Problema 8

$$Z = \frac{x - \mu}{\sigma}$$

x: variable aleatoria
μ: $E(x)$ media X
σ: $\sqrt{Var(x)}$ desviación estándar X.

$$E(Z) = \frac{E(x-\mu)}{E\sigma} = \frac{E(x)-\mu}{E\sigma} = \frac{E(x)-E(x)}{E\sigma} = \frac{0}{E\sigma} = 0$$

$\mu = E(x)$

$$Var\left(\frac{x-\mu}{\sigma}\right) = \frac{Var(x-\mu)}{\sigma^2} = \frac{\sigma^2}{\sigma^2} = 1$$

$Var(x-\mu) = \sigma^2$

Problema 9

Demostrar: $\sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$

$(x_i - \bar{x})^2 = x_i^2 + 2x_i\bar{x} + \bar{x}^2$

$\sum_{i=1}^{n} x_i = n\bar{x}$

x^2 = es una constante

$$\sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - 2\sum_{i=1}^{n} x_i\bar{x} + \sum_{i=1}^{n} \bar{x}^2$$

$$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + n\bar{x}^2$$

$$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}(n\bar{x}) + n\bar{x}^2$$

$$= \sum_{i=1}^{n} x_i^2 - 2n\bar{x}^2 + n\bar{x}^2$$

$$\sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$

Problema 10

Modelo de regresión lineal simple:

$y_i = \beta_0 + \beta_1 x_i + u_i$ → Encontrar β_0 y β_1 use SEE

Usamos las derivadas para encontrar valores β_0 y β_1

$$S(\beta_0, \beta_1) = \sum_{i=1}^{n} (y_i - (\beta_0 + \beta_1 X_i))^2$$

* Derivada β_0

$$\frac{\partial S}{\partial \beta_0} = -2 \sum_{i=1}^{n} (Y_i - (\beta_0 + \beta_1 X_i))$$

$$\frac{\partial S}{\partial \beta_0} = 0 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 X_i)$$

$$\sum_{i=1}^{n} y_i = n\beta_0 + \beta_1 \sum_{i=1}^{n} x_i$$

$$\bar{y} = \beta_0 + \beta_1 \bar{x}$$

$$\beta_0 = \bar{y} - \beta_1 \bar{x}$$

* Derivada β_1

$$\frac{\partial S}{\partial \beta_1} = -2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 X_i) X_i$$

$$\sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 X_i) X_i = 0 \qquad \boxed{\beta_0 = \bar{y} - \beta_1 \bar{x}}$$

$$\sum_{i=1}^{n} (y_1 - (\bar{y} - \beta_1 \bar{x}) - \beta_1 X_i) X_i = 0$$

$$\sum_{i=1}^{n} ((Y_i - \bar{Y}) - \beta_1 (X_i - \bar{x}))(x_i - \bar{x}) = 0$$

$$\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x}) - \beta_1 \sum_{i=1}^{n} (x_i - \bar{x})^2 = 0$$

$$\beta_1 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(X_i - \bar{x})}{\sum_{i=1}^{n} (X_i - \bar{x})^2}$$

$$\beta_0 = \bar{Y} - \beta_1 \bar{X}$$

$$\beta_0 = \bar{y} - \left(\frac{\sum_{i=1}^{n} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{n} (X_i - \bar{x})^2} \right) \bar{X}$$