Mecânica Clássica

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EXEMPLO 1 – Uma mola com duas massas iguais m vinculadas a um aro de raio R

$$T = \frac{m}{2}(R\dot{\theta})^2 + \frac{m}{2}(R\dot{\varphi})^2$$

$$V = -mgR(1 - cos\theta) - mgR(1 - cos\varphi) + \frac{k}{2}[2Rsen\left(\frac{\varphi - \theta}{2}\right)]^2$$

$$\mathcal{L} = \frac{m}{2}R^2\left(\dot{\theta}^2 + \dot{\varphi}^2\right) + mgR(2 - cos\theta - cos\varphi) - 2kR^2sen^2(\frac{\varphi - \theta}{2})$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\theta}} = mR^2\dot{\theta} \qquad \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\theta}} = mR^2\ddot{\theta}$$

$$\frac{\partial\mathcal{L}}{\partial\theta} = mgRsen\theta + kR^2sen(\varphi - \theta)$$

$$mR^2\ddot{\theta} = mgRsen\theta + kR^2sen(\varphi - \theta)$$

$$\ddot{\theta} = \frac{g}{R}sen\theta + \frac{k}{m}sen(\varphi - \theta)$$

$$\ddot{\varphi} = \frac{g}{R}sen\varphi + \frac{k}{m}sen(\varphi - \theta)$$

EXEMPLO 2 – Um pendulo de conprimento 2l com uma massa m e uma articulação em l

$$V = -mgl(cos\theta + cos\varphi)$$

$$T = \frac{ml^2}{2}[\dot{\theta}^2 + \dot{\varphi}^2 + 2\dot{\theta}\dot{\varphi}\cos(\theta - \varphi)]$$

$$\mathcal{L} = \frac{ml^2}{2}\left[\dot{\theta}^2 + \dot{\varphi}^2 + 2\dot{\theta}\dot{\varphi}\cos(\theta - \varphi)\right] + mgl(cos\theta + cos\varphi)$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = ml^2[\dot{\varphi} + \dot{\theta}\cos(\theta - \varphi)]$$

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = ml^2\ddot{\varphi} + ml^2\ddot{\theta}\cos(\theta - \varphi) - ml^2\dot{\theta}sen(\theta - \varphi)(\dot{\theta} - \dot{\varphi})$$

$$\frac{\partial\mathcal{L}}{\partial\varphi} = 2\dot{\theta}\dot{\varphi}sen(\theta - \varphi)\frac{ml^2}{2} - mglsen\varphi$$