



Modulation instability of broad optical beams in unbiased photorefractive pyroelectric crystals

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ABSTRACT

We present a study of the one dimensional modulation instability due to a broad optical beam in pyroelectric photorefractive crystals where the space charge field is formed due to solely the pyroelectric effect. The one-dimensional growth rate of the modulation instability depends upon the intensity of the incident beam and the magnitude of the temperature change. Relevant example of a Strontium Barium Niobate crystal is taken to illustrate the theoretical analysis.

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1. Introduction

Photorefractive solitons have attracted much attention in the past two decades due to their applications in optical switching and routing. They have been observed as screening photorefractive solitons [1], screening photovoltaic solitons [2], photovoltaic solitons [3] and in centrosymmetric photorefractive media. Screening solitons require an external electric field which screens the space charge field. In photovoltaic crystals, the bulk photovoltaic field is responsible for the formation of the space charge field. There have been extensive studies on various characteristics of solitons in photorefractive media [4–15]. Recently, there have been studies on soliton formation in pyroelectric photorefractive media [16–18]. In [17], the authors have predicted the existence of solitons due to solely the pyroelectric effect in SBN crystals. We have, recently studied the incoherently coupled soliton pairs in such pyroelectric photorefractive media [19]. In a ferroelectric crystal, the net electric field inside the crystal is zero. This is because the charge distribution on the crystal faces compensates the field due to spontaneous polarization. A temperature change causes a spontaneous polarization change and hence, a transient electric field E_{py} . This is called as the pyroelectric field. This field is not compensated immediately and consequently, a drift current can be set up mimicking the effect of an external electric field applied to the crystal. Now, this field is locally screened due to the space charge field formed due to the photorefractive effect and hence, a self trapped beam results. We emphasize that as compared to the formation

of screening solitons, replacing the external electric field with the temperature change induced pyroelectric field for soliton formation has clear advantages. Firstly, we need not know the direction of the crystal c-axis since the pyroelectric field is always along the c-axis, i.e., in one direction for heating and in the reverse direction for cooling. Secondly, no electrodes are required on the crystal [20,21].

Modulation instability is a common phenomenon which is inherent in many non-linear systems [22–26]. It occurs in the same parameter space in which solitons are observed and is considered as a precursor to soliton formation. It manifests as a growth of spatial frequency sidebands in a broad optical beam. There have been investigations of the modulation instability of screening solitons and screening photovoltaic solitons [27–29], photorefractive solitons in centrosymmetric media [30]. The modulation instability in unbiased photorefractive crystals in which the space charge field is formed due to the pyroelectric effect alone has not been studied as yet. Hence, it is of interest to study the modulation instability of such photorefractive solitons which form due to solely the pyroelectric effects. We shall study this in detail in the current paper taking a relevant example of an SBN crystal.

2. Theory

Let us consider a broad optical beam propagating in a photorefractive crystal having finite pyroelectric coefficient. The beam is allowed to diffract only along the x-direction and propagates along the z-direction. The crystal is kept in contact with a metallic plate which has its temperature accurately controlled by an external agency. The crystal is duly covered by an insulating plastic cover. In our analysis, we shall take the photorefractive pyroelectric crystal to be Strontium Barium Niobate, $\text{Sr}_{0.6}\text{Ba}_{0.4}\text{Nb}_2\text{O}_6$ which

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is abbreviated as SBN henceforth. The incident beam is expressed as a slowly varying envelope $\vec{E} = \hat{x}A(x, z) \exp(ikz)$ where $k = k_0 n_e$, $k_0 = \frac{2\pi}{\lambda_0}$. Here, n_e is the unperturbed extraordinary refractive index and λ_0 is the free space wavelength. Under these conditions, the evolution of the optical beams is given by [1],

$$\left(i \frac{\partial}{\partial z} + \frac{1}{2k} \frac{\partial^2}{\partial x^2} + \frac{k}{n_e} \Delta n\right) A(x, z) = 0 \quad (1)$$

$$\Delta n = -\frac{1}{2} n_e^3 r_{eff} E_{sc} \quad (2)$$

where r_{eff} is the electro-optic coefficient and $E_{sc} = E_{pysc}$ is the space charge field in the medium resulting from the transient pyroelectric field and is [17],

$$E_{pysc} = -E_{py} \frac{I}{I + I_d} \quad (3)$$

E_{py} is the transient pyroelectric field and is given by [18],

$$E_{py} = -\frac{1}{\varepsilon_0 \varepsilon_r} \frac{\partial P}{\partial T} \Delta T \quad (4)$$

where $\frac{\partial P}{\partial T}$ is the pyroelectric coefficient and ΔT is the magnitude of the temperature change of the crystal and I_d is the dark irradiance, i.e., the intensity at constant illumination region of the crystal, $I(x \rightarrow \infty)$

From the expression for the space charge field (3), we can infer that the value and sign of the space charge field depends upon E_{py} which in turn can be varied by controlling the change in temperature, i.e., by the requisite heating, or cooling effects. The total optical power density for the two mutually incoherent beams can be obtained by the Poynting flux,

$$I = \frac{n_e}{2\eta_0} (|A|^2) \quad (5)$$

with $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$.

Substituting E_{pysc} and Δn in (1), one gets the following equation,

$$iU_z + \frac{1}{2} U_{xx} + \beta E_{py} \frac{|U|^2}{1 + |U|^2} U = 0 \quad (6)$$

where we have written,

$$A = \left(\frac{2\eta_0 I_d}{n_e}\right)^{1/2} U, \quad U_z = \frac{\partial U}{\partial z}, \quad U_{xx} = \frac{\partial^2 U}{\partial x^2}, \quad I = I_d |U|^2,$$

$$\beta = \frac{1}{2} (k_0 n_e^3 r_{eff})$$

The plane wave broad beam solution of (6) takes the form,

$$U = r^{1/2} \exp \left[i\beta E_{py} \left\{ \frac{r}{1+r} \right\} z \right] \quad (7)$$

For studying the modulation instability of the plane wave, we express,

$$U = [r^{1/2} + \sigma(x, z)] \exp \left[i\beta E_{py} \left\{ \frac{r}{1+r} \right\} z \right] \quad (8)$$

where $\sigma(x, z)$ contains two sideband plane waves and is the weak modulation term added to the steady state solution (7). This weak perturbation satisfies

$$|\sigma(x, z)| \ll r^{1/2} \quad (9)$$

and is assumed,

$$\sigma = a(z) \exp(ipx) + b(z) \exp(-ipx) \quad (10)$$

We want to investigate the possible exponential growth of this perturbation. For this purpose, substituting (8) in (6) and using the linearizing condition (9), we get the following evolution equation for the perturbation $\sigma(x, z)$,

$$i \frac{\partial \sigma}{\partial z} + \frac{1}{2k} \frac{\partial^2 \sigma}{\partial x^2} + \beta E_{py} \frac{r}{(1+r)^2} (\sigma + \sigma^*) = 0 \quad (11)$$

Substituting (10) in (11), we obtain the following coupled equations,

$$i \frac{da}{dz} - \frac{1}{2k} p^2 a + \beta E_{py} \frac{r}{(1+r)^2} (a + b^*) = 0 \quad (12)$$

$$i \frac{db}{dz} - \frac{1}{2k} p^2 b + \beta E_{py} \frac{r}{(1+r)^2} (a^* + b) = 0 \quad (13)$$

By decoupling Eqs. (12) and (13), we get simply,

$$\frac{d^2 a}{dz^2} = \left[\beta E_{py} \frac{r}{(1+r)^2} \frac{p^2}{k} - \frac{p^4}{4k^2} \right] a \quad (14)$$

$$\frac{d^2 b}{dz^2} = \left[\beta E_{py} \frac{r}{(1+r)^2} \frac{p^2}{k} - \frac{p^4}{4k^2} \right] b \quad (15)$$

Eqs. (14) and (15) are of the form of a linear second order differential equation. Hence, their solutions are of the form $\exp(\omega z)$ with

$$\omega = \left[\beta E_{py} \frac{r}{(1+r)^2} \frac{p^2}{k} - \frac{p^4}{4k^2} \right]^{1/2} \quad (16)$$

Hence, the local modulation instability gain is given as,

$$g = \text{Re} \left\{ \left[\beta E_{py} \frac{r}{(1+r)^2} \frac{p^2}{k} - \frac{p^4}{4k^2} \right]^{1/2} \right\} \quad (17)$$

From (17), we can find the maximum modulation instability gain as,

$$g_{\max} = \left[\frac{1}{2} k_0 E_{py} n_e^3 r_{eff} \frac{r}{(1+r)^2} \right] \quad (18)$$

and the associated spatial frequency is,

$$p_{\max} = \frac{k_0 n_e^2}{1+r} [r_{eff} E_{py} r]^{1/2} \quad (19)$$

3. Results and Discussion

We consider the SBN crystal and hence, we take the following parameters [17,31,32], $n_e = 2.35$, $\lambda_0 = 532 \text{ nm}$, $r_{eff} = 237 \times 10^{-12} \text{ m/V}$, $\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$, $\varepsilon_r = 3400$, $\frac{\partial P}{\partial T} = -3 \times 10^{-4} \text{ Cm}^{-2} \text{K}^{-1}$, $r = 10$.

For this set of values, we get, $\beta = 0.018$ and $k = 2.774 \times 10^7$. Fig. 1 shows the MI gain g as a function of p/k at $\Delta T = 20^\circ \text{C}$. p/k is the angle at which the plane wave components of the $\sigma(x, z)$ perturbation propagate with respect to the broad optical beam. We note that the MI gain g first increases with an increase in p/k , and then after reaching the peak value, g decreases and finally becomes zero after a certain value of p/k .

Now, it is important to note that the MI gain g depends upon E_{py} and hence ΔT which is the change in temperature of the photorefractive pyroelectric crystal. Hence, it is interesting to study the MI gain g with varying ΔT and p/k . Fig. 2 shows the dependence of g simultaneously with ΔT and p/k for $r = 1$. We can see that as the value of ΔT increases, the peak MI gain g_{\max} increases. Also, as the value of ΔT increases, the value of p/k after which g becomes zero also increases, i.e., the range of values of p/k for which there is a finite MI gain increases with an increase in ΔT .

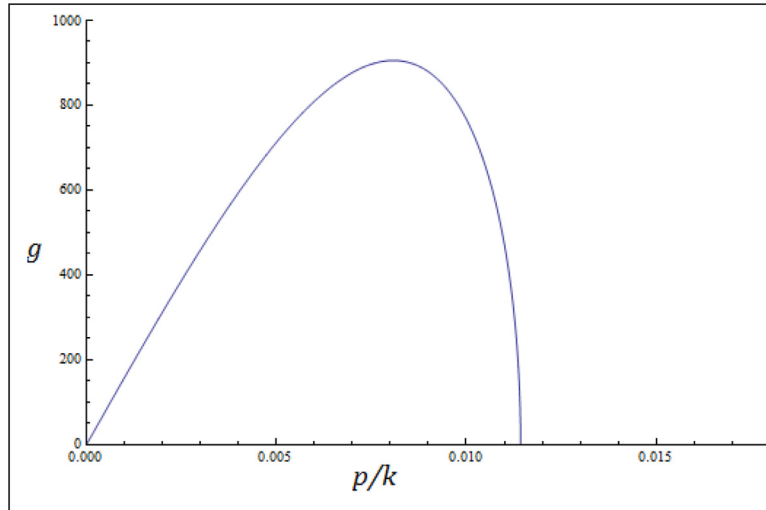


Fig. 1. MI gain g as a function of p/k with $\Delta T = 20^\circ\text{C}$.

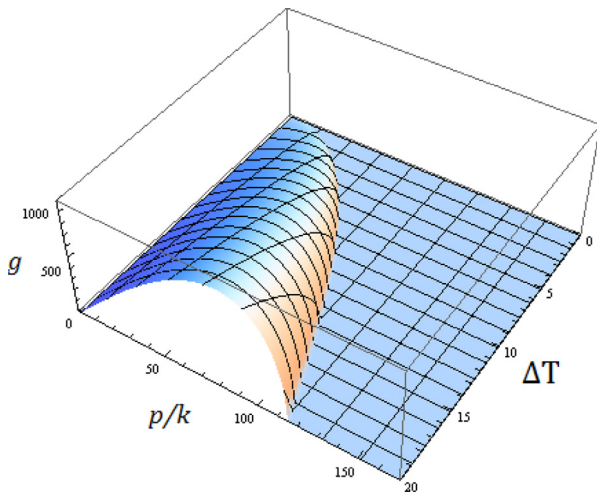


Fig. 2. MI gain with respect to p/k and ΔT when $r=1$.

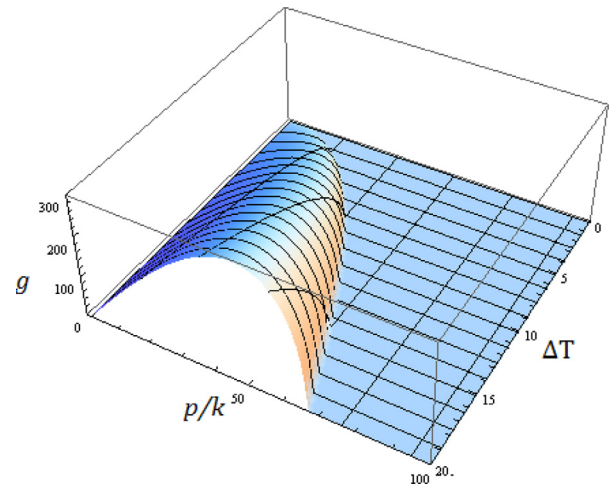


Fig. 3. MI gain with respect to p/k and ΔT when $r=10$.

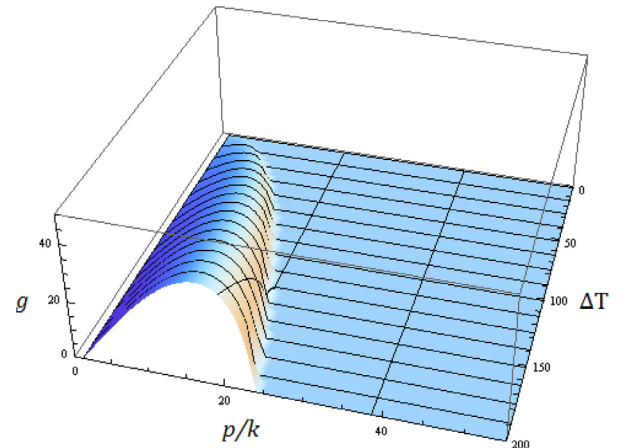


Fig. 4. MI gain with respect to p/k and ΔT when $r=100$.

To study the MI gain dependence on r , the ratio of the maximum intensity to the dark irradiance ($= \frac{I_{\max}}{I_d}$), we plot the variation of MI gain with respect to p/k and ΔT for $r=10$ and $r=100$ in Figs. 3 and 4 respectively. As r increases, we find that the peak MI gain g_{\max} decreases progressively. We also find that, for a given value of temperature change ΔT , the range of values of p/k for which there is a finite MI gain also decreases as the value of r increases.

Finally, we plot the dependence of the peak MI gain g_{\max} upon the temperature change in Fig. 5. The maximum MI gain g_{\max} increases with an increase in ΔT . The rate of increase is rapid for $\Delta T < 5$ and then moderates slightly.

4. Conclusions

We have investigated the modulation instability for planar beams of light in an unbiased photorefractive crystal in steady state where the space charge field is formed solely due to the pyroelectric effect. We study the dependence of the MI gain in detail. We find that the magnitude of temperature change causing the transient pyroelectric field and the ratio of the maximum in-

tensity to the dark irradiance affect the MI gain. The maximum MI gain and its dependence is also discussed while the corresponding spatial frequency is calculated.

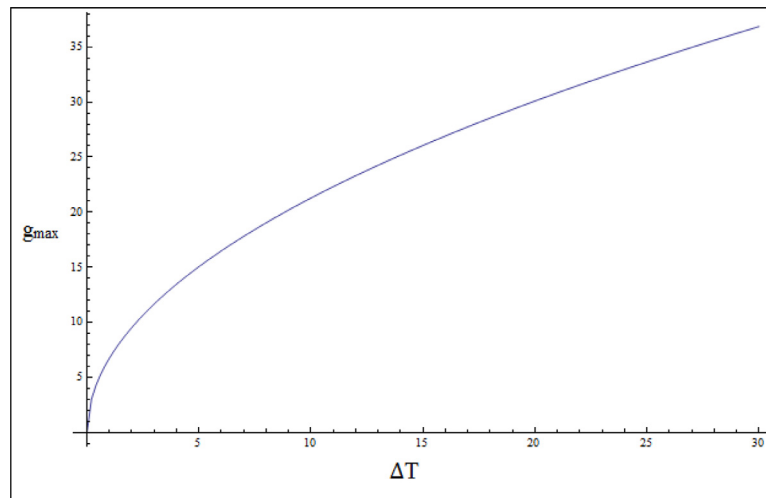


Fig. 5. Peak MI gain variation with ΔT (in $^{\circ}\text{C}$) when $r=1$.

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References

- [1] Christodoulides DN, Carvalho MI. Bright, dark, and gray spatial soliton states in photorefractive media. *J Opt Soc Am B* Sep. 1995;12(9):1628.
- [2] Liu JS, Lu KQ. Screening-photovoltaic spatial solitons in biased photovoltaic-photorefractive crystals and their self-deflection. *J Opt Soc Am B-Opt Phys* 1999;16(4):550–5.
- [3] Valley GC, Segev M, Crosignani B, Yariv A, Fejer MM, Bashaw MC. Dark and bright photovoltaic spatial solitons. *Phys Rev A* 1994;50(6):R4457.
- [4] Katti A, Yadav RA. Coupled spatial solitons in photorefractive multiple quantum well planar waveguides. In: 13th international conference on fiber optics and photonics, OSA Technical Digest (online) (Optical Society of America); 2016. p. Tu3D.3.
- [5] Zhan K, Hou C, Tian H, Zhang Y. Incoherently coupled Gaussian soliton pairs in biased centrosymmetric photorefractive crystals. *Opt Laser Technol* 2010;42(7):1176–9.
- [6] Morin M, Duree G, Salamo G, Segev M. Waveguides formed by quasi-steady-state photorefractive spatial solitons. *Opt Lett* 1995;20(20):2066–8.
- [7] Katti A, Yadav RA, Singh DP. Theoretical investigation of incoherently coupled solitons in centrosymmetric photorefractive crystals. *Optik - Int J Light Electron Opt* 2017;136:89–106.
- [8] Katti A, Yadav RA. Coupling effects for grey separate spatial solitons in a biased series photorefractive crystal circuit with both the linear and quadratic electro-optic effects. *Opt Quantum Electron* 2017;49(1):36.
- [9] Ji X, Jiang Q, Liu J. Separate spatial holographic and two-photon-Hamiltonian soliton pairs in an unbiased series photorefractive crystal circuit. *Optik - Int J Light Electron Opt* 2012;123(14):1223–7.
- [10] Liu JS, Hao ZH. Evolution of separate screening soliton pairs in a biased series photorefractive crystal circuit. *Phys Rev E* 2002;65(6):12.
- [11] Hao Z, Liu J. Interaction between separate spatial soliton pairs in a biased serial photovoltaic photorefractive crystal circuit and its application. *Photonics Asia* 2002 2002:326–35.
- [12] Keqing L, Yanpeng Z, Tiantong T, Bo L. Incoherently coupled steady-state soliton pairs in biased photorefractive-photovoltaic materials. *Phys Rev E - Stat Nonlinear, Soft Matter Phys* 2001;64(5 Pt 2):056603/1–056603/9.
- [13] Hao L, Wang Q, Hou C. Incoherently coupled spatial soliton pairs due to both the linear and quadratic electro-optic effects. *J Mod Opt* 2015;62(3):205–11.
- [14] Chun-feng H, Bin L, Yong-yuan SXJ, Ke-bin X. Incoherently coupled screening-photovoltaic soliton families in biased photovoltaic photorefractive crystals. *Chinese Phys Apr.* 2001;10(4):310–13.
- [15] Hou C, Zhou Z, Sun X, Yuan B. Incoherently coupled grey-grey screening-photovoltaic soliton pairs in biased photovoltaic-photorefractive crystals. *Optik - Int J Light Electron Opt Jan.* 2001;112(1):17–20.
- [16] Katti A, Yadav RA. Spatial solitons in biased photovoltaic photorefractive materials with the pyroelectric effect. *Phys Lett Sect A Gen At Solid State Phys* 2017;381(3):166–70.
- [17] Su Y, Jiang Q, Ji X. Photorefractive spatial solitons supported by pyroelectric effects in strontium barium niobate crystals. *Optik - Int J Light Electron Opt* 2015;126(18):1621–4.
- [18] Safioui J, Devaux F, Chauvet M. Pyroliton: pyroelectric spatial soliton. *Opt Express* 2009;17(24):22209–16.
- [19] Katti A, Yadav RA. Incoherently coupled photorefractive spatial solitons supported by pyroelectric effects. *J Nonlinear Opt Phys Mater Mar.* 2017;26(1):1750002.
- [20] Popescu ST, Petris A, Vlad VI. Recording of self-induced waveguides in lithium niobate at 405 nm wavelength by photorefractive-pyroelectric effect. *J Appl Phys Jun.* 2013;113(21):213110.
- [21] Popescu ST, Petris A, Vlad VI. Fast writing of soliton waveguides in lithium niobate using low-intensity blue light. *Appl Phys B Sep.* 2012;108(4):799–805.
- [22] Van Simaey G. Observation of modulational instability recurrence in optical fibers. In: *Nonlinear guided waves and their applications*; 2001. p. 441–3.
- [23] Wen S, Wang Y, Su W, Xiang Y, Fu X, Fan D. Modulational instability in nonlinear negative-index material. *Phys Rev E - Stat Nonlinear, Soft Matter Phys* 2006;73(3):36617.
- [24] Zakharov VE, Ostrovsky LA. Modulational instability: the beginning. *Phys D Nonlinear Phenom* 2009;238(5):540–8.
- [25] Akhmediev NN, Korneev VI. Modulational instability and periodic solutions of the nonlinear schrödinger equation. *Theor Math Phys* 1986;69(2):1089–93.
- [26] Kivshar Y, Peyrard M. Modulational instability in discrete lattices. *Phys Rev A* 1992;46(6).
- [27] Carvalho M, Singh S, Christodoulides D. Modulational instability of quasi-plane-wave optical beams biased in photorefractive crystals. *Opt Commun* 1996;126(1–3):167–74.
- [28] Ke-Qing L, Wei Z, Yan-Long Y. Modulational instability in biased photorefractive-photovoltaic crystals. *Chinese Phys* 2004;21(6):1086–8.
- [29] Lu KQ, Zhao W, Yang YL, Zhu XP, Li JP, Zhang YP. Modulational instability of quasi-plane-wave optical beams in biased photorefractive-photovoltaic crystals. *Chinese Phys* 2004;13(12):2077–81.
- [30] Zhan K, Hou C. One-dimensional modulational instability of broad optical beams in biased centrosymmetric photorefractive crystals. *Phys Lett Sect A Gen At Solid State Phys* 2009;374(2):169–72.
- [31] Savchenkov AA, Matsko AB, Ilchenko VS, Solomatine I, Seidel D, Maleki L. RF-induced change of optical refractive index in strontium barium niobate. *Photonics west, laser resonators microresonators, beam control XV* (February 22, 2013), 8600; 2013. 860000-1–860000-9.
- [32] Kos K, Meng H, Salamo G, Shih M, Segev M, Valley GC. One-dimensional steady-state photorefractive screening solitons. *Phys Rev E Stat Phys Plasmas Fluids Relat Interdiscip Top May* 1996;53(5):R4330–3.