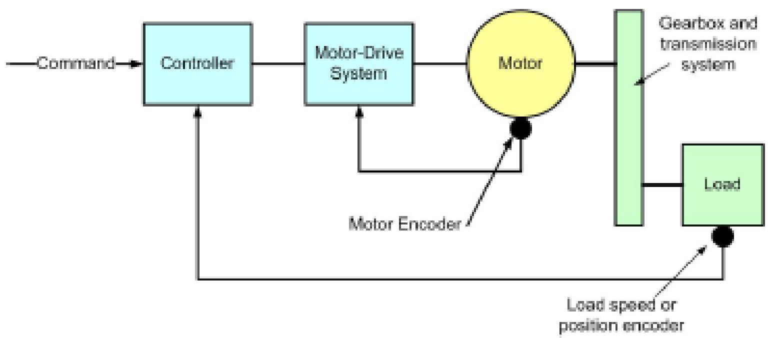
ELEC2208: Power Electronics and **Motor Drives**:

Chapter 1: Relevant Electrical and Mechanical Basic Concepts

C. S. Lim

Introduction

- A motion control system consists of five elements - need to be carefully determined in order to get the optimum performance out from the whole system.
  - Motor and drive's power electronic package
  - Control algorithm
  - Transmission system
  - Encoders and/or transducers
  - Load



Electric motor drives – what are the applications?

Hard Drive
Elevator/lift
Electrical bikes
Electrical cars
Flywheel
Milling and Turning (lathe)

Paper mills
Cement plant

***ABB Azipod's* electric propulsion**

- https://www.youtube.com/watch?v=dv4C_LdoDaE

***NASA's* Electric Aircraft**

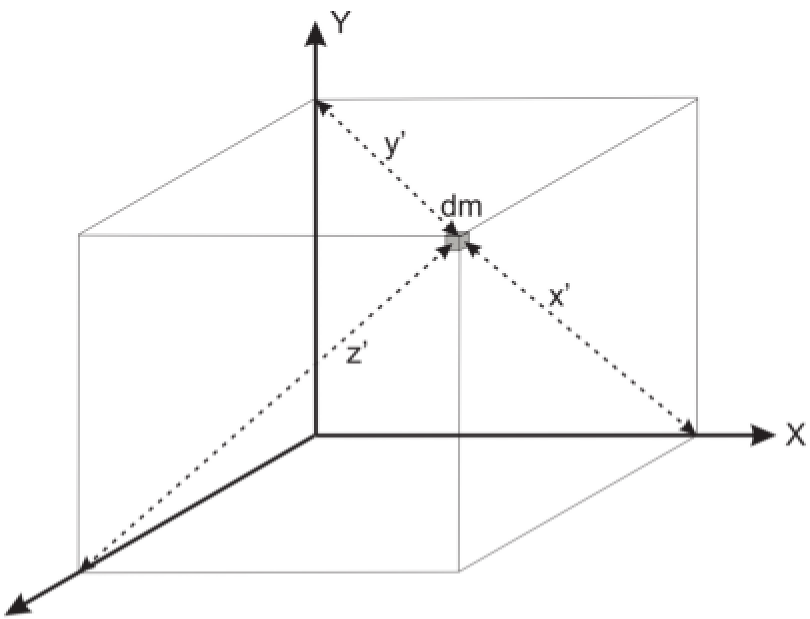
- https://www.youtube.com/watch?v=wpOzuGQtwkc

Same underlying principle:
wind turbine doubly-fed induction generator speed control
Microturbine
etc.

Moment of inertia, friction, and path control

Moment of inertia

- Moment of inertia, or rotational inertia is defined as the mass x perpendicular distance squared ($I = mr^2$). Mass and moment of inertia of non-point mass object is obtained by integration.

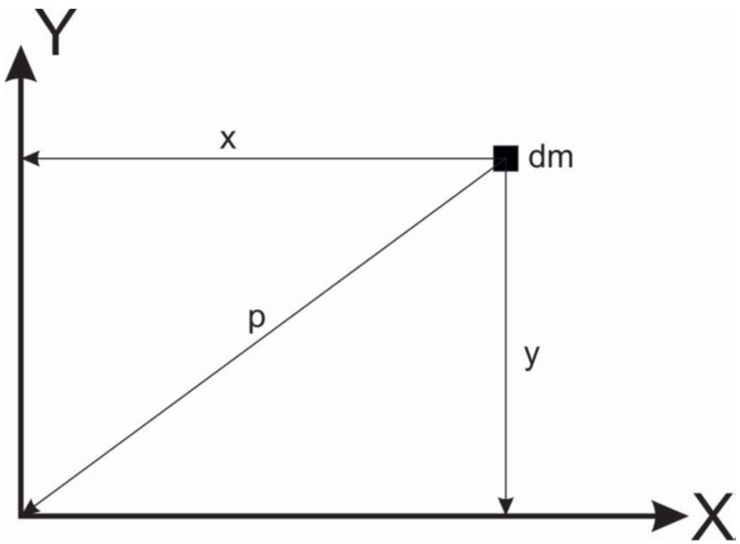


$$I_{xx} = \int x'^2 dm = \int (y^2 + z^2) dm$$

$$I_{yy} = \int y'^2 dm = \int (x^2 + z^2) dm$$

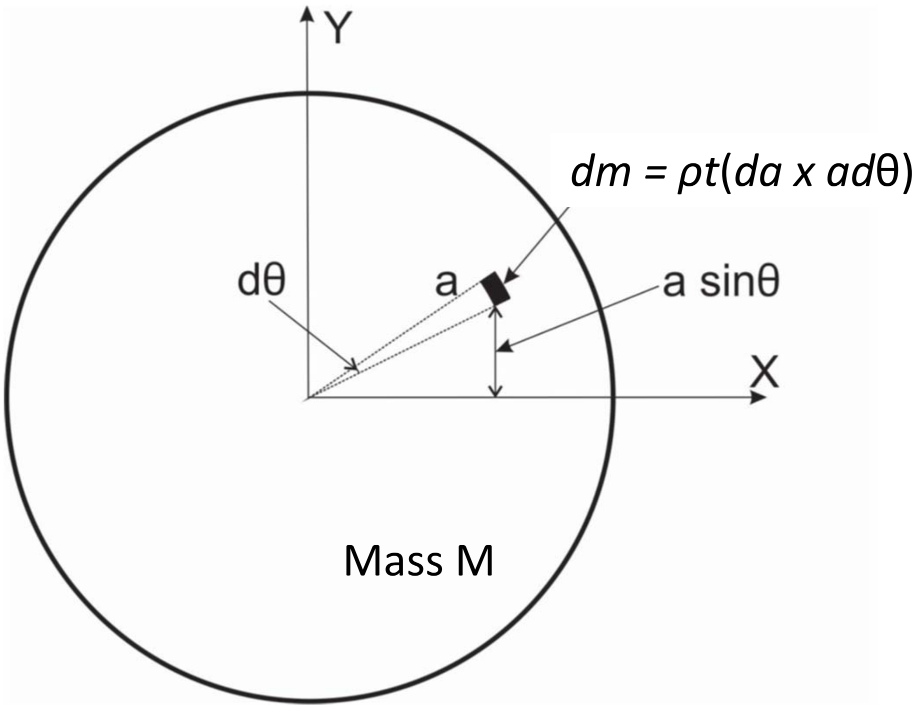
$$I_{zz} = \int z'^2 dm = \int (y^2 + x^2) dm$$

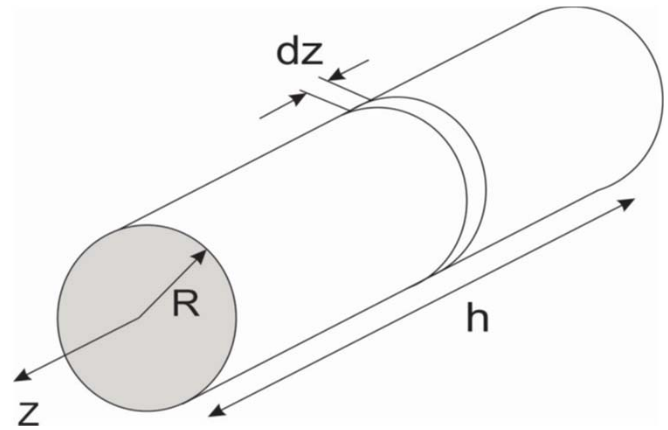
Inertia on the XY-plane



- *z=0*
- $I_x = \int (y^2) dm$
- $I_y = \int (x^2) dm$
- $I_{zz} = \int p^2 dm = \int (y^2 + x^2) dm = I_x + I_y$

Inertia of a thin disc and a cylinder





- Consider the cylinder to be constructed of thin discs, *dz* thick.

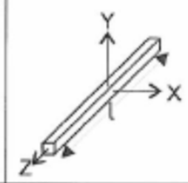
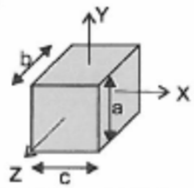
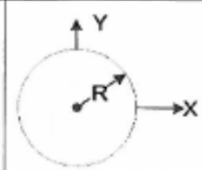
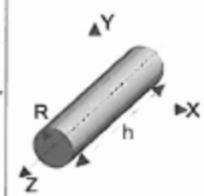
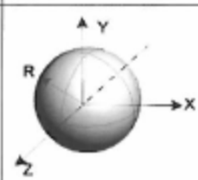
$$I_z = \frac{1}{2} M R^2$$

Total volume = $\pi R^2 t$

total mass M = ρ x total volume

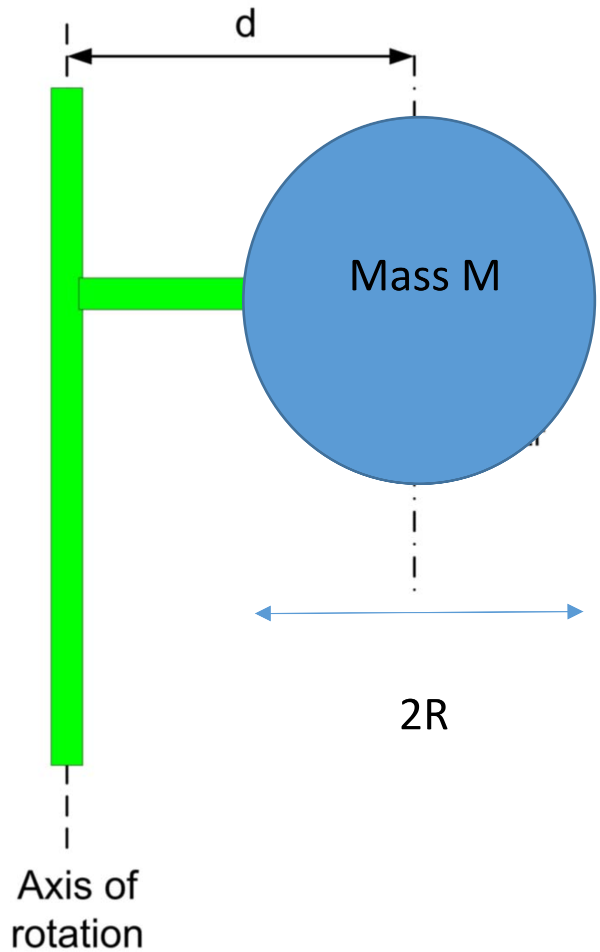
$$I_z = I_x + I_y = \frac{1}{2} MR^2$$

Other common bodies

Body		I_{xx}	I_{yy}	I_{zz}
Slender bar		$\frac{ml^2}{12}$	$\frac{ml^2}{12}$	–
Cuboid		$\frac{m}{12}(a^2+b^2)$	$\frac{m}{12}(b^2+c^2)$	$\frac{m}{12}(a^2+c^2)$
Thin disc*		$\frac{mR^2}{4}$	$\frac{mR^2}{4}$	$\frac{mR^2}{2}$
Cylinder		$\frac{m}{12}(3R^2+h^2)$	$\frac{m}{12}(3R^2+h^2)$	$\frac{mR^2}{2}$
Sphere		$\frac{2}{5}mR^2$	$\frac{2}{5}mR^2$	$\frac{2}{5}mR^2$

* A thin disc is considered a special case of a cylinder where $h = 0$.

Moment of inertia



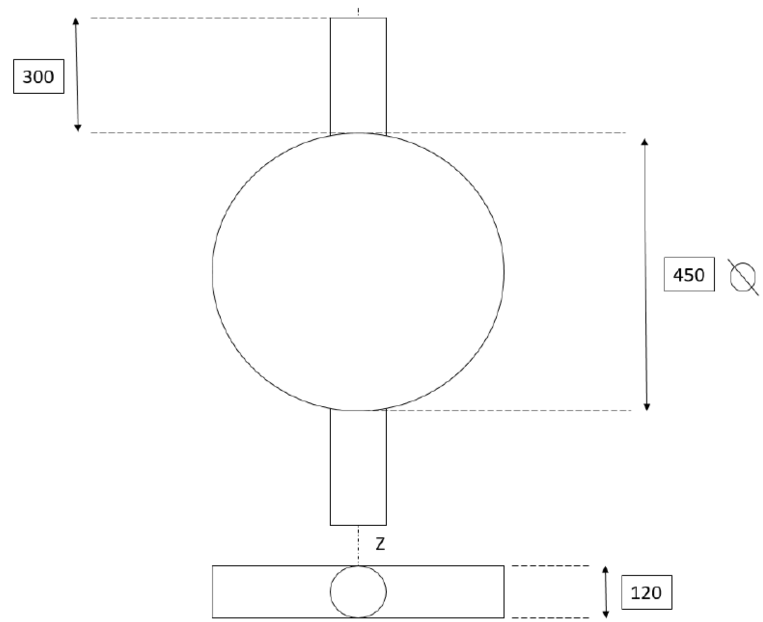
- The moment of inertia of any object about an axis through its center of mass is the minimum moment of inertia for an axis in that direction in space. The moment of inertia about any axis parallel to that axis through the center of mass is given by

$$J_{thin\ disc,new} = J_{thin\ disc} + Md^2$$

- E.g.: the thin disc's new moment of inertia about the new axis of rotation is

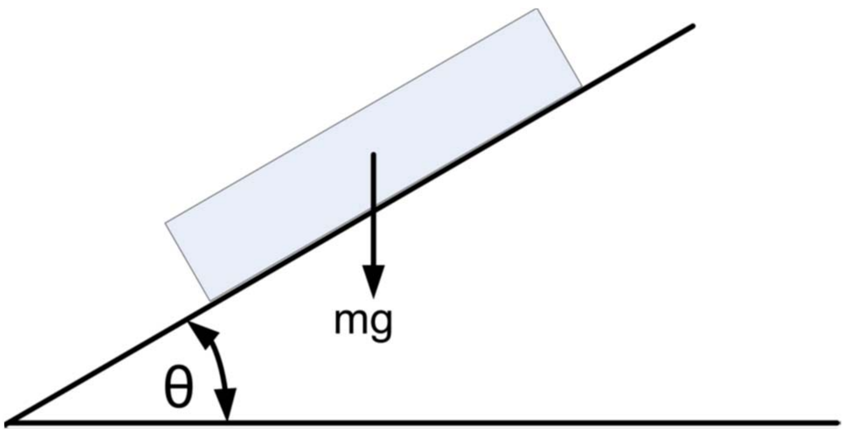
$$J_{thin\ disc,new} = \frac{MR^2}{4} + Md^2$$

Exercise – Inertia calculation



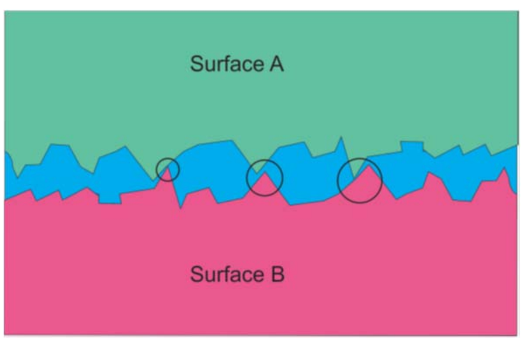
- All dimensions are in mm.
- This object has a uniform density of 5000 kg.m^{-3} and will be spun about the z-z axis, determine its moment of inertia.

Friction



- Static or dynamic friction is governed by the value of μ, where μ_s is the coefficient of static friction and μ_k is the coefficient dynamic/kinetic friction.

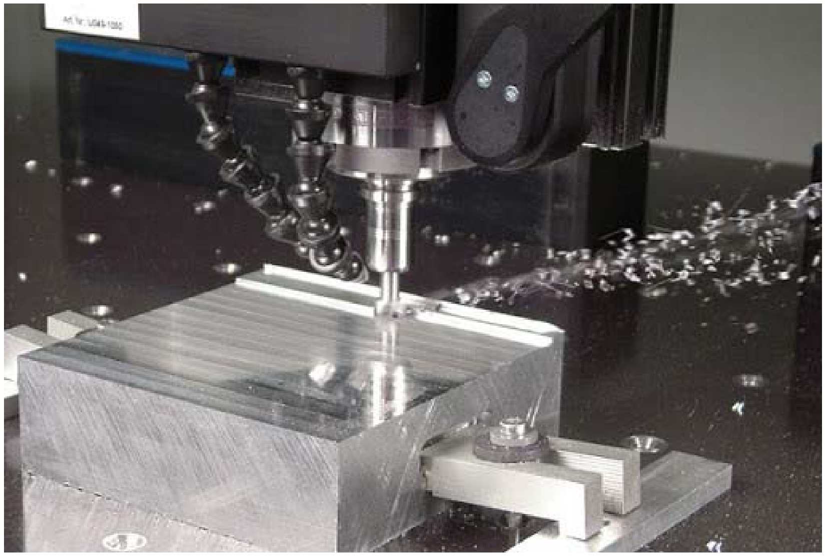
$$F_{friction} = \mu . F_{normal}$$
$$F_{friction} = \mu . mg . cos\theta$$

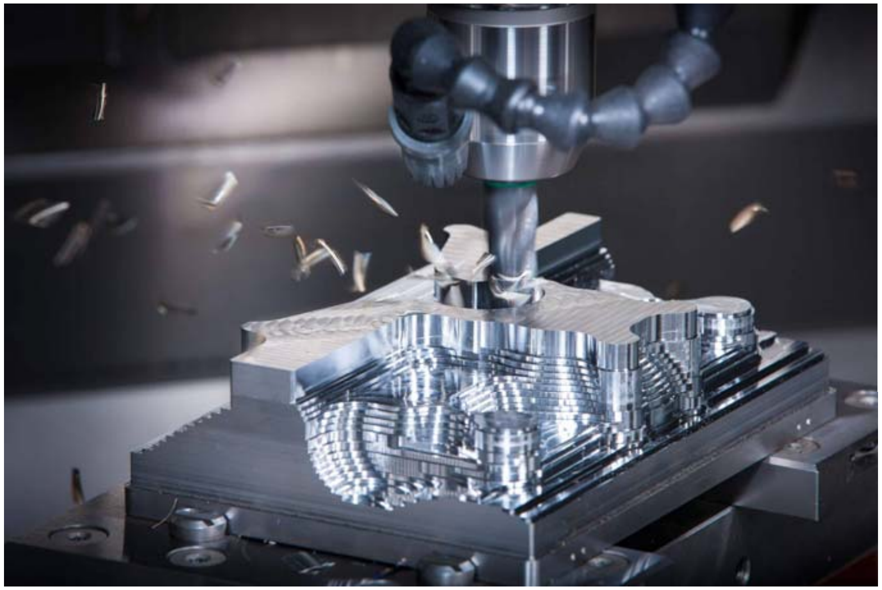


- Example values of μ
  - Steel on PTFE: 0.18
  - Steel on brass: 0.24
  - Steel on steel: 0.65

Path control

- Path/position control (on the Cartesian plane, not machine's angular position as in typical electrical drive's internal control) is required for automation.
- Three basic strategies for path control can be employed:
  - Straight cut (single axis)
  - Point-to-Point (two axes)
  - Contouring (multiple axes)

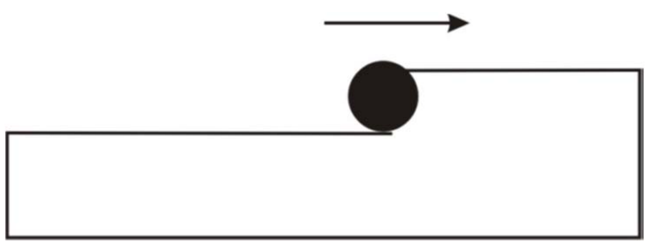


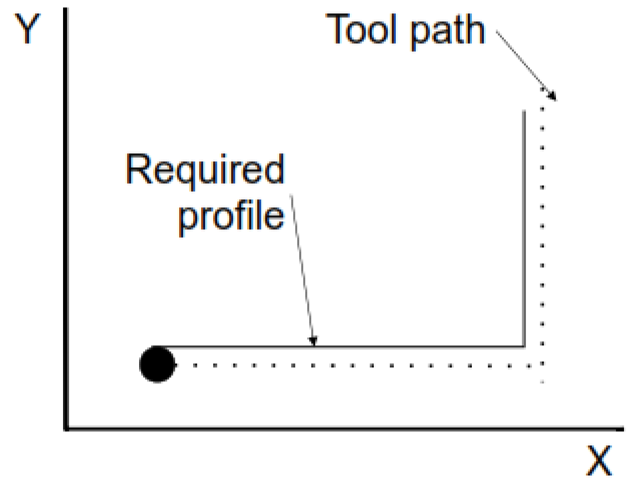


Figures are from 3erp and WMT CNC Industrial

Straight cut

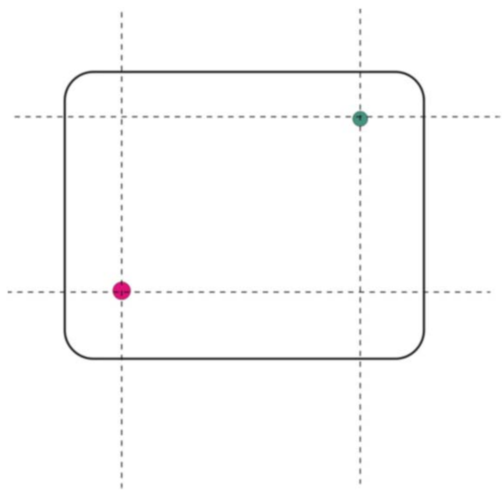
- E.g. Milling processes' feed rate.

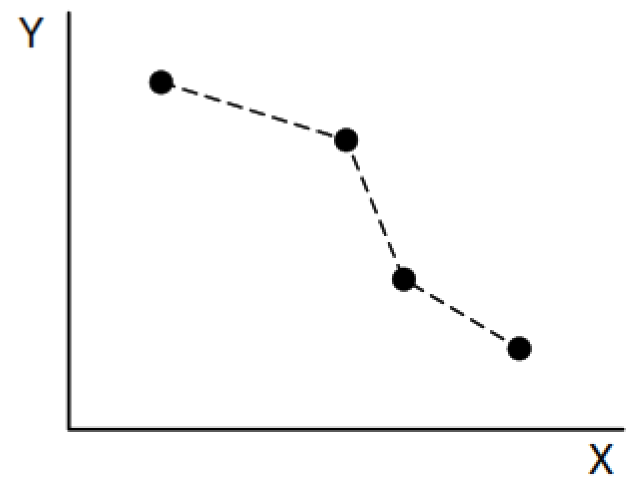




Point-to-point

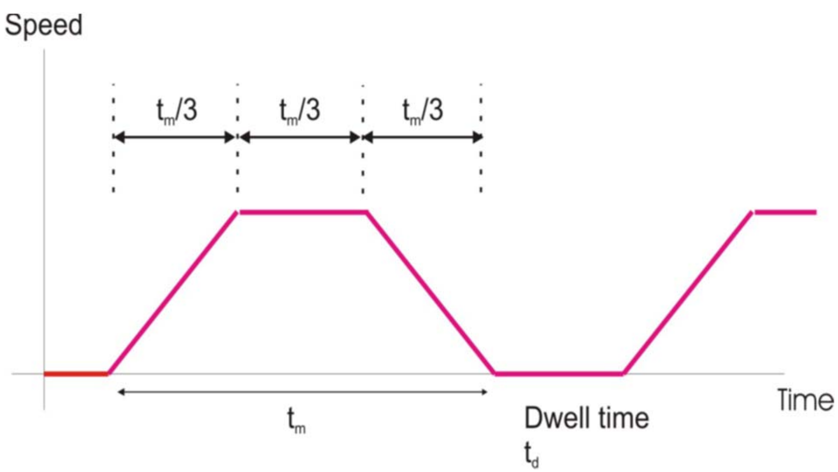
- E.g CNC machine





- Two types of movements:
  - Trapezoidal – more energy efficient (proof?)
  - Triangular
- In order to know the power requirement to the drive system, the following needs to be known:
  - Peak speed
  - Peak acceleration
  - Acceleration/deceleration duty cycle

Trapezoidal point-to-point (assumed equal intervals)



- For a rotary move of θ (angle) in the whole t_m:
- Peak speed of the movement:

$$\omega_{peak} = \frac{3\theta}{2t_m}$$

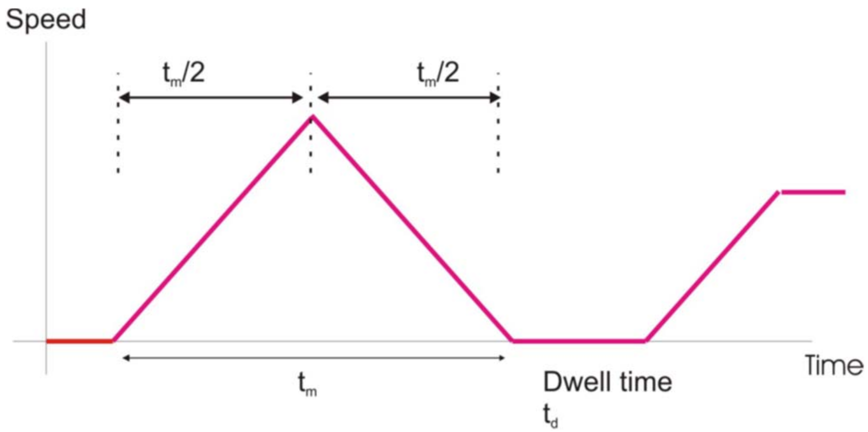
- Acceleration (during the slopes)

$$\alpha_{slope} = \frac{3\omega_{peak}}{t_m}$$

- Acceleration/deceleration duty cycle

$$d = \frac{2t_m}{3(t_m + t_d)}$$

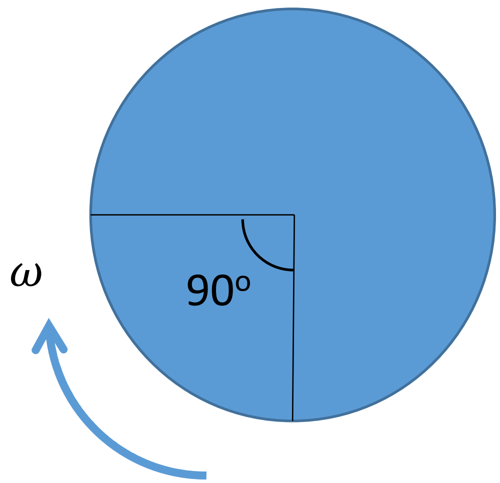
Triangular point-to-point



- For a rotary move of θ (angle) in the whole t_m:
- Peak speed of the movement:
$$\omega_{peak} = \frac{2\theta}{t_m}$$
- Acceleration (during the slopes)
$$\alpha_{slope} = \frac{2\omega_{peak}}{t_m}$$
- Acceleration/deceleration duty cycle
$$d = \frac{t_m}{t_m + t_d}$$

Example

- A disk is required to move from standstill, 90 degrees in 1 sec and then stop there, based on (a) a trapezoidal path; and (b) a triangular path. Find the peak speed and slope's acceleration.



(a) Trapezoidal path

$$\omega_{peak} = \frac{3\theta}{2t_m} = \frac{3\left(\frac{\pi}{2}\right)}{2 \times 1} = 2.365\ rad.s^{-1}$$

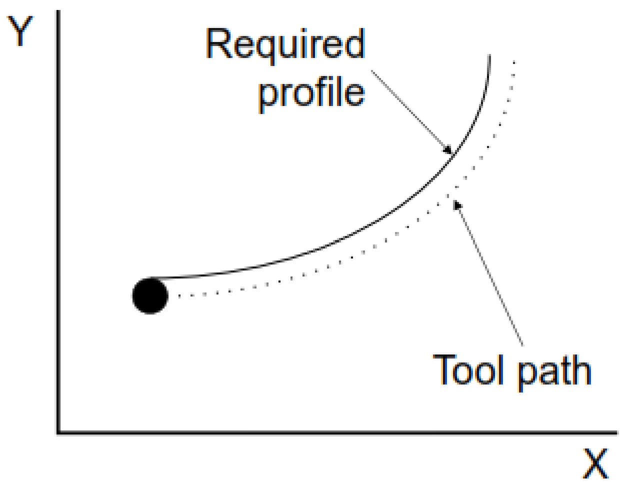
$$\alpha_{slope} = \frac{3\omega_{peak}}{t_m} = \frac{3 \times 2.36}{1} = 7.1\ rad.s^{-2}$$

(b) Triangular path

$$\omega_{peak} = \frac{3\theta}{2t_m} = \frac{2\left(\frac{\pi}{2}\right)}{1} = 3.142\ rad.s^{-1}$$

$$\alpha_{slope} = \frac{2\omega_{peak}}{t_m} = \frac{2 \times 3.142}{1} = 6.282\ rad.s^{-2}$$

Contouring



- A smooth contour can be generated by a polynomial's path.
  - Cubic; or
  - Higher order polynomial.

- E.g. a cubic path has the following form:
  $$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

- There are four unknown parameters.
  - If there are four known points (of the angular position, speed, or acceleration), then these parameters can be found.
  - Two examples of these four-points sets are:

Set 1: $\theta(0) = \theta_1;\ \theta(t_f) = \theta_2;\ \dot{\theta}(0) = 0;\dot{\theta}(t_f) = 0;$

Set 2: $\theta(0) = \theta_1;\ \theta(t_f) = \theta_2;\ \dot{\theta}(0) \neq 0;\dot{\theta}(t_f) \neq 0;$

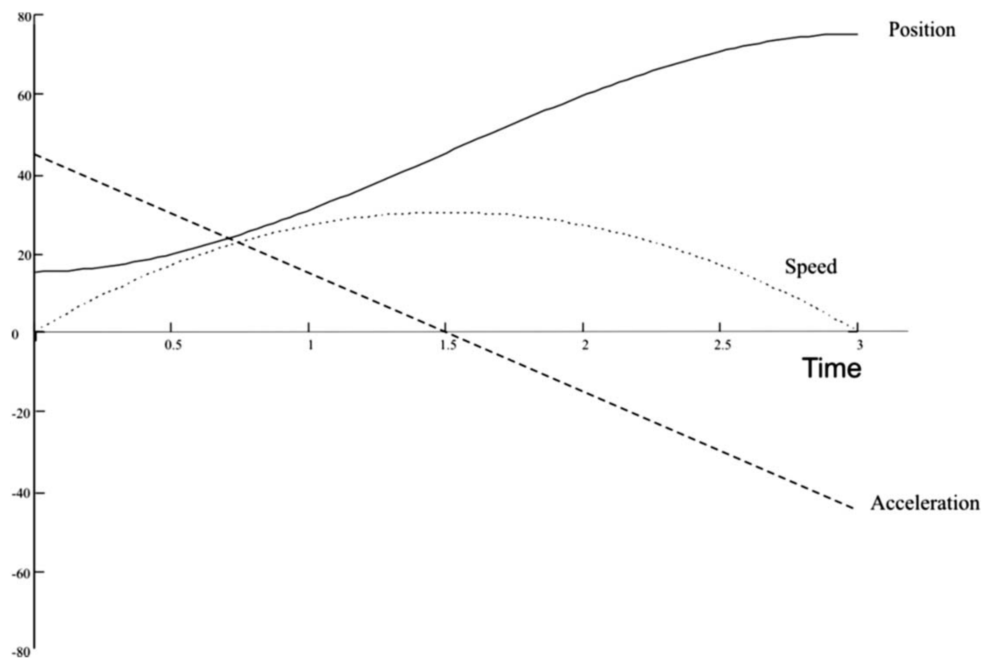
Cubic polynomial contouring

- Angular position profile:

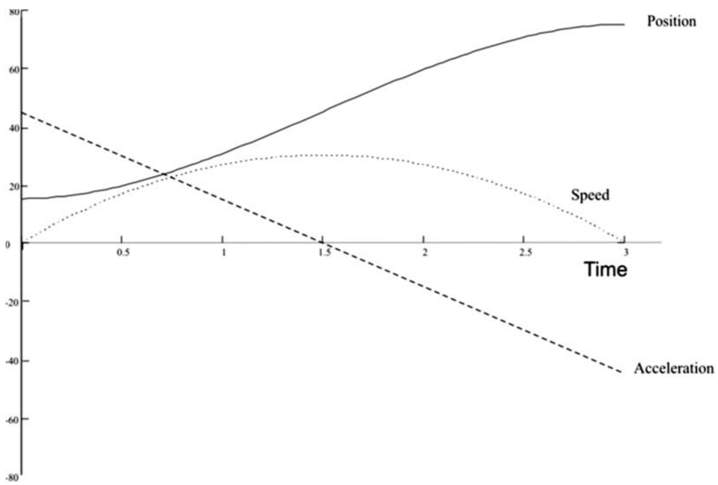
$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

- The corresponding speed and acceleration profiles can be derived as:

$$\dot{\theta}(t) = a_1 + 2a_2 t + 3a_3 t^2$$
$$\ddot{\theta}(t) = 2a_2 + 6a_3 t$$



Cubic polynomial contouring



- **Set 1:** $\theta(0) = \theta_1$; $\theta(t_f) = \theta_2$; $\dot{\theta}(0) = 0$;$\dot{\theta}(t_f) = 0$; where t_f is the time required to complete the path from θ_1 to θ_2.

- Initial angle θ_1 is known, i.e. $\theta(0) = a_0 = \theta_1$
- Final angle θ_2 is known, i.e. $\theta(t_f) = \theta_2$
- Initial and final speed: 0, i.e. $\dot{\theta}(0) = a_1 = 0$, $\dot{\theta}(t_f) = 0$

- Solve for a_2 and a_3 : in terms of the four known points (but effectively only terms θ_1 and θ_2, since speeds at $t = 0$ and t_f are zero.

$$a_2 = \frac{3}{t_f^2}(\theta_2 - \theta_1) \text{ and } a_3 = -\frac{2}{t_f^3}(\theta_2 - \theta_1)$$

Cubic polynomial contouring

- The four unknown coefficients have been found, expressed in terms of θ_1, θ_2 and final time/duration t_f:
  - $a_0 = \theta_1$
  - $a_1 = 0$
  - $a_2 = \frac{3}{t_f^2}(\theta_2 - \theta_1)$
  - $a_3 = -\frac{2}{t_f^3}(\theta_2 - \theta_1)$

- Therefore, the cubic profile is

$$\theta(t) = \theta_1 + \frac{3}{t_f^2}(\theta_2 - \theta_1)t^2 - \frac{2}{t_f^3}(\theta_2 - \theta_1)t^3$$

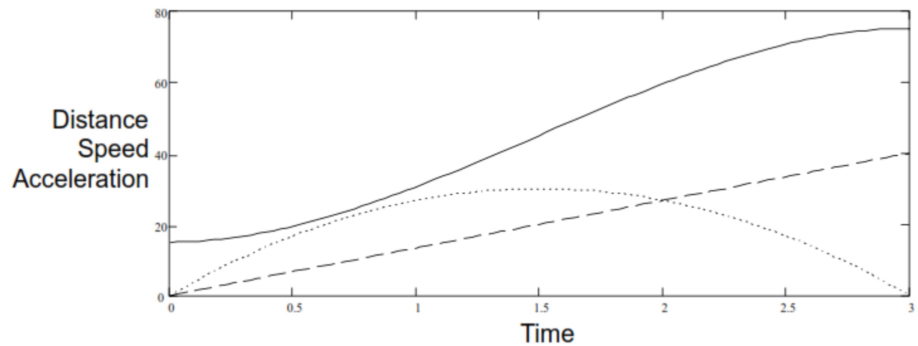
- The corresponding speed and acceleration profiles are:

$$\dot{\theta}(t) = \frac{6}{t_f^2}(\theta_2 - \theta_1)t - \frac{6}{t_f^3}(\theta_2 - \theta_1)t^2$$

$$\ddot{\theta}(t) = \frac{6}{t_f^2}(\theta_2 - \theta_1) - \frac{12}{t_f^3}(\theta_2 - \theta_1)t$$

Exercise

- If a joint is at rest at $\theta(0) = 15^o$, and is required to move to $\theta(t_f) = 75^o$ in $3s$ based on the cubic profile with the zero starting and ending speeds. Find the angular position, speed and acceleration profiles.
  - $a_0 = \theta_1 = 15.0$
  - $a_1 = 0$
  - $a_2 = \frac{3}{t_f^2}(\theta_2 - \theta_1) = 20.0$
  - $a_3 = -\frac{2}{t_f^3}(\theta_2 - \theta_1) = -4.44$

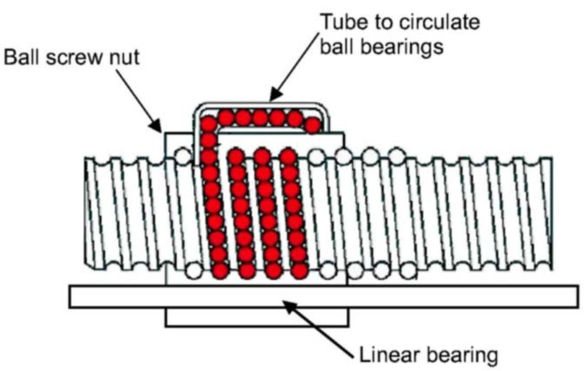


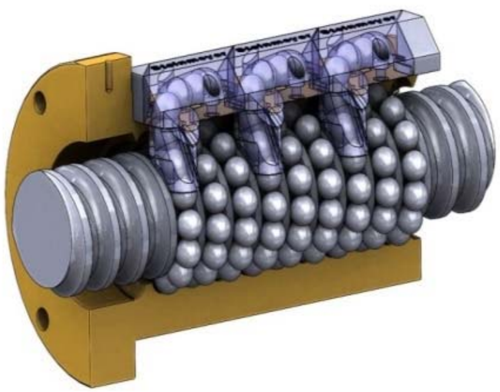
- The corresponding angular position, speed and acceleration profiles are:

$$\theta(t) = 15.0 + 20.0t^2 - 4.44t^3$$
$$\dot{\theta}(t) = 40.0t + 13.33t^2$$
$$\ddot{\theta}(t) = 40.0 - 26.66t$$

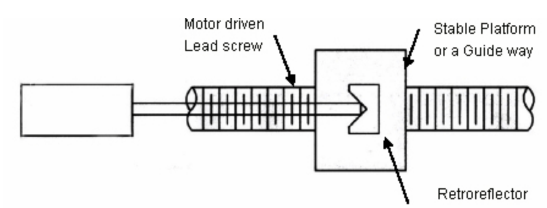
Power Transmission (linkage)
- Leadscrew and ballscrew mechanism

Power transmission: Ballscrew and leadscrew

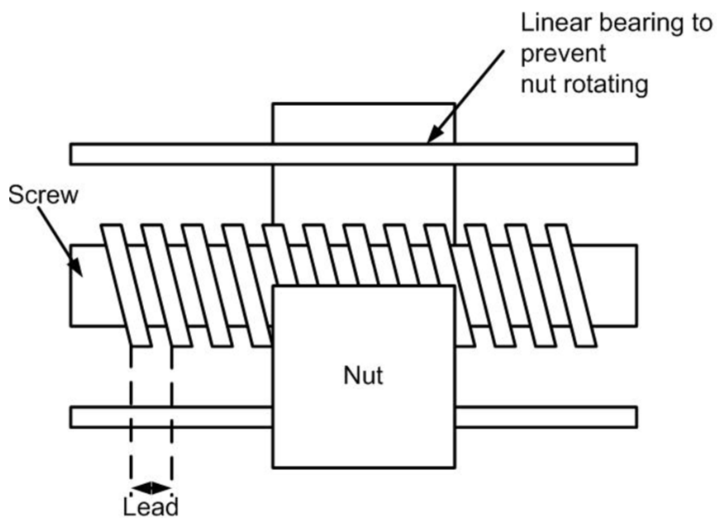


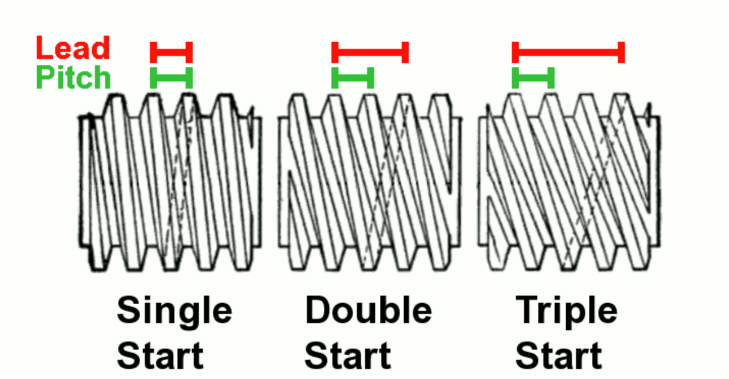


- Both convert rotary motion efficiently into linear motion.
- Ballscrews have low friction and hence good dynamic response.
- Leadscrews have direct contact between the screw and the nut and this leads to a relatively high friction (hence less efficient).



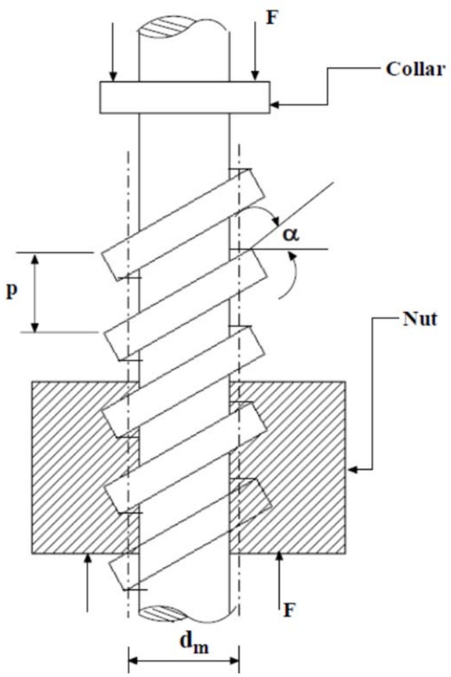
Definition of lead

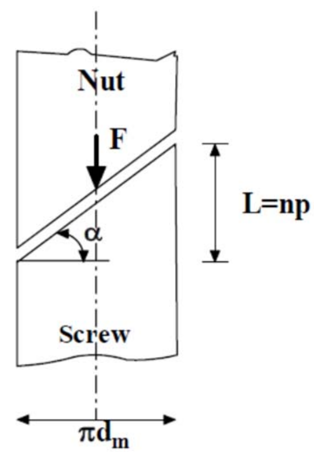


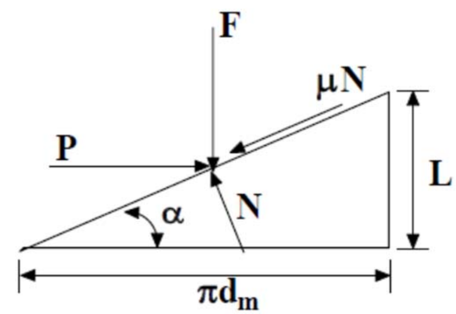


- The distance moved by one turn of the leadscrew is termed the **lead**.
- **Lead** should not be confused with the **pitch** of the screw, which is the distance between the threads.

Proof - free body diagram

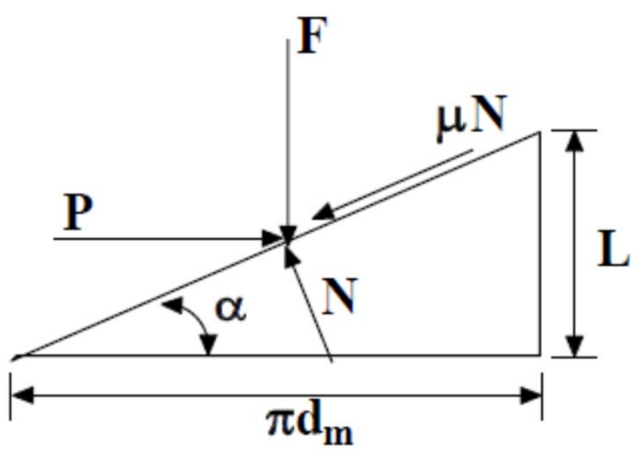






Proof - screw-raising torque

Other source, note the difference in symbols.



At equilibrium:

$$P - \mu N\cos\alpha - N\sin\alpha = 0$$
$$F + \mu N\sin\alpha - N\cos\alpha = 0$$

This gives:

$$N = \frac{F}{\cos\alpha - \mu\sin\alpha}$$
$$P = \frac{F(\mu\cos\alpha + \sin\alpha)}{\cos\alpha - \mu\sin\alpha}$$

Torque transmitted during raising of the load is given by:

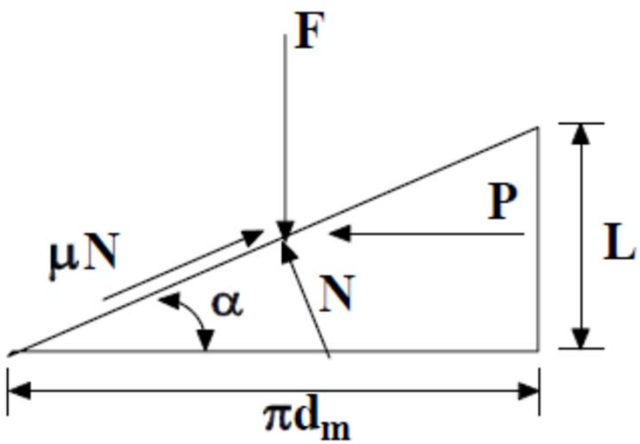
$$T_R = P\frac{d_m}{2} = \frac{d_m}{2}\frac{F(\mu\cos\alpha + \sin\alpha)}{(\cos\alpha - \mu\sin\alpha)} = \frac{d_m}{2}\frac{F(\mu + \tan\alpha)}{(1 - \mu\tan\alpha)}$$

Since $\tan\alpha = \frac{L}{\pi d_m}$, we have

$$T_R = \frac{d_m}{2}\frac{F(\mu\pi d_m + L)}{(\pi d_m - \mu L)}$$

Proof - screw-lowering torque

Other source, note the difference in symbols.



At equilibrium:

$$P - \mu N\cos\alpha + N\sin\alpha = 0$$
$$F - \mu N\sin\alpha - N\cos\alpha = 0$$

This gives:

$$N = \frac{F}{\cos\alpha + \mu\sin\alpha}$$
$$P = \frac{F(\mu\cos\alpha - \sin\alpha)}{\cos\alpha + \mu\sin\alpha}$$

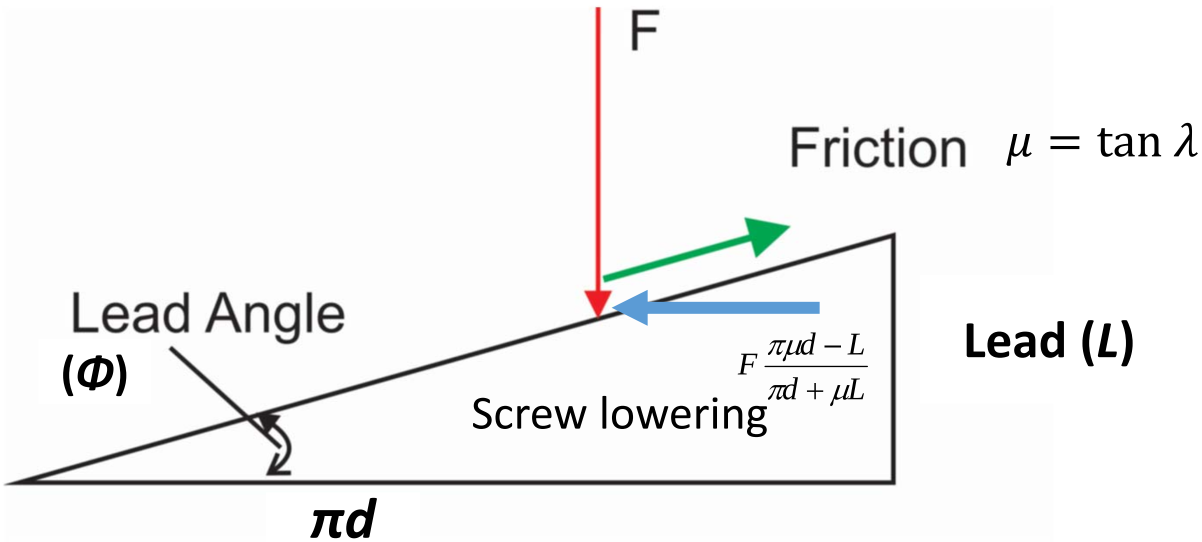
Torque transmitted during lowering of the load is given by:

$$T_R = P\frac{d_m}{2} = \frac{d_m}{2}\frac{F(\mu\cos\alpha - \sin\alpha)}{(\cos\alpha + \mu\sin\alpha)} = \frac{d_m}{2}\frac{F(\mu - \tan\alpha)}{(1 + \mu\tan\alpha)}$$

Since $\tan\alpha = \frac{L}{\pi d_m}$, we have

$$T_R = \frac{d_m}{2}\frac{F(\mu\pi d_m - L)}{(\pi d_m + \mu L)}$$

Torques with "screw" as the power transmission mechanism



If the friction is NOT ignored:

$$T_{raise} = \frac{Fd}{2}\left(\frac{\pi\mu d + L}{\pi d - \mu L}\right) = \frac{Fd}{2}tan(\lambda + \Phi)$$

$$T_{lower} = \frac{Fd}{2}\left(\frac{\pi\mu d - L}{\pi d + \mu L}\right) = \frac{Fd}{2}tan(\lambda - \Phi)$$

If the friction is ignored:

$$T_{raise} = \frac{Fd}{2}\frac{L}{\pi d} = \frac{FL}{2\pi}$$

$$T_{lower} = \frac{Fd}{2}\left(\frac{-L}{\pi d}\right) = -\frac{FL}{2\pi}$$

Displacement and speed relations

- Lead $L = \pi d \tan\phi = 2\pi r \tan\phi$

- Angular speed [*rps*]:

$$N = \frac{V_{Linear}}{L}$$

unit for V_L is ms^{-1}. RPM is $60N$.

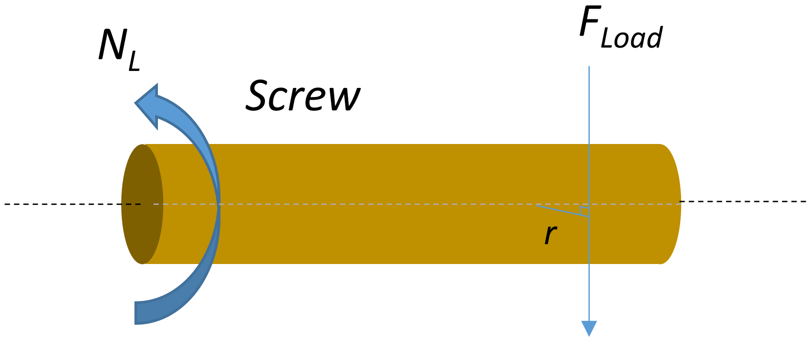
- Linear displacement step $S[m]$ to angular displacement step $\theta[rad]$:

$$S = \frac{\theta L}{2\pi}$$

- E.g. a linear displacement step (sometimes refer to as resolution) of 10^{-5}m with a ballscrew of lead L=0.01m, has a angular displacement step of 0.00628rad

- Number of steps (e.g. no. of pulses in an optical encoder) per revolution given the required linear displacement resolution $S_{resolution}$:

$$Steps\ per\ revolution = \frac{L}{S_{resolution}}$$

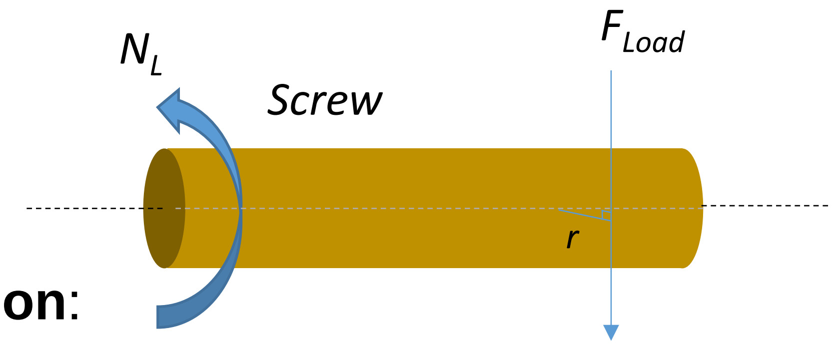


- Φ is the lead angle.

Torque and inertia relations

- Total inertia: $J_{total} = J_{screw} + J_{Load}$,

$J_{screw} = \frac{m_{screw} r^2}{2}$ and $J_{Load} = m_{Load}(\frac{L}{2\pi})^2$.



- **Ideal linear force to torque conversion**:

$$T_{Load} = \frac{LF_{Load}}{2\pi}$$

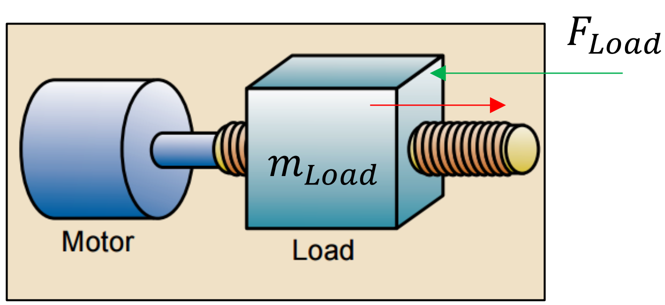
- m_{Load} is assumed as point mass on the screw peripheral.

- Effective torque required (efficiency e=0.3 to 0.9):

$$T_{eff} = \frac{T_{Load}}{e}$$

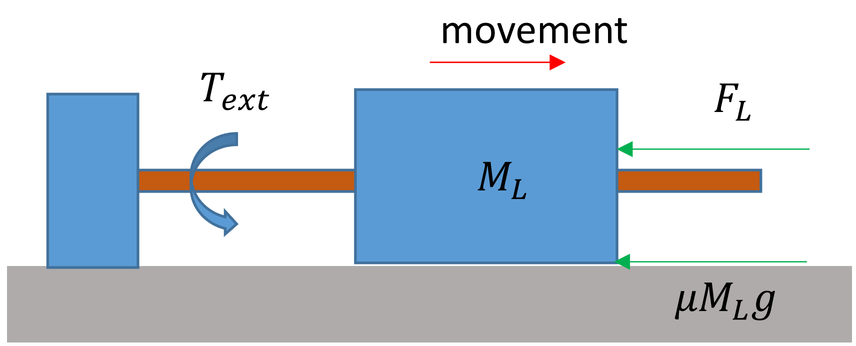
Table 3.1. Typical efficiencies for lead and ball screws

System type	Efficiency
Ball screw	0.95
Lead screw	0.90
Rolled-ball lead screw	0.80
ACME threaded lead screw	0.40



Exercise 1

- Determine the acceleration and torque requirements for the following leadscrew application:
  - The length L_s of a leadscrew is $1m$, its radius R_s is $20mm$, and it manufactured from steel ($\rho = 7850\ kgm^{-3}$).The lead L is $6mm\ rev^{-1}$. The efficiency e of the leadscrew is 0.85.
  - The total linear mass M_L to be moved is $150\ kg$. The coefficient of friction μ between the mass and its slipway is 0.5. A $50\ N$ linear force F_L is being applied to the mass.
  - The maximum speed of the load V_L has to be $6m\ min^{-1}$ and the time t the system is required to reach this speed is $1s$.



Exercise 1

Solution:

- The mass of the lead screw and its inertia are calculated first:

$$M_s = \rho\pi R_s^2 L_s = (7850)\pi(0.02)^2(1) = 9.865\ kg$$

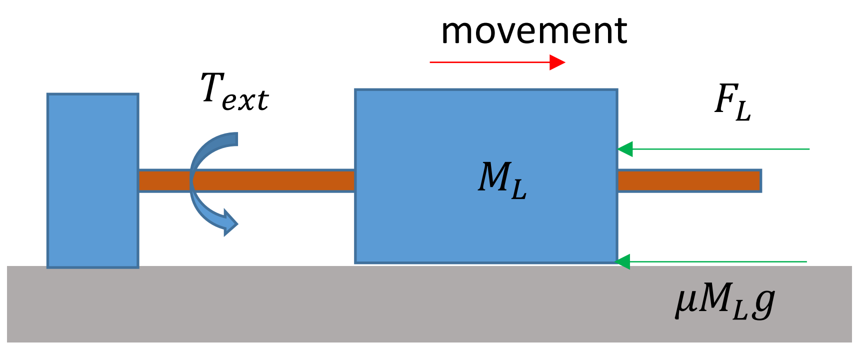
$$J_s = \frac{M_s R_s^2}{2} = 1.97 \times 10^{-3}\ kgm^2$$

- The equivalent load inertia:

$$J_L = M_L \left(\frac{L}{2\pi}\right)^2 = 1.368 \times 10^{-4}\ kgm^2$$

- The torque required to drive the load against the external and frictional forces, accounting for the efficiency of the lead screw, is given by:

$$T_{ext} = \frac{1}{e}\left(\frac{LF_L}{2\pi} + \frac{L(\mu M_L g)}{2\pi}\right) = 0.88\ Nm$$



Exercise 1

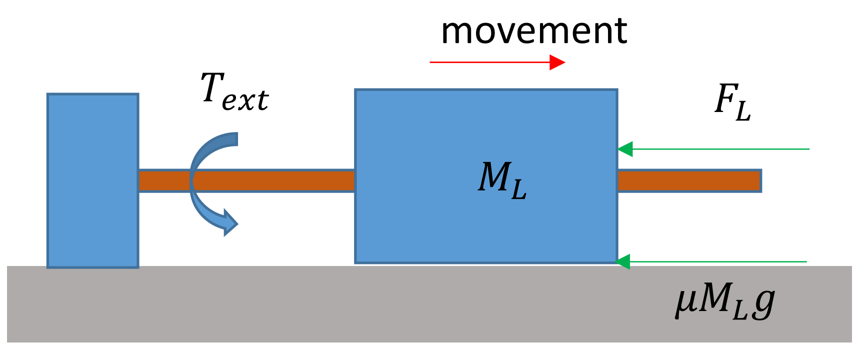
- The angular speed and the "constant" acceleration required are given by:

$$N_{L,final} = \frac{V_L}{L} = \frac{6m\ min^{-1}}{0.006} = 1000rpm$$

$$\alpha_{req} = \frac{N_{L,final} - N_{L,initial}}{t_{req}} = \frac{1000\left(\frac{2\pi}{60}\right)}{1} = 104.7\ rads^{-1}$$

- The input torque required:

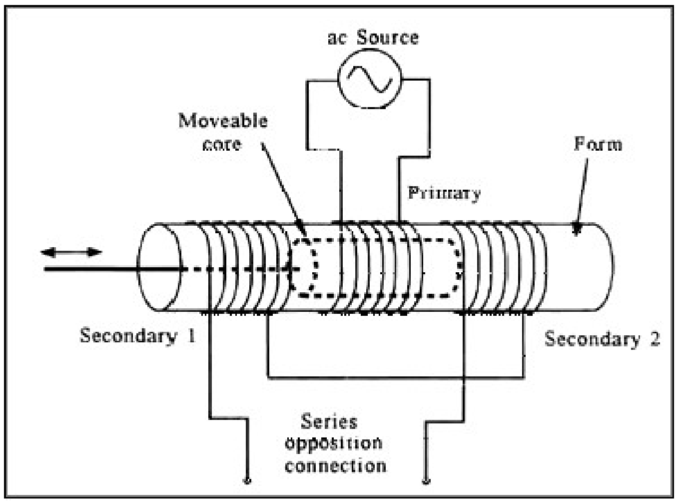
$$T_{req} = J_{screw}\alpha_{req} + \frac{J_L\alpha_{req}}{e} + T_{ext} = \left(1.97\times10^{-3} + \frac{1.368\times10^{-4}}{0.85}\right)(104.7) + 0.88 = 1.1Nm$$

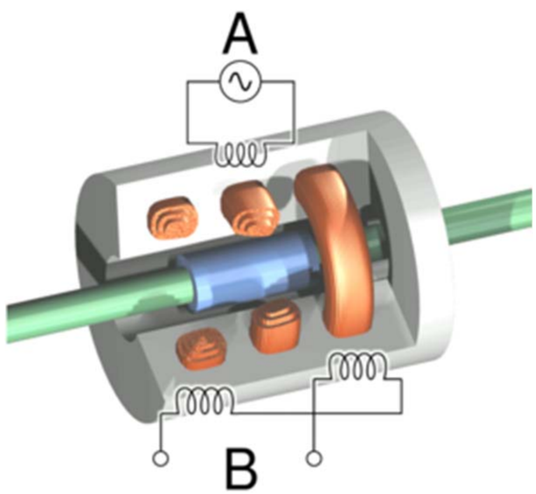


Encoders, basic power sizing, and other fundamental concepts

Types of encoders (1)

- Types:
  - Linear (rotary) variable different transformer LVDT
  - Resolver
  - Inductosyn
  - Digital encoder
- Linear (rotary) variable differential transformer LVDT – based on transformer principle with static primary and secondary but moving magnetic core
  - Advantage: robust to application environment due to absence of electronic parts
  - Disadvantage: AC source operated





Figures from National Instrument

Types of encoders (2)

- Resolvers – similar principle as LVDT but with static secondary winding and moving primary winding
  - Advantage: robust to application environment
  - Disadvantage: bulky, give “absolute” reading only over one revolution

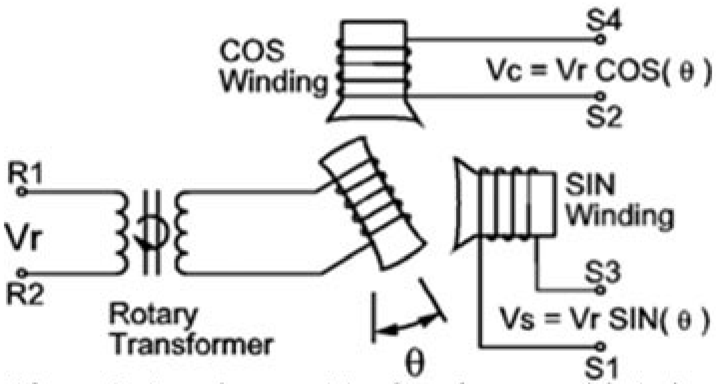
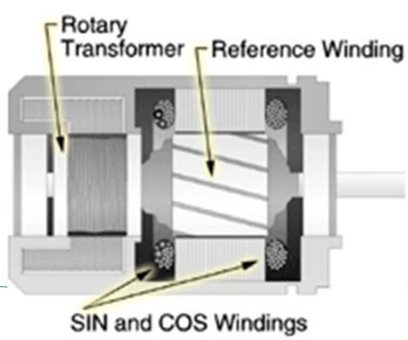
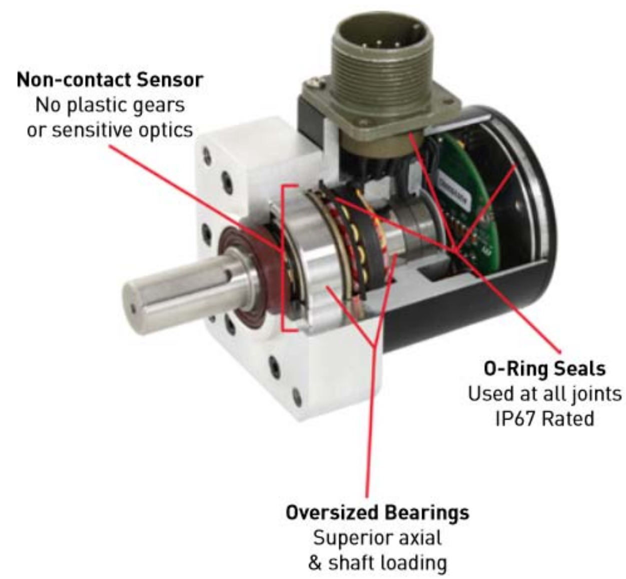


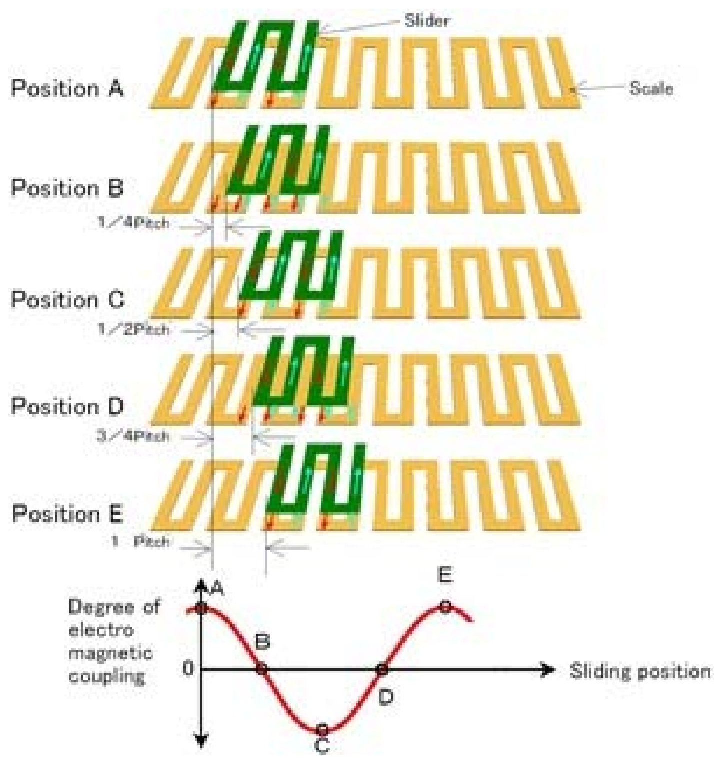
Figure 1: A resolver consists of a reference coil (rotor) and a pair of orthogonally positioned stator coils. As the energized reference coil turns, it induces voltages in the stator coils that can be processed to yield angular position.





Types of encoders (3)

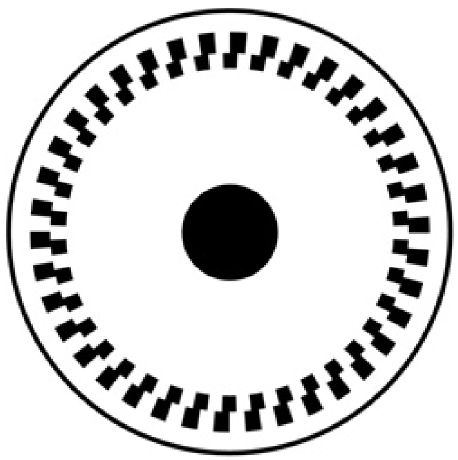
- Inductosyn – inductive sensor. A “planner” resolver with flat, disk-shaped stator and rotor.
  - Advantage: extremely robust to application environment
  - Disadvantage: give “absolute” reading only over one revolution



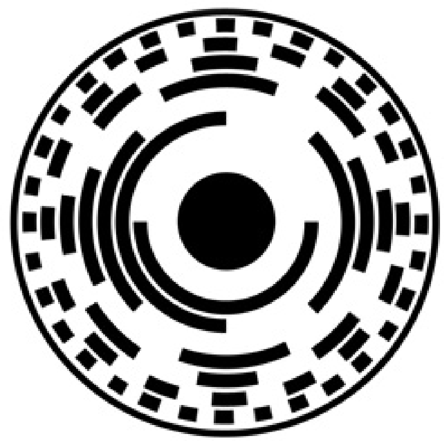
Figures from Mitsubishi and Zettlex

Types of encoders (4)

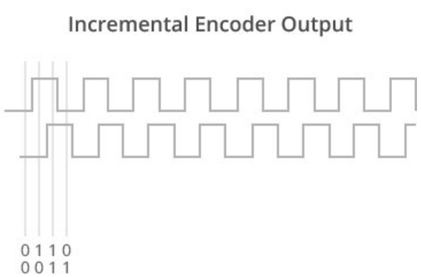
- Digital rotary encoder (incremental and absolute) – consists of three elements: optical, a light source, and a code wheel.
  - Advantage: Typically cheap
  - Disadvantage: Require delicate handling and excessive vibration and shocks may damage the signal and integrity

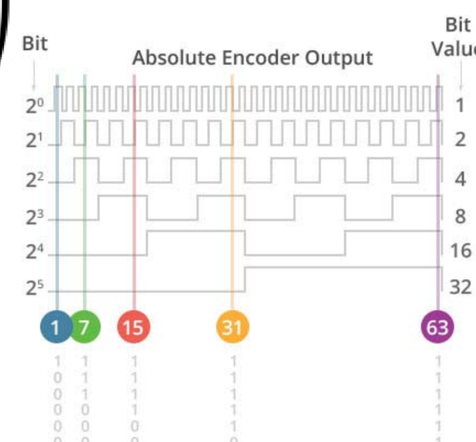


Incremental



Absolute





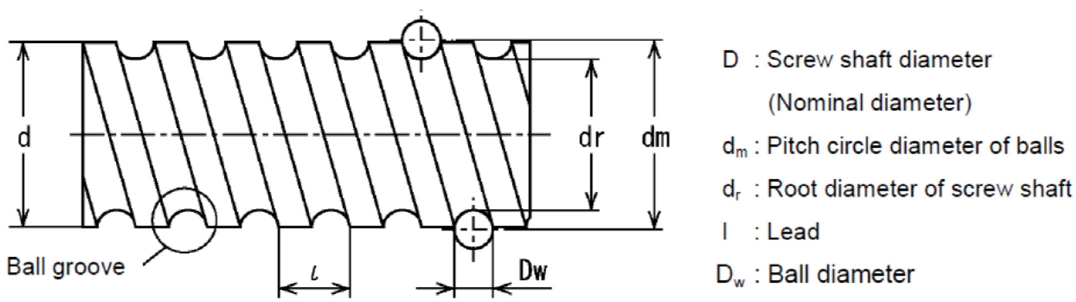
Exercise 2

- A linear positioning system has a high performance ballscrew with a lead of 0.02m (as shown in the figure), and is required to position the load to a resolution of 10^{-5}m.

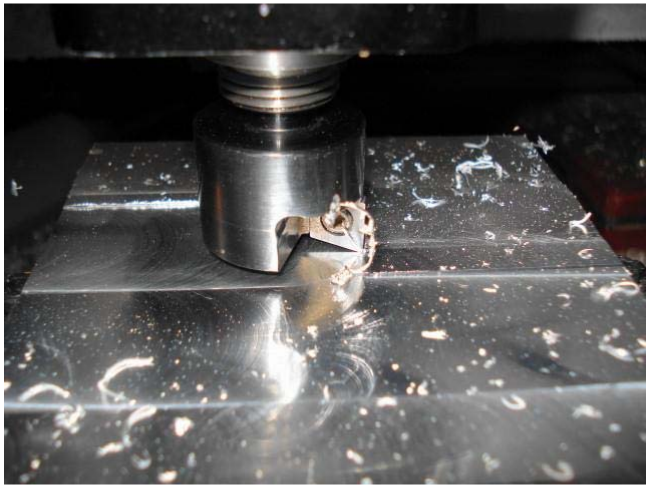
  (a) Determine the angular displacement step/resolution.

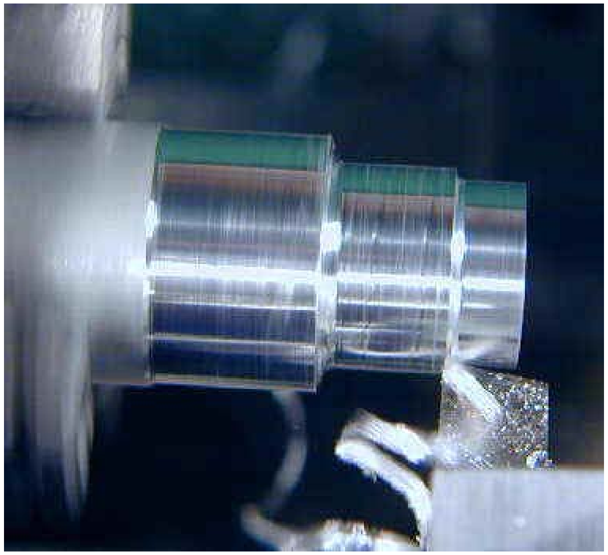
  (b) If an incremental rotary position transducer is coupled directly to the ballscrew, determine the minimum pulses per revolution (PPR) required to meet the resolution requirement?

  (c) If the maximum linear speed of the load (linear) is $10ms^{-1}$, determine the encoder's maximum speed and maximum output pulse frequency?



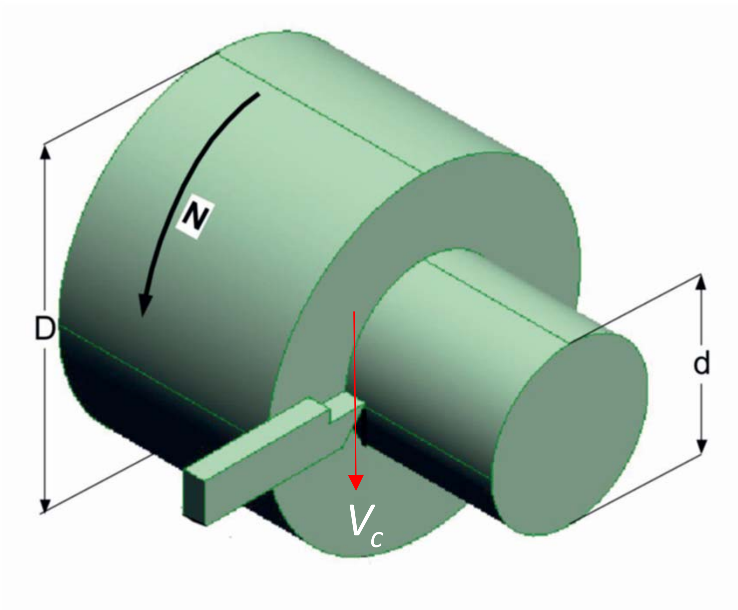
Power sizing of Turning and Milling Processes





Figures from Internet

Turning process



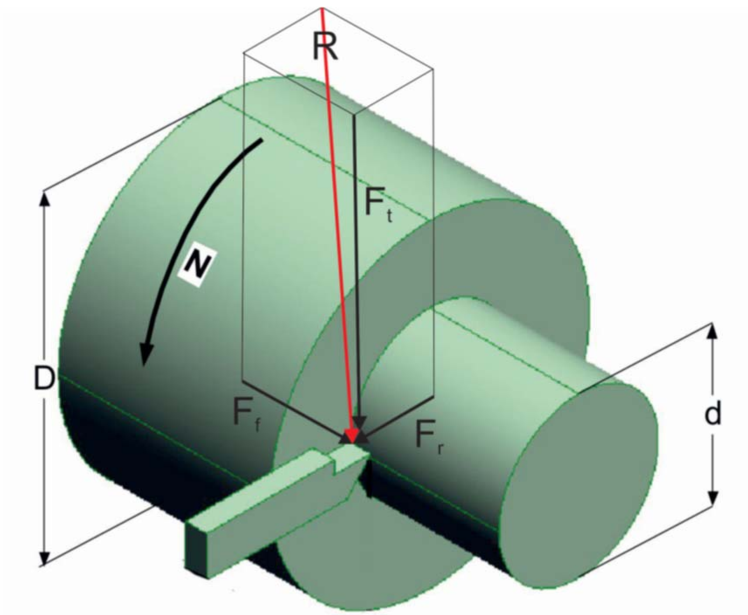
- In order to ensure that the drive system will not stall, the power and torque required by the turning operation are to be known.
- Cutting speed/machined surface speed:

$$V_c = d\pi N$$

with unit: $m.min^{-1}$ and *N* is the angular speed in rpm.

- Note: *d* can be changed to *D* for max. speed, or to the average diameter for the average speed.

Free body diagram



R - Resultant force
F_t - Tangential force
F_f - Feed force
F_r - Radial force
N – RPM

- Metal removal rate:

$$MRR = feed \text{ x } depth \text{ x } D_{avrg}\pi N$$
$$MRR = feed \text{ x } N \text{ x} \frac{\pi}{4}(D^2 - d^2)$$
$$MRR = f_{rate} \text{ x } depth \text{ x } D_{avrg}\pi$$

unit : $m^3 min^{-1}$,where feed is the linear feed per rev. $m\, rev^{-1}$, $depth = \frac{1}{2}(D - d)$, and N is angular speed, unit $rev.min^{-1}$, $D_{avrg} = \frac{1}{2}(d + D)$, $f_{rate} = feed \text{ x } N$.

- Spindle power $P_{spindle} = MRR \text{ x } U_p$

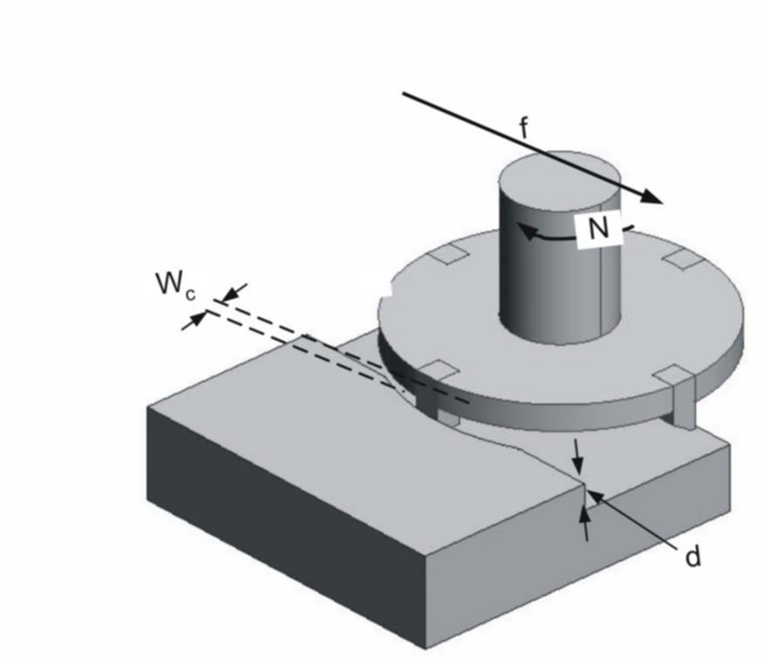
where U_p is the empirical constant (usually obtained from table. E.g. U_p for stainless steel has $4\ Ws/mm^3$ value).

- Spindle tangential force/torque requirement:

$$F_t = \frac{P_{spindle}}{V_c} \text{ or } T = \frac{60 P_{spindle}}{2\pi N}$$

- V_c is calculated based on diameter at the contact point.

Milling process



N: RPM

f_{rate} : feed rate $m.min^{-1} = feed \times N$

W_c : width of the cut

d: depth of the cut

U_p: constant defined by the material

- In the milling operation the work piece is moved relative to the cutting tool.

- Feed rate:

$$f_{rate} = feed \times N$$

- Metal removal rate:

$$MRR = f_{rate} \times depth \times width$$

where $width = W_c$ and $depth = d$ (in diagram).

- Spindle power requirement:

$$P_{spindle} = MRR \times U_p$$

- Given the spindle speed N, the spindle's torque requirement is:

$$T_{spindle} = \frac{P_{spindle}}{\omega_{spindle}} \text{ or } T_{spindle} = \frac{60Pspindle}{2\pi N}$$

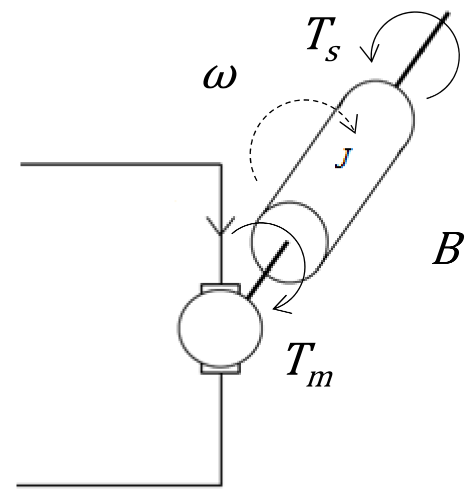
Example - Turning

- A 6m long, 1cm diameter 304 stainless steel rod is being reduced in diameter to 0.95cm in by turning on a lathe. The spindle rotates at *N*=400 rpm , and the tool is traveling at an axial speed of 0.2 m/min. Calculate, the cutting speed, material removal rate, cutting time, and power. [U_p for stainless steel has $3.5\ Ws/mm^3$]

Torque balanced equation

- Basic torque balanced equation:

$$T_m = T_s + J\dot{\omega} + B\omega$$



T_m - Torque developed by the motor, in *Nm*.

T_s - Torque required to drive the load (referred to the motor shaft), in *Nm.*

J - System's total moment of inertia, in *kgm*^{2}.

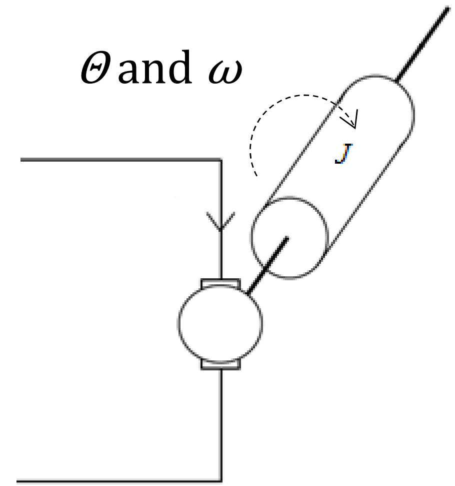
B - Damping constant, in *Nm*(*rad/s*)$^{-1}$.

ω - Angular velocity of the motor shaft, in *rads*$^{-2}$.

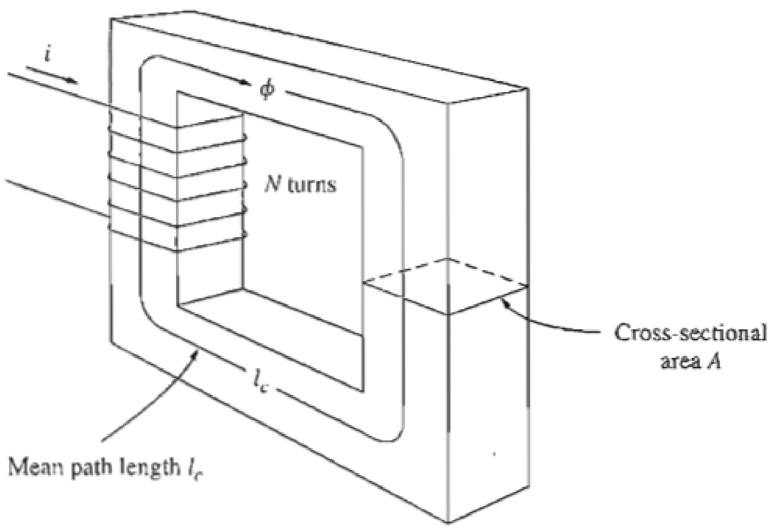
Kinetic energy change

- An acceleration or deceleration leads to a change in the kinetic energy of the rotating mass.
- The change in kinetic energy for a speed change from $\dot{\theta}_1$ from $\dot{\theta}_2$ is given by:

$$\Delta E_k = \frac{1}{2} J(\,\omega_2^2 - \omega_1^2)$$



Magnetic field and magnetomotive force



- Ampere's Law

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{net}$$

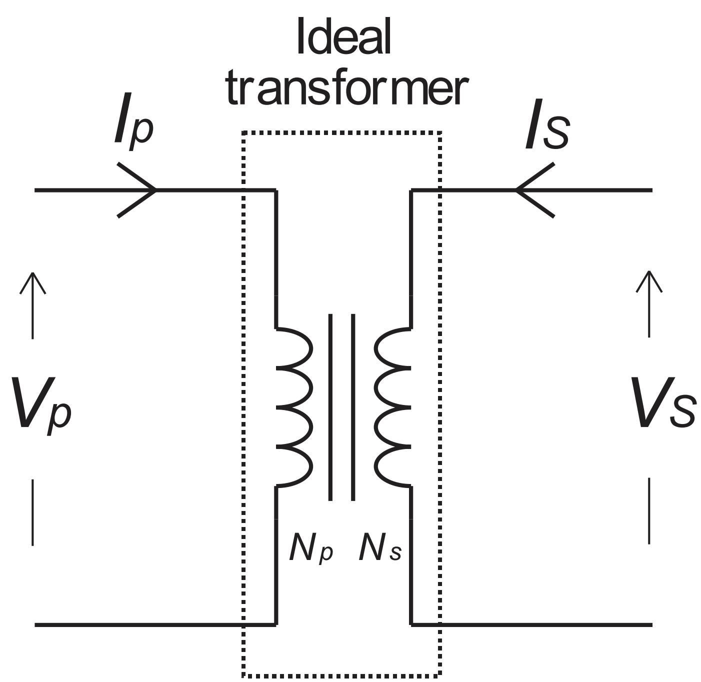
- Two quantities to characterize the "amount" of magnetic flux:

$$\mathbf{B} = \mu \mathbf{H}$$

- Magnetomotive force MMF:

$$MMF = \phi \Re = Ni$$

Ideal transformer

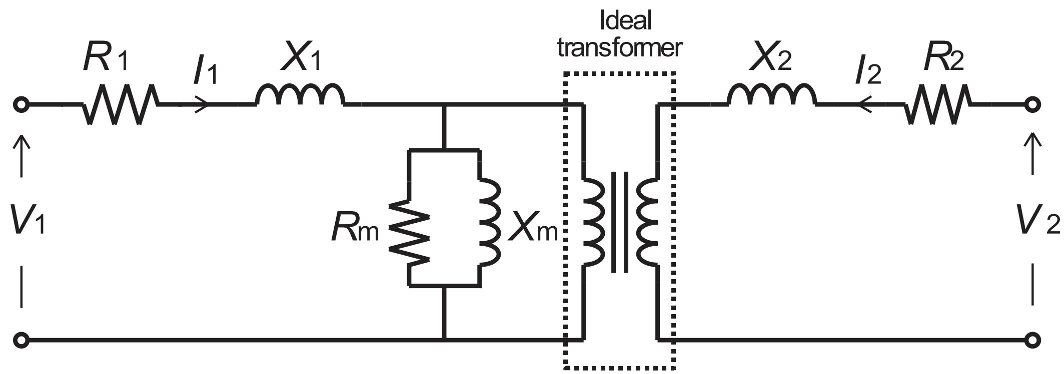


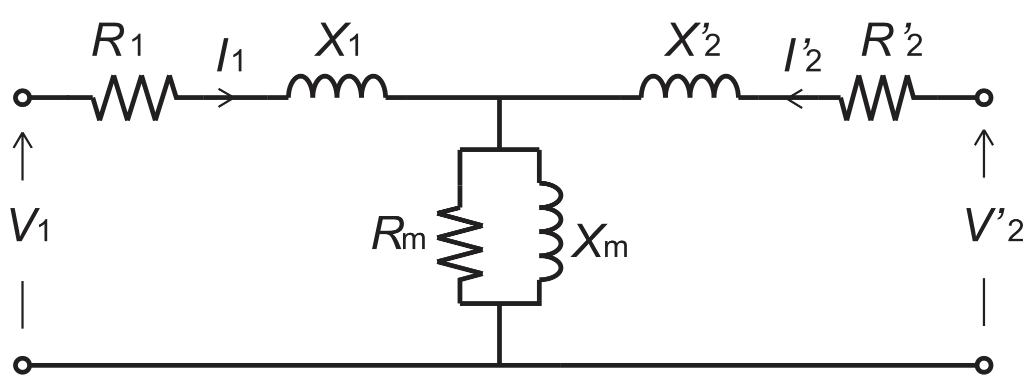
- Transformers convert AC electrical energy at one voltage level to another voltage level
- N_p turns of wire on its primary side and N_s turns of wire on its secondary side

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = a$$

$$\frac{I_p}{I_s} = \frac{1}{a}$$

A realistic representation and variable transformation





- Major imperfections are accounted for: the copper loss (by including R_1 and R_2), leakage flux (by including X_1 and X_2), the magnetizing current (by including X_m) and the core loss (by including R_m)

- To make the circuit useful for circuit analysis, the secondary circuit is "referred" to the primary circuit, entailing the following scaling of variables:

$$R'_2 = a^2 R_2 \qquad V'_2 = aV_2$$

$$X'_2 = a^2 X_2 \qquad I'_2 = \frac{I_2}{a}$$

Southampton (UNIVERSITY OF)

Exercise 3

- A single phase circuit consisting a 400V-50Hz generator supplying a load Z_{load} = (4 + j3) Ω through a transmission line of impedance Z_{line} = (0.18 + j0.24) Ω.
  - Calculate load voltage and the transmission line ohmic losses [378∠-0.9^{o}; 1.03kW];
  - If a 1:10 step-up transformer is placed at the generator end of the transmission line and a 10:1 transformer is placed at the load end of the transmission line. Calculate again the new load voltage and the transmission line ohmic lossess [≈400∠-0.03 V; 11.5W].