Quantum Pirates

A Quantum Game-Theory Approach to The Pirate's Game

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Vogal:

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I can live with doubt and uncertainty and not knowing. I think it is much more interesting to live not knowing than to have answers that might be wrong. If we will only allow that, as we progress, we remain unsure, we will leave opportunities for alternatives. We will not become enthusiastic for the fact, the knowledge, the absolute truth of the day, but remain always uncertain â¦ In order to make progress, one must leave the door to the unknown ajar.

Richard P. Feynman

Acknowledgments

I would like to thank the Academy and Teresa Guilherme!

Abstract

Resumo em inglês

Keywords

palavrachave1, palavrachave2, palavrachave3 em inglês

Resumo

Abstract in portuguese.

Palavras Chave

keyword1, keyword2, keyword3, etc, in portuguese.

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Abbreviations

(Examples below:)

BeKi - Benchmark Kinect

OpenGL - Open Graphics Library

SHREC - SHape REtrieval Contest

TREC - Text REtrieval Contest

List of Symbols

1

Introduction

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Super motivacao

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Background

Contents

Present the chapter content.

2.1 Von Neumann Probability

In the beginning on the 20th century the nature of light was once again in the spotlight. The question whether light would be a particle (corpuscular theory), or a wave (undulatory theory), was posed throughout History. Newton, notoriously, considered light to be a particle and presented arguments such as the fact that light travels in a straight like, not bending when presented with obstacles, unlike waves, and gave an interpretation of the diffraction mechanism by resorting to a special medium (aether), where the light corpuscles could create a localized wave[1].

The idea of light as a particle stood up until the 18th century as many scientists (Robert Hooke, Christian Huygens and Leonhard Euler to name a few) tried to explain contradictions found in corpuscular theory. This brought back the idea that light behaves like a wave.

One of the most famous experiments that corroborates the undulatory theory is the Young's experiments (19th century), or the double-slit interferometer.
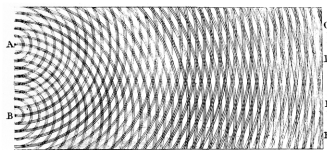
Figure 2.1: Thomas Young's sketch of two-slit diffraction of light.

The apparatus for the double-slit experiment can be seen in Figure 2.1. A light source is placed in such a way that "two portions" of light arrive at same time at the slits. Behind the barrier is a "wall" placed to intercept the light. The light captured at a wall will sport an interference pattern similar to the pattern when two waves interfere. The double-slit experiment was considered for a while the full stop on the discussion on the nature of light. However with experiments on the spectres of the light emitted by diverse substances and its relation with temperature, a new problem was posed.

The black body radiation problem was the theoretical problem where a body that absorbs light in all the electromagnetic spectrum, this makes the body acting as an natural vibrator, where each mode would have the same energy, according to the classic theory.

When a black body is at a determined temperature the frequency of the radiation it emits depends on the temperature. The classic theory predicted that most of the energy of the body would be in the high frequency part of the spectrum (violet part) where most modes would be found, this led to a

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[0]Source: Young, Thomas: Probability. `http://en.wikipedia.org/wiki/File:Young_Diffraction.png`(1803)

prediction called the ultraviolet catastrophe. According to the classic theory the black body would emit radiation with an infinite power for temperatures above approximately 5000K. Max Plank(1901), provided an explanation where the light was exchanged in discrete amounts called quanta, so that each frequency would only have specific levels of energy. Plank also determined through experimentation the value of the energy of the quanta that became known as photons later, that value became the physical constant called Plank constant:

$$h = 6.62606957(29) \times 10^{-34} J.s \tag{2.1}$$

In 1905, Einstein used the concept of quanta (photons) to explain the photoelectric effect. De Broglie(1924), suggested that all the matter had a wave-particle duality. This prediction was confirmed by studying the interference patterns caused by electron diffraction.

2.1.1 Mathematical Foundations of Quantum Probability

As previously explained, Quantum Theory is a branch of physics that has arised from the need to explain certain phenomena that could not be explained with the current classical theory. In the beginning of the 20th century Dirac and von Neumann helped to create the mathematical formalisms for this theory [2][3].

Von Neumann's contributions were focused in the mathematical rigor, as is framework is strongly based in Hilbert's theory of operators. Dirac's concerns were more of a practical nature. Their combined contributions were invaluable to establish this area.

From Dirac it is important to point the Dirac's notation (also known as Bra-ket notation or $\langle Bra|c|ket \rangle$) (introduced in 1939 [2]), that is widely used in literature based on quantum theory. This notation uses angle brackets and vertical bars to represent quantum states (or abstract vectors) as it can be seen in the formulas (2.2) and (2.3).

$$\langle z| = \begin{bmatrix} z_1^* & z_2^* & ... & z_n^* \end{bmatrix} \tag{2.2}$$

$$|z\rangle = (\langle z|)^* = \begin{bmatrix} z_1 \\ z_2 \\ ... \\ z_n \end{bmatrix} \tag{2.3}$$

This notation provides for an elegant representation of the inner product (2.4) and verifies the has lin-

earity as it follows in equation (2.5). While the Bra-ket notation can be useful in terms of condensing information, using vectors and matrices to represent the states turns out to be a more approachable way to understand and manipulate data.

$$\langle z|z \rangle = \sum_{i=1}^{n} \bar{z}_i z_i \tag{2.4}$$

where $\bar{z}_i$ is the complex conjugate of z_i.

$$\langle z|(\alpha|x\rangle + \beta|y\rangle) = \alpha\langle z|x\rangle + \beta\langle z|y\rangle \tag{2.5}$$

2.1.1.A Born rule

The Born rule was formulated by Born in 1926. This law allows to predict the probability that a measurement on a quantum system will yield a certain result. This law provides a link between the mathematical foundation of Quantum Mechanics and the experimental evidence[4][5].

A quantum system is represented by a n-dimensional Hilbert Space, a complex vector space in which the inner product is defined.

A two-dimensional Hilbert Space (corresponding to a qubit for example), can be represented by a Bloch Sphere.

The Born rule states that if there is a system is in a state $|v\rangle$ (in a given n-dimensional Hilbert Space H), and an Hermitian operator A is applied then the probability of measuring a specific eigenvalue λ_i associated with the i-th eigenvector of A (ψ_i), will be given by[4]:

$$P_v(\lambda_i) = \langle v|Proj_i|v\rangle \tag{2.6}$$

where $Proj_i$ is a projection matrix corresponding to ψ_i :

$$Proj_i = |\psi_i\rangle\langle\psi_i| \tag{2.7}$$

Given the properties of A, the set of eigenvectors $\{\psi_1, \psi_2, ..., \psi_i, ..., \psi_n\}$ forms a orthogonal basis of the n-dimensional Hilbert Space considered. Thus the state $|v\rangle$ can be written as a linear combination of

the eigenvectors of A:

$$|v\rangle = \alpha_1\psi_1 + \alpha_2\psi_2 + ... + \alpha_i\psi_i + ... + \alpha_n\psi_n \tag{2.8}$$

The coefficients α_i are complex numbers called probability amplitudes, and their squared sum is equal to 1:

$$\sum_{i=0}^{n} |\alpha_i|^2 = 1 \tag{2.9}$$

this brings us to:

$$P_v(\lambda_i) = \langle v|\psi_i\rangle\langle\psi_i|v\rangle = |\langle v|\psi_i\rangle|^2 = |\alpha_i^*\alpha_i|^2 \tag{2.10}$$

So the determination of the probability of an event ($P(A)$), is made by projecting the quantum state on the eigenvectors corresponding to the operator A of the Hilbert Space and measuring the squared length of the projection.[6]

$$P(A) = (Proj_A|z\rangle)^2$$

Applying an operator can be seen as applying a rotation matrix on the system ($|v\rangle$) and measuring the projection of $|v\rangle$ onto the imaginary axis and the real axis (Figure **??**), or considering that we have a determined state vector and rotating the orthogonal basis of the Hilbert Space according to an operator and then to do a projection on the new chosen orthogonal basis.

According to Leiffer[7] "quantum theory can be thought of as a non-commutative, operator-valued, generalization of classical probability theory".

As in the classical probability theory where from a random variable it is possible to establish a probability distribution, also known as density function, in the Hilbert space there is a equivalent density operator. The density operator (ρ) is a Hermitian operator that has the particularity of having its trace equal to 1[4].

$$\rho = \sum_{i=0}^{n} \alpha_i|\psi_i\rangle\langle\psi_i| \tag{2.11}$$

$$tr(\rho) = 1 \tag{2.12}$$

2.1.1.B Example of the double-slit experiment with electrons

Like the Young's Experiment with light created an interference pattern similar to a wave, firing electrons one at the time produces a similar pattern. The unobserved fired electron behaved like a wave and after passing the slits the wavelets interfered with one another to create a interference pattern. However if a measuring device was active while the electron was fired the interference pattern wasn't registered.
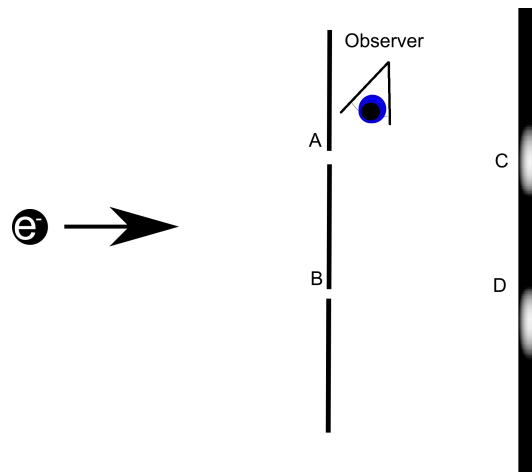
Figure 2.2: Double-slit experiment where there is a measuring device that allows to know through which slit the electron passed.
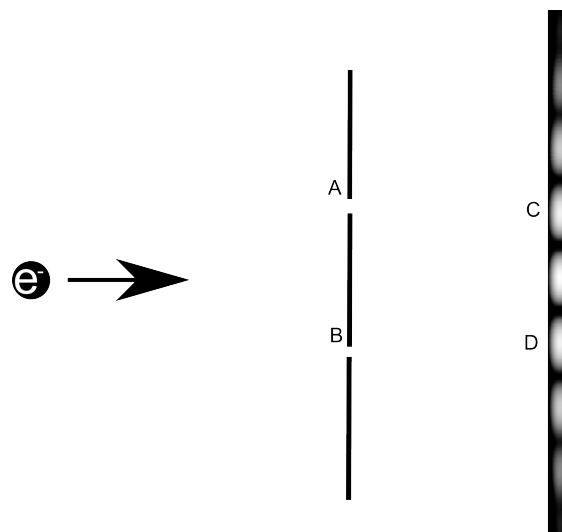
Figure 2.3: Double-slit experiment, where electrons exhibit the interference pattern characteristic in waves

The fact that the electron was measured while passing through a slit produced a particle behaviour, explained by the classical theory (Figure 2.2).

In this experiment a single electron is shoot at a time. So in the start of the experiment (S), we know the initial position of the electron.

A final measurement (F) is made when the electron hits the wall behind the slits, where we know the

final position of the electron. [1]

If this experiment is observed, there is an intermediate measure that tells us whether the electron went throught the slit A or B. The corresponding probability amplitudes related to this measurement are ω_A and ω_B, and:

$$\omega_A = \langle F|A\rangle\langle A|S\rangle \tag{2.13}$$

$$\omega_B = \langle F|B\rangle\langle B|S\rangle \tag{2.14}$$

If we consider the intermediate measurement the probability $P(F|S)$ will be:

$$P(F|S) = |\langle F|A\rangle\langle A|S\rangle|^2 + |\langle F|B\rangle\langle B|S\rangle|^2 \tag{2.15}$$

But if we only measure the position of the electron at the end of the experiment that probability will be:

$$P(F|S) = |\langle F|A\rangle\langle A|S\rangle + \langle F|B\rangle\langle B|S\rangle|^2 \tag{2.16}$$

The latter equation will be dependent on a interference coefficient that will be responsible the interference pattern observed in the unobserved experiment.

2.1.1.C Example of the Polarization of Light

The photons in a beam of light don't vibrate all the same direction in most of the natural sources of light. To filter the light polaroids are used. A polaroid only allows the passage of light in a well-defined direction and thus reducing the intensity of the light. In the Figure 2.4 we can observe that the introduction of the oblique polaroid in the third situation led to a passage of light. Although there is a classical explanation to this phenomenon if we consider waves when we are considering a beam of light, if our light source emits one photon at the time a quantum mechanical explanation is needed[8].

To model the polarization of the photon in a quantum setting, we will use a vector $|v\rangle$ in a two-dimensional Hilbert Space:

$$|v\rangle = a \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2.17}$$

where $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ would represent the vertical direction (could also be represented by the state vector $|\uparrow\rangle$), and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ the horizontal one (another possible representation to this basis could be $|\rightarrow\rangle$). We can consider the Figure **??** as a graphical representation for this system.

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[1]Mohrhoff, U.: Two Slits. `http://thisquantumworld.com/wp/the-mystique-of-quantum-mechanics/two-slit-experiment/#fn1back`
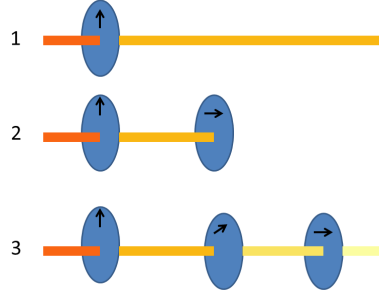
Figure 2.4: 1. With one vertical polaroid the unpolarized light is attenuated by a half. 2. Vertical polarization followed by a horizontal polarization will block all the passing light. 3. Inserting a oblique polaroid between the vertical and horizontal polaroids will allow light to pass.

In the first situation if $a = \frac{1}{\sqrt{2}}$, that would mean that the probability of passing the vertical polaroid would be $a = (\frac{1}{\sqrt{2}})^2 = 0.5$, that would light to the expected reduction of a half of the intensity of light. After passing through the vertical polaroid the photon will have a polarization of $|v\rangle = a \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Considering now the second situation after we have our photon polarized vertically (like in the end on the first situation), the probability of being vertically polarized is 1, thus making the probability of passing through the horizontal polaroid 0. In the third situation, after being vertically polarized the photon will pass through an oblique polaroid that makes its direction

$$|v\rangle = i.sen(\theta) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + cos(\theta) \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2.18}$$

θ being the angle of the polaroid. The photon filtered by the vertical polaroid will pass this second polaroid with a probability of $(i.sen(\theta))^2$, becoming polarized according to the filter, as we can observe depending on the value of θ we will now have a horizontal component in the vector that describes the state of the photon. This will make the photon pass the horizontal polaroid with a probability of $cos(\theta)^2$.

2.2 Section A

2.2.1 Subsection A

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2.2.2 Subsection B

Figure 2.5: Dummy Figure Caption.

Remember you can change the reference style. Another dummy citation [**?**].

2.3 Section B

2.3.1 Subsection A

The model described can also be represented as

$$\dot{\mathbf{x}}(t) = \mathbf{T}\mathbf{z}(y),\ \mathbf{y}(0) = \mathbf{y}_0,\ z \geq 0 \qquad (2.19)$$

where

$$\mathbf{A} = \left[\begin{array}{cc} -(a_{12} + a_{10}) & a_{21} \\ a_{12} & -(a_{21} + a_{20}) \end{array} \right],\ \mathbf{x} = \left[\begin{array}{c} x_1 \\ x_2 \end{array} \right] \qquad (2.20)$$

2.3.2 Subsection B

Table 2.1: Dummy Table.

Vendor Name	Short Name	Commercial Name	Manufacturer
	ABC	ABC®	ABC SA
Text in Multiple Row	DEF	DEF®	DEF SA
	GHF	GHF®	GHF SA
Text in Single Row	IJK	IJK®	IJK SA
Frescos SA	LMN	LMN®	LMN SA
Carros Lda.	Text in Multiple Column		

3

Related Work

Contents

Present the chapter content.

3.1 Quantum Walks

3.1.1 Quantum Walk on a Line

A Quantum Walk is the quantum version of random walks, which are a mathematical formalism to describe a path constructed by random steps. This processes can be described by a Markov Chain.

We can define the Discrete Quantum Walk on a Line as a series of Left/Right decisions. Understanding this algorithm is important towards being able to define and design more complex algorithms.

We followed an approach suggested by [**?**] towards simulating n-steps of a Quantum Walk on a Line, the algorithm can be consulted on A. In a discrete quantum walk in a line we want to preserve some properties, namely: the probability of turning left should be equal to the probability of turning right. To represent a state in this algorithm we will need the number of the node and a direction (identified as L,R) 3.1.

$$|\psi\rangle = |n, L\rangle \tag{3.1}$$

Having two equally possible choices we can resort to a coin metaphor[**?**][**?**]. We toss a coin and go either Left or Right depending on the result. In a quantum version we need to define a Coin Operator (Coin Matrix) which is responsible to imprint a direction to our state with equal probability. This operator is a unitary matrix in a 2-dimension Hilbert space. Some examples of Coin Operators are the Hadamard matrix3.2 and a symmetric unitary matrix 3.3.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{3.2}$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{3.3}$$

Taking the Hadamard matrix as an example3.2, the coin matrix will operate on the state in the following way 3.43.5[**?**].

$$C|n, L\rangle = \frac{1}{\sqrt{2}}|n, L\rangle + \frac{1}{\sqrt{2}}|n, R\rangle \tag{3.4}$$

$$C|n, R\rangle = \frac{1}{\sqrt{2}}|n, L\rangle - \frac{1}{\sqrt{2}}|n, R\rangle \tag{3.5}$$

The Coin Matrix obtains its name by being the quantum equivalent of flipping a classic coin. After tossing a coin comes an operator that will move the node in the direction assigned. The operator responsible for this modification is commonly referred as Shift Operator 3.63.7.

$$S|n, L\rangle = \frac{1}{\sqrt{2}}|n - 1, L\rangle \tag{3.6}$$

$$S|n, R\rangle = \frac{1}{\sqrt{2}}|n + 1, R\rangle \tag{3.7}$$

These matrices (Coin Matrix and Shift Operator) are referred conceptually ubiquitously throughout the literature [sources], therefore it is important to be familiar with them. A single step of the algorithm A is illustrated in Figure 3.1.

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Depending on the Coin Matrix we can get different distributions. In Figures 3.3 and 3.2

Starting on the middle of a line we can shift one unit left or right. If we took the classical approach in which we tossed a fair coin, and after n-steps we measured the final node repeatedly, by the Central Limit Theorem (CLT) the final distribution would converge to a normal distribution.

3.2 Section A

3.2.1 Subsection A

This would be a citation [**?**].

Each discrete node in the line is represented $|n, d\rangle$
Here we have. $|0, R\rangle$

$|s\rangle = |0, R\rangle$

$|s\rangle = C|0, R\rangle$
$= \frac{1}{\sqrt[2]{2}}|0, R\rangle + \frac{1}{\sqrt[2]{2}}|0, L\rangle$

Aplying the Coin Operator

$|s\rangle = SC|0, R\rangle$
$= S\left[\frac{1}{\sqrt[2]{2}}|0, R\rangle + \frac{1}{\sqrt[2]{2}}|0, L\rangle\right]$
$= \frac{1}{\sqrt[2]{2}}|1, R\rangle + \frac{1}{\sqrt[2]{2}}|-1, L\rangle$

Shifting
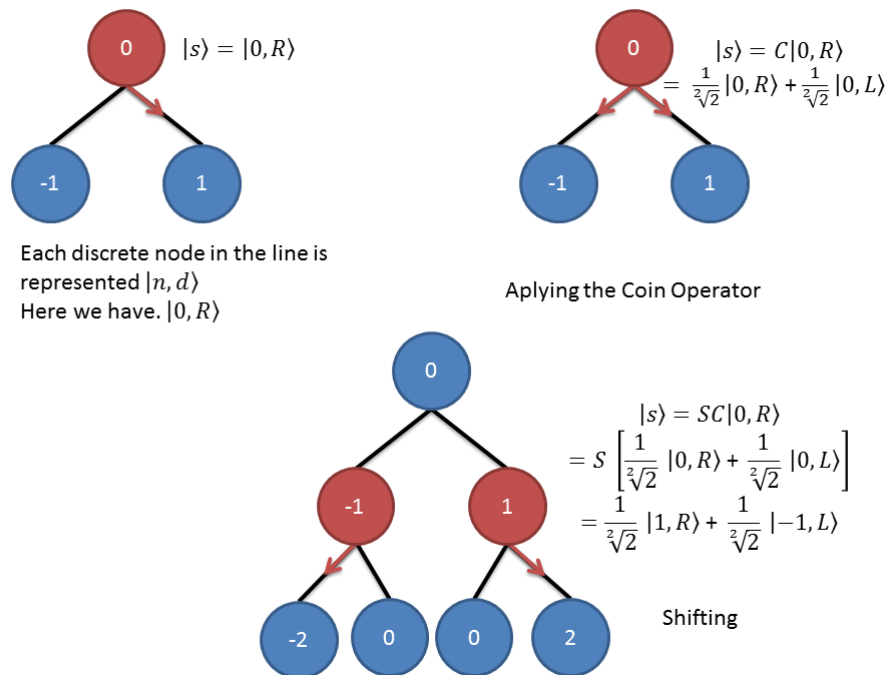
Figure 3.1: Simulating a step of a discrete quantum walk on a line. In the beginning we have a state characterized by the position (0) and a direction (either Left or Right).

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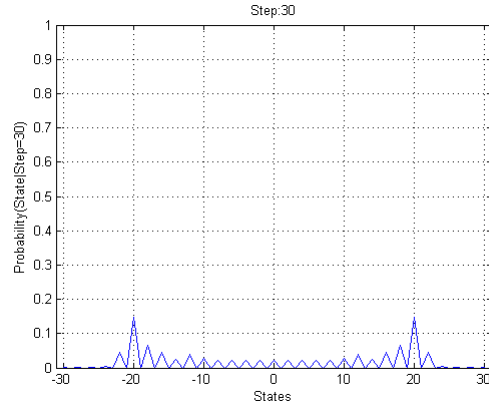
3.2.2 Subsection B

Figure 3.4: Dummy Figure Caption.

Remember you can change the reference style. Another dummy citation [**?**].
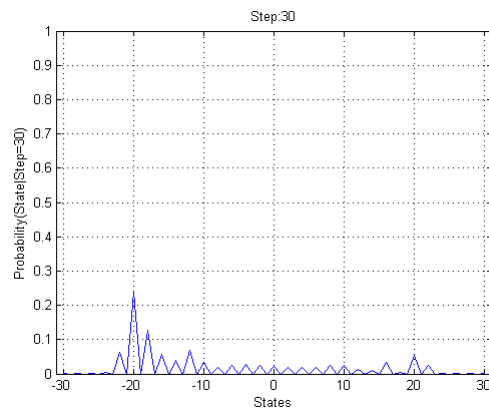
3.3 Section B

3.3.1 Subsection A

The model described can also be represented as

Figure 3.2: 30 Step of the Simulation A using Matrix 3.2 as a Coin Operator.

Figure 3.3: 30 Step of the Simulation A using a Hadamard Matrix 3.2 as a Coin Operator.

$$\dot{\mathbf{x}}(t) = \mathbf{T}\mathbf{z}(y), \ \mathbf{y}(0) = \mathbf{y}_0, \ z \geq 0 \tag{3.8}$$

where

$$\mathbf{A} = \left[\begin{array}{cc} -(a_{12} + a_{10}) & a_{21} \\ a_{12} & -(a_{21} + a_{20}) \end{array} \right], \ \mathbf{x} = \left[\begin{array}{c} x_1 \\ x_2 \end{array} \right] \tag{3.9}$$

3.3.2 Subsection B

Table 3.1: Dummy Table.

Vendor Name	Short Name	Commercial Name	Manufacturer
	ABC	ABC®	ABC SA
Text in Multiple Row	DEF	DEF®	DEF SA
	GHF	GHF®	GHF SA
Text in Single Row	IJK	IJK®	IJK SA
Frescos SA	LMN	LMN®	LMN SA
Carros Lda.	Text in Multiple Column		

4

Your Solution here

Contents

Present the chapter content.

4.1 Section A

4.1.1 Subsection A

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4.1.2 Subsection B

Figure 4.1: Dummy Figure Caption.

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4.2 Section B

4.2.1 Subsection A

The model described can also be represented as

$$\dot{\mathbf{x}}(t) = \mathbf{T}\mathbf{z}(y), \ \mathbf{y}(0) = \mathbf{y}_0, \ z \geq 0 \tag{4.1}$$

where

$$\mathbf{A} = \begin{bmatrix} -(a_{12} + a_{10}) & a_{21} \\ a_{12} & -(a_{21} + a_{20}) \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{4.2}$$

4.2.2 Subsection B

Table 4.1: Dummy Table.

Vendor Name	Short Name	Commercial Name	Manufacturer
Text in Multiple Row	ABC	ABC$^{®}$	ABC SA
	DEF	DEF$^{®}$	DEF SA
	GHF	GHF$^{®}$	GHF SA
Text in Single Row	IJK	IJK$^{®}$	IJK SA
Frescos SA	LMN	LMN$^{®}$	LMN SA
Carros Lda.	Text in Multiple Column		

5

Evaluation

Contents

Present the chapter content.

5.1 Section A

5.1.1 Subsection A

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5.1.2 Subsection B

Figure 5.1: Dummy Figure Caption.

Remember you can change the reference style. Another dummy citation [**?**].

5.2 Section B

5.2.1 Subsection A

The model described can also be represented as

$$\dot{\mathbf{x}}(t) = \mathbf{T}\mathbf{z}(y),\ \mathbf{y}(0) = \mathbf{y}_0,\ z \geq 0 \tag{5.1}$$

where

$$\mathbf{A} = \begin{bmatrix} -(a_{12} + a_{10}) & a_{21} \\ a_{12} & -(a_{21} + a_{20}) \end{bmatrix},\ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{5.2}$$

5.2.2 Subsection B

Table 5.1: Dummy Table.

Vendor Name	Short Name	Commercial Name	Manufacturer
Text in Multiple Row	ABC	ABC$^®$	ABC SA
	DEF	DEF$^®$	DEF SA
	GHF	GHF$^®$	GHF SA
Text in Single Row	IJK	IJK$^®$	IJK SA
Frescos SA	LMN	LMN$^®$	LMN SA
Carros Lda.	Text in Multiple Column		

6

Conclusions and Future Work

Conclusions Chapter

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A

Matlab Simulation: Discrete Quantum Walk on a Line

```matlab
1
2
3   %── number of steps in the simulation
4   steps= 30;
5
6   %── hadamard matrix
7   H = [1/sqrt(2) 1/sqrt(2); 1/sqrt(2) −1/sqrt(2) ];
8   %── symetric matrix
9   M = [1/sqrt(2) 1i/sqrt(2); 1i/sqrt(2) 1/sqrt(2) ];
10  %── coin flip unitary operator
11  C = H;
12  C=M;
13
14  %── coin flip matrix
15  CM= zeros((steps*2+1),2);
16  %── shift matrix
17  SM= zeros(steps*2+1,2);
18
19  %── middle index
20  i0= steps+1;
21
22  %── initialize flip probability amplitudes (coin has 1/2 chance of going +1 or −1)
23  CM(i0,1)=1/sqrt(2);
24  CM(i0,2)=1/sqrt(2);
25
26  for i=1:steps
27      %── clean SM
28      SM= zeros(steps*2+1,2);
29
30      for j=1:(steps*2+1)
31          if CM(j, 1)≠0
32              SM(j−1,1)=CM(j, 1);
33          end
34          if CM(j, 2)≠0
35              SM(j+1,2)=CM(j, 2);
36          end
37      end
38      SM;
39      %disp('──────');
40      %── clean CM
41      CM= zeros(steps*2+1,2);
42      for j=1:(steps*2+1)
43          if SM(j, 1)≠0
44              CM(j, 1)= CM(j, 1)+C(1,1)*SM(j, 1);
45              CM(j, 2)= CM(j, 2)+C(1,2)*SM(j, 1);
46          end
47          if SM(j, 2)≠0
48              CM(j, 1)= CM(j, 1)+C(2,1)*SM(j, 2);
49              CM(j, 2)= CM(j, 2)+C(2,2)*SM(j, 2);
50          end
51      end
52      CM;
53
54      %Display
55
56      figure
57
58      probability=zeros(steps*2+1);
59
60      for j=1:steps*2+1
61          probability(j)=abs(SM(j,1)).^2+ abs(SM(j,2)).^2;
62      end
63      axisP = −steps : steps;
64      plot(axisP,probability)
65      title(strcat('Step: ' ,num2str(i)))
66      axis([−steps−1 , steps+1, 0, 1]);
67      grid on
68      ylabel(strcat('Probability(State|Step=',num2str(i), ')'));
69      xlabel('States');
70
71      %Display
72  end
```