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Bright pyroelectric quasi-solitons in a photorefractive waveguide

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ABSTRACT

Characteristics of propagation of bright pyroelectric quasi-solitons are studied in a photorefractive waveguide. These solitons considered are supported solely by the pyroelectric effect. The investigation is performed under Wentzel-Kramers-Brillouin-Jefferys approximation and a Gaussian ansatz for the soliton is used instead of numerical solutions. The planar waveguide structure intensifies the self-focussing while decreasing the minimum or threshold power required for self trapping. The waveguide structure embedded in the crystal can self trap a soliton of power lower than the threshold power. As the waveguide parameter increases, minimum required power to self trap the beam decreases. The existence of bistable states is also predicted. Four regimes of power are identified and investigated in which the quasi-solitons behaviour is distinct.

1. Introduction

Optical solitons are an attractive area for research because of their potential applications in optical switching, routing, waveguiding and optical communications and signal processing [1–9]. Optical solitons can be of three types, spatial, temporal and spatio-temporal [1–3]. Spatial solitons are localized solitary waves and they are confined in space. Temporal solitons are confined in time while spatio-temporal solitons are confined in both space and time.

In spatial solitons, the diffraction effects are balanced by the self focussing induced due to the non-linearity while in temporal solitons, the dispersion is balanced by the non-linearity.

Optical spatial solitons were first predicted in photorefractive materials in 1992 by Segev et al. and discovered experimentally by Duree et al. [10,11]. These solitons are unique because they are formed at very low laser powers(of the order of a few mW). Photorefractive non-linearity constitutes a saturable non-linearity and hence, catastrophic collapse of $(2+1)$D solitons as in case of Kerr solitons is avoided. When a photorefractive crystal is illuminated by a beam of light, it causes generation of charge carriers. These charge carriers migrate either due to an external electric field, bulk photovoltaic field or a transient pyroelectric field resulting in a space charge field. This space charge field induces a refractive index change through the Pockels' electro-optic effect.

The migration of charge carriers can take place due to various mechanisms, like the external electric field, bulk photovoltaic field or a combination of both of these. It results in the build up of a space charge field which causes the refractive index change. Hence, we classify the photorefractive solitons as, (i) screening solitons [12], which require an external elec-

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tric field, (ii) photovoltaic solitons [13], which require a finite bulk photovoltaic coefficient, and (iii) screening photovoltaic solitons [14], which are based on the combination of the external electric field and the bulk photovoltaic field. In addition, pyroelectricity alone and in conjunction with the external electric field and the photovoltaic field has been shown to support the self trapping in photorefractive media [15–18]. At equilibrium in a ferroelectric crystal, the net internal electric field is zero since the field due to spontaneous polarisation is compensated by charges accumulated on the crystal faces. A temperature change induces a spontaneous polarisation change and hence a transient electric field E_{py}. This field is not immediately compensated and a drift current can consequently take place as if an external voltage was applied to the crystal. This initial homogeneous pyroelectric electric field E_{py} is locally screened by the photorefractive effect and beam self-trapping can be induced [15].

Photorefractive solitons were originally demonstrated in photorefractive media where the change of refractive index is due to the linear electro-optic effect [10–18]. Again, Segev et al. proved that photorefractive solitons can also be supported in the centrosymmetric photorefractive media where the change of refractive index is due to the quadratic electro-optic effect [19].

While there have been extensive investigations on varied aspects of photorefractive solitons [4–18,20–24], there is scope for investigation of characteristics of spatial solitons in photorefractive waveguides. The diffraction of an optical beam can be countered due to the waveguiding effect. Hence, in a photorefractive waveguide, the minimum or threshold power for soliton formation will be much lower as compared to that in bulk photorefractive media. It is notable that the bright screening-photovoltaic solitons in photorefractive-photovoltaic waveguide [25] and the bright optical spatial solitons in centrosymmetric photorefractive waveguides [26] have been investigated recently. To our knowledge, nobody has yet studied the characteristics ofpyroelectric solitons in a photorefractive waveguide. In this paper, we investigate the propagation characteristics of bright pyroelectric solitons in a planar waveguide embedded in an unbiased photorefractive crystal having a finite pyroelectric coefficient.

2. Mathematical formulation

We consider an optical beam propagating along the z-direction in a waveguide which is embedded in a photorefractive crystal. The optical c-axis of the photorefractive waveguide is along the x-direction. The photorefractive crystal is placed between an insulating cover and a metallic plate whose temperature is accurately controlled by a Peltier cell. The beam is polarized along the x-direction and we assume that it is allowed to diffract along that direction only. Hence, a space charge field $\vec{E}_{SC} = \hat{x}E_{SC}$ is set up in the photorefractive waveguide.

The electric field $\vec{E}$ of the incident beam travelling through the photorefractive waveguide satisfies the following wave equation [25,26],

$$\nabla^2\vec{E} + \left(k_0 n_e'\right)^2\vec{E} - gx^2\vec{E} = 0 \tag{1}$$

where k_0 is the free space wave number and n_e' is the perturbed extraordinary index of refraction.n_e' is given by [12],

$$n_e'2 = n_e^2 - n_e^4 r_{33} E_{sc} \tag{2}$$

r_{33} is the linear electro-optic coefficient, g is the waveguide parameter, which we take as positive. Here, $E_{sc} = E_{pysc}$ which is the space charge field induced solely by the transient pyroelectric field. The slowly varying envelope of the electric field of the incident beam is expressed as,

$$\vec{E}(x,z) = \hat{x}\phi(x,z)e^{ikz} \tag{3}$$

where $\Phi(x,z)$ is the slowly varying envelope of the wave. Substitute (2), (3), in (1) and making an approximation of slowly varying envelope, we obtain

$$i\frac{\partial\Phi}{\partial z} + \frac{1}{2k_0 n_e}\frac{\partial^2\Phi}{\partial x^2} - \frac{1}{2}k_0 n_e^3 r_{eff} E_{pysc}\Phi - gx^2\Phi = 0 \tag{4}$$

The expression for the space charge field build-up in a photorefractive material due to solely the pyreoelectric effect and neglecting the effect of diffusion has been derived elsewhere in detail as [27],

$$E_{pysc} = -E_{py}\frac{I}{I + I_d} \tag{5a}$$

where I represents the power density profile of the optical beam. I_d is the so-called dark irradiance. E_{py} is the transient pyroelectric field which is expressed as [15,27],

$$E_{py} = -\frac{1}{\varepsilon_0\varepsilon_r}\frac{\partial P}{\partial T}\Delta\mathrm{T} \tag{5b}$$

In (5b), ε_0 is the vacuum permittivity, ε_r is the dielectric constant, $\frac{\partial P}{\partial T}$ is the pyroelectric coefficient, and ΔT is the change in temperature of the photorefractive crystal.

Now, we shall transform to the following dimensionless coordinates,

$$\xi = \frac{z}{kx_0^2}, I = \frac{n_e}{2\eta_0}|\Phi|^2, \quad \Phi = \sqrt{\frac{2\eta_0 I_d}{n_e}}\mathrm{U}, \ s = \frac{x}{x_0}$$

where x_0 is an arbitrary spatial width and $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

Using these dimensionless coordinates, the space charge field becomes,

$$E_{pysc} = -E_{py}\frac{|U|^2}{1+|U|^2} \tag{6}$$

Substituting (6) in (4) and using the dimensionless co-ordinates as defined above, the evolution equation becomes,

$$i\frac{\partial U}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U}{\partial s^2} + \alpha\frac{|U|^2}{1+|U|^2}U - \delta s^2 U = 0 \tag{7}$$

where

$$\alpha = \frac{(k_0x_0)^2 n_e^4 r_{33}}{2}E_{py} \text{ and } \delta = gk_0x_0^4 n_e.$$

The non-linear contribution to the refractive index of the photorefractive crystal comes from the third term in Eq. (7). Eq. (7) cannot be solved to obtain a closed form solution, but there are several methods to approximately solve this equation. Segev's method [10], Akhmanov's paraxial method [28], Anderson's variational method [29] and Vlasov's moment method [30] can be used to solve [7]. In the present study, we shall employ the variational method alongwith the paraxial approximation. Also, we shall study the bright solitons in our investigation. Assume the slowly varying beam envelope to be expressed as,

$$U\left(\xi, s\right) = U_0\left(\xi, s\right) \mathrm{e}^{-i\Omega(\xi,s)} \tag{8}$$

$U_0(\xi, s)$ is a purely real quantity and $\Omega(\xi, s)$ represents the phase. Substituting this ansatz in (7) gives,

$$\left(i\frac{\partial U_0}{\partial \xi} + U_0\frac{\partial \Omega}{\partial \xi}\right) + \frac{1}{2}\left\{\frac{\partial^2 U_0}{\partial s^2} - 2i\frac{\partial U_0}{\partial s}\frac{\partial \Omega}{\partial s} - iU_0\frac{\partial^2 \Omega}{\partial s^2} - U_0\left(\frac{\partial \Omega}{\partial s}\right)^2\right\} + \alpha\frac{{U_0}^2}{(1+U_0^2)}U_0 - \delta s^2 U_0 = 0 \tag{9}$$

Equating the real and imaginary parts,

$$\frac{\partial U_0}{\partial \xi} - \frac{\partial U_0}{\partial s}\frac{\partial \Omega}{\partial s} - \frac{1}{2}U_0\frac{\partial^2 \Omega}{\partial s^2} = 0 \tag{10}$$

$$U_0\frac{\partial \Omega}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U_0}{\partial s^2} - \frac{1}{2}U_0\left(\frac{\partial \Omega}{\partial s}\right)^2 + \alpha\Theta_1(\xi, s)U_0 - \delta s^2 U_0 = 0 \tag{11}$$

where

$$\Theta_1(\xi, s) = \frac{|U_0|^2}{1+|U_0|^2} \tag{12}$$

In Eq. (11), $\Theta_1(\xi, s)$ represents the non-linearity due to space charge field induced refractive index change due to the linear and quadratic electro-optic effect respectively.

The last term in (11) is due to the planar waveguide structure in the photorefractive material. The last two terms control the diffraction effects and lead to a self trapping mechanism.

3. Results and discussion

We shall look for self similar spatial soliton solutions for which the electromagnetic field energy is confined to the centre of the beam. The Gaussian solution gives good analytical results and can be taken as an acceptable approximation to the actual solution which can be found from numerical methods. So the solution can be termed as a quasi-soliton. Hence, we shall take the following ansatz in our investigation,

$$U_0(\xi, s) = \frac{U_{00}}{\sqrt{f(\xi)}}e^{-s^2/2r^2f^2(\xi)} \tag{13}$$

$$\Omega(\xi, s) = \frac{s^2}{2}\Gamma(\xi) \tag{13}$$

Table 1
Various parameters taken for SBN crystal in our theoretical investigation [27,31,32].

Parameter	Value	Parameter	Value
n_e	2.35	ΔT	20 °C
x_0	20 μm	ε_0	8.85×10^{-12} F/m
λ_0	532 nm.	ε_r	3400
r_{eff}	237×10^{-12} m/V	α	40.2
$\frac{\partial P}{\partial T}$	-3×10^{-4} Cm^{-2} K^{-1}		

$$\Gamma(\xi) = -\frac{1}{f(\xi)}\frac{df(\xi)}{d\xi} \tag{15}$$

where r is a positive constant, U_{00} is the square root of normalized peak power of the quasi-soliton, $f(\xi)$ is the variable beam width parameter such that the product $rf(\xi)$ gives the spatial width of the soliton. (13) is the assumed variational solution with $f(\xi)$, $\Gamma(\xi)$ and r as the variational parameters. We assume that the quasi-soliton beam is not diffracting when it enters the photorefractive crystal, i.e, $\frac{df}{d\xi} = 0$ at $\xi = 0$. Also, without loss of any generality, we take $f = 1$ at $\xi = 0$. The non-linear contribution Θ_1 can be expanded in Taylor series and approximating to first order, we get,

$$\Theta_1(\xi, s) \approx \frac{\frac{U_{00}^2}{f(\xi)}}{\left(1 + \frac{U_{00}^2}{f}\right)} + s^2\frac{\frac{U_{00}^2}{r^2 f^3(\xi)}}{\left(1 + \frac{U_{00}^2}{f(\xi)}\right)^2} \tag{16}$$

Substituting (14)–(16) in (11) gives us an equation in various powers of s^2. Equating the coefficients of s^2 in LHS and RHS, we get the evolution equation for the beam width parameter $f(\xi)$,

$$\frac{d^2 f(\xi)}{d\xi^2} = \frac{1}{r^4 f^3(\xi)} - 2\alpha\frac{\frac{P_0}{r^2 f^2(\xi)}}{\left(1 + \frac{P_0}{f(\xi)}\right)^2} - 2\delta f(\xi) \tag{17}$$

It is notable that the first order approximation in (16) works because we derive the evolution Eq. (17) by equating coefficients of s^2 on both sides after substituting (14)–(16) in (11). Since we do not use the higher powers of s^2, hence it is quite acceptable to approximate (16) as above. We would like to mention that we have followed Refs. [25] and [26] for the aforementioned approach for the first order approximation to the nonlinear contribution in (16).

$P_0 = U_{00}^2$ is the normalized power of the soliton in the photorefractive waveguide. Depending upon the power P_0 and the parameter α, the optical solitary wave may diverge, be compressed or travel stably after self trapping. A self trapped solitary wave,i.e, a quasi-soliton will be formed when the beam width $f(\xi)$ remains constant. Hence, in (17), we put LHS equal to zero and get,

$$\frac{1}{r^4} = 2\delta + \frac{2\alpha P_{0t}}{r^2(1 + P_{0t})^2} \tag{18}$$

This is an equilibrium condition containing four roots and tells us the threshold power P_{0t} for stationary propagation. It is an existence equation for pyroelectric optical spatial quasi-solitons propagating through the photorefractive waveguide. If we examine the solutions of the above equation, we find that out of the four roots, two are imaginary, one is negative and one is positive. Since r certainly cannot be imaginary or negative for spatial solitons, only one solution is physically acceptable for the case of solitons.

For illustration of the quasi-soliton characteristics, we shall consider an unbiased Strontium Barium Niobate (SBN) crystal in which a temperature change leads to a transient pyroelectric field. The parameters taken for a SBN crystal in our investigation are shown lucidly in Table 1. We have plotted r versus the threshold power P_{0t} for different values of the waveguide parameter in Fig. 1. We can infer the fact that there exist two threshold powers for a particular value of r. This clearly shows the existence of bistable states of stable self trapped spatial quasi-solitons in photorefractive waveguides. We shall name these two threshold powers (at which the quasi-soliton of a specific width can form) as P_{0t1} and P_{0t2}. From the figure, we can see that the width of the quasi-soliton state decreases with an increase in power in the low power regions while the width of the quasi-soliton state increases with an increase in power in the high power regions.

We shall investigate the behaviour of spatial quasi-solitons in absence of the waveguiding effect. For clear illustration, we take four different values of power, $P_1(=0.0999) < P_{0t1}$, $P_2(=0.2435) = P_{0t1}$, $P_{0t1} < P_3(=1) < P_{0t2}$, $P_4(=24.35) > P_{0t2}$. We plot the variation in f with the normalized propagation distance ξ in Fig. 2. From the figure, we can see that at P_1, the beam width diverges to a very large value. This is because the peak power of the quasi-soliton is much less than the threshold power and hence the solitary wave cannot be self-trapped. At the threshold power, $P = P_{0t1}$, the spatial quasi-soliton travels and does not change its shape. This can be inferred from the constant soliton beam width parameter $f = 1$. The spatial quasi-soliton beam width oscillates with an amplitude less than unity when the quasi-soliton's peak power is between the two threshold powers P_{0t1} and P_{0t2}. If the quasi-soliton's peak power is greater than both of the threshold power values, then again we find that the spatial quasi-soliton beam width oscillates but with an oscillation amplitude much greater than unity.

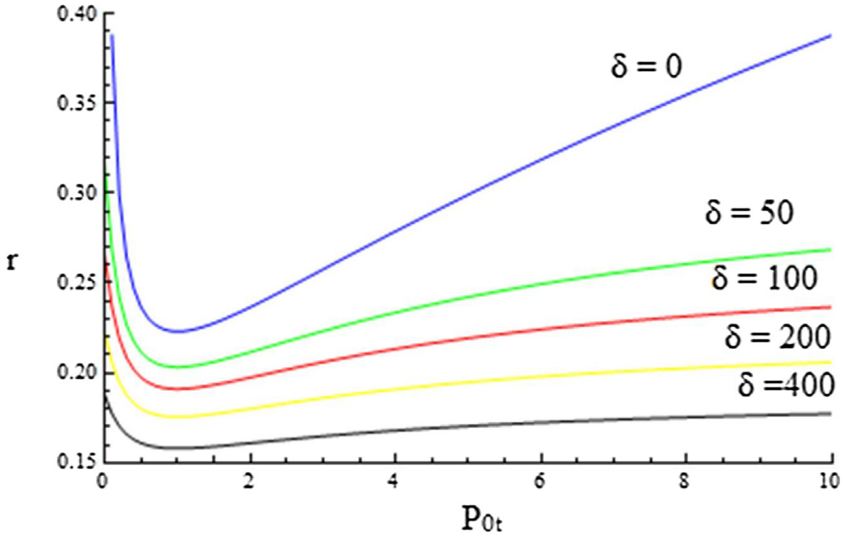

Fig. 1. The variation of equilibrium spatial width r with threshold peak power P_{0t} of the spatial solitons. $\alpha = 40.2$.

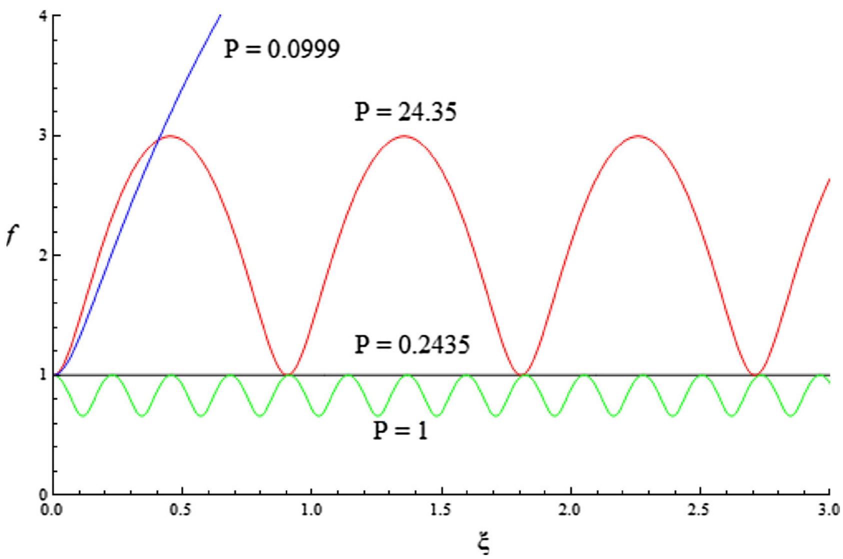

Fig. 2. The variation of variable beam width parameter $f(\xi)$ with normalized distance of propagation ξ at four different soliton peak powers $\alpha = 40.2$, $r = 0.2810$.

Now, we will investigate the effect of waveguiding due to the waveguide structure embedded in the photorefractive crystal on the propagation of the quasi-soliton. In Fig. 3, we plot the variation of the quasi-soliton width parameter f with the normalized distance of propagation for different waveguide parameter δ. If there is no waveguiding, then we have, $\delta = 0$ and this is shown in the top curve in Fig. 3 which demonstrates the divergence of the solitary wave as the available power is less than the threshold power. The variation of f with normalized distance travelled changes when $\delta \neq 0$. As δ increases beyond a certain value (>50), f becomes an oscillatory function with the amplitude of oscillation less than one. This indicates formation of a quasi-soliton at peak powers less than the threshold power. As we increase the value of δ, the power required to trap the quasi-soliton decreases even more.

4. Conclusions

We have presented the propagation characteristics of hitherto uninvestigated case of pyroelectric quasi-solitons in a planar waveguide embedded in an unbiased photorefractive crystal. We would like to mention here that our work differs considerably from the previous researches [25,26]. Firstly, while in Ref. [25], the authors consider both the external electric field and the bulk photovoltaic field to be the cause of the buildup of space charge field and hence their results pertain to the screening photovoltaic quasi-solitons, we consider the solitons self trapping due solely to the pyroelectric effect in a

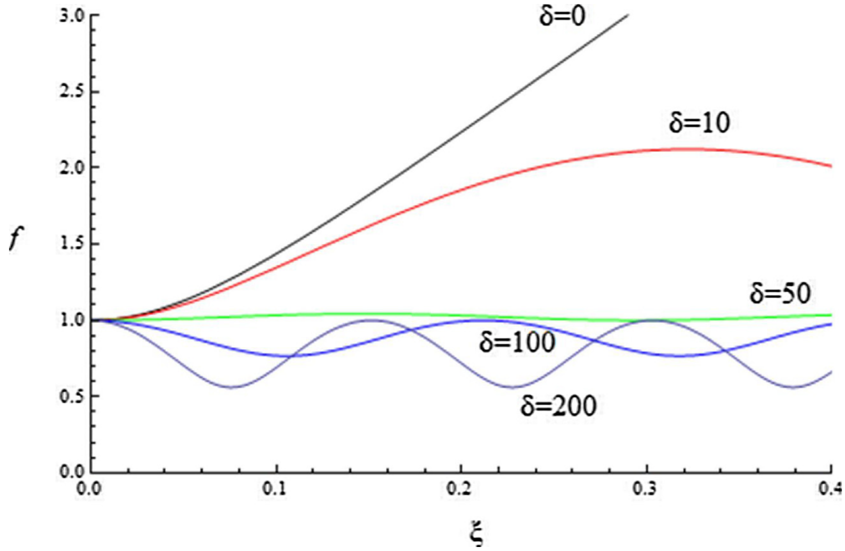

Fig. 3. The variation of variable beam width parameter $f(\xi)$ with normalized distance of propagation ξ for five different values of waveguide parameter δ, $\alpha = 40.2$, $P_0 = 0.055$, $r = 0.2810$.

photorefractive crystal waveguide. Hence our results pertain to a totally different class of photorefractive quasi-solitons. Secondly, we consider the photorefractive effect to be originated from the linear (Pockels') electro-optic effect and not the quadratic(Kerr) electro-optic effect as considered in Ref. [26].

The waveguide structure supports and increases the self focussing effect of the photorefractive material and hence the minimum power for self trapping of a solitary wave reduces. Spatial quasi-solitons with a very low peak power can be formed with a finite value of the waveguiding parameter. These could not have formed otherwise in case the waveguiding structure was absent. As the value of the waveguide coefficient increases, the power required to trap the quasi-soliton decreases. We have shown four different power regimes and investigated the quasi-solitons behaviour in each of the power regimes. In addition, we also show the existence of bistable states of the solitons in a photorefractive waveguide.

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