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# NOTE ON OPEN BOUNDARY CONDITIONS FOR ELLIPTIC FLOWS

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***Four different open (outflow) boundary conditions are compared and evaluated by using the incompressible Poiseuille/Benard flow in a two-dimensional rectangular duct as a test case. The conditions are: simple upwinding, linearly and quadratically weighted upwinding, and vanishing of a linearized convective derivative. The upwinding conditions generate significant reflection of outgoing waves back into the computational domain, while the convective condition presents little reflection. This last condition, which is a Sommerfeld-type radiation condition, is recommended for use at boundaries where a net outflow of fluid occurs.***

## INTRODUCTION

Although the issue of open (also referred to as outflow) boundary conditions in the simulation of incompressible fluid flows is a fundamental one, little attention has, judging from the available literature, been devoted to it until now. At an open boundary of a computational domain the fluid must be allowed to exit the domain, as well as enter it. In the latter case it is usually not possible to provide information on the nature of the fluid entering the calculation domain from the outside. The standard practice of vanishing streamwise gradients (the Neumann condition) at the open boundary is found inadequate and produces errors that propagate and alter the solution throughout the computational domain. A better approach that involves a convective estimate of the flow variables was first proposed by Dawson and Marcus in a not-much-noticed paper [1].

In this article we compare some conditions applied to a problem ideally suited for showing the upstream reflection due to open boundary conditions (OBC). The physical situation under consideration, illustrated in Fig. 1, is that of mixed convection inside a two-dimensional rectangular duct. The problem was first examined by Gage and Reid [2] through stability analysis for the cases of stable and unstable vertical stratification. The focus here is on the unstable stratification case, a combination of Poiseuille flow and Rayleigh-Benard convection, in the parameter space corresponding to an aspect ratio $A = 5$ or 10, $\mathrm{Gr} = 1.5 \times 10^4$, $\mathrm{Re} = 10$, and $\mathrm{Pr} = \frac{2}{3}$ (equations and scales are reported in Fig. 1). With these parameters the flow structure is that of steady traveling waves [2].

The problem considered was proposed by Philip M. Gresho and Robert L. Sani as a test case for an Outflow Boundary Condition Minisymposium that took place in Swansea, UK, 10 July 1989, and it is there that this material was first presented. The author benefited greatly from discussions with the participants to the minisymposium and with Inge Ryhming, Gino Moretti, Spyros Gavrilakis, Michael R. von Spakovsky, and Jan Vos. The computations have been carried out on the Convex C2 of the Paul Sherrer Institute, Villigen, Switzerland.

## NOMENCLATURE

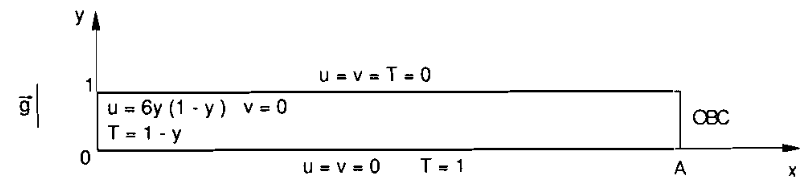
| | | | |
|---|---|---|---|
| $A$ | aspect ratio ($= L/H$) | $\alpha$ | thermal diffusivity |
| $\mathbf{c}$ | phase velocity ($= [c, 0]$) | $\beta$ | thermal expansion coefficient |
| $g$ | gravitational acceleration | $\lambda$ | wavelength |
| Gr | Grashof number ($= g\beta(T_H - T_C)H^3/\nu^2$) | $\nu$ | kinematic viscosity |
| $i, j$ | Cartesian indices | $\tau$ | period of the oscillations |
| $IL$ | last grid point index along $x$ | $\Phi$ | generic physical variable |
| $L$ | length of the duct | | |
| $H$ | height of the duct | **Subscripts** | |
| $p$ | pressure | | |
| Pr | Prandtl number ($= \nu/\alpha$) | $C$ | cold |
| Re | Reynolds number ($= UH/\nu$) | $H$ | hot |
| $t$ | time | max | maximum value |
| $T$ | temperature | $o$ | corresponding to a minimum in the time signal of temperature at point (2.5, 0.5) |
| $\mathbf{u}$ | Cartesian velocity vector $[=(u, v)]$ | $t, x, y$ | derivative with respect to $t, x, y$ |
| $\mathbf{U}$ | average velocity vector $[= (U, V)]$ | | |
| $U$ | average horizontal velocity component ($= \int_0^1 u\, dy$) | **Superscript** | |
| $V$ | average vertical velocity component ($= \int_0^1 v\, dy$) | | |
| $x, y$ | Cartesian coordinates | $n$ | time index |

## NUMERICAL TECHNIQUE AND OBCs ADOPTED

The two-dimensional Navier-Stokes/Boussinesq equations have been solved by a finite volume method that employs primitive variables on a staggered mesh. Diffusive and convective fluxes are discretized by central differencing, while the pressure coupling is dealt with through the SIMPLER algorithm [3]. Solutions are obtained by fully implicit marching in time. With a time step of $10^{-2}$, we typically perform four internal iterations to make the residuals acceptably low. The grid employed is uniform and consists of 21 nodes along $y$ and 80 along $x$ (on the longer domain), resulting in 1482 internal control volumes and 198 volumes of zero thickness at the four boundaries (this procedure is useful for easily enforcing boundary conditions). At the open boundary, where a net outflow of fluid occurs, the following conditions were tested:

OBC1, simple upwinding
OBC2, linearly weighted upwinding
OBC3, quadratically weighted upwinding
OBC4, the vanishing of linearized convective derivatives

It should be noted here that no boundary conditions are needed for the pressure and pressure correction equations, because the boundaries of the domain coincide with the control volume faces where the velocity components (or functions of them) are to be specified. The pressure is staggered with respect to the velocity components and is computed in the interior (up to a constant) from the velocity field [3]. Sometimes pressure boundary conditions are necessary (e.g., when solving a Poisson equation for pressure on a grid that is nonstaggered), and, in these cases, Neumann boundary conditions are appropriate at Dirichlet-type boundaries for the velocity (such as no-slip boundaries) [4]. At the open boundary, the question of a pressure condition is unresolved. A hydrostatic




$$u_x + v_y = 0$$

$$u_t + u\,u_x + v\,u_y = -p_x + \frac{1}{\mathrm{Re}}\,(u_{xx} + u_{yy})$$

$$v_t + u\,v_x + v\,v_y = -p_y + \frac{1}{\mathrm{Re}}\,(v_{xx} + v_{yy}) + \frac{\mathrm{Gr}}{\mathrm{Re}^2}\,T$$

$$T_t + u\,T_x + v\,T_y = \frac{1}{\mathrm{Re\ Pr}}\,(T_{xx} + T_{yy})$$

Scales:

| | |
|---|---|
| time | $H U^{-1}$ |
| length | $H$ |
| pressure | $\rho U^2$ |
| temperature | $T_H - T_C$ |
| velocity | $U$ |

**Fig. 1** Sketch of the problem considered, governing equations in dimensionless form, and scales.

pressure boundary condition, recently proposed by Gresho (communicated at the OBC Minisymposium in Wales), awaits further testing.

The term upwinding in the definitions of OBC1, 2, and 3 refers to the net flow direction. A weighted extrapolation means that after doing a two- or three-point extrapolation to the open boundary (corresponding to linear or quadratic upwinding), the velocity components are multiplied by a constant factor such that global mass balance is satisfied. The weighting is, as is verified numerically, necessary for both two- and three-point extrapolations to prevent a rapid divergence of the iterative procedure due to not satisfying the equation for the conservation of mass. In the simple upwinding case, mass is automatically satisfied at each station in the streamwise direction because of incompressibility so that a weighting factor is superfluous.

The OBC4 results from a desire to impose a condition that varies with time. It assumes that the fluid is steadily transported at a speed that is the average speed $\mathbf{U} = (U, V)$ in the duct. This speed is the free stream velocity for external flows. Thus, the momentum and energy equations are

$$(u - U)u_x + (v - V)u_y + p_x = \frac{1}{\mathrm{Re}}\,(u_{xx} + u_{yy}) \tag{1}$$

$$(u - U)v_x + (v - V)v_y + p_y = \frac{\mathrm{Gr}}{\mathrm{Re}^2}\,T + \frac{1}{\mathrm{Re}}\,(v_{xx} + v_{yy}) \tag{2}$$

$$(u - U)T_x + (v - V)T_y = \frac{1}{\text{Re Pr}}(T_{xx} + T_{yy}) \tag{3}$$

Subtracting Eqs. (1)–(3) from the Navier-Stokes/Boussinesq equations, it is found that the linearized convective derivatives of the unknowns vanish. These can then be used as boundary conditions at the open boundary and correspond to expressing the advection of the frozen vector field, that is,

$$u_t + Uu_x + Vu_y = 0 \tag{4}$$

$$v_t + Uv_x + Vv_y = 0 \tag{5}$$

$$T_t + UT_x + VT_y = 0 \tag{6}$$

It was pointed out by Lugt and Haussling [5] that using the local velocity $\mathbf{u}$ in Eqs. (4)–(6) instead of the average velocity $\mathbf{U}$ is equivalent to neglecting the pressure gradients, the diffusive term, and the buoyancy term, which in general is unacceptable. Even so, Nataf [6] was able to produce results with little reflection for the case of flow past an ellipse at a zero angle of attack, by employing the local speed at the open boundary. Equations (4)–(6) are also called the Sommerfeld radiation conditions when an appropriate phase velocity $\mathbf{c}$ is used instead of $\mathbf{U}$. Orlanski [7] evaluated the time derivatives to second order and the space derivatives to first order (over the last three time levels) and estimated the phase velocity $\mathbf{c}$ by the use of the Sommerfeld conditions at the grid point, in the streamwise direction, just prior to the boundary. Hence, he obtained a phase velocity $\mathbf{c}$ in the vicinity of the boundary, which he then inserted into the Sommerfeld condition evaluated at the open boundary. Such a boundary condition produces no reflection for single wave propagation. The Sommerfeld condition has been proposed for unbounded hyperbolic problems but is also thought to be acceptable for elliptic and parabolic cases [7]. The use of $\mathbf{U}$ in Eqs. (4)–(6) in place of some estimate of $\mathbf{c}$ or $\mathbf{u}$ simplifies the formulation and is acceptable in that there are no correct open boundary conditions for bounded elliptic problems. The convective boundary conditions (4)–(6) have also been used successfully in the numerical simulation of the transition to turbulence [8].

For the present case, we have $V = 0$, while $U$ is the average streamwise speed in the duct. Evaluating Eqs. (4)–(6) numerically, explicitly, to first order in time and space (on the generic variable $\Phi$) results in

$$\frac{\Phi^{n+1}(IL, j) - \Phi^n(IL, j)}{t^{n+1} - t^n} + U\frac{\Phi^n(IL, j) - \Phi^n(IL - 1, j)}{x(IL) - x(IL - 1)} = 0 \tag{7}$$

which leads to

$$\Phi^{n+1}(IL, j) = \Phi^n(IL, j) - U\frac{t^{n+1} - t^n}{x(IL) - x(IL - 1)}[\Phi^n(IL, j) - \Phi^n(IL - 1, j)] \tag{8}$$

Notice that in Eq. (8) the outflow value at the time level $n + 1$ is based only on information coming from the previous time level.

## RESULTS

### OBC1

In numerical terms, OBC1 is equivalent to setting $\Phi^{n+1}(IL, j) = \Phi^{n+1}(IL - 1, j)$, thus expressing the vanishing of the streamwise derivative to first order. In Figs. 2 and 3 we have reported the solutions to the test problem chosen ($A = 10$) in terms of (from top to bottom) velocity vectors, streamlines, isovorticity lines, isotherms, and isobars at a fixed time $t_0$ that corresponds to a minimum in the periodic time signal of temperature at $x = 2.5$ and $y = 0.5$ and half a period later. It should be pointed out here that the results presented correspond to an established state, characterized by well-defined sinusoidal oscillations in time. This implies that the solution is free from disturbances once the initial transient state (perturbed by reflection) is finished. In fact, these disturbances would result in erratic or aperiodic signals.

The traveling wave solution predicted by the analysis of Gage and Reid [2] is clearly visible in these figures. About six roll pairs are found in the domain at each time, and they are carried downstream because of forced convection. At the open boundary, the fluid tends to leave (and enter) with a direction parallel to the side walls ($v$ at the open boundary is zero for all time), which is a phenomenon induced by the fact that setting $u_x = v_x = 0$ results in $v_y = 0$ (continuity), so that $v = 0$ at the open boundary. The dimensionless vertical velocities at $x = 2.5$, 5, and 7.5, at a constant $y = 0.5$, oscillate between $-4.9$ and $+4.9$ with a period $\tau$ of about 1.22. The horizontal velocity components at the same positions have a sinusoidal signal of period 0.5 $\tau$ and are between the values of 1.5 and 1.7, while at $x = 10$ the oscillations around an average value of 0.3 have an amplitude of about 0.6 (three times larger than in the interior points). The nondimensional temperature signals at the same four points vary in time between the values of 0.3 and 0.7 with period $\tau$. For OBC1 the wavelength increases as the fluid approaches the exit boundary. From Figs. 2 and 3 it is observed that the wavelength $\lambda$ changes from 1.29 at half domain to 1.45 close to the exit.

The results are comparable to those obtained by Evans and Paolucci [9] with the same OBC. They solved the problem by a finite volume technique on a domain with $A = 20$ and by using a finer staggered mesh of $40 \times 400$ nodes. They reported $\lambda = 1.512$, $\tau = 1.322$, and maximum vertical velocity component 5.03 at $y = 0.5$. Our results, although obtained on a coarser mesh, do not seem to suffer from insufficient resolution since the isovorticity lines, a good indicator of accuracy, do not show any wiggles. Lines at constant vorticity deviate from the quasi-periodic pattern as the exit is approached. The isotherms show periodic plumes rising from the hot wall and being transported away by forced convection, while the isobars reflect the wavy nature of the situation considered. Also these two scalar fields are affected by the OBC used close to the duct exit.

### OBC2

The two-point extrapolation procedure produces results qualitatively similar to the one-point extrapolation technique on the same domain length. The most important findings in this case can be summarized as follows: The $u$ velocity at the exit has an amplitude of oscillation about fourteen times larger than in the interior, while $v$, although nonzero,

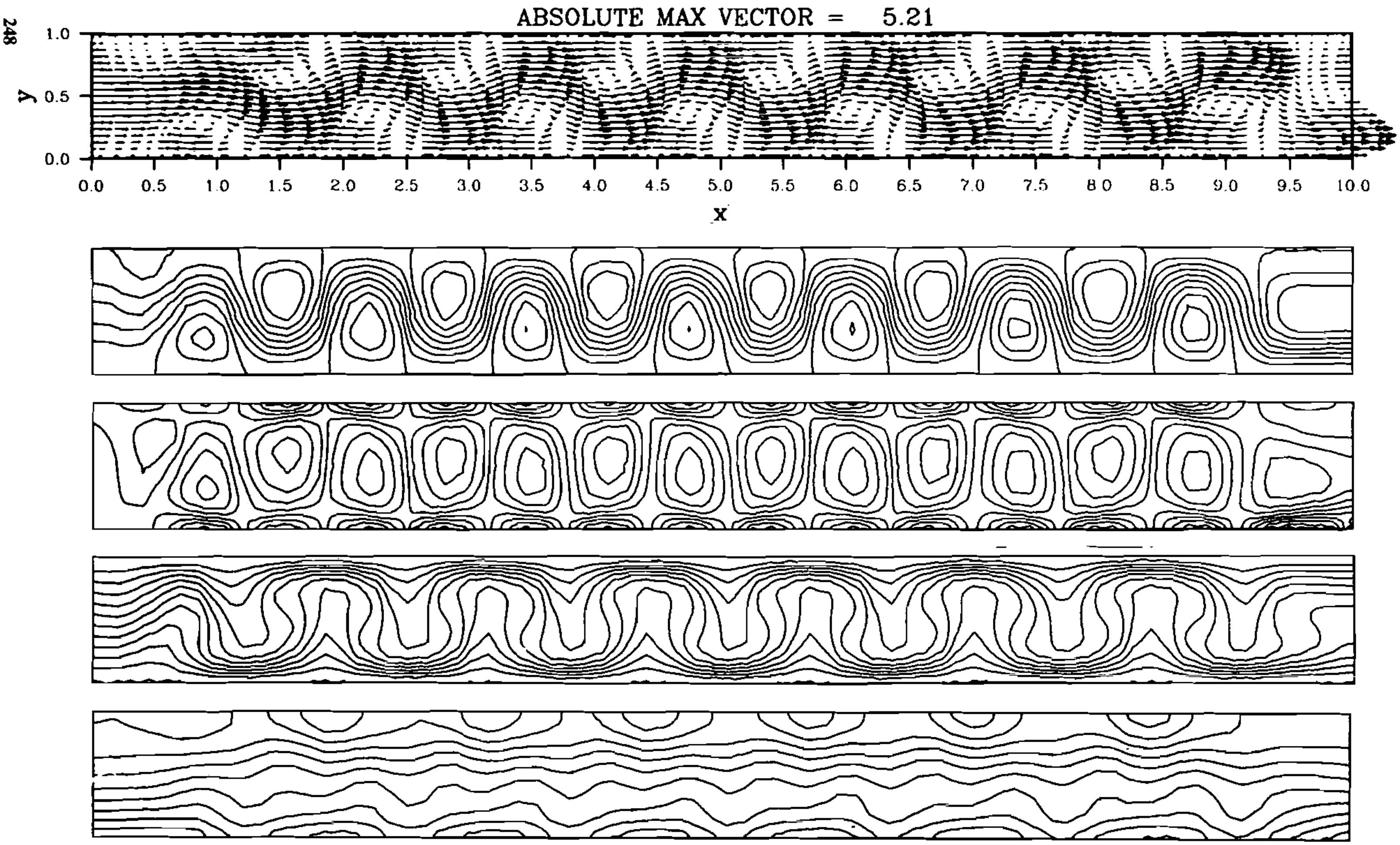



**Fig. 2** From the top: velocity field, streamlines, isovorticity lines, isotherms, and isobars at time $t_0$ obtained by adopting OBC1.

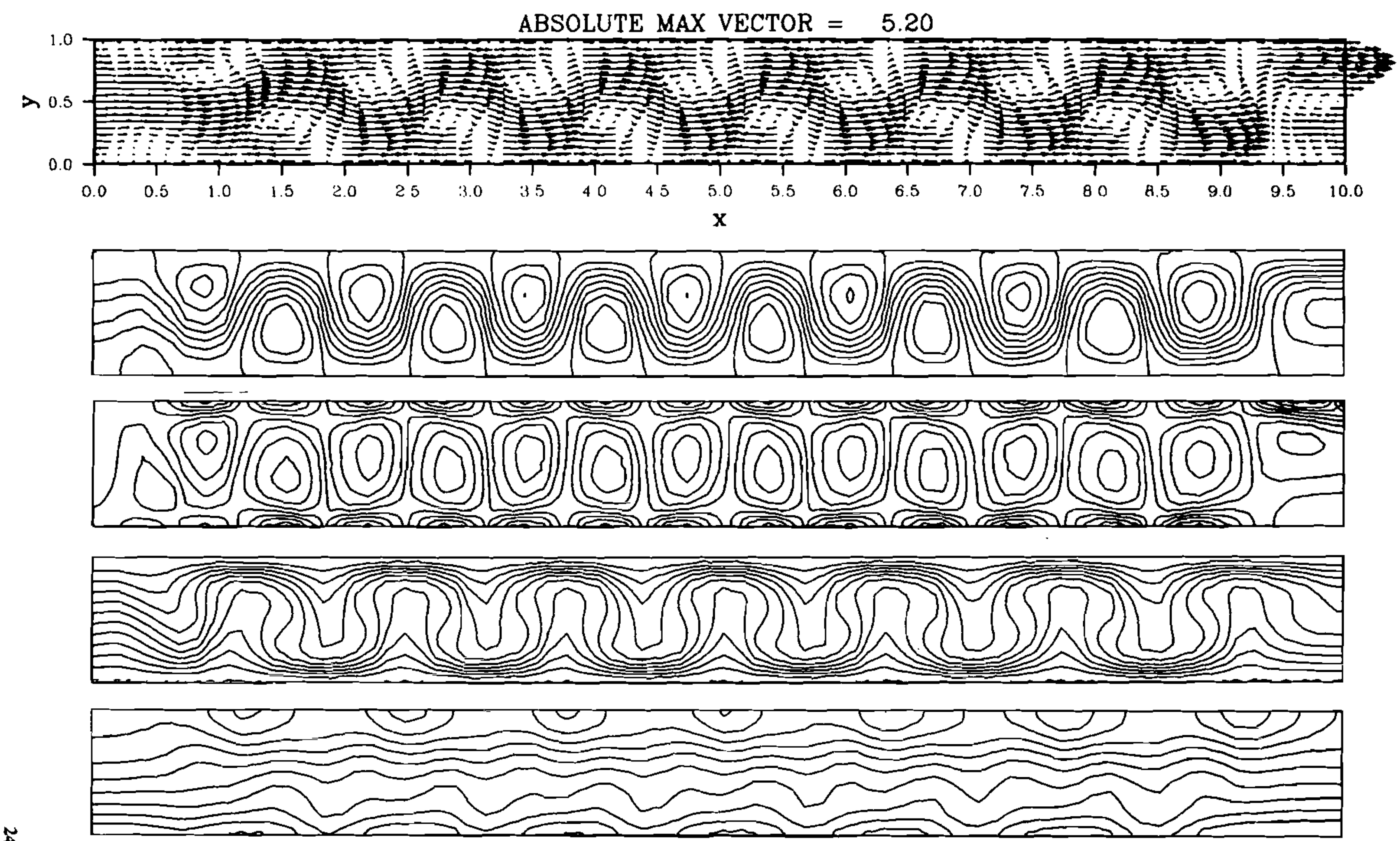



Fig. 3 Same as Fig. 2, half a cycle later.

has a lower amplitude than in the interior points by a factor of 1.25 and oscillates around the zero value. The temperature at the exit has twice the amplitude of oscillation than in the interior, a phenomenon presumably caused by the numerical distortion of the exit velocity field.

## OBC3

The quadratically weighted upwinding technique was tested on a domain of aspect ratio $A = 5$ and with a mesh of $21 \times 40$ node points. The different fields at time $t_o$ are pictured in Fig. 4. In the first place, the maximum velocity vector in the domain is found to be considerably larger than for OBC1. This is caused by the large variation in the exit horizontal velocity, which is between $-1.0$ and 1.2. At (1.25, 0.5), $u$ oscillates between 2.10 and 2.24, while at $x = 2.5$ and 3.75, $u$ is between 1.58 and 1.72. This means that

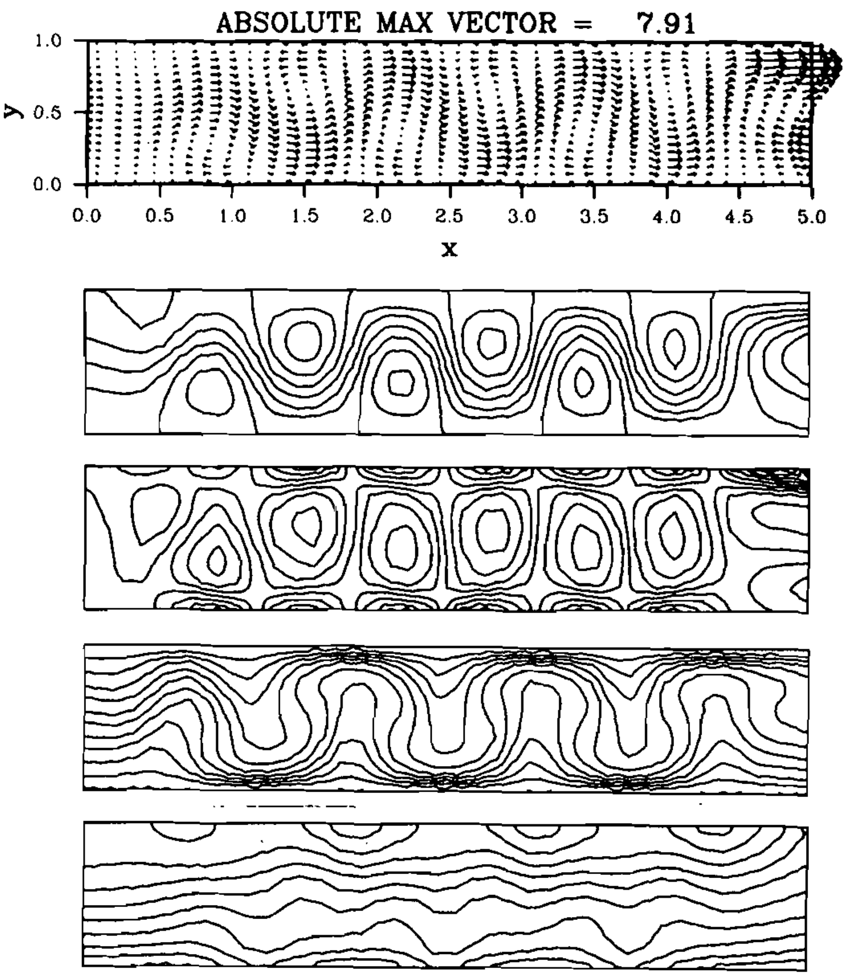



**Fig. 4** Solution on the short domain at $t_0$, using weighted quadratic extrapolation boundary conditions.

by performing a parabolic extrapolation to the open boundary we have increased the magnitude of the variation of $u$ at the exit of the duct by approximately 15 times as compared to the interior points. The vertical velocity components oscillate in phase between $-5$ and 5, while at $x = 5$ they oscillate around zero with an amplitude of about 7.6. The temperature time signals vary in phase between 0.3 and 0.7 except at $x = 5$ where it swings between 0.15 and 0.85. For this case, $\tau$ and $\lambda$ are found to be about 1.2. Notice how the pressure isolines at the duct exit are distorted as a consequence of the continuous adjustment of velocity to satisfy continuity. Although this case is a good example of how poor such a choice of open boundary conditions is, its numerical results should be looked at carefully. In fact, the domain is probably too short for a fully developed region to be present (i.e., there is competition between an entry length of about 2 and a numerical reflection generated by the OBC of at least the same magnitude).

## OBC4

The results obtained with OBC4 are reported in Figs. 5 and 6 at times $t_o$ and $t_o + \tau/2$ on the longer domain ($A = 10$). We may immediately notice how well behaved the streamlines, isovorticity lines, isotherms, and isobars are near the duct exit. Time signals of the four equidistant points at $y = 0.5$ are all in phase and almost perfectly overlapped with the exception that, at $x = 10$, $v$ has a slightly smaller amplitude and $u$ oscillates around a different average value of 1.15, although with the same amplitude. The remaining results can be summarized as follows: over $u$ in the interior, oscillations about a value of 1.65 with an amplitude of 0.13; over $v$, oscillations around 0 with amplitude of almost 10; and over $T$, oscillations about 0.5 with a minimum at 0.32 and a maximum at 0.68. The period $\tau$ is about 1.21 except for $u$, for which it is half this value. The wavelength $\lambda$ is approximately 1.24, and it is constant throughout the duct for $x > 2$. The solution presents about seven traveling roll pairs.

When the shorter domain is used, the following differences with OBC3 are found: $u$ at $x = 5$ oscillates about an average value of 1.15 with the same amplitude as the interior points. The oscillations of $v$ are everywhere in phase and with the same amplitude (a little less than 10). Temperature signals at four equidistant points at $y = 0.5$ are also in phase, and the exit point exhibits the same signal as the interior points. The trends obtained by using OBC4 on short and long ducts as compared to extrapolated results (OBC1, 2, and 3) seem remarkably more coherent.

A comparison between values of variables at $x = 5$ and $y = 0.5$, as computed on the two different length domains with OBC4, yields the results shown in Figs. 7 and 8. Velocity components and temperature are in phase, and the periods of the oscillations are the same. The vertical velocity computed on the short domain exhibits a smaller amplitude of oscillation, while the horizontal velocity of the longer duct varies about a higher average value. Temperature signals are almost exactly overlapped. The small differences found are not unexpected: Since the system of equations is elliptic, the longer channel has a portion of flow between $x = 5$ and $x = 10$ that influences the first half of the channel. The values found at $x = 5$ on two different length ducts are expected to be different; however, some reflection, confined to a distance of one wavelength from the exit, still appears, and it may be partially responsible for the differences.

Some test is needed to judge the quality of the convective open boundary condition. This is provided by the particular nature of the flow field under study. A cell that is

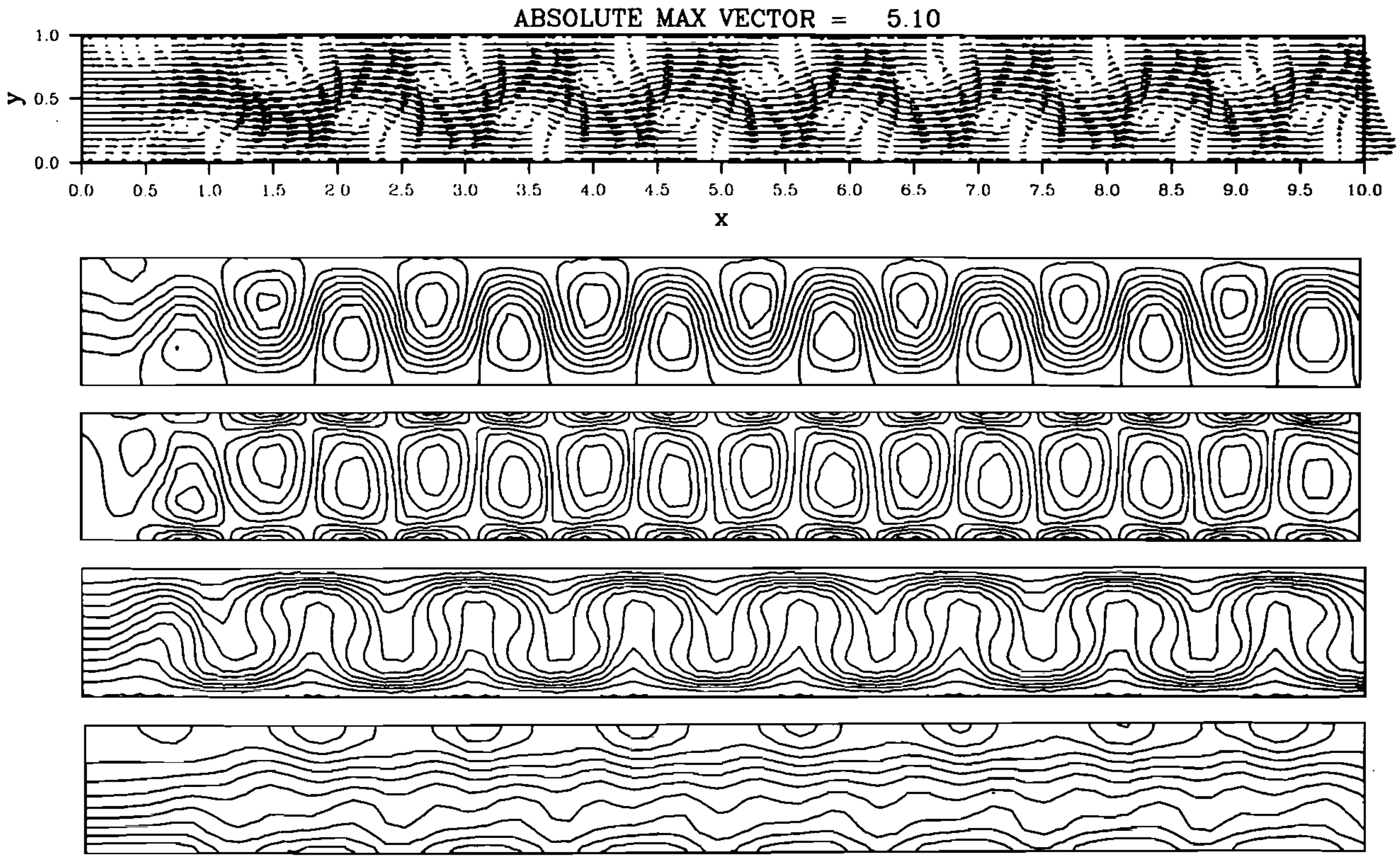



**Fig. 5** Solution on the long domain at $t_0$ with the convective OBC.

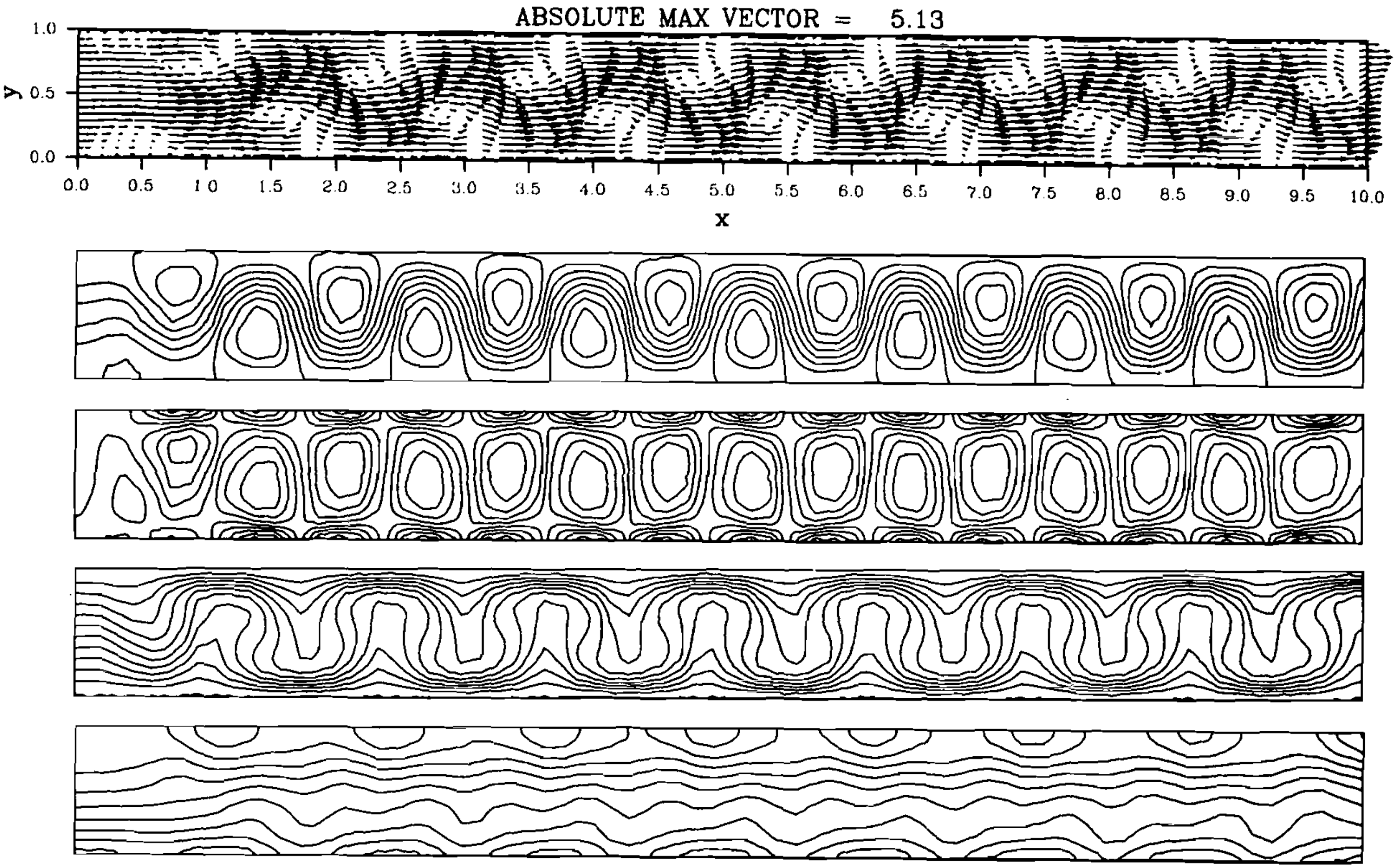



**Fig. 6** Same as Fig. 5, half a cycle later.

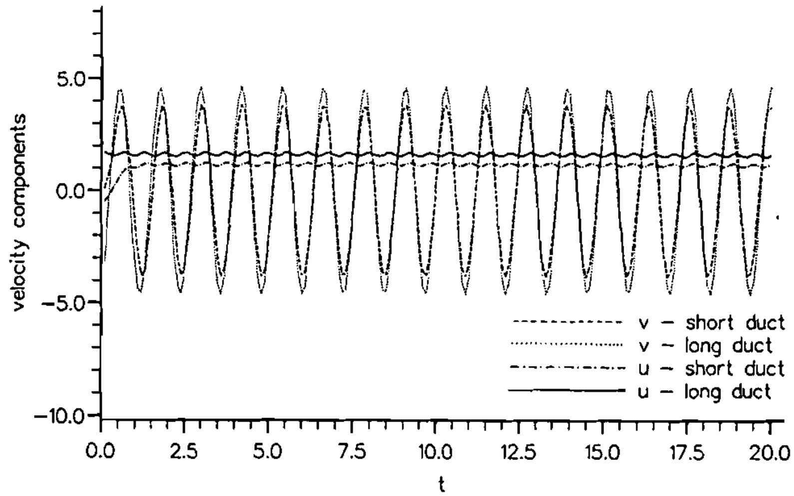



**Fig. 7** Time signal of the two velocity components at $x = 5$ and $y = 0.5$, computed on two domain lengths ($A = 5$ and $A = 10$) with the same convective OBC.

located at $x$, near the hot boundary, at time $t_o$ will travel half a wavelength to $x + \lambda/2$ and still be close to the hot boundary at time $t_o + \tau/2$. This means that flow structures at $t_o$ are symmetric about the $x$ axis with flow structures at $t_o + \tau/2$. This is valid for streamlines, isotherms, and isovorticity lines. For isobars, the isolines computed at a half period difference should be perfectly overlapped when considering half a wavelength shift. The measure by which these properties are not satisfied is an indication of the amount of reflection that the OBC introduces into the domain. It was found that for the simple upwinding technique (OBC1), an overlapping of, for example, the streamline plots at $t_o$ and half a period later (taking care to reflect this last plot about $x$) is such that

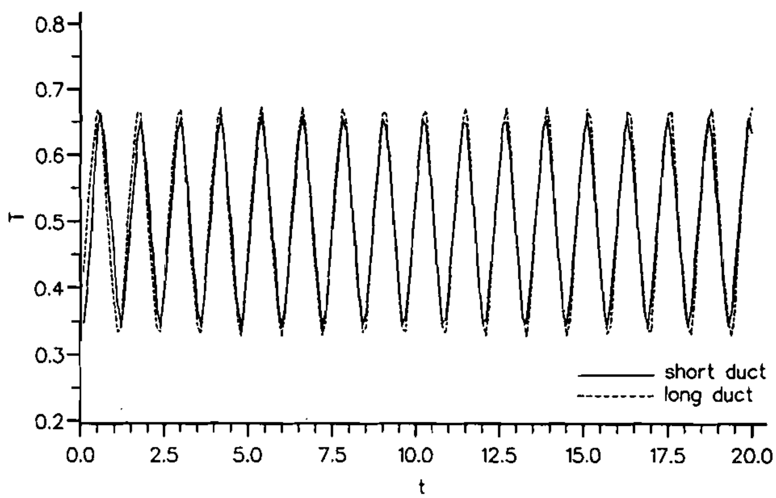



**Fig. 8** Temperature oscillations at (5, 0.5) for the two different aspect ratio ducts.

significant differences appear after half the duct length and become more and more pronounced as the exit boundary is approached. This means that $\lambda$ not only increases with $x$ but also oscillates in time for $x$ larger than about 5. On the other hand, by using OBC4, a good overlapping of lines may be found, and wavelengths are preserved, which is a sign of little reflection affecting the inherent symmetric properties of this flow.

## CONCLUSIONS

To summarize our conclusions, in Table 1 we have reported wavelengths, maximum temperature, and maximum velocity components at time $t_o$ and $y$ equal to 0.5, for the last four pairs of rolls in the domain, for OBC1 and OBC4. Although variations about average values for each quantity reported are contained, a monotonic trend with $x$ is discernable from the data related to OBC1, the variation in $\lambda$ being the most significant. This fact is caused by the open boundary condition and affects the numerical solution on the last two roll pairs. The distortion caused by upwind boundary conditions is even more pronounced when OBC2 and OBC3 are adopted. On the contrary, OBC4 gives rise to small nonmonotonic variations about average values, which should mainly be ascribed to the horizontal resolution of the computed flow within each wavelength.

Another indication that significant reflection inside the domain takes place when adopting OBC1, 2, or 3 is provided by the properties of this particular problem. It is found that upwinding techniques at the open boundary destroy symmetry, at least in the second half of the channel. It has been shown that, as a consequence, oscillation amplitudes and wavelengths are poorly estimated. Any upwind method, simple or several points extrapolation, amplifies the oscillation of the streamwise component of the velocity at the open boundary and decreases the amplitude of the crosswise component. The simple upwind case produces a $y$ component of velocity that vanishes at the exit of the duct. When more than one point is used in the extrapolation, unrealistic pressure oscillations are produced at the duct exit boundary. This is caused by the continuous (and necessary) adjustment of the exit velocity to satisfy the global mass balance. Thus, when no reflection of waves inside the computational domain is desired, Neumann and Dirichlet-type boundary conditions should be avoided. Convective techniques of the Sommerfeld radiation condition type produce little reflection [7] and are better at expressing the physical

**Table 1** Comparison between OBC1 and OBC4 at $t = t_o$ and $y = 0.5$ for the Last Four Developed Pairs of Rolls in the Duct

| | $\lambda$ | $u_{max}$ | $v_{max}$ | $T_{max}$ |
|---|---|---|---|---|
| OBC1 | | | | |
| Last pair ($n$) | 1.45 | 1.703 | 5.006 | 0.700 |
| Pair $n$-1 | 1.34 | 1.686 | 4.929 | 0.690 |
| Pair $n$-2 | 1.29 | 1.690 | 4.905 | 0.689 |
| Pair $n$-3 | 1.29 | 1.689 | 4.899 | 0.688 |
| OBC4 | | | | |
| Last pair ($m$) | 1.23 | 1.597 | 4.833 | 0.687 |
| Pair $m$-1 | 1.26 | 1.605 | 4.785 | 0.676 |
| Pair $m$-2 | 1.24 | 1.614 | 4.667 | 0.679 |
| Pair $m$-3 | 1.25 | 1.628 | 4.831 | 0.687 |

convection of vector and scalar fields outside the domain of integration. For the test problem considered here it was shown that the use of the convective boundary condition gives

1. Time signals of $u$, $v$, and $T$ at different $x$ positions that are overlapped or of comparable amplitude
2. Preservation of symmetry and wavelength; as a consequence, reflection is minimal

A final remark pertains to the nature of Eqs. (4)–(6). They are wave equations and express well the flow behavior as the fluid approaches the exit of the channel because the problem considered is that of a traveling wave. A more significant test would be that of a flow without a specified mean direction, with significant inflow and outflow over the same open boundary. In this case, the way to proceed could be outlined as follows:

1. Search at each time step for the area of inflow and the area of outflow on the open boundary.
2. Then, over each one of these two areas, compute $U$ and $V$ (average components of the velocity) and apply Eqs. (4)–(6).

## REFERENCES

1. C. Dawson and M. Marcus, DMC—A Computer Code to Simulate Viscous Flow and Arbitrarily Shaped Bodies, in T. Sarpkaya, ed., *Proc. 1970 Heat Transfer and Fluid Mech. Inst.*, pp. 323–338, 1970.
2. K. S. Gage and W. H. Reid, The Stability of Thermally Stratified Plane Poiseuille Flow, *J. Fluid Mech.*, vol. 33, pp. 21–32, 1968.
3. S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*, chap. 6, Hemisphere, Washington, DC, 1980.
4. P. M. Gresho and R. L. Sani, On Pressure Boundary Conditions for the Incompressible Navier-Stokes Equations, *Int. J. Num. Methods Fluids*, vol. 7, pp. 1111–1145, 1987.
5. H. J. Lugt and H. J. Haussling, Laminar Flow Past an Abruptly Accelerated Elliptic Cylinder at 45° Incidence, *J. Fluid Mech.*, vol. 65, pp. 711–734, 1974.
6. F. Nataf, An Open Boundary Condition for the Computation of the Steady Incompressible Navier-Stokes Equations, in R. Gruber, J. Periaux, and R. P. Shaw (eds.), *Proc. Fifth Int. Symposium Num. Methods in Eng.*, vol. 1, pp. 749–756, 1989.
7. I. Orlanski, A Simple Boundary Condition for Unbounded Hyperbolic Flows, *J. Comp. Physics*, vol. 21, pp. 251–269, 1976.
8. P. S. Lowery and W. C. Reynolds, Numerical Simulation of a Spatially-Developing, Forced, Plane Mixing Layer, *Stanford Rep. TF-26*, Palo Alto, Calif. Sept. 1986.
9. G. Evans and S. Paolucci, Private communication with P. M. Gresho.

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